BLED WORKSHOPS
IN PHYSICS
VOL. 3, NO. 4

Proceedings to the 5th Workshop
***What comes beyond** . . . (p. 163)*
Bled, Slovenia, July 13-24, 2002

Neutrino Oscillations in Vacuum on the Large Distance: Influence of the Leptonic CP-phase.

D.A. Ryzhikh* and K.A. Ter-Martirosyan**

ITEP, Moscow, 117259, B.Cheremushkinskaya, 25

Abstract. Vacuum neutrino oscillations for three generations are considered. The influence of the leptonic CP-violating phase (similar to the quarks CP-phase) on neutrino oscillations is taken into account in the matrix of leptons mixing. The dependence of probabilities of a transition of one kind neutrino to another kind on three mixing angles and on the CP-phase is obtained in a general form. It is pointed that one can reconstruct the value of the leptonic CP-phase by measuring probabilities for a transition of one kind neutrino to another kind averaging over all oscillations. Also it is noted that the manifestation of the CP-phase in deviations of probabilities of forward neutrino transitions from probabilities of backward neutrino transitions is an effect practically slipping from an observation.

1 Introduction.

It is unclear up to now in spite of great number of papers devoted to the investigation of neutrino oscillations, what is the real precision of experimental values of three mixing angles and masses of neutrino from different generations? And consequently do neutrino really oscillate? The central values of these angles and values of errors obtained in different papers and given in our paper change from author to author and from paper to paper. Therefore these data are very suspicious. Below in this paper we gave the value of this precision approximately because it is defined very roughly. Nevertheless the investigations of neutrino look rather encouraging since the set of large perspective devices (K2K in Japan [1,2], CERN-GRAND Sacco (CNGS) [3] in Europe and Fermilab-Soudan in USA) and some small but also perspective devices in another regions of the Earth began to work recently or will begin to work in the near future. In particular, the precision of defining of the values of ν_τ and ν_μ masses and also the values of sines of the neutrino mixing angles will be appreciably improved in the nearest future (in one or two years). Furthermore, we believe that attempts to obtain the value of the CP-phase from experimental data will be made in the future in spite of apparent present-day hopelessness.

The present work is devoted to neutrino oscillations and, in particular, to a possible manifestation of the leptonic CP-phase in neutrino oscillations. At the second section of this paper we consider the standard theory of the neutrino oscillations with regard for the leptonic CP-violating phase. Then we give the

* ryzhikh@heron.itep.ru
** termarti@heron.itep.ru

formulas for the probabilities of the conservation of the neutrino kind and for the probabilities of the neutrino transition to another neutrino kind with some examples of the possible manifestation of the leptonic CP-phase using modern experimental data. And then we investigate the difference between the $\nu_\alpha \to \nu_\beta$ transitions probability and the $\nu_\beta \to \nu_\alpha$ transitions probability and the possible influence of the leptonic CP-phase on this difference.

2 Standard theory of neutrino oscillations with regard for leptonic CP-violating phase.

In this section we describe the standard theory of neutrino oscillations including the leptonic CP-phase. So, neutrino $(\nu_e)_L, (\nu_\mu)_L, (\nu_\tau)_L$ which were born in the decay reactions or in collisions do not have definite masses. They are superpositions of neutrino states ν_1, ν_2, ν_3 with definite masses, and their wave functions are:

$$\nu_\beta(x,t) = \sum_{k=1}^{3} (\hat{V}^l)_{\beta k} \nu_k(x,t), \quad \beta = e, \mu, \tau; \quad k = 1,2,3. \tag{1}$$

Here it is supposed that $\nu_k = (\nu_1, \nu_2, \nu_3)$ are the wave functions of the neutrino with definite masses moved in a beam along the axis OX with not small momentum $|\mathbf{p}_\nu| \gg m_\nu$ and ultrarelativistic energy $E_k = \sqrt{\mathbf{p}_\nu^2 + m_k^2} \simeq |\mathbf{p}_\nu| + m_k^2/2p_\nu$, $k = 1,2,3$. Thus their wave functions look like:

$$\nu_k(x,t) = e^{i\mathbf{p}_\nu x} e^{-iE_k t} \nu_k(0) = e^{-i\frac{m_k^2}{2p_\nu}t} \nu_k(0) \tag{2}$$

Mixing of leptons, i.e. mixing of neutrino when the mass matrix of "electrons" of three-generations is diagonal, is defined by a unitary 3×3 matrix $\hat{V}^l = \hat{V}^l_{MNS}$ Maki–Nakagava–Sakata. This matrix depends on three mixing angles of the leptons $\vartheta_{12}, \vartheta_{13}$ and ϑ_{23}. It is similar to CKM matrix of quarks mixing and has a well-known form:

$$\hat{V}^l = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_l} \\ -s_{12}c_{13} - c_{12}s_{23}s_{13}e^{i\delta_l} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_l} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_l} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_l} & c_{23}c_{13} \end{pmatrix} \tag{3}$$

Note that the $\hat{V}^l$ can be represented in the form of the product of three matrices of rotations (or of mixing of two generations in pairs) $\hat{O}_{12}, \hat{O}_{13}(\delta_l)$ and $\hat{O}_{23}$. It is easy to verify that $\hat{V}^l \equiv \hat{O}_{12}\hat{O}_{13}(\delta_l)\hat{O}_{23}$, where:

$$\hat{O}_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \hat{O}_{13}(\delta_l) = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_l} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_l} & 0 & c_{13} \end{pmatrix},$$
$$\hat{O}_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}; \tag{4}$$

here and in (3) δ_l is the leptonic CP-violating phase. Its value is not known up to now, sometimes, for example, it is considered to be equal to 0 whereas its analogue – the quark CP-phase δ_q seems to be close to $\pi/2$ [4]. Acting by matrix (3) on column $\hat{\nu} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$ we obtain according to (1):

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} (t) = \hat{V}_l \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\begin{cases} \nu_e(t) = [c_{12}c_{13}\nu_1(0) + s_{12}c_{13}\nu_2(0)e^{-i\varphi_{21}} \\ \qquad\qquad + s_{13}\nu_3(0)e^{-i\varphi_{31}-i\delta_l}]e^{-i\frac{m_1^2}{2p_\nu}t} \\ \nu_\mu(t) = [-(s_{12}c_{23} + c_{12}s_{23}s_{13}e^{i\delta_l})\nu_1(0) \\ \qquad\qquad + (c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_l})\nu_2(0)e^{-i\varphi_{21}} \\ \qquad\qquad + c_{13}s_{23}\nu_3(0)e^{-i\varphi_{31}}]e^{-i\frac{m_1^2}{2p_\nu}t} \\ \nu_\tau(t) = [(s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_l})\nu_1(0) \\ \qquad\qquad - (c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta_l})\nu_2(0)e^{-i\varphi_{21}} \\ \qquad\qquad + c_{13}c_{23}\nu_3(0)e^{-i\varphi_{31}}]e^{-i\frac{m_1^2}{2p_\nu}t} \end{cases} \tag{5}$$

where, using dependence (2) of neutrino states on the time $t = L/c$ we have:

$$\varphi_{ij} = \frac{(m_i^2 - m_j^2)}{2p_\nu}t = 1.27\frac{(m_i^2 - m_j^2)(\mathrm{eV}^2)}{E_\nu(\mathrm{MeV})}L(\mathrm{m}) \tag{6}$$

where $E_\nu \simeq cp_\nu$ is an energy of the neutrino beam: $E_\nu \gg m_3 > m_2 > m_1$. Neutrino states with definite masses are mutually orthogonal and are normalized to unity. Using these statements we can easy obtain expressions for probabilities of a transition in vacuum of one kind neutrino to neutrino of another kind during the time t.

For probabilities of conservation of e, μ, τ–neutrino kind we have, respectively:

$$\begin{cases} P(\nu_e\nu_e) = |c_{12}^2c_{13}^2 + s_{12}^2c_{13}^2e^{i\varphi_{21}} + s_{13}^2e^{i\varphi_{31}}|^2 \\ P(\nu_\mu\nu_\mu) = ||c_{13}s_{12} + c_{12}s_{13}s_{23}e^{i\delta_l}|^2 + |c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_l}|^2e^{i\varphi_{21}} \\ \qquad\qquad + c_{13}^2s_{23}^2e^{i\varphi_{31}}|^2 \\ P(\nu_\tau\nu_\tau) = ||s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_l}|^2 + |c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta_l}|^2e^{i\varphi_{21}} \\ \qquad\qquad + c_{13}^2c_{23}^2e^{i\varphi_{31}}|^2 \end{cases} \tag{7}$$

And for probabilities of transitions of ν_α neutrino to neutrino of another kind ν_β we obtain:

$$\begin{cases} P(\nu_e\nu_\mu) = |c_{12}c_{13}(c_{13}s_{12} + c_{12}s_{23}s_{13}e^{i\delta_l}) \\ \qquad - c_{13}s_{12}(c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_l})e^{i\varphi_{21}} \\ \qquad - s_{13}c_{13}s_{23}e^{i(\delta_l+\varphi_{31})}|^2 \\ P(\nu_e\nu_\tau) = |c_{12}c_{13}(s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta_l}) \\ \qquad - c_{13}s_{12}(c_{12}s_{23} + c_{23}s_{12}s_{13}e^{i\delta_l})e^{i\varphi_{21}} \\ \qquad + s_{13}c_{13}c_{23}e^{i(\delta_l+\varphi_{31})}|^2 \\ P(\nu_\mu\nu_\tau) = |(c_{13}s_{12} + c_{12}s_{13}s_{23}e^{i\delta_l})(s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta_l}) \\ \qquad + (c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta})(c_{12}s_{23} + c_{23}s_{12}s_{13}e^{i\delta})e^{i\varphi_{21}} \\ \qquad - c_{23}c_{13}^2s_{23}e^{i\varphi_{31}}|^2 \end{cases} \tag{8}$$

3 Probabilities of the change of the neutrino kind $1 - P(\nu_\alpha\nu_\alpha)$ and of neutrino ν_α transition to neutrino ν_β of another kind: $P(\nu_\alpha\nu_\beta)$.

After not complicated, but cumbersome transformations of the formulas (7),(8) we have complete expressions for the probability of the change of neutrino kind $1-P(\nu_\alpha\nu_\alpha)$ and for the probabilities of transition of one kind neutrino to neutrino of another kind. But these formulas are very complex for an analysis and because of experimental peculiarities of the neutrino registration it is more convenient to use probabilities averaging over oscillations, i.e. over phases (6) of neutrino of the continuous spectra. Therefore we adduce these complete formulas only for references.

$$\begin{aligned} 1 - P(\nu_e\nu_e) = {} & c_{12}^2\sin^2(2\vartheta_{13})\sin^2(\varphi_{31}/2) + c_{13}^4\sin^2(2\vartheta_{12})\sin^2(\varphi_{21}/2) \\ & + s_{12}^2\sin^2(2\vartheta_{13})\sin^2(\varphi_{32}/2) \end{aligned}$$

$$\begin{aligned} 1 - P(\nu_\mu\nu_\mu) = {} & \{c_{23}^4\sin^2(2\vartheta_{12}) + s_{12}^4s_{13}^2\sin^2(2\vartheta_{23}) + s_{23}^4s_{13}^4\sin^2(2\vartheta_{12}) \\ & + c_{12}^4s_{13}^2\sin^2(2\vartheta_{23}) + \cos\delta_l\sin(4\vartheta_{12})\sin(2\vartheta_{23})(s_{13}c_{23}^2 - s_{13}^3s_{23}^2) \\ & - \cos^2\delta_l s_{13}^2\sin^2(2\vartheta_{23})\sin^2(2\vartheta_{12})\}\sin^2(\varphi_{21}/2) \\ & + \{s_{12}^2c_{13}^2\sin^2(2\vartheta_{23}) + c_{12}^2s_{23}^4\sin^2(2\vartheta_{13}) \\ & + \cos\delta_l s_{23}^2c_{13}\sin(2\vartheta_{12})\sin(2\vartheta_{23})\sin(2\vartheta_{13})\}\sin^2(\varphi_{31}/2) \\ & + \{c_{12}^2c_{13}^2\sin^2(2\vartheta_{23}) + s_{12}^2s_{23}^4\sin^2(2\vartheta_{13}) \\ & - \cos\delta_l s_{23}^2c_{13}\sin(2\vartheta_{12})\sin(2\vartheta_{23})\sin(2\vartheta_{13})\}\sin^2(\varphi_{32}/2) \end{aligned}$$

$$\begin{aligned} 1 - P(\nu_\tau\nu_\tau) = {} & \{s_{23}^4\sin^2(2\vartheta_{12}) + s_{12}^4s_{13}^2\sin^2(2\vartheta_{23}) \\ & + c_{23}^4s_{13}^4\sin^2(2\vartheta_{12}) + c_{12}^4s_{13}^2\sin^2(2\vartheta_{23}) \\ & + \cos\delta_l\sin(4\vartheta_{12})\sin(2\vartheta_{23})(s_{13}^3c_{23}^2 - s_{13}^2s_{23}^2) \\ & - \cos^2\delta_l s_{13}^2\sin^2(2\vartheta_{23})\sin^2(2\vartheta_{12})\}\sin^2(\varphi_{21}/2) \\ & + \{s_{12}^2c_{13}^2\sin^2(2\vartheta_{23}) + c_{12}^2c_{23}^4\sin^2(2\vartheta_{13}) \\ & - \cos\delta_l c_{23}^2c_{13}\sin(2\vartheta_{12})\sin(2\vartheta_{23})\sin(2\vartheta_{13})\}\sin^2(\varphi_{31}/2) \\ & + \{c_{12}^2c_{13}^2\sin^2(2\vartheta_{23}) + s_{12}^2c_{23}^4\sin^2(2\vartheta_{13}) \\ & + \cos\delta_l c_{23}^2c_{13}\sin(2\vartheta_{12})\sin(2\vartheta_{23})\sin(2\vartheta_{13})\}\sin^2(\varphi_{32}/2) \end{aligned}$$

$$
\begin{aligned}
P(\nu_e\nu_\mu) = \frac{1}{4}\{&\sin^2(2\vartheta_{13})(s_{23}^2 + c_{12}^4 s_{23}^2 + s_{12}^4 s_{23}^2)\\
&+ \frac{1}{2}c_{13}\sin(2\vartheta_{13})\sin(2\vartheta_{23})\sin(4\vartheta_{12})\cos\delta_l)\\
&- 2c_{13}^2\sin^2(2\vartheta_{12})(c_{23}^2 - s_{13}^2 s_{23}^2)\cos(\varphi_{21})\\
&- 2s_{23}^2\sin^2(2\vartheta_{13})(c_{12}^2\cos(\varphi_{31}) + s_{12}^2\cos(\varphi_{32}))\\
&+ c_{13}\sin(2\vartheta_{12})\sin(2\vartheta_{13})\sin(2\vartheta_{23})\\
&\quad\cdot(s_{12}^2\cos(\delta_l + \varphi_{21}) - c_{12}^2\cos(\delta_l - \varphi_{21}))\\
&+ c_{13}\sin(2\vartheta_{12})\sin(2\vartheta_{13})\sin(2\vartheta_{23})\\
&\quad\cdot(\cos(\delta_l + \varphi_{32}) - \cos(\delta_l - \varphi_{31}))\\
&+ 2c_{13}^2 c_{23}^2\sin^2(2\vartheta_{12})\}
\end{aligned}
$$

$$
\begin{aligned}
P(\nu_e\nu_\tau) = \frac{1}{4}\{&\sin^2(2\vartheta_{13})(c_{23}^2 + c_{12}^4 c_{23}^2 + s_{12}^4 c_{23}^2)\\
&- \frac{1}{2}c_{13}\sin(2\vartheta_{13})\sin(2\vartheta_{23})\sin(4\vartheta_{12})\cos\delta_l)\\
&+ 2c_{13}^2\sin^2(2\vartheta_{12})(s_{23}^2 - c_{13}^2 s_{23}^2)\cos(\varphi_{21})\\
&\quad - 2c_{23}^2\sin^2(2\vartheta_{13})(c_{12}^2\cos(\varphi_{31}) + s_{12}^2\cos(\varphi_{32}))\\
&+ c_{13}\sin(2\vartheta_{12})\sin(2\vartheta_{13})\sin(2\vartheta_{23})\\
&\quad\cdot(c_{12}^2\cos(\delta_l - \varphi_{21}) - s_{12}^2\cos(\delta_l + \varphi_{21}))\\
&+ c_{13}\sin(2\vartheta_{12})\sin(2\vartheta_{13})\sin(2\vartheta_{23})\\
&\quad\cdot(\cos(\delta_l + \varphi_{31}) - \cos(\delta_l + \varphi_{32}))\\
&+ 2c_{13}^2 s_{23}^2\sin^2(2\vartheta_{12})\}
\end{aligned}
$$

$$
\begin{aligned}
P(\nu_\mu\nu_\tau) = \frac{1}{4}\{&2s_{13}^2\sin^2(2\vartheta_{12})\cos^2(2\vartheta_{23})\\
&+ (c_{13}^4 + c_{12}^4 + s_{12}^4 + (c_{12}^4 + s_{12}^4)s_{13}^4)\sin^2(2\vartheta_{23})\\
&- [2s_{13}^2(c_{23}^4 + s_{23}^4)\sin^2(2\vartheta_{12}) + [2s_{13}^2(c_{12}^4 + s_{12}^4)\\
&\quad - (1 + s_{13}^4)\sin^2(2\vartheta_{12})]\sin^2(2\vartheta_{23})]\cos(\varphi_{21})\\
&- [2c_{13}^2(s_{12}^2 + c_{12}^2 s_{13}^2)\sin^2(2\vartheta_{23})\\
&\quad - \frac{1}{2}c_{13}\sin(2\vartheta_{12})\sin(2\vartheta_{13})\sin(4\vartheta_{23})\cos\delta_l]\cos(\varphi_{31})\\
&+ [2c_{13}^2\sin^2(2\vartheta_{23})(s_{12}^2 s_{13}^2 - c_{12}^2)\\
&\quad - \frac{1}{2}c_{13}\sin(2\vartheta_{12})\sin(2\vartheta_{13})\sin(4\vartheta_{23})\cos\delta_l]\cos(\varphi_{32})\\
&+ 2c_{13}\sin(2\vartheta_{12})\sin(2\vartheta_{13})\sin(2\vartheta_{23})\\
&\quad\cdot\sin\delta_l\sin(\varphi_{21}/2)\cos(\frac{\varphi_{31} + \varphi_{32}}{2})\\
&+ s_{13}\sin(4\vartheta_{12})\sin(4\vartheta_{23})\cos\delta_l[1 + s_{13}^2]\sin^2(\varphi_{21}/2)\\
&- c_{13}\sin(2\vartheta_{12})\sin(2\vartheta_{13})\sin(2\vartheta_{23})\sin\delta_l\sin(\varphi_{21})\\
&- 2s_{13}^2\sin^2(2\vartheta_{12})\sin^2(2\vartheta_{23})\cos(2\delta_l)\sin^2(\varphi_{21}/2)\}
\end{aligned}
$$

Note that the probability of $\nu_e\nu_e$ oscillations does not depend on the leptonic CP-phase in contrast to another probabilities of the neutrino oscillations. After averaging these formulas over all phases φ_{ij} and taking into account $\langle\cos(\varphi_{ij} \pm \delta_l)\rangle = 0$, $\langle\sin^2\varphi_{ij}\rangle = \langle\cos^2\varphi_{ij}\rangle = 1/2$ we have:

$$
\left\{
\begin{aligned}
\langle 1 - P(\nu_e\nu_e)\rangle &= A_{ee}\\
\langle 1 - P(\nu_\mu\nu_\mu)\rangle &= A_{\mu\mu} + B_{\mu\mu}\cos\delta_l + C_{\mu\mu}\cos^2\delta_l\\
\langle 1 - P(\nu_\tau\nu_\tau)\rangle &= A_{\tau\tau} + B_{\tau\tau}\cos\delta_l + C_{\tau\tau}\cos^2\delta_l\\
\langle P(\nu_e\nu_\mu)\rangle &= A_{e\mu} + B_{e\mu}\cos\delta_l\\
\langle P(\nu_e\nu_\tau)\rangle &= A_{e\tau} + B_{e\tau}\cos\delta_l\\
\langle P(\nu_\mu\nu_\tau)\rangle &= A_{\mu\tau} + B_{\mu\tau}\cos\delta_l + C_{\mu\tau}\cos(2\delta_l)
\end{aligned}
\right. \tag{9}
$$

$$
\begin{cases}
A_{ee} = \frac{1}{2}[c_{13}^4 \sin^2(2\vartheta_{12}) + \sin^2(2\vartheta_{13})] & \\
A_{\mu\mu} = \frac{1}{2}[(c_{13}^2 + (c_{12}^4 + s_{12}^4)s_{13}^2)\sin^2(2\vartheta_{23}) & B_{\mu\mu} = \frac{1}{2}(c_{23}^2 - s_{23}^2 s_{13}^2)s_{13} \\
\quad +(s_{13}^4 \sin^2(2\vartheta_{12}) + \sin^2(2\vartheta_{13}))s_{23}^4 & \quad \cdot \sin(2\vartheta_{23})\sin(4\vartheta_{12}) \\
\quad +c_{23}^4 \sin^2(2\vartheta_{12})] & C_{\mu\mu} = -\frac{1}{2}s_{13}^2 \sin^2(2\vartheta_{23}) \\
& \quad \cdot \sin^2(2\vartheta_{12}) \\
A_{\tau\tau} = \frac{1}{2}[(c_{13}^2 + (c_{12}^4 + s_{12}^4)s_{13}^2)\sin^2(2\vartheta_{23}) & B_{\tau\tau} = -\frac{1}{2}s_{13}\sin(2\vartheta_{23}) \\
\quad +(s_{13}^4 \sin^2(2\vartheta_{12}) + \sin^2(2\vartheta_{13}))c_{23}^4 & \quad \cdot (s_{23}^2 - c_{23}^2 s_{13}^2)\sin(4\vartheta_{12}) \\
\quad +s_{23}^4 \sin^2(2\vartheta_{12})] & C_{\tau\tau} = -\frac{1}{2}s_{13}^2 \sin^2(2\vartheta_{23}) \\
& \quad \cdot \sin^2(2\vartheta_{12}) \\
A_{e\mu} = \frac{1}{4}[(1 + c_{12}^4 + s_{12}^4)s_{23}^2 \sin^2(2\vartheta_{13})] & B_{e\mu} = \frac{1}{8}c_{13}\sin(2\vartheta_{13})\sin(2\vartheta_{23}) \\
\quad +2c_{13}^2 c_{23}^2 \sin^2(2\vartheta_{12})] & \quad \cdot \sin(4\vartheta_{12}) \\
A_{e\tau} = \frac{1}{4}[(1 + c_{12}^4 + s_{12}^4)c_{23}^2 \sin^2(2\vartheta_{13})] & B_{e\tau} = -\frac{1}{8}c_{13}\sin(2\vartheta_{13}) \\
\quad +2c_{13}^2 s_{23}^2 \sin^2(2\vartheta_{12})] & \quad \cdot \sin(2\vartheta_{23})\sin(4\vartheta_{12}) \\
A_{\mu\tau} = \frac{1}{4}[2s_{13}^2 \sin^2(2\vartheta_{12})\cos^2(2\vartheta_{23}) & B_{\mu\tau} = \frac{1}{8}(1 + s_{13}^2)s_{13}\sin(4\vartheta_{12}) \\
\quad + \sin^2(2\vartheta_{23})\{(c_{12}^4 + s_{12}^4)s_{13}^4 & \quad \cdot \sin(4\vartheta_{23}) \\
\quad +c_{13}^4 + c_{12}^4 + s_{12}^4\}] & C_{\mu\tau} = -\frac{1}{4}s_{13}^2 \sin^2(2\vartheta_{12}) \\
& \quad \cdot \sin^2(2\vartheta_{23})
\end{cases}
\tag{10}
$$

These expressions are organized in such a way for to emphasize the influence of the leptonic CP-phase on the averaging probabilities of the neutrino oscillations.

Note that the probabilities of the change of neutrino kind and the probabilities of transitions to another two neutrino states obviously obey the following rules:

$$1 - P(\nu_\alpha \nu_\alpha) = P(\nu_\alpha \nu_\beta) + P(\nu_\alpha \nu_\gamma), \text{ where } \alpha, \beta, \gamma = e, \mu, \tau.$$

4 Some examples of the manifestation of the leptonic CP-phase

In this section we give some examples demonstrating a possible dependence of the probabilities (9) on CP-phase. But first, for the convenience we introduce new designations:

$$b_{ik} = B_{ik}/A_{ik}, \; c_{ik} = C_{ik}/A_{ik}$$

The first set of possible values for mixing angles taken from experimental data [5] is:
example a)

$$\vartheta_{12} = (42 \pm 2)^\circ, \; \vartheta_{13} = (4.0 \pm 0.5)^\circ, \; \vartheta_{23} = (43.6 \pm 0.5)^\circ \tag{11}$$

(small mixing of 1,3 generations was obtained from experimental data of nuclear reactor CHOOZ [3]). Here and below in our paper an average error is taken from tables adduced in papers [5,6]. These values are preliminary and are used below for our estimations. In this case all coefficients b_{ik}, c_{ik} in formulas (9) have very small values, in particular because of smallness of mixing angle $s_{13} = \sin\vartheta_{13}$. The table of all coefficients of formulas (9) for this case is:

$$\begin{cases} A_{ee} = 0.499; \\ A_{\mu\mu} = 0.636,\ b_{\mu\mu} = 0.0058,\ c_{\mu\mu} = -0.0038; \\ A_{\tau\tau} = 0.613,\ b_{\tau\tau} = 0.0055,\ c_{\tau\tau} = -0.0040; \\ A_{e\mu} = 0.261,\ b_{e\mu} = 0.014; \\ A_{e\tau} = 0.238,\ b_{e\tau} = -0.015; \\ A_{\mu\tau} = 0.373,\ b_{\mu\tau} = 0.0005,\ c_{\mu\tau} = -0.0032. \end{cases} \tag{12}$$

As we can see, the ratio of the number of the μ-neutrino to the number of the τ-neutrino produced in the initial beam of electron neutrino ν_e at a large distance from the source is:

$$\frac{\langle P(\nu_e\nu_\mu)\rangle}{\langle P(\nu_e\nu_\tau)\rangle} = \frac{A_{e\mu}}{A_{e\tau}} \cdot \frac{(1 + b_{e\mu}\cos\delta_l)}{(1 + b_{e\tau}\cos\delta_l)} \simeq \frac{A_{e\mu}}{A_{e\tau}}(1 + (b_{e\mu} - b_{e\tau})\cos\delta_l) \tag{13}$$

where $b_{e\mu} - b_{e\tau} \simeq 2b_{e\mu} \simeq 2.8\%$, (as $b_{e\tau} \simeq -b_{e\mu}$), with $\frac{A_{e\mu}}{A_{e\tau}} \simeq 1.1$. Thus, the contribution of terms containing $\cos\delta_l$ to the ratio of probabilities $\langle P(\nu_e\nu_\mu)\rangle$ and $\langle P(\nu_e\nu_\tau)\rangle$ is of order of 3%. Therefore an experimental observation of the CP-violating phase manifestation is very difficult for this set of mixing angles (see Fig.1).

The second set of possible values for mixing angles taken from experimental data is:

example b)

$$\vartheta_{12} = (42.0 \pm 2.0)^\circ,\ \vartheta_{13} = (14.0 \pm 1.0)^\circ,\ \vartheta_{23} = (43.6 \pm 0.5)^\circ, \tag{14}$$

(this example is not in a good agreement with experimental data [3] because, although it gives appropriate values for ϑ_{12} and ϑ_{23} (11), the value of ϑ_{13} is rather large here [7]).

In this case, i.e. at $\sin\vartheta_{13} = 0.24$, coefficients b_{ik}, c_{ik} have larger values in comparison with the previous case. The values of these coefficients are of the order of several percent, in particular, $b_{e\mu} - b_{e\tau} \simeq 2b_{e\mu} \simeq 8.4\%$. So, we have in this case:

$$\begin{cases} A_{ee} = 0.548; \\ A_{\mu\mu} = 0.645,\ b_{\mu\mu} = 0.019,\ c_{\mu\mu} = -0.044; \\ A_{\tau\tau} = 0.627,\ b_{\tau\tau} = 0.017,\ c_{\tau\tau} = -0.045; \\ A_{e\mu} = 0.283,\ b_{e\mu} = 0.041; \\ A_{e\tau} = 0.265,\ b_{e\tau} = -0.043; \\ A_{\mu\tau} = 0.348,\ b_{\mu\tau} = 0.0077,\ c_{\mu\tau} = -0.0409. \end{cases} \tag{15}$$

Since $b_{e\mu} - b_{e\tau} \simeq 8.4\%$, the contribution of terms containing $\cos\delta_l$ to the ratio of probabilities $\langle P(\nu_e\nu_\mu)\rangle$ and $\langle P(\nu_e\nu_\tau)\rangle$ is of order of 8.4%. The results of corresponding measurements seem to be very interesting (see Fig.2).

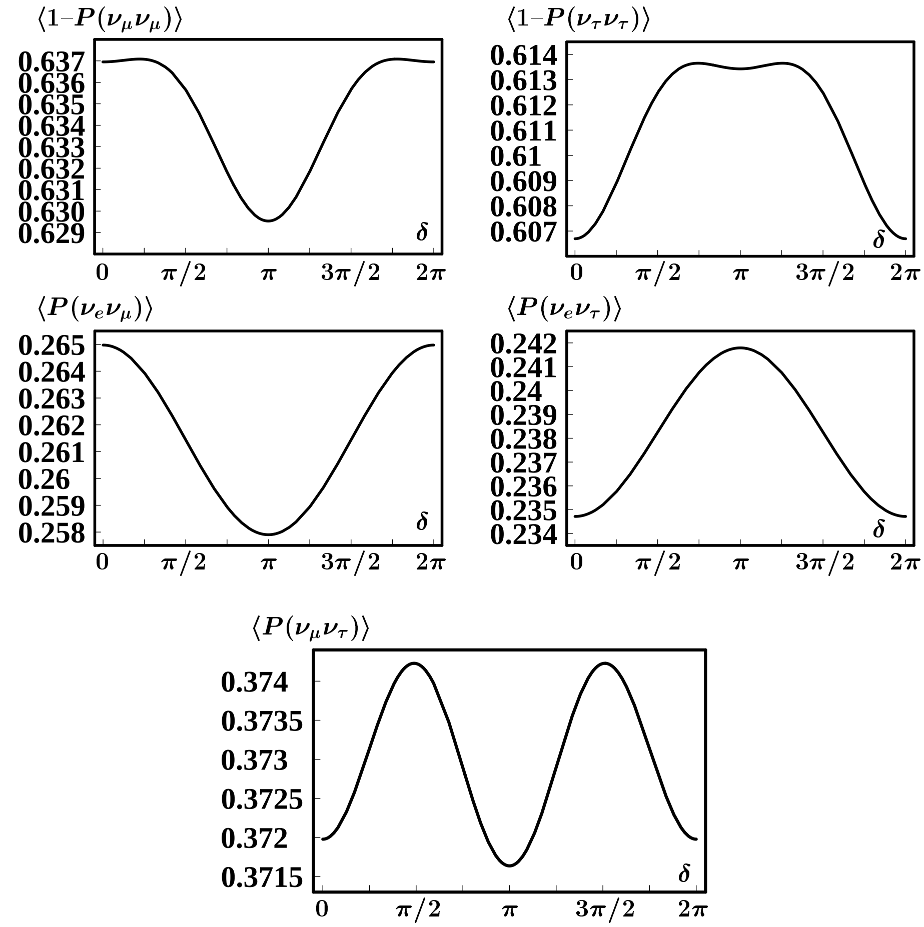

Fig. 1. In this figure the dependences of the probabilities averaging over neutrino oscillations on the leptonic CP-phase are shown. These dependences are small and difference between maximal and minimal values of probabilities equals approximately 0.7% in the $\nu_\mu\nu_\mu$ and $\nu_\tau\nu_\tau$ cases, 0.8% in the $\nu_e\nu_\mu$ and $\nu_e\nu_\tau$ transitions and 0.03% in the $\nu_\mu\nu_\tau$ transitions.

5 The difference between the $\nu_\alpha \to \nu_\beta$ transitions probability and the $\nu_\beta \to \nu_\alpha$ transitions probability and leptonic CP-phase

Let us consider $\nu_\alpha \to \nu_\beta$ transitions in neutrino oscillations (denote them ″forward″ transitions) and compare them with $\nu_\beta \to \nu_\alpha$ transitions (denote them ″backward″ transitions). At $\delta_l \neq 0$ the probabilities of forward transitions differ from probabilities of backward transitions. Note right away that the probability of the forward transition coincides with the probability of the backward transition after averaging over all oscillations, while before averaging these probabilities are different provided CP-violating, i.e. at $\delta_l \neq 0$. And the difference between these

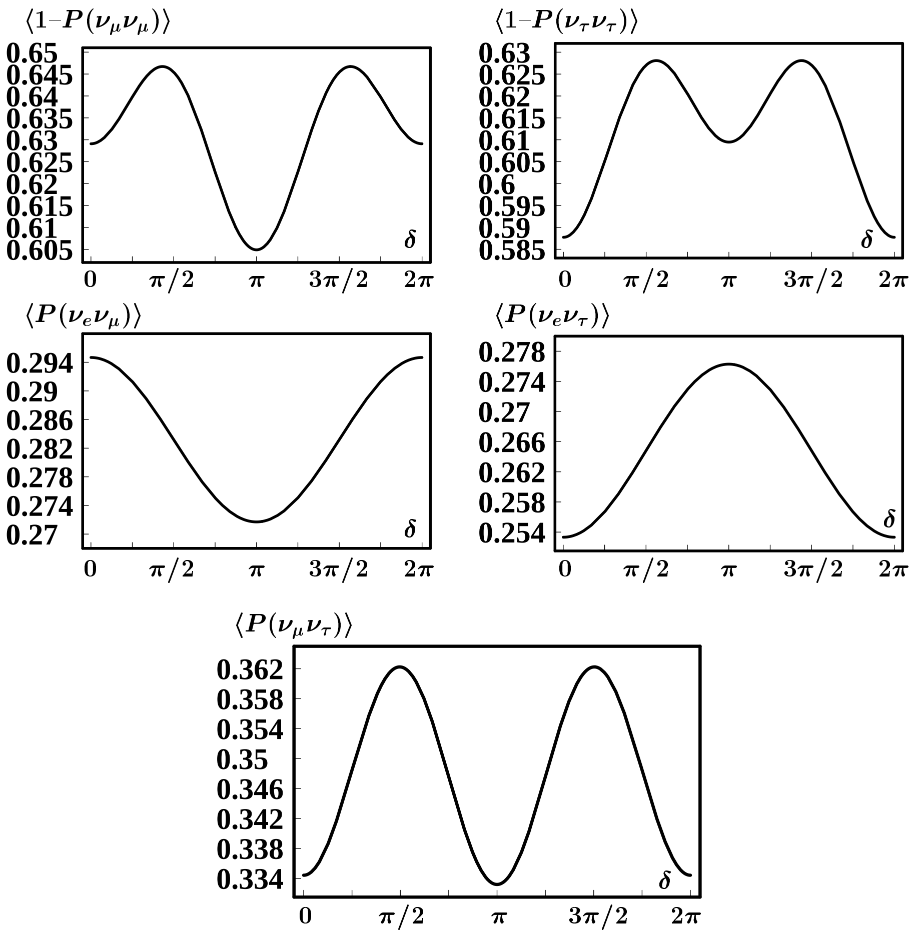

Fig. 2. In this figure the dependences of the probabilities averaging over neutrino oscillations on the leptonic CP-phase are not so small as in the previous case (Fig.1.) because ϑ_{13} is not small and difference between maximal and minimal values of probabilities equals approximately 4% in the $\nu_\mu\nu_\mu$ and $\nu_\tau\nu_\tau$ cases, 2.5% in the $\nu_e\nu_\mu$ and $\nu_e\nu_\tau$ transitions and 2.8% in the $\nu_\mu\nu_\tau$ transitions. Thus the role of ϑ_{13} in the manifestation of leptonic CP-phase in neutrino oscillations is very important.

probabilities is proportional to $\sin\delta_l$. These statements are direct consequences of general formulas for $P(\nu_e\nu_\mu), P(\nu_e\nu_\tau)$ and $P(\nu_\mu\nu_\tau)$ which given above. So, on obtaining probabilities of backward transitions by replacement $\delta_l \to -\delta_l$ we subtract probabilities of forward transitions from them. As a result we have:

$$\begin{cases} P(\nu_\mu\nu_e) - P(\nu_e\nu_\mu) = a_0(\sin\varphi_{21} + \sin\varphi_{32} - \sin\varphi_{31})\sin\delta_l \\ P(\nu_\tau\nu_e) - P(\nu_e\nu_\tau) = -a_0(\sin\varphi_{21} + \sin\varphi_{32} - \sin\varphi_{31})\sin\delta_l \\ P(\nu_\tau\nu_\mu) - P(\nu_\mu\nu_\tau) = a_0(\sin\varphi_{21} - 2\sin\frac{\varphi_{21}}{2}\cos\frac{(\varphi_{31}+\varphi_{32})}{2})\sin\delta_l \end{cases} \tag{16}$$

where $a_0 = \frac{1}{2}c_{13}\sin 2\vartheta_{12}\sin 2\vartheta_{13}\sin 2\vartheta_{23}$. Here phases $\varphi_{21}, \varphi_{32}, \varphi_{31}$ depend on the time of neutrino flight in vacuum t (see (6)) or, in other words, on distance (base-line) L between points of neutrino birth and neutrino absorption and also on difference of squared masses $\Delta m_{ij}^2 = m_i^2 - m_j^2$. For the most possible values of neutrino masses and theirs average errors taken from experiment [5,6]:

$$m_3 = (1/17 \pm 1/50)eV, \; m_2 = (1/175 \pm 1/300)eV, \; m_1 \ll m_2 \qquad (17)$$

We can see that the distance of the neutrino oscillations, i.e. the base-line for the experimental devices must not be less 10^3m. The experimental definition of the CP-phase based on the correlation (16) would be the most natural, however now it is practically impossible because beams of different types (for example ν_e and ν_μ) of neutrino (that is, obtained in different reactions) but with the same energy are required for the experiment. This problem possibly will be solved in the future, but until now all experimental data were obtained only for beams of the neutrino with the continuous energy spectra. The cause of this problem consists, in particular, in very small cross-sections of neutrino interactions.

In current experiments we deal only with probabilities of transitions of neutrino with continuous energy spectra in the initial beam, i.e. with all phases (6) averaging over oscillations (9),(10) which also depend on the CP-violating phase δ_l. The main idea of my talk consists in a suggestion to find the value of the CP-phase using data of experiments with large base-line and formulas (9),(10).

Note that coefficient a_0 defining the value of the effect of $t-$symmetry violating (16) is not too small in both $a)-$ and $b)-$cases:

$$a)\; a_0 = 0.07 \qquad b)\; a_0 = 0.23$$

Moreover in b-case it is large. Therefore measurements of this effect are possible although they are difficult.

6 Conclusions.

So, the main results of our work are the following:

- The expressions for the probabilities of neutrino oscillations were obtained in the explicit form with regard for the leptonic CP-phase.
- The manifestation of the leptonic CP-phase in neutrino oscillations was investigated by the example of the probabilities averaging over oscillations. Using modern experimental data the model calculations and numerical estimates were done.
- The question of the $t-$symmetry violation for neutrino oscillation was analyzed. And there was established that the difference between the "forward" probabilities and the "backward" probabilities was proportional to sine of the leptonic CP-phase.

References

1. Y. Fukuda *et al.,* Phys. Lett. **B433**(1998)9; Phys. Lett. **B436**(1998)33;
   Y. Fukuda *et al.,* Phys. Lett. **B467**(1999)185; Phys. Rev. Lett. **82**(1999)2644;
   H. Sobel, talk at XIX International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada, June 2000 (*http://nu2000.sno.laurentian.ca*); T. Toshito, talk at the XXXth international Conference on High Energy Physics, July 27 – August 2, 2000 (ICHEP 2000) Osaka, Japan (*http://www.ichep2000.rl.ac.uk*);
   C. Athanassopoulos *et al.,* (LSND Collaboration) Phys. Rev. Lett. **81**(1998)1774.
2. Y. Suzuki, talk at XIX International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada, June 2000 (*http://nu2000.sno.laurentian.ca*); T. Takeuchi, talk at the XXXth International Conference on High Energy Physics, July 27 – August 2, 2000 (ICHEP 2000) Osaka, Japan (*http://www.ichep2000.rl.ac.uk*);
   B.T. Cleveland *et al.,* Astrophys. J. **496**(1998)505; R. Davis, Prog. Part. Nucl. Phys. **32**(1994)13; K. Lande, talk at XIX International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada, June 2000 (*http://nu2000.sno.laurentian.ca*);
   SAGE Collaboration, J.N. Abdurashitov *et al.,*, Phys. Rev. **C60**(1999)055801; V. Garvin, talk at XIX International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada, June 2000 (*http://nu2000.sno.laurentian.ca*);
   GALLEX Collaboration, W. Hampel *et al.*, Phys. Lett. **B447**(1999)127;
   E. Belloti, talk at XIX International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada, June 2000 (*http://nu2000.sno.laurentian.ca*);
   F. Ronda, MACRO Collaboration, hep–ex/0001058.
3. CHOOZ Collaboration, M. Apollino *et al.,* Phys. Lett. **B420**(1998)397, F. Boehm *et al.,* hep–ex/9912050.
4. A. Ali, D. London, DESY'99–042 and 00–026, or hep–ph/9903535 and hep–ph/0002167.
5. M.C. Gonzalez–Garsia, M. Maltoni, C. Peña–Garay and J.W.F. Valle, hep–ph/0009350.
6. M. Campanelli, hep–ex/0010006.
7. K. Hagivara, N. Okamuta, Nucl. Phys. **B548**(1999)60;
   Z. Berezhiani and A. Rossi, Phys. Lett. **B367**(1999)219.