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THE GREEN FUNCTION OF NEUTRAL GLUONS IN COLOR MAGNETIC BACKGROUND FIELD AT FINITE TEMPERATURE

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In SU(2) gluodynamics, the structure of the exact Green function of neural gluons in an Abelian homogeneous magnetic field at finite temperature is derived. It is expressed through 10 tensors, which form an algebra with respect to anticommutation operation, and corresponding form factors. The structure constants of the algebra are calculated. The spectrum of gluons is derived from the pole positions of this Green function for the form factors calculated in one-loop order. The high temperature limits for these form factors are computed. The spectra of different gluon states are obtained.

1 Introduction

Recent investigations of QCD at high temperature revealed an important role of colored magnetic fields. In particular, it has been elucidated in gluodynamics that color magnetic fields are spontaneously created at high temperature [1, 2, 6]. It is reasonable to suppose that the spontaneous generation of magnetic fields is also responsible for producing of seed magnetic fields in the early universe. From the analysis of the lattice simulations [4], and using the perturbative daisy resummations in the external field at high temperature [6] it was discovered that Abelian chromomagnetic fields of order $gB \sim g^4T^2$, where g is a gauge coupling constant, are spontaneously created.

To investigate physics in this case one has to calculate the spectra of quarks and gluons in the background field and at finite temperature. As a first step, the operator structure of the gluon Green function and the spectra for this background should be calculated.

Let us divide the gauge field potential $A^a_\mu(x)$ into the background Abelian homogeneous magnetic field $B^a_\mu(x)$ and the quantum fluctuations $Q^a_\mu(x)$,

$$A^a_\mu(x) = B^a_\mu(x) + Q^a_\mu(x). \quad (1)$$

The background field $B^a_\mu(x)$ is directed along the third axis in both the color and the configuration spaces. Its vector potential is

$$B^a_\mu(x) = \delta^{a3}\delta_{\mu 2}x_1 B \quad (2)$$

and the corresponding field strength tensor equals to

$$F^a_{ij} = \delta^{a3}F_{ij} = \delta^{a3}B\epsilon^{3ij}, \quad (3)$$

where only the spatial components $(i, j = 1, 2, 3)$ are nonzero. In the field presence it is convenient to turn to the so-called "charged basis" $W^\pm_\mu = (Q^1_\mu \pm iQ^2_\mu)/\sqrt{2}$, $Q_\mu = Q^3_\mu$, with the interpretation of $W^\pm_\mu$ as color charged fields ("charged" gluons) and Q_μ as color neutral fields ("neutral" gluons). The neutral gluon has continuous momentum, whereas charged one forms the discrete Landau levels in the perpendicular with respect to the field direction. In Ref. [5] the gluon polarization tensor at zero temperature was derived. In Ref. [6] that has been done for the finite temperature case. In the present paper we investigate the properties of the exact neutral gluon Green function in the external field at zero and finite temperature.

We use the Feynman gauge where the propagator of neutral gluon in Euclidean's metric with a momentum k_μ is

$$D^{(0)}_{\mu\nu} = \frac{\delta_{\mu\nu}}{k^2}. \quad (4)$$

In a tree approximation, the spectrum can be determined from the pole position of $D^{(0)}_{\mu\nu}$, that is from the equation $k^2 = 0$. The aim of the present paper is to determine the gluon spectrum in the field $B^a_\mu(x)$ derived from the pole position of the exact Green function. First we describe the tensor structure of the neutral gluon Green function at zero and finite temperature.

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2 The structure of the polarization tensor

The exact Green function $D_{\mu\nu}$ of neutral gluons in the field $B^a_\mu(x)$ is a function of two vectors formed from momentum components: $h_\lambda = (k_1, k_2, 0, 0)$, $l_\mu = (0, 0, k_3, k_4)$ and the field induction B. It is given by the operator Schwinger-Dyson equation

$$D = \frac{1}{(k^2 - \Pi)}, \tag{5}$$

were Π is the polarization tensor (PT).

As it was shown in Ref. [5], in a magnetic field the PT is not transversal. This means that the condition $k_\mu \Pi_{\mu\nu} = 0$ does not hold. So, a weaker condition was used

$$k_\mu \Pi_{\mu\nu} k_\nu = 0. \tag{6}$$

In Ref. [6] the following tensor structure of the neutral gluon PT at finite temperature was derived

$$\Pi_{\mu\nu} = \sum_{i=1}^{10} \Pi^{(i)} T^{(i)}_{\mu\nu} \tag{7}$$

with

$$\begin{aligned} T^{(1)}_{\lambda\lambda'} &= l^2 \delta^{||}_{\lambda\lambda'} - l_\lambda l_{\lambda'}, & T^{(2)}_{\lambda\lambda'} &= h^2 \delta^{\perp}_{\lambda\lambda'} - h_\lambda h_{\lambda'} = d_\lambda d_{\lambda'}, \\ T^{(3)}_{\lambda\lambda'} &= h^2 \delta^{||}_{\lambda\lambda'} + l^2 \delta^{\perp}_{\lambda\lambda'} - l_\lambda h_{\lambda'} - h_\lambda l_{\lambda'}, & T^{(4)}_{\lambda\lambda'} &= h^2 \delta^{||}_{\lambda\lambda'} - l^2 \delta^{\perp}_{\lambda\lambda'}, \\ T^{(5)}_{\lambda\lambda'} &= i(l_\lambda d_{\lambda'} - d_\lambda l_{\lambda'}) + i l^2 F_{\lambda\lambda'}, & T^{(6)}_{\lambda\lambda'} &= i F_{\lambda\lambda'}, \end{aligned} \tag{8}$$

where we use the notation

$$\delta^{||}_{\lambda\lambda'} = \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1 \end{pmatrix}, \quad \delta^{\perp}_{\lambda\lambda'} = \begin{pmatrix} 1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix}, \quad F_{\lambda\lambda'} = \begin{pmatrix} 0&1&0&0\\-1&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix},$$

$$l_\lambda = (0, 0, k_3, k_4), \quad h_\lambda = (k_1, k_2, 0, 0), \quad d_\lambda = (k_2, -k_1, 0, 0). \tag{9}$$

The first four tensors $T^{(i)}$ are transversal, $k_\mu T_{\mu\nu} = 0$, whereas the last two obey the Eq. (6), only. At finite temperature, we have to take into consideration the additional vector $u = (0, 0, 0, 1)$ – the thermostat velocity. Using the vector u one can construct four additional tensors:

$$\begin{aligned} T^{(7)}_{\lambda\lambda'} &= (uk)(u_\lambda l_{\lambda'} + l_\lambda u_{\lambda'}) - \delta^{||}_{\lambda\lambda'} (uk)^2 - u_\lambda u_{\lambda'} l^2, & T^{(8)}_{\lambda\lambda'} &= (uk)(u_\lambda h_{\lambda'} + h_\lambda u_{\lambda'}) - \delta^{\perp}_{\lambda\lambda'} (uk)^2 - u_\lambda u_{\lambda'} h^2, \\ T^{(9)}_{\lambda\lambda'} &= i(u_\lambda d_{\lambda'} - d_\lambda u_{\lambda'}) + i F_{\lambda\lambda'} (uk), & T^{(10)}_{\lambda\lambda'} &= k^2 \delta_{\lambda\lambda'} - \frac{(k^2)^2 u_\lambda u_{\lambda'}}{(uk)^2}. \end{aligned} \tag{10}$$

Here the scalar product $(uk) = k_4$ is the fourth component of the momentum. The tensors $T^{(7)}$, $T^{(8)}$, and $T^{(9)}$ are transversal, whereas $T^{(10)}$ satisfies only the weaker condition (6).

It is possible to check that the set of tensors (8)–(10) together with the identity matrix $T^{(0)}_{\mu\nu} = k^2 (\delta^{||}_{\mu\nu} + \delta^{\perp}_{\mu\nu})$ forms an algebra

$$\{T^{(i)}, T^{(j)}\} = 2 C^{ij}_k T^{(k)}. \tag{11}$$

Its structure constants C^{ij}_k were calculated from explicit expressions for tensors $T^{(i)}$, where the indices run the values $i, j = 0, 1, \ldots, 10$. This is assumed below. Due to completeness of the set of operators $T^{(i)}$, one can obtain D as a linear combination

$$D_{\mu\nu} = \sum_{i=0}^{10} D^{(i)} T^{(i)}_{\mu\nu}, \tag{12}$$

where $D^{(i)}$ are some scalar functions of the form factors $\Pi^{(j)}$.They will be calculated in the next section.

3 The gluon Green function at finite temperature

First we notice that $T^{(i)}$ are the functions of $h_\mu = (k_1, k_2, 0, 0)$, $l_\mu = (0, 0, k_3, k_4)$ and $u_\mu = (0, 0, 0, 1)$. The convolution of $T^{(i)}$ and some linear combination of h_μ, l_μ and u_μ is again a linear combination of these vectors with other coefficients, $(\alpha l_\mu + \beta l_\mu + \gamma u_\mu) T^{(i)}_{\mu\nu} = x l_\nu + y h_\nu + z u_\mu$. Let us consider a tensor

$$P(\alpha, \beta, \gamma, x, y, z)_{\mu\nu} \equiv (\alpha l_\mu + \beta h_\mu + \gamma u_\mu)(x l_\nu + y h_\nu + z u_\nu) \tag{13}$$

and its convolution with D. From Eq. (12) we obtain

$$P(\alpha,\beta,\gamma,x,y,z)_{\mu\nu}D_{\mu\nu} = (\alpha l_\mu + \beta h_\mu + \gamma u_\mu)D_{\mu\nu}(x l_\nu + y h_\nu + z u_\nu) = \sum_{i=0}^{10} D^{(i)}\,(\alpha l_\mu + \beta h_\mu + \gamma u_\mu)T^{(i)}_{\mu\nu}(x l_\nu + y h_\nu + z u_\nu). \tag{14}$$

On the other hand, we can substitute $(k^2-\Pi)^{-1}_{\mu\nu}$ for $D_{\mu\nu}$ in Eq. (14) and get some functions which depend on the form factors $\Pi^{(i)}$,

$$P(\alpha,\beta,\gamma,x,y,z)_{\mu\nu}\left[\frac{1}{(k^2-\Pi)}\right]_{\nu\mu} = \frac{1}{k^2}\sum_{t=0}^{\infty}\frac{1}{k^{2t}}\left[(\alpha l_\mu + \beta h_\mu + \gamma u_\mu)\Pi^t_{\mu\nu}\right](x l_\nu + y h_\nu + z u_\nu). \tag{15}$$

Here we expressed the function of Π in the form of series to find

$$(\alpha l_\mu + \beta h_\mu + \gamma u_\mu)\Pi_{\mu\nu} = \alpha_1 l_\nu + \beta_1 h_\nu + \gamma_1 u_\nu. \tag{16}$$

In the operator form we get

$$A\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} = \begin{pmatrix}\alpha_1\\ \beta_1\\ \gamma_1\end{pmatrix}, \tag{17}$$

where A is a transformation matrix. Obviously that

$$(\alpha l_\mu + \beta l_\mu + \gamma u_\mu)\Pi^t_{\mu\nu} = \alpha_t l_\nu + \beta_t h_\nu + \gamma_t u_\nu, \tag{18}$$

where

$$\big(\alpha_t,\beta_t,\gamma_t\big)^T = A^t\ (\alpha,\beta,\gamma)^T. \tag{19}$$

So, if we have a function of Π we can replace it by A:

$$P(\alpha,\beta,\gamma,x,y,z)_{\mu\nu}\left[\frac{1}{(k^2-\Pi)}\right]_{\nu\mu} = \big(\alpha' l_\mu + \beta' h_\mu + \gamma' u_\mu\big)\big(x l_\mu + y h_\mu + z u_\mu\big), \tag{20}$$

where $\big(\alpha',\beta',\gamma'\big)^T = (k^2-A)^{-1}\ (\alpha,\beta,\gamma)^T$.

In our case, the matrix A has the following elements:

$$\begin{aligned}
A_{11} &= h^2(\Pi^{(3)}+\Pi^{(5)}) + (uk)^2(\Pi^{(7)}-\Pi^{(8)});\\
A_{12} &= -l^2\Pi^{(3)} + (uk)^2\Pi^{(8)}; \qquad A_{13} = -h^2\Pi^{(8)};\\
A_{21} &= -h^2\Pi^{(3)}; \qquad A_{22} = l^2(\Pi^{(3)}-\Pi^{(5)}); \quad A_{23} = h^2\Pi^{(8)};\\
A_{31} &= (uk)^2\Pi^{(1)}; \qquad A_{32} = (uk)^2(\Pi^{(3)}+\Pi^{(8)});\\
A_{33} &= l^2\Pi^{(1)} + h^2(\Pi^{(3)}+\Pi^{(5)}) + ((uk)^2-h^2)\Pi^{(8)}.
\end{aligned} \tag{21}$$

By specifying the values of coefficients $\alpha,\beta,\gamma,x,y,z$ we can derive the factors $D^{(i)}$:

$$\begin{aligned}
D^{(0)} &= \frac{B_{11}+B_{12}+B_{21}+B_{22}}{\psi\,k^2},\\
D^{(1)} &= -\frac{\omega + (uk)^2\Pi^{(7)}\delta}{k^2 - l^2\Pi^{(1)} - h^2[\Pi^{(3)}+\Pi^{(5)}] + (uk)^2\Pi^{(7)}},\\
D^{(2)} &= \frac{-k^2\Pi^{(2)} + h^2\Pi^{(2)}[\Pi^{(3)}+\Pi^{(5)}] + h^2\Pi^{(3)2} + (uk)^2\Pi^{(8)}[\Pi^{(2)}-\Pi^{(8)}]}{\psi\left[k^2 - h^2\Pi^{(2)} - l^2(\Pi^{(3)}-\Pi^{(5)}) + (uk)^2\Pi^{(8)}\right]},\\
D^{(3)} &= -\frac{B_{12}+B_{32}}{\psi\,h^2[l^2+(uk)^2]},\\
D^{(5)} &= \frac{(uk)^2-k^2}{\psi\,k^2h^2[(uk)^2-l^2]}\left[B_{11}+B_{13}+B_{31}+\frac{h^2}{(uk)^2-k^2}B_{32}\right],\\
D^{(7)} &= -\frac{\omega + (uk)^2\Pi^{(7)}\delta}{k^2 - l^2\Pi^{(1)} - h^2(\Pi^{(3)}+\Pi^{(5)}) + (uk)^2\Pi^{(7)}} - \delta,\\
D^{(8)} &= \frac{1}{\psi\,h^2[l^2+(uk)^2]}\left[B_{12}+\frac{l^2}{(uk)^2}B_{32}\right],\\
D^{(10)} &= \frac{B_{21}+B_{31}+B_{22}+B_{32}\,l^2/(uk)^2}{\psi\,k^4[1-l^2/(uk)^2]},
\end{aligned} \tag{22}$$

where we denoted $\psi = \det[k^2 - A]$, and B_{ij} are the matrix elements of $B = (k^2 - A)^{-1}\psi$, $\delta = (B_{31} + B_{32})[l^2 - (uk)^2]^{-1}\psi^{-1}$, $\omega = [k^2\Pi^{(1)}D_0 + h^2\Pi^{(1)}(D_3 + D_5) + h^2\Pi^{(3)}D_1]$. Having calculated the factors in Eqs. (22) we derive the tensor structure of the exact neutral gluon Green function in the external magnetic field at finite temperature. Note that the coefficient $\Pi^{(i)}$ are arbitrary functions of their arguments. In principle, they can be calculated in loop expansion or in a nonperturbative way.

To obtain the spectral equations for the neutral gluons we have to consider the pole positions of the Green function. There are three spectral equations, two of them are linear with respect to k^2, and one is the cubic equation in k^2:

$$k^2 - h^2\Pi^{(2)} - l^2(\Pi^{(3)} - \Pi^{(5)}) + (uk)^2\Pi^{(8)} = 0, \tag{23}$$

$$k^2 - l^2\Pi^{(1)} - h^2(\Pi^{(3)} + \Pi^{(5)}) + (uk)^2\Pi^{(7)} = 0, \qquad \psi = 0. \tag{24}$$

The next step is to calculate the form factors $\Pi^{(i)}$ in order to determine the spectra in a chosen approximation.

4 Form factors in one-loop order

In Ref. [6] the form factors $\Pi^{(i)}$ have been calculated as the two-parametric integrals of the form:

$$\Pi^{(i)}(k) = \sum_{N=-\infty}^{\infty} \int_0^{\infty} ds\, dt\, M^{(i)}(s,t)\Theta_T, \tag{25}$$

where the functions $M^{(i)}(s,t)$ are

$$\begin{aligned}
M^{(1)} &= 4 - 2\left(\frac{\xi}{q}\right)^2 \cosh(2q), \qquad M^{(2)} = 4\frac{1-\cosh(q)\cosh(\xi)}{\sinh^2 q} - 2 + 8\cosh(q)\cosh(\xi),\\
M^{(3)} &= -2\cosh(2q)\frac{\xi\sinh\xi}{q\sinh q} - 2 + 6\cosh(\xi)\cosh(q),\\
M^{(4)} &= -2 + 2\cosh(q)\cosh(\xi), \qquad M^{(5)} = 2\frac{\xi}{q}\left[\sinh(2q) - \frac{\cosh q - \cosh\xi}{\sinh q}\right] - 6\cosh(q)\sinh(\xi),\\
M^{(6)}_{(1)} &= 2\left[\frac{\xi}{q}\coth(q)(1 - 3\sinh^2 q) + \sinh(\xi)\cosh(q)\right] l^2\\
&+ 2\left[\frac{\sinh\xi}{\sinh q}\coth(q)(1 - 3\sinh^2 q) + 2\sinh(\xi)\cosh(q)\right] h^2,\\
M^{(6)}_{(2)} &= 2\,\frac{iN}{qT}k_4\left(\sinh(2q) - \coth q\right),\\
M^{(7)} &= -2\,\frac{iN}{qT}\frac{1}{k_4}\frac{\xi}{q}\cosh(2q), \qquad M^{(8)} = \frac{iN}{qT}\frac{1}{k_4}\left[-2\frac{\sinh\xi}{\sinh q} - 4\sinh(q)\sinh(\xi)\right],\\
M^{(9)} &= \frac{iN}{qT}2\left[\frac{\cosh q - \cosh\xi}{\sinh q} - \sinh(2q) - 2\sinh(q)\cosh(\xi)\right], \qquad M^{(10)} = 0,
\end{aligned} \tag{26}$$

and $\xi = s - t$, $q = s + t$. The symmetric form factors have to be multiplied by

$$\Theta_T^s = \frac{1}{2}\langle\Theta(s,t)\rangle\left[\exp\left(\frac{ik_4N}{qT}t\right) + \exp\left(\frac{ik_4N}{qT}s\right)\right]\exp\left(-\frac{N^2B}{4T^2q}\right), \tag{27}$$

and the antisymmetric ones – by

$$\Theta_T^a = \frac{1}{2}\langle\Theta(s,t)\rangle\left[\exp\left(\frac{ik_4N}{qT}t\right) - \exp\left(\frac{ik_4N}{qT}s\right)\right]\exp\left(-\frac{N^2B}{4T^2q}\right). \tag{28}$$

The function $\langle\Theta\rangle$ equals to

$$\langle\Theta\rangle = \frac{1}{(4\pi)^2(s+t)\sinh(s+t)}\exp\left[-\frac{k}{B}\left(\delta^{||}\frac{st}{s+t} + \delta^{\perp}\frac{ST}{S+T}\right)k\right], \tag{29}$$

where $S \equiv \tanh s$ and $T \equiv \tanh t$.

In this paper we are interested in the spectrum at high temperature $\sqrt{B}/T \ll 0$ in the limit of $k_4 = 0$, $\vec{k} \to 0$. For this case we calculate the following asymptotic form for the form factors,

$$\Pi^{(n)}(k) = \frac{T}{\sqrt{B}(4\pi)^{3/2}}\left(a_n - \frac{k_3^2}{B}b_n - \frac{h^2}{B}c_n\right) - \theta_n. \tag{30}$$

Table 1. The coefficients a, b, c in Eq. (30)

n	a_n	b_n	c_n
1	$10.56832 - 0.59082i$	$1.85028 + 0.08862i$	$1.64935 + 0.29541i$
2	$-5.79894 - 7.08982i$	$-4.16625 + 3.54491i$	$-4.63238 - 1.77245i$
3	$1.04427 - 8.86227i$	$-4.16625 + 3.54491i$	$-2.84292 - 3.10179i$
4	0	0	0
5	$-4.21405 - 1.77245i$	$-1.60873 + 0.88622i$	$-1.58031 + 0.44311i$
6	0	0	0
7	$-1.40468 - 0.59082i$	$-0.10712 + 0.08862i$	$0.13310 + 0.29541i$
8	$1.71341 - 3.54491i$	$-1.90805 - 1.77245i$	$0.38174 - 1.77245i$
9	0	0	0

The corresponding coefficients a, b, c are shown in Table 1, and θ are found to be

$$\theta_n = \frac{10}{3(4\pi)^2} \ln \frac{T^2}{B}, \qquad n = 1, 2, 3; \qquad\qquad \theta_n = 0, \qquad n \neq 1, 2, 3. \tag{31}$$

The imaginary part is signaling the instability of the state because of the tachyonic mode presenting in the spectrum of charged gluons (see, for instance, Ref. [5]), and the real part is responsible for the screening of transversal gluon fields. It is important to note that at finite temperature all the states are unstable because of the Landau damping. So the ratio of the imaginary and the real parts, ρ, is an important parameter characterizing the stability of a state. If this ratio is less then 1, we consider corresponding state as a quasi-stable one. And in the opposite case this state is unstable. In other words, the tachyonic instability is not distinguishable from the usual instability of quasiparticles at finite temperature. As one can see from Table 1, for different form factors these ratios are different, smaller or larger than unit. This has an important role for resummation of perturbation series in order to improve the infrared behaviour of the corresponding state. In case of small ρ the form factor could not be resummed. For $\rho > 1$ the form factor should be resummed.

In the same way we can calculate the form factors with the fourth momentum component $k_4 = 2\pi n_k T$, $n_k \neq 0$. In this case we obtain the following results

$$\Pi^{(m)}(k) = \frac{T}{(4\pi)^{3/2}} \left(\frac{1}{k_4}\, \tilde{a}_m - \frac{\vec{k}^2}{k_4^3}\, \tilde{c}_m + \frac{\sqrt{B}}{k_4^2}\, \tilde{b}_m \right) - \theta_m, \quad m = 1, 2, 3, 7, 8,$$

$$\Pi^{(5)}(k) = \frac{T\sqrt{B}}{4\pi} \left[(1+i)\frac{1}{k_4^2} - (1-i)\frac{B}{k_4^4} - \frac{1+i}{2}\,\frac{\vec{k}^2}{k_4^4} \right], \qquad \Pi^{(4)}(k) = \Pi^{(6)}(k) = \Pi^{(9)}(k) = 0. \tag{32}$$

For the coefficients $\tilde{a}_m$, $\tilde{b}_m$ and $\tilde{c}_m$ the following expressions have been obtained:

$$\tilde{a}_1 = \tilde{a}_2 = \tilde{a}_3 = 21.7315 + \frac{2}{(4\pi)^2}\frac{k_4}{T} \int_0^1 dx\, [1 + 4x(1-x)] \sum_{N \neq 0, -n_k} \left(\frac{2\pi T}{\sqrt{k_4^2 x(1-x) + (k_4 x + 2\pi N T)^2}} - \frac{1}{|N|} \right),$$

$$\tilde{a}_7 = \tilde{a}_8 = -1.8906 + \frac{1}{4\pi}\frac{k_4}{T} \int_0^1 dx\, (1-2x) \sum_{N \neq 0, -n_k} \frac{k_4 x + 2\pi N T}{\sqrt{(k_4(1-x) + 2\pi N T)^2 + k_4^2 x(1-x)}};$$

$$\tilde{b}_1 = 1.0248 + 7.0898i, \qquad \tilde{b}_2 = 20.2447 + 14,1796i, \qquad \tilde{b}_3 = 11.6595 + 17.7245i, \qquad \tilde{b}_7 = \tilde{b}_8 = 0;$$

$$\tilde{c}_1 = \tilde{c}_2 = \tilde{c}_3 = 6.8873 + k_4^3\sqrt{\pi} \sum_{N \neq 0, -n_k} \int_0^1 \frac{dx[1 + 4x(1-x)]}{[k_4^2 x(1-x) + (k_4 x + 2\pi N T)^2]^{3/2}},$$

$$\tilde{c}_7 = \tilde{c}_8 = 0.2708 + k_4^2\, 2\sqrt{\pi} \int_0^1 dx \sum_{N \neq 0, -n_k} \frac{(1-2x)x(1-x)(k_4 x + 2\pi N T)}{[(k_4(1-x) + 2\pi N T)^2 + k_4^2 x(1-x)]^{3/2}}. \tag{33}$$

Function θ_m is the same as in Eq. (31). Substituting $B = 0$ in Eq. (32) we obtain the polarization tensor in the high temperature limit at zero field. In this case non-transversal form factor $\Pi^{(5)}$ equals to zero, the form factors $\Pi^{(1)}$, $\Pi^{(2)}$, $\Pi^{(3)}$ and $\Pi^{(7)}$, $\Pi^{(8)}$ become equal to each other. This is an expected result.

5 Conclusions

In the framework of SU(2) gluodynamics, we derived the tensor structure of the exact neutral gluon Green function in an Abelian homogeneous magnetic field at finite temperature. It is presented as the linear combination of ten tensors $T^{(i)}$ introduced already in Ref. [6]. It was discovered that these tensors form an algebra with

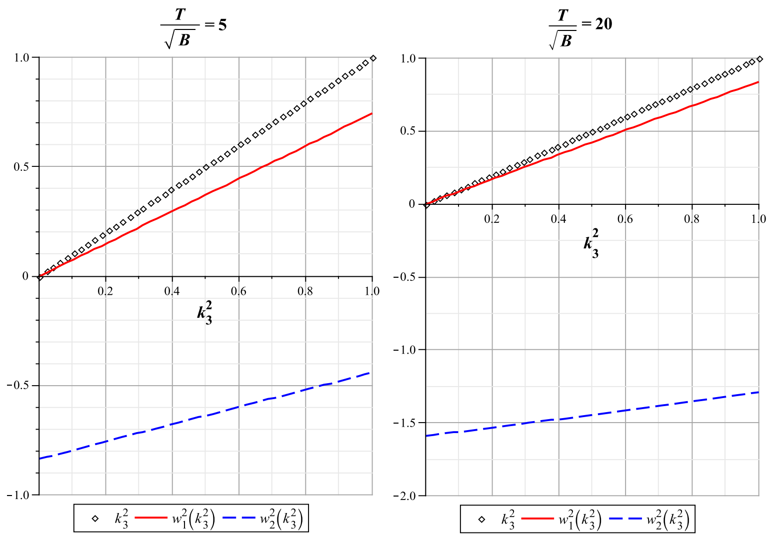

Figure 1. Dispersion relations for the transversal modes in case of motion along the field $h^2 \to 0$. Two cases are considered: $T/\sqrt{B} = 5$ and $T/\sqrt{B} = 20$. Curves represent dependence between square of the gluon frequency w^2 and square of the momentum $\vec{k}^2 = k_3^2$. Dot line is a trivial spectrum $w^2 = k_3^2$. Solid line is a first solution of the Eq. (24), dash line is the other.

respect to the operation of anticommutation, which structure constants have been calculated. To obtain the coefficients at the tensors in the Green function we have applied the method which can be useful in the more complicated case of SU(3) gluodynamics. For the one-loop form factors we obtain the explicit formulas in case of the motion along the field direction. The spectrum of gluons is derived from the pole position of the exact Green function with the one-loop form factors been accounted for. Spectral equations were obtained. In the case of $T \neq 0$ the high temperature limit for the form factors was computed. It is found that in this approximation all the form factors contain imaginary parts. Therefore a resummation of perturbation series should be carry out in order to obtain a stable spectrum. That can be done on the base of solution of the Schwinger-Dyson equation where, to calculate the polarization tensor, as the neutral gluon propagator the derived Green function should be substituted.

References

[1] K.Enqvist and P. Olesen, Phys.Lett. **B329**, 195 (1994).
[2] A.O. Starinets, A.S. Vshivtsev and V.Ch. Zhukovskii, Phys.Lett. **B322**, 287 (1994).
[3] V. Skalozub and M. Bordag, Nucl.Phys. **B576**, 430 (2000).
[4] N.O. Agasian, Phys.Lett. **B562**, 257 (2003).
[5] M. Bordag, V.Skalozub, Eur.Phys.J. **C45**, 159 (2006).
[6] M. Bordag, V.Skalozub, Phys.Rev. **D75**, 1250003 (2007).