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# Torsion mass generation induced by Einstein–Cartan gravity with a Barbero–Immirzi Higgs field analogue

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**Abstract** It is well known that torsion waves in teleparallel Einstein gravity may induce gravitational waves (GWs). In this paper we show that a modification to Holst gravity induces low-frequency torsion waves of torsional frequency $\sim 10^{12} Hz = 1 THz$ as GWs sourced from astrophysical black holes. Of course, since here we use the almost Riemann-flat manifold approximation, there are no Riemannian GWs as in teleparallel spacetime but only torsionful gravitational waves. This result in Holst gravity allows us to determine an upper bound for the Barbero–Immirzi (BI) parameter as $\gamma <<< 10^{-58}$, which agrees with the BI bound obtained by Aliberti and Lambiase by considering matter–anti-matter asymmetry in Holst gravity. Unfortunately, this very interesting result suffers from the usual torsion ambiguity. To remedy this situation, we proceed with the BI parameter promoted to a field ore BI scalarization. Again we find a BI parameter with a bound of $10^{-58}$ from a terahertz-frequency torsion wave. The BI parameter is promoted to a field, and with specific approximation one obtains a BI wave equation with a plane wave solution, similar to those obtained by Taveras and Yunes. From this wave equation constant the BI parameter implies zero torsion, and consequently general relativity (GR), but the converse is not true. Higgs field variation is regarded as responsible for massive torsion. This comes from an analogy between the Higgs field and BI scalar.

## 1 Introduction

Several attempts to numerically determine the Barbero–Immirzi (BI) parameter [1–3] have been made from well-known physical phenomena such as matter–anti-matter asymmetry [4] or earlier using particle physics results at the Large Hadron Collider (LHC) at Cern [5,6] and the attempt by Perlov [7] to determine the BI parameter from the parameterized post-Newtonian (PPN) framework in satellite measurement. The important issue is having an experimental base for the determination of the BI parameter. Mercuri [8,9] and collaborators have undertaken intensive studies of the Peccei–Quinn CP asymmetry violation using a Nieh–Yan (NY) gravity term and not only the Holst term, which is another topological invariant which we shall discuss in the Sect. 2 revision. In Sects. 3 and 4 we illustrate these studies by two examples: The first represents an extension of a model used by Dombriz et al. [10] to propose a dark torsion candidate for dark matter (DM) using a Poincaré gauge theory of gravity (PGT) by performing a modification to this PGT similar to that by Minxi He et al. [11], and Garcia de Andrade [12–14] of a modified Holst model of Einstein–Cartan (EC) gravity to investigate the inflatons and massive torsion of trace and axial modes. This model is shown in Sect. 2 to lead naturally to a torsion propagation in the form of a very-low-frequency torsion wave. In Sect. 3 we address the problem of the scalarization of the BI parameter without the Nieh–Yan (NY) term; we show that when the BI parameter coincides with the imaginary unit as in Ashtekar gravitational theory, the torsion wave frequency depends upon the axial torsion mass, which yields a negative frequency suitable to dark torsion waves candidates. Holst-modified EC gravity also leads to torsion waves from axial mass estimated at the CERN LHC as investigated by Almeida et al. [5,6]. In comparison with gravitational waves, torsion waves of $10^{-3} Hz$ are well within the band of frequencies between the GW ripples of $10^{-17} Hz$ up to the $10^{3} Hz$ produced from neutron stars. The third example illustrates the propagation of torsion in sev-

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eral models with Holst–Nieh–Yan (HNY) gravity and PGT with modifications. In Sects. 4 and 5 we address the scalarization of the BI parameter in the realm of PGT. Application to phantoms with torsion as dark radiation is also investigated in Sect. 6. DM with dark torsion radiation of phantoms represents possible sources with negative frequency of DM. Section **??** is left for conclusions.

## 2 Massless torsion propagation in Poincaré gauge gravity with modifications

We recently [15] investigated LIGO (Laser Interferometer, Gravitational-Wave Observatory) signs of gravitational waves from teleparallel NY gravity and gravitational anomalies [16]. On the other hand, in this paper, we shall assume a non-vanishing torsion mass investigated at LHC in an almost Riemannian-flat manifold spacetime. The Holst gravity Lagrangian is given by

$$\mathcal{L}_{PGT} = a_0 R + \frac{1}{2}{m^2}_s S^2 + \frac{1}{2}{m^2}_T T^2 - \alpha\phi^2 + 2\alpha\phi\mathcal{H}^2. \quad (1)$$

We shall start the illustrations of DM in PGT of gravity with massless torsion modes, where $m_s = 0 = m_T$. This greatly simplifies the Lagrangian. Other simplification we can do in this first pedestrian model is to consider that $\alpha$ is a constant parameter. Since this parameter is similar to the BI parameter in the last section, we shall address the more complicated case of a dynamical $\alpha(x)$ as in the case of the BI parameter, which undergoes a scalarization as investigated by Mercuri. The main difference between the present paper and that of Dombriz and Mota [17] is that we extend the Lagrangian (1) by a NY term

$$I_{NY} = -\partial S. \quad (2)$$

We recall that the metric used here is approximately Minkowskian. The other important addition, generally used for Higgs inflation by He et al. [17], is the case of special types of inflation. In our case we write the correction like $X = (S, T)$ as

$$-\frac{1}{4}[\partial X]. \quad (3)$$

This Lagrangian correction term endows the spacetime with a propagation along X torsion modes. Let us start with the simplest modified Lagrangian (1)

$$\mathcal{L}_{PGT} = \alpha\partial S - \alpha\phi^2 + 2\alpha\phi\mathcal{H} + \frac{1}{2}(\partial\phi)^2, \quad (4)$$

where we consider that the Holst term is $\mathcal{H} = -TS$, which is written in linear order, contrary to Dombriz et al. We shall see that for a constant $\alpha$, the NY term does not affect the results. This Lagrangian does not present the correction term and is one of the simplest for which we could extract some dynamics. A simple application of the Euler–Lagrange equation would lead to $T = S = 0$, so the massless torsion modes vanish and one is left with just a scalar wave like the Klein–Gordon equation

$$\ddot{\phi} + \alpha\phi = 0, \quad (5)$$

where the d'Alembertian operator is reduced to second-order time-derivative due to the presence of a homogeneous scalar field $\phi$. Now let us meaningfully modifying the last Lagrangian to extract the dynamics of massless torsion. The second example is then made by adding the correction term for S and torsion trace vector T. This Lagrangian is

$$\mathcal{L}_{PGT} = -\frac{1}{4}(\partial S)^2 - \frac{1}{4}(\partial T)^2 - \alpha\phi^2 - 2\alpha\phi TS + \frac{1}{2}(\partial\phi)^2. \quad (6)$$

Although correction terms are present, we disregard the NY term since it will not contribute to this section where the BI parameter like $\alpha$ is not scalarized as in the next sections. Variation of this Lagrangian with respect to T and axial torsion pseudo-vector S yields

$$\Box\phi + 2\alpha\phi = -2\alpha(TS), \quad (7)$$

which shows that the product of both modes of torsion is the source of the scalar field for dilatons, Higgs, axion-like particles (ALPs), or phantons. The remaining equations of torsion mode propagation are self-dual in the sense that they both obey the following Klein–Gordon equation as

$$\Box T - 4\alpha\phi S = 0 \quad (8)$$

and

$$\Box S - 4\alpha\phi T = 0. \quad (9)$$

Here, we do not necessarily have an ambiguity of Holst type, where a pseudo-vector S would be proportional to a trace vector. This ambiguity can be avoided by various tools, such as the introduction of fermions or scalar fields. To solve the scalar Klein–Gordon equation we shall use a technique of differential equations with a sum of the two solutions, one of which is a general solution of the homogeneous and the other a particular solution of the non-homogeneous equation to solve the complete non-homogeneous differential equation. Let us now apply this method to the $\phi$ solution as

$$\phi = \phi_1 + \phi_2, \quad (10)$$

where $\phi_1$ satisfies the homogeneous equation where either T or S is zero, which implies that $\phi_1$ is an oscillatory solution as in the axion case given by

$$\phi = -2\alpha(TS) + \phi^0{}_1 \exp i(\omega_\phi t). \quad (11)$$

Of course, $\phi_2$ is assumed to obey the homogeneous equation

$$\Box\phi_2 = 0, \quad (12)$$

which represents an almost massless scalaron or ALPs. By taking the torsion trace and oseudo-vector axial torsion propagation equations above, we note that although we already obtain that the scalaron is induced by the product of T and S modes, by assuming the ansatz for example that $S = S_0 \exp[f(t)]$, one obtains by substituting only the homogeneous equation solution for weak torsion $TS <<< 1$ is given that the torsion pseudo-vector propagation equation is

$$\ddot{\phi}_1 - \phi^0{}_1 \exp i(\omega_\phi t)\phi_1 = 0. \quad (13)$$

Taking $S = S_0 \exp[f(t)]$, in this equation one obtains

$$[\ddot{f} + \dot{f}^2]e^{f(t)} = \phi^0{}_1 \exp i(\omega_\phi t), \quad (14)$$

where $\omega_\phi$ is the oscillation frequency of the ALP scalaron. This results in the solution

$$f(t) = \frac{f_0}{\omega_\phi{}^2} e^{[i\omega_\phi t]}, \quad (15)$$

which determines axial torsion. In the next section we present a different model Lagrangian in Holst gravity with quantum correction and BI scalarization [19].

## 3 BI scalarization with correction to generate terahertz torsion waves

In this section we aim to obtain ultrahigh-frequency (UHF) torsion waves in Holst gravity in several models: The first will be in Holst gravity with massive torsion modes without the NY term but with correction as in Einstein–Cartan. In the second we add the NY term. In addition to the correction in these two cases, we consider the scalarization of the BI parameter as a field in the non-Riemannian manifold. UHF torsion waves beyond gigahertz frequency are found. The simplest model even without the scalarization of the BI parameter allows us to determine the BI parameter on the order of $10^{-26}$, which coincides with the range obtained by Aliberti and Lambiase when investigating the BI effect in matter–anti-matter asymmetry in Holst gravity. Let us start by simply introducing the torsion mass expression

$$\mathcal{L}_{Holst+m} \supset \mathcal{L}_{Holst} + \frac{1}{2}m^2{}_s S^2 + \frac{1}{2}m^2{}_T T^2 + \frac{1}{2}m^2{}_\phi \phi^2 + \frac{1}{2}(\partial\phi)^2, \quad (16)$$

where the Lagrangian $\mathcal{L}_{Holst} \supset \gamma TS$, where $\gamma$ is the BI non-dynamical parameter. Here it is important to assume that to associate torsion to scalaron axion, to have $S = \partial\phi$. This axion torsion transmutation is like that in Duncan et al. [18] to ALPs. The last Lagrangian is written as

$$\mathcal{L}_{Holst+m} \supset m^2{}_P \gamma T \partial\phi + \frac{1}{2}m^2{}_s(\partial\phi)^2 + \frac{1}{2}m^2{}_T T^2 + \frac{1}{2}m^2{}_\phi \phi^2. \quad (17)$$

Notice that this last Lagrangian is fully dynamical in the ALP, and one may obtain by variation of field $(T, \phi)$. Namely, the constraint of ALP transmutation reduces the number of variables from three to two, which greatly simplifies the computations. Variation on torsion trace $\delta T$ yields

$$T(x) = -\gamma \frac{m^2{}_s}{2m^2{}_T}\partial\phi. \quad (18)$$

Note that at $\phi = constant$, we would end up with vanishing torsion in GR. A fundamental question is that in the case where the Higgs field does not vary, the torsion vanishes, so the Higgs would be constant and the presence of Higgs field variation would give rise to torsion mass. Therefore we again have the problem of ambiguity between torsion modes. The interesting fact on this example is that when we substitute the last expression for T into the Lagrangian and perform the variation $\partial\phi$, one is able to determine the BI parameter in terms of the spectra of Planck $m_P$ and torsion masses. This can be done by obtaining the scalaron ALP field equation

$$\left[m^2{}_s - \frac{\gamma m^4{}_P}{m^2{}_T}\right]\Box\phi + m^2{}_\phi \phi = 0. \quad (19)$$

This equation turns out to be in terms of the torsion mass and obeys the equation

$$\Box\phi + \frac{m^2{}_\phi}{\left[m^2{}_s - \frac{\gamma m^4{}_P}{m^2{}_T}\right]}\phi = 0, \quad (20)$$

$$\left[1 + m^2{}_s - \frac{\gamma m^4{}_P}{m^2{}_T}\right]\Box\phi + m^2{}_\phi \phi = 0. \quad (21)$$

Comparison of the last two expressions yields a constraint on the BI parameter $\gamma$ as

$$\gamma = \frac{m^2{}_s m^2{}_T}{m^4{}_P}. \quad (22)$$

Taking the Planck mass as $\sim 10^{16} GeV$ into expression (23), we obtain a BI parameter on the order of $10^{-58}$, which is the order obtained by Rufrano and Lambiase for the case of matter–anti-matter asymmetry. To have a propagating torsion, we need as before to introduce the quantum correction to the last Lagrangian. The trace torsion is not corrected, only the axial torsion S. Therefore, from this Lagrangian without the NY term, one obtains by variation of S the following Therefore, taking the appropriated computations and bounds on torsion mode masses, we are able to obtain the frequency resonances of scalarons. The first step is to observe that from the last equation, the scalaron frequency is $\omega = \sqrt{2} m_s$, and by taking the lower bound for the axial torsion mass as in Dombriz et al., we use $m_s = 1.63 \times meV$. By changing units of eV to Hz, one finds that the frequency is very high, on the order of the $3THz$, which is present in astrophysical settings such as close to black holes. However, the problem of torsion ambiguities is still present. Therefore, to gain more confidence in our results, one should consider the scalarization of the BI parameter. Now we shall examine the problem of resonance of torsion waves in spacetime, and the anti-commutation of the covariant derivatives in Riemann spacetime, where one obtains

$$\Box A^k - R^{*k}{}_i A^i - g_{a\gamma} \tilde{F}^{lk} \partial_l a = 0. \tag{23}$$

Therefore, by assuming that the dual electromagnetic field tensor is in the expression interacting with the axion gradient, one obtains a very elegant equation for the wave equation of the Riemannian spacetime. This equation can be used in appropriate media for dynamo equation amplification as we shall see in the non-Riemannian case. Accordingly to Schwarz et al., one obtains the Riemannian geometrical optics by assuming the ansatz $A^k = \mathcal{A}^k \exp[i\psi]$ and using the assumption of large gradients in space and time, described by the real phase $\psi$. The wave vector is defined by the expression

$$k_i := \partial_i \psi. \tag{24}$$

It is easy to see that the commutativity of the flat spacetime of $\partial_i k_j = \partial_j k_i$ can be reproduced in the case of the Riemannian covariant derivative but also with the torsionful covariant derivative operator, as we shall see in what follows. Now, by substitution of the ansatz above for the wave phase on the equation for the magnetic wave potential, they obtained

$$k_i k^i = 0, \tag{25}$$

$$k_i \mathcal{A}^i = 0. \tag{26}$$

Let us now obtain the magnetic wave equation in our non-Riemannian case by following the same path but using the above Ricci–Cartan identity in the presence of torsion. We shall prove in what follows that, for example, their geodesic equation

$$k^i \nabla k_i = 0 \tag{27}$$

shall be substituted by the autoparallels which appear in Riemann–Cartan spacetime. In our case, actually, when the torsion 0-component is constant in the axionic background, we shall obtain the geodesic equations, and only when the torsion is a dynamical degree of freedom does one obtain the autoparallels which deviate from the geodesics. Now contracting indices i and k in Eq. (4), one obtains the Ricci–Cartan identity

$$[D_k, D_j] A_k = -R_{jl} A^l - 2 S_{kj}{}^l D_l A^k. \tag{28}$$

Now recall that we may actually substitute the Riemann–Cartan (RC) covariant derivative operator as

$$D_i = \partial_i - S_i, \tag{29}$$

where $S^i$ is the axial torsion vector. By substitution of this definition into the Ricci–Cartan identity, one obtains

$$\Box A^k - R^k{}_i A^i - 2 S^{kip} [\partial_p A_i - \partial_p (ln\sqrt{-g}) A_i] g_{a\gamma} \tilde{F}^{lk} \partial_l a = 0. \tag{30}$$

Note that this is the extension of the Schwarz et al. equation for the magnetic potential wave, where the wave operator is given by

$$\Box A^k - R^k{}_i A^i - 2 S^{kip} [\partial_p [1 - (ln\sqrt{-g})]] A_i - g_{a\gamma} \tilde{F}^{lk} \partial_l a = 0. \tag{31}$$

Let us now proceed to compute the difference or non-commutativity in principle of the covariant derivatives endowed with torsion as

$$\begin{aligned} 2D_{[i} k_{j]} &= D_i k_j - D_j k_i = (\nabla_i k_j - S_{ij}{}^l k_l) - (\nabla_j k_i - S_{ji}{}^l k_l) \\ &= -2 S_{ij}{}^l k_l. \end{aligned} \tag{32}$$

To complete the proof that for some constraints these commutators vanish, one simply has to show that the last torsion expression vanishes. However, expanding this equation in terms of relation $S_{ijl} k^l = \epsilon_{ijlp} S^p$, with the aid of this relation, we obtain that the last expression on the right-hand side of the last equation reads

$$2D_{[i} k_{j]} = D_i k_j - D_j k_i = -2\epsilon_{ijlp} S^p k^l = 0. \tag{33}$$

A simple and immediate solution of this expression is given by $S^p = \beta k^p$, where $\beta$ is a constant of proportionality. Therefore, for the RC covariant derivative operator D, to commute when applied to the wave vector k, it must have an axial torsion vector proportional to the wave vector. Note that as $\psi$ is a scalar field, and they do not couple with torsion, then

$$k_i k^i = (D_i\psi)(D^i\psi) = (\partial_i\psi\partial^i\psi) = 0, \tag{34}$$

as already demonstrated in the case of Riemannian background of axions. This allows one to say that under the assumption that the axial torsion vector is proportional to the wave vector k, the light in the non-Riemannian background in this case propagates according to the null equation (8). We will now show that in the special case of constant torsion, one may say that this null equation in RC spacetime does not necessarily imply that we will obtain that light and shall follow the geodesic equation as in the Riemannian case. To achieve this task, we shall make use of the ansatz above for the phase and the Eq. (14), which yields the equation

$$\mathcal{A}[\partial_i k^i + \dot{S}_0] = \left(S_0 - \frac{1}{2}g_{a\gamma}\dot{a}\right)k\mathcal{A}, \tag{35}$$

which is equivalent to

$$[\partial_i k^i + \dot{S}_0] = \left(S_0 - \frac{1}{2}g_{a\gamma}\dot{a}\right)k, \tag{36}$$

where to simplify matters, we assume that the vector magnetic potential is orthogonal to the wave vector k. The remaining equation obtained as the coefficients of $cos\psi$ and $sin\psi$ yields

$$\frac{\dot{\mathcal{A}}}{\mathcal{A}} = k\left(S_0 - \frac{1}{2}g_{a\gamma}\dot{a}\right). \tag{37}$$

Since we have assumed here that torsion is constant, which by the way favors Lorentz violation [19], this equation can be written as

$$\mathcal{A} = \mathcal{A}_0 \exp\left[kS_0 t - \frac{1}{2}g_{a\gamma}a(t)\right]. \tag{38}$$

Since one may express the magnetic field as $B = ikA$, one may notice that the magnetic dynamo amplification can be obtained from the last expression if we assume the inequality

$$kS_0 t \geq \frac{1}{2}g_{a\gamma}a(t). \tag{39}$$

Therefore, we notice that torsion evolution in time has a lower bound dependent on the axion scale. We can say that the axion scale has an upper bound which depends upon torsion and which scales we are. Numerical examples could be obtained if one knows at which parsec (pc) quantity we have a determined torsion and the $g_{a\gamma}$ which is very small. In the next section we shall show that even in flat spacetime, we can determine the dynamo equation from axion electrodynamics. By the way, there are cases where the axion and torsion potential are equivalent, which we have not assumed in this paper.

## 4 Bending light in Riemannian and non-Riemannian chiral media

In this section we shall address a very interesting aspect of torsion. This is the gravitational lenses. In the previous sections we studied the chiral light bending as it passes through an axion clump. Optical gravitational property analogues can be understood, and analogue optical gravitational lens systems were investigated by Leonhardt [20]. In this section we show that axion-torsion semi-minimal coupling is so rich that under strong curvature constraints on the RC curvature it may induce a gravitational lens effect. This can be understood as follows. Let us start with the electric and magnetic wave equations added by a chiral current proportional to the magnetic field in the chiral plasma by

$$\Box\mathbf{E} + \nabla(\nabla.\mathbf{E}) + \dot{\mathbf{J}}_P + \dot{\mathbf{J}}_a + \dot{\mathbf{J}}_5 = 0. \tag{40}$$

The currents are as follows: The plasma current

$$\dot{\mathbf{J}}_P = \omega_P\mathbf{E}, \tag{41}$$

the axion current is given by

$$\dot{\mathbf{J}}_a = g_{a\gamma}\dot{a}\mathbf{B}, \tag{42}$$

and the chiral current is

$$\dot{\mathbf{J}}_5 = \mu_5\mathbf{B}, \tag{43}$$

where $\mu_5$ is the chiral chemical potential. Similarly, the magnetic wave equation is

$$\Box\mathbf{B} - \nabla\times\mathbf{J}_a - \nabla\times\mathbf{J}_P - \nabla\times\mathbf{J}_5 = 0, \tag{44}$$

where here we have adopted that the axion scale a is uniform, and moreover, the magnetic fields are helical. Now, following the McDonald–Ventura paper [21], we shall employ the Hamiltonian formalism to obtain the trajectory of light beams, and verify here in Riemannian geometrical optics whether they bent when entering the axion clump, or followed the same trajectory as in optically transparent media.

We shall now assume the geometrical optics approximation which yields

$$\mathbf{E}=\mathbf{E}_0 e^{iS} \qquad \mathbf{B}=\mathbf{B}_0 e^{iS}, \tag{45}$$

where frequency and momentum are identified as $\omega=-\dot{S}$ and $\mathbf{k}=\nabla S$, where S is a surface of propagation. This eikonal approximation leads to the equations

$$\Box\mathbf{E}+\nabla(\nabla.\mathbf{E})+\sigma\dot{\mathbf{E}}[-i\omega k-ig_{a\gamma}\dot{a}\omega\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{B}=-\mu_5\lambda\dot{\mathbf{B}}, \tag{46}$$

where the parameter $\lambda$ is the magnetic helicity parameter from $curl\mathbf{B}=\lambda\mathbf{B}$ and

$$\Box\mathbf{B}-\sigma\dot{\mathbf{B}}-g_{a\gamma}\nabla\times[\dot{a}\mathbf{B}]=-\mu_5\mathbf{B}, \tag{47}$$

which become

$$\Box\mathbf{E}+{\omega_P}^2\mathbf{E}+g_{a\gamma}\frac{\mathrm{d}}{\mathrm{d}t}\dot{a}\mathbf{B}=-\mu_5\lambda\dot{\mathbf{B}}, \tag{48}$$

where the parameter $\lambda$ is the magnetic helicity parameter from $curl\mathbf{B}=\lambda\mathbf{B}$ and

$$\Box\mathbf{B}+{\omega_P}^2\mathbf{B}-g_{a\gamma}[\dot{a}\nabla\times\mathbf{B}]=-\mu_5\mathbf{B}. \tag{49}$$

The second derivatives of the axion were neglected in accordance with geometrical optics approximation. The system of differential equations (??) and (45) with the eikonal form of electric and magnetic fields in terms of S allows us to express this system in the form

$$\mathbf{M}(\omega,\mathbf{k}).(\mathbf{E},\mathbf{B})^T=0, \tag{50}$$

which is an eigen-matrix problem with eigenvalues of zero. The matrix $\mathbf{M}$ is such that the structure of the wave equations can be read from the matrix. Therefore, explicitly, the axion-photon-torsion with magnetic helicity eigenvalue may be expressed as

$$\begin{aligned}D^{\pm}=(k^2-\omega^2+{\omega_P}^2)+[(\mu_5-\lambda g_{a\gamma}\dot{a})\lambda\\ -\sigma\omega-\sigma(i+\omega)S_0+i\omega k]\pm b_0\sqrt{b_0},\end{aligned} \tag{51}$$

where $b_0$ is simply the first term inside the square brackets. Now we shall apply the wave electric and magnetic equations to a simple problem where no magnetic helicity and chiral currents are present, just ALP and torsion. Then the equations become

$$\Box\mathbf{B}_0-\omega\sigma\mathbf{B}_0-g_{a\gamma}[-(\omega+S_0)]\mathbf{E}_0=0, \tag{52}$$

where we have used the covariant derivative $D_0=\partial_0-iS_0$ into the d'Alembertian wave operator $\Box=\partial^2-\nabla^2$. Now the equation for the electric field is

$$[\omega^2-k^2-{\omega_P}^2-S_0\omega]\mathbf{E}_0-g_{a\gamma}\dot{a}\mathbf{B}_0=0. \tag{53}$$

Thus, these two equations for E and B within the eikonal equation can induce the matrix M and the eigenvalue problem. This gives rise to

$$D^{\pm}=(\omega^2-k^2-{\omega_P}^2)\pm\left[-\frac{3i}{2}S_0k\omega g_{a\gamma}\right]. \tag{54}$$

Therefore, by computing $D^{\pm}$ in terms of the world line parameter $\tau$, we have that this $D^{\pm}$ vanishes at the world line parameter.

$$\frac{\mathrm{d}}{\mathrm{d}\tau}D^{\pm}=\partial_k D^{\pm}\frac{\mathrm{d}x^i}{\mathrm{d}\tau}+\partial_x D^{\pm}\frac{\mathrm{d}x^i}{\mathrm{d}\tau}. \tag{55}$$

Note that here the expression $\partial_k D^{\pm}=\frac{\partial_k D^{\pm}}{\partial k}$. Therefore, from this chain rule of differential calculus, we obtain the Hamiltonian equation

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\tau}=-\partial_k D^{\pm} \qquad \frac{\mathrm{d}k^i}{\mathrm{d}\tau}=\partial_x D^{\pm}. \tag{56}$$

Since $k^i=\omega$, and $\mathbf{k}$ and $k_i=(\omega-\mathbf{k})$, we then eliminate $\tau$ by $t$ as cosmic time world line. This is simply understood by the expressions

$$\frac{\mathrm{d}\mathbf{x}}{\mathbf{d}t}=-\frac{\partial_{\mathbf{k}}D^{\pm}}{\partial_\omega D^{\pm}}. \tag{57}$$

Now let us apply the $D^{\pm}$ of Eq. (57). This yields

$$\partial_{\mathbf{k}}D^{\pm}=-2k\pm\left[-\frac{3i}{2}S_0\omega g_{a\gamma}\right], \tag{58}$$

$$\partial_\omega D^{\pm}=-2k^2\pm\left[-\frac{3i}{2}S_0 k g_{a\gamma}\right]. \tag{59}$$

Therefore, from the last two equations, one obtains

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t}=\frac{-2k\pm\frac{3i}{2}S_0\omega g_{a\gamma}}{2\omega\pm\frac{3i}{2}S_0 k g_{a\gamma}}. \tag{60}$$

Note that if $\omega=k$, we have that

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t}=-1, \tag{61}$$

and obviously this expression respects the geodesic equation in flat spacetime, since

$$\frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2}=0. \tag{62}$$

But note that if frequency coincides with the wave vector k, this simply states that $c = 1$ or the velocity of light coincides with the velocity at vacuum; this actually means that the light has found in ALP a transparent media so it could ignore the torsionful ALP clumps, since if the light traverses a nontransparent chiral media, the frequency and wave vector should be distinct so the ratio does not reduce to unity. Let us now approximate this expression by

$$\frac{dx}{dt} = -\left[4k\omega \mp 3i S_0 \omega^2 g_{a\gamma} + \frac{9}{8} {S_0}^2 k g_{a\gamma} \dot{a}^2 \omega\right]. \tag{63}$$

But note that the last term on the right-hand side of this equation can be neglected for weak torsion, keeping only terms of the first order in the axion–photon coupling. Therefore, one obtains

$$\frac{dx}{dt} = -\left[4k\omega \mp 3i S_0 \omega^2 g_{a\gamma}\right]. \tag{64}$$

Then, if we apply the time derivative once more, we obtain

$$\frac{d^2x}{dt^2} = -\left[3\omega^2 k g_{a\gamma} \frac{dS^0}{dt}\right], \tag{65}$$

which can be expressed in terms of the phase velocity, and $v_{ph} = \omega k$ becomes

$$\frac{d^2x}{dt^2} = -3\omega v_{ph} k g_{a\gamma} \frac{dS^0}{dt}, \tag{66}$$

where we also have that the frequency is high. Therefore, we show that dynamical torsion is fundamental to bending of light when passing through an APL torsion clump. Actually, if one considers the geodesic equation in Riemannian spacetime, we obtain a very well-known expression

$$\frac{d^2x}{dt^2} = -{\Gamma^k}_{il} \frac{dx^i}{dt} \frac{dx^l}{dt}. \tag{67}$$

From the right-hand-side terms of these last two equations, even with the presence of phase velocity, they describe very similar equations. But since we are dealing with a flat metric plus weak torsion, where the Riemann–Christoffel symbols $\Gamma$ vanish, as the term on the right-hand side indicates, it seems to be an equation where the spinning particle passes by a Riemann–Cartan spacetime background. Clearly in our case we have a geodetic deviation in RC spacetime. Accordingly to Hehl and Trautman [22], the geodesic deviation equation possesses a term which is proportional to the Riemann–Cartan curvature tensor and components of the spin-angular momentum and velocity. In addition, in a Riemann–Cartan curvature tensor there are components of torsion.

## 5 Dynamical BI parameter and planar waves

For a Lagrangian of the type

$$\mathcal{L}_{BI} \supset (\partial\gamma)^2 - \frac{{m^2}_P}{\gamma} TS + \frac{1}{2}{m^2}_s S^2 + \frac{1}{2}{m^2}_T T^2, \tag{68}$$

we assume that the BI field $\gamma(x)$ propagates in vacuum and the matter Lagrangian $\mathcal{L}_m$ can be disregarded. The variation of this Lagrangian with respect to $\gamma$ or $\delta_\gamma \mathcal{L}_{BI}$ then yields the equation

$$\Box\gamma - \frac{{m^2}_P}{2\gamma^2} T.S = 0. \tag{69}$$

From this wave equation sourced by torsion, one notes that for a constant BI parameter, it is not necessary for both but only for at least one torsion mode to vanish. This does not imply GR. Let us therefore perform the variation of T and S of the Lagrangian $\mathcal{L}_{BI}$. One obtains

$$\frac{{m^2}_P}{\gamma} S - {m^2}_T T = 0 \tag{70}$$

and

$$\frac{{m^2}_P}{\gamma} T - {m^2}_s S = 0. \tag{71}$$

Note that from these two last equations, when T.S vanish, T and also S vanish at the same time, and consequently the lemma that a constant BI field implies GR is valid here as well. Note that the converse of this lemma is not true. For example, assume that the torsion modes vanish, meaning that one is on a GR background; nevertheless, the BI field is not constant, since the BI field satisfies a wave equation of the type

$$\Box\gamma = 0, \tag{72}$$

which is a planar BI wave in GR.

## 6 Conclusions

In this paper we have studied the behavior of physical quantities and mechanisms such as photons and dynamos in the presence of torsionful axion-like particles in a clump. Several previously reported results agree, and we show that they are consistent, that in flat torsionless spacetimes the clumps of APLs are unable to bend light. Nevertheless, in addition to to investigating several consequences of axion electrodynamics and the non-Riemannian geometrical optics we have shown in the last section of the paper that in the presence of dynamical torsion, the chiral bending of light actually seems to be possible in first-order torsion and the axion–photon conversion parameter. We hope that in the near future, this paper will help others to generalize these ideas, for example, by investigating how light can passes by a spin-polarized structure where certainly we would have non-dynamical torsion degrees of freedom. Recently, non-dynamical degrees of freedom have been used to obtain four-fermion EC gravity with constant BI parameters [23]. Carroll and Field [23] used the parameter b here, as an indication of propagation of spin-0 fields which could be used as a torsion scalar in quantum electrodynamics (QED) such as in photon propagation by de Sabbata and Gasperini [24]. Piani et al. [25] recently found oscillons at the preheating phase of EC cosmology. The idea of a torsion scalar degree of freedom as an ALP was taken from Duncan et al. [26]. A great deal of the geometrical optics part of the paper was inspired by previous works [27–30].

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