

# Covariant flavour effects in semileptonic $K$ and $D$ decay

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**Abstract.** We present constraints on the left-handed dimension-6 interactions that lead to semileptonic and leptonic decays of  $K$ ,  $D$  mesons. We employ the flavour covariant description of the effective couplings, identify universal CP phases of New Physics and derive constraints from decay rates and CP-odd quantities. As a result, we can predict the maximal effects of such flavoured NP in  $D$  decays from stringent  $K$  decay constraints and vice-versa.

## 1. Introduction

The standard model (SM) has a unique way of incorporating CP violation (CPV) and suppressing flavor changing neutral currents (FCNCs) in the quark sector. In particular, the lightness of the first two generation quark masses and the approximate diagonality of the CKM matrix severely suppress FCNC transitions involving the first two generation quarks.

In fact, no significant deviations from the SM predictions related to first two generation quark flavor violation have been observed in experiments to date, placing stringent limits on possible beyond SM (BSM) effects in this sector. On the other hand, observed patterns of deviations from SM predictions in both charged current (CC) and FCNC mediated semileptonic b-hadron decays, which still await experimental clarification, have triggered many studies throughout the last decade (see e.g. Refs. [1, 2]).

Intriguingly, the most straightforward and successful BSM proposals addressing these so-called B anomalies, involve new interactions of left-handed quarks (and leptons) [3, 4, 5, 6, 7, 8, 9, 10], which imply novel flavor breaking sources (besides SM Yukawas) of the  $SU(3)_Q$  flavour symmetry, respected by the SM gauge interactions. This already motivated a reconsideration of BSM effects in (rare) (semi)leptonic decays of kaons [11, 12], D-mesons [13, 14, 15], and also top quarks [16].

Experimentally, there has been recent progress in the search for the rare leptonic  $D^0 \rightarrow \mu^+ \mu^-$  [17] decay as well as the analysis of non-resonant regions of the differential rate for  $D^+ \rightarrow \pi^+ \mu^+ \mu^-$  [18] by the LHCb collaboration. BESIII collaboration has also recently reported results from a first dedicated search for  $D^0 \rightarrow \pi^0 \nu \bar{\nu}$  decay [19]. Similarly, new results have been recently reported on semileptonic  $s \rightarrow d$  transitions in both charged [20, 21] and neutral [22] kaon decays by the NA62 and KOTO collaborations, respectively. Further significant improvements in these measurements and searches are expected from these and the next generation of flavor experiments [23].



Motivated by these developments, we investigate the interplay of possible NP effects in semileptonic charged and flavour changing neutral currents involving purely left-handed first- and second-generation quarks.

In particular, it has been shown previously [24] that the peculiar structure of  $SU(3)_Q$  breaking in the SM implies that possible BSM sources of CPV in this sector affect rare charm and kaon decays in a universal way (see also Ref. [25]). For this work we assume for simplicity lepton flavor universality, whereas lepton-specific study results are presented in [26].

## 2. Framework

We are interested in BSM effects in semileptonic transitions involving exclusively left-handed quarks of first two generations. Working within the SM effective field theory (SMEFT) [27], we thus supplement the SM Lagrangian by terms constructed out of semileptonic effective operators with left-chiral quarks

$$\mathcal{L}_{\text{SMEFT}} \supset \frac{X_{ij}^{(3,\ell)}}{\Lambda^2} (\bar{Q}_i \gamma_\mu \sigma^a Q_j) (\bar{L}_\ell \gamma^\mu \sigma_a L_\ell) + \frac{X_{ij}^{(1,\ell)}}{\Lambda^2} (\bar{Q}_i \gamma_\mu Q_j) (\bar{L}_\ell \gamma^\mu L_\ell). \quad (1)$$

We set  $\Lambda = 1 \text{ TeV}$ . Here  $Q_i$  is the  $i$ -th generation left-handed quark doublet, which we write in the down-quark mass basis as  $Q_i = (u'_{Li}, d_{Li})^T$ . The up quark fields in this basis are related to their mass eigenstates via the CKM matrix  $V$  as  $u'_i = V_{ji}^* u_j$ . For leptons we choose the charged lepton mass basis:  $L_i = (U_{ji}^* \nu_{Lj}, \ell_{Li})^T$ , where  $U$  is the PMNS matrix. Pauli matrices  $\sigma^a$ ,  $a = 1, 2, 3$ , act in the  $SU(2)_L$  space. We assume in Eq. (1) that lepton flavour is conserved, whereas the BSM quark flavour conversion is parametrized by Hermitian matrices  $X^{(\ell)}$ . The resulting Lagrangian containing FCNCs reads

$$\mathcal{L}_{\text{FCNC}} = \frac{1}{\Lambda^2} X_{ij}^{(+)} \left[ (\bar{u}'_i \gamma^\mu P_L u'_j) (\bar{\nu} \gamma_\mu P_L \nu) + (\bar{d}_i \gamma^\mu P_L d_j) (\bar{\ell} \gamma_\mu P_L \ell) \right] \quad (2)$$

$$+ \frac{1}{\Lambda^2} X_{ij}^{(-)} \left[ (\bar{u}'_i \gamma^\mu P_L u'_j) (\bar{\ell} \gamma_\mu P_L \ell) + (\bar{d}_i \gamma^\mu P_L d_j) (\bar{\nu} \gamma_\mu P_L \nu) \right], \quad (3)$$

where  $P_{R,L} = (1 \pm \gamma_5)/2$ . Above, we have introduced the matrices  $X^{(\pm)} = X^{(1)} \pm X^{(3)}$  and suppressed explicit lepton flavour index for clarity. On the other hand, the charged currents stemming from Eq. (1) are only due to the  $X^{(3)}$

$$\mathcal{L}_{\text{CC}} = \frac{1}{\Lambda^2} 2X_{ij}^{(3)} (\bar{u}'_i \gamma^\mu P_L d_j) (\bar{\ell} \gamma_\mu P_L \nu) + \text{h.c.} \quad (4)$$

Next we focus exclusively on the first two generations and use the fact that any two-dimensional hermitian matrix can be decomposed in terms of the identity and Pauli matrices, thus we can write<sup>1</sup>

$$X_{ij}^{(\pm)} = \lambda^{(\pm)} \delta_{ij} + c_a^{(\pm)} (\sigma^a)_{ij}, \quad (5)$$

where  $\lambda$  and  $c_a$  are real. It is only the traceless part ( $c_a$ ) that plays a role in FCNC processes. In contrast,  $\lambda$ 's contribute to flavour-diagonal neutral currents (which we do not consider, as well as to charged current processes via  $X^{(3)}$ )

$$2X_{ij}^{(3)} = (\lambda^{(+)} - \lambda^{(-)}) \delta_{ij} + (c_a^{(+)} - c_a^{(-)}) (\sigma^a)_{ij}. \quad (6)$$

<sup>1</sup> Note that in isolating the first two generations, in the following we are neglecting possible additional BSM effects due to mixing with the third quark generation. However, the resulting modifications of our results are in general severely suppressed due the hierarchical structure of the SM quark Yukawas. See Ref. [24] for in depth discussion on this point.

Notice that a unique parameter encodes CP violation  $c_2^{(\pm)}$ , while the remaining three couplings are real. The traceless part of the coupling matrix offers an intuitive geometrical interpretation [28] since it spans a 3-dimensional space. Each such matrix  $A$  is thus equivalent to a real 3-dimensional vector  $\mathbf{a}$  such that  $A = \mathbf{a} \cdot \boldsymbol{\sigma}$  holds. In the remainder of this work, we describe the traceless part of the couplings by the cylindrical coordinates  $c_R, c_I$ , and  $\theta_d$ , which are related to the cartesian ones as

$$-c_3 = c_R \cos \theta_d, \quad c_1 = c_R \sin \theta_d, \quad c_2 = c_I. \quad (7)$$

For later convenience, we introduce normalised vectors

$$\hat{\mathcal{A}}_d = -\sigma_3, \quad (8)$$

$$\hat{\mathcal{A}}_u = -\cos(2\theta_c)\sigma_3 + \sin(2\theta_c)\sigma_1, \quad (9)$$

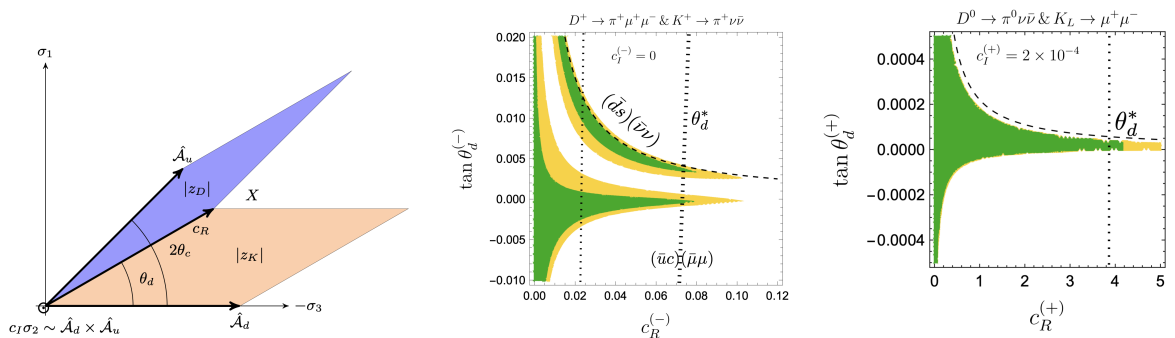
which are shown in left panel in Fig. 1, where it is also evident how  $\mathcal{A}_{u,d}$  are related to cylindrical coordinates.

Now we focus on the allowed size of the couplings  $c_{R,I}$  depending on the alignment angle  $\theta_d$ , when we rely on the CP-even experimental upper bounds:  $|z_K^{\text{exp}}|, |z_D^{\text{exp}}|$ , defined as

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{(\pm)} &= \frac{z_K^{(-)}}{\Lambda^2} (\bar{d}_L \gamma^\mu s_L) (\bar{\nu}_L \gamma_\mu \nu_L) + \frac{z_D^{(-)}}{\Lambda^2} (\bar{u}_L \gamma^\mu c_L) (\bar{\ell}_L \gamma_\mu \ell_L) + \\ &+ \frac{z_K^{(+)}}{\Lambda^2} (\bar{d}_L \gamma^\mu s_L) (\bar{\ell}_L \gamma_\mu \ell_L) + \frac{z_D^{(+)}}{\Lambda^2} (\bar{u}_L \gamma^\mu c_L) (\bar{\nu}_L \gamma_\mu \nu_L) + \text{h.c.} \end{aligned} \quad (10)$$

If we only have CP even information about these couplings at our disposal, namely upper bounds on  $|z_{K,D}|$ , then the optimal alignment angle  $\theta_d^*$ , that allows for largest  $|X|$ , is given by  $\tan \theta_d^*|_{c_I=0} = \frac{r_{KD} \sin 2\theta_c}{1 + r_{KD} \cos 2\theta_c}$ , where  $r_{KD} \equiv |z_K^{\text{exp}}|/|z_D^{\text{exp}}|$  is a ratio of upper experimental bounds. The above expression is valid in the  $c_I = 0$  scenario, for a general expression see [26].

### 3. Results and discussion



**Figure 1.** Left: basis vectors for couplings  $X$ . Center: Magnitudes of NP contribution to  $K \rightarrow \pi \nu \bar{\nu}$  and  $D \rightarrow (\pi) \ell \bar{\ell}$  rare semileptonic processes depend on the alignment angle  $\theta_d^{(-)}$ . Right: Magnitudes of NP contribution to  $K \rightarrow (\pi) \ell \bar{\ell}$  and  $D \rightarrow \pi \nu \bar{\nu}$  rare semileptonic processes depend on the alignment angle  $\theta_d^{(+)}$ .

Here we present the results on the  $X^{(-)}$  and  $X^{(+)}$  sectors, which are only weakly correlated by charged current constraints [26]. The  $X^{(-)}$  couplings cause correlations between  $s \rightarrow d \nu \bar{\nu}$

and  $c \rightarrow u\ell^+\ell^-$  processes. The results are shown in the center panel in Fig. 1, where the hyperbolically shaped constraints are from  $K^+ \rightarrow \pi^+\nu\bar{\nu}$ , whereas thick dotted vertical line corresponds to  $\text{Br}(D \rightarrow \pi\mu^+\mu^-)$  upper bound that cuts away runaway direction at small  $\theta_d^{(-)}$ . The upper bound from  $\text{Br}(K^+ \rightarrow \pi^+\nu\nu)$  is  $c_I^{(-)} < 2 \times 10^{-4}$ . For further details see [26]. On the other hand, the  $X^{(+)}$  couplings are limited by  $K \rightarrow (\pi)\ell^+\ell^-$  and also by a recent BESSIII result  $\text{Br}(D^0 \rightarrow \pi^0\nu\bar{\nu})$ . Here, from the CP-violating decay mode  $K_L \rightarrow \pi^0 e^+ e^-$  we infer  $c_I^{(+)} < 2 \times 10^{-4}$ . The results are shown in the right panel of Fig. 1. Notice that the results on the  $c_R$  strongly depend on the alignment angle  $\theta_d$  in both sectors, whereas the bounds on the CPV coupling  $c_I$  are invariant to  $\theta_d$ . For further predictions, charged current effects and lepton specific analysis cf. [26].

We have shown in this work how the new physics effects from left-handed semileptonic SMEFT operators are correlated among the up- and down-quark sectors. While the interplay between the size of CP-conserving effects in kaon and charm processes can be tuned by the alignment angle, the CP-violating coupling is universal for left-handed up- and down-quark operators. The presented features allow to efficiently constraint such New physics scenarios and underline the importance of experimental progress in kaon, as well as on charm, rare decays.

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