

# STUDY OF HADRONIC EVENT PROPERTIES WITH NEXT-TO-LEADING LOGARITHM PARTON-SHOWER MODEL

Yoshihide Sakai  
(AMY Collaboration)  
*National Laboratory for High Energy Physics  
Tsukuba, Ibaraki 305 Japan*

## ABSTRACT

The general properties of multi-hadron final states produced by  $e^+e^-$  annihilation are compared with predictions of the Next-to-Leading Logarithm Parton-Shower (NLL PS) model. The NLL PS model with parameters tuned for AMY data reproduces hadronic event shapes as nicely as Leading-Logarithm (LL) PS model. The QCD scale parameter  $\Lambda_{\overline{MS}}$  is extracted from three different quantities: event shapes, three-jet fraction, and Energy-energy correlation.

## Introduction

In principle, hadronic process in  $e^+e^-$  annihilation should be described completely by Quantum Chromodynamics (QCD), the non-Abelian gauge theory of quarks and gluons. However, simulation of events is usually separated into two steps, since we do not know how to apply QCD to large-distance processes. First, a set of partons are generated using perturbative QCD calculations and, second, the partons are converted into observable particles using a phenomenological hadronization model. These two steps are separated by an energy scale parameter  $Q_0$ .

In the parton-generation process, some approximations are needed since it is not possible to sum all of the perturbative diagrams. Two different approximation methods are used: the Matrix-Element (ME) method [1] and the Parton-Shower (PS) model based on renormalization-group [2]. In the ME method, diagrams up to a fixed order in the QCD coupling strength,  $\alpha_s$  - currently  $O(\alpha_s^2)$ , are calculated exactly; in the PS method all-order diagrams are summed in terms of logarithm divergent orders - currently leading logarithm (LL) is commonly used.

There are advantages and disadvantages to both schemes. The ME method allows one to fix the renormalization scheme to determine the QCD scale parameter  $\Lambda_{\overline{MS}}$ , and provides a good approximation for the production of clean three-jet events. However, the number of partons that can be generated is limited, and it is impossible to generate partons with small transverse momentum relative to the radiating quark. On the other hand, in the LL PS method, it is possible to generate large number of partons; the  $Q_0$  cutoff determines the maximum number. In ad-

dition, it provides a good approximation for the production of small transverse-momentum gluons. However,  $\Lambda_{\overline{MS}}$  is undefined in the LL PS method.

When one goes to the Next-to-Leading Logarithm (NLL), Parton-Shower scheme has the advantages of both methods. In NLL PS method,  $\Lambda_{\overline{MS}}$  can be fixed and, in addition, partons can be precisely generated with both small and large transverse momentum values.

Based on the theoretical framework on the NLL [3,4], Kato and Munehisa developed an event generation program for the NLL PS model [5] including detailed considerations of the kinematics. This generator is combined with the LUND String-Fragmentation model [6] to generate the hadrons.

The NLL PS program contains following parameters. The three NLL jet parameters:  $\Lambda_{\overline{MS}}$ ;  $Q_0$ , the minimum virtuality; and  $\delta$ , cut-off virtuality between three-jet and two-jet at primary vertex. The three LUND String-Fragmentation parameters:  $\sigma_q$ , the width in the transverse momentum of hadrons; and the  $a$  and  $b$  of the symmetric fragmentation function.

Here, reported is a study of multi-hadron final states observed in the AMY experiment using the NLL PS model program. Similar studies using the NLL PS model were also previously reported [7].

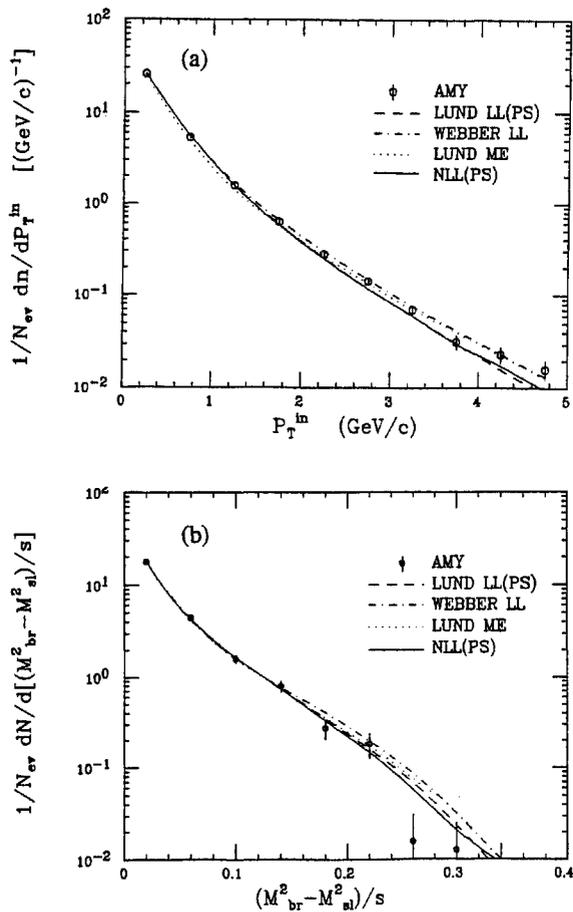
## Tuning the NLL with Event Shapes

Hadronic events are selected with the AMY standard hadronic event selection [8]. In addition, to ensure a large acceptance for particles in jets, only

events with  $|\cos\theta_{thrust}| < 0.7$  are considered, where  $\theta_{thrust}$  is the angle between the thrust axis of the event and beam axis.

A total of 1911 hadronic events, at center-of-mass energies between 52 and 57 GeV, satisfy all of the selection criteria; the average center-of-mass energy is 55.2 GeV and the total luminosity is  $18.6 \text{ pb}^{-1}$ . The corrected data for various global event-shape variables, together with comparisons with the LUND 6.3 LL PS [9] the LUND 6.2 ME [10] and the Webber LL PS [11] models, have already been reported [12].

The NLL generator was tuned using AMY data by varying the parameters  $\Lambda_{\overline{MS}}$ ,  $\sigma_q$  and  $a$ . Since the parameters  $a$  and  $b$  in the LUND symmetric fragmentation function are strongly correlated, we fixed  $b$  to be 0.9 and only allowed  $a$  to vary. The  $\delta$  parameter is fixed at 0.1, since dependence of  $\delta$  is cancelled internally and does not appear in final hadrons [5]. We choose a small value of 1.0 GeV for  $Q_0$  in order to reduce the dependence on the hadronization model.



**Fig. 1.** The acceptance-corrected distribution and predictions of the models: for (a)  $p_T^{in}$  with respect to the sphericity-axis-defined event plane for charged particles and (b)  $\Delta M^2 = (M_{BR}^2 - M_{SL}^2)/s$ .

For tuning, we fit to the four distributions: the scaled charged particle momentum  $X_p = 2p/\sqrt{s}$ ; the charged particle transverse momentum in and out of the event plane,  $p_T^{in}$  and  $p_T^{out}$ , where the event plane is defined by the two Sphericity axes with the highest eigenvalues; and the jet mass difference  $\Delta M^2 = (M_{BR}^2 - M_{SL}^2)/E_{vis}^2$ , where the  $M_{BR}$  ( $M_{SL}$ ) is the largest (smallest) mass determined by dividing each event by a plane perpendicular to the Sphericity axis and calculating the invariant mass of the particles in each hemisphere.  $\Lambda_{\overline{MS}}$  is sensitive to  $p_T^{in}$  and the jet mass difference  $\Delta M^2$ ;  $\sigma_q$  is sensitive to  $p_T^{out}$ ; and  $a$  is sensitive to  $X_p$ .

A minimum value of  $\chi^2 = 67.4$  is found at  $a = 0.73 \pm 0.07$ ,  $\Lambda_{\overline{MS}} = 0.31 \pm 0.02 \text{ GeV}$ , and  $\sigma_q = 0.41 \pm 0.01 \text{ GeV}$ .

In Figs. 1 (a) and (b), the corrected AMY results for the  $p_T^{in}$  and  $\Delta M^2$  distributions, taken from Ref. 12, are shown together with the results from the tuned NLL PS model, shown as solid line curves. The total  $\chi^2$  values for all 22 (fitted 4) distributions are 386(67.4), 371(60.6), 2783(553.8), and 601(102.0) for the tuned NLL PS, the LUND LL PS, LUND ME, and Webber model, respectively. The tuned NLL PS model has almost same  $\chi^2$  values as LUND LL PS model.

**Table 1.** Summary of tuned values of parameters for various  $Q_0$  cutoff for NLL PS model. The tuned (total)  $\chi^2$  is the sum of  $\chi^2$  of the four distributions used for tuning (all 22 distributions in Ref. 12.)

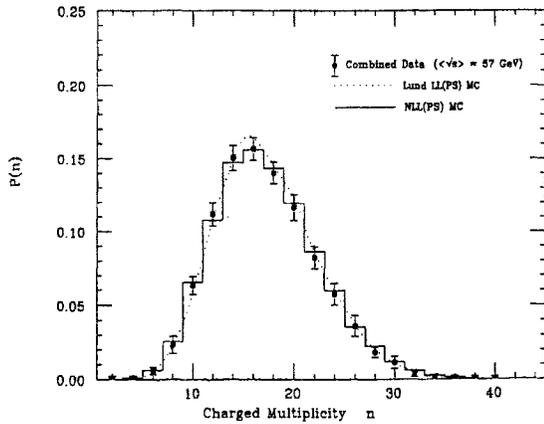
Parameter	a) $Q_0 = 1 \text{ GeV}$	b) $Q_0 = \sqrt{2} \text{ GeV}$	c) $Q_0 = 2 \text{ GeV}$	d) $Q_0 = 3 \text{ GeV}$	e) $Q_0 = 4 \text{ GeV}$
$a$	0.73	0.97	1.17	1.37	1.53
$\Lambda_{\overline{MS}}$	0.31	0.31	0.34	0.42	0.49
$\sigma_q$	0.41	0.41	0.42	0.42	0.43
tuned $\chi^2$	67.4	65.6	73.4	79.8	178.5
total $\chi^2$	386	552	566	756	1563

The effect of the  $Q_0$  cutoff values is examined using values of  $Q_0$ . The parameters  $a$ ,  $\Lambda_{\overline{MS}}$  and  $\sigma_q$  are adjusted for each  $Q_0$ . Table 1 shows the total  $\chi^2$ , together with the best values for  $a$ ,  $\Lambda_{\overline{MS}}$ , and  $\sigma_q$  for each value of the  $Q_0$  cutoff. The best value of the  $Q_0$  cutoff is found to be 1 GeV and the total  $\chi^2$  doubles at  $Q_0 = 3 \text{ GeV}$ . We use variation of parameters with this range of  $Q_0$  as a preliminary estimate

of the systematic error:  $a$  from 0.66 to 1.4,  $\sigma_q$  from 0.40 to 0.43 GeV, and  $\Lambda_{\overline{MS}} = 0.31 \pm 0.02(\text{stat}) \pm 0.11(\text{sys})$  GeV

### Charged Particle Multiplicities

Shown in Fig. 2 is the fully corrected charged particle multiplicity distribution for energies from 50 to 61.4 GeV (corresponding luminosity of  $30 \text{ pb}^{-1}$ ,  $\langle\sqrt{s}\rangle = 57 \text{ GeV}$ ) observed in AMY experiment [13], together with the LUND LL and tuned NLL PS model predictions. There is excellent agreement between the data and both of the model predictions.



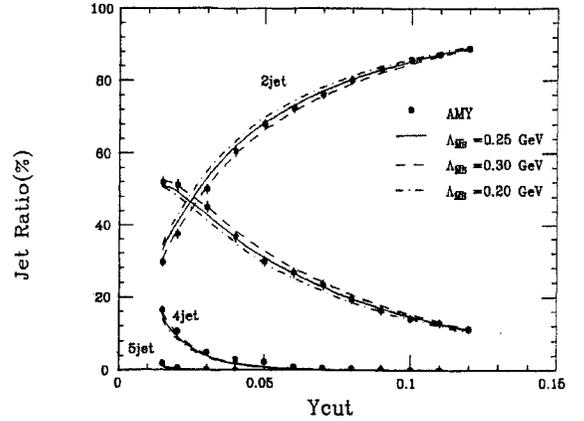
**Fig. 2.** The acceptance-corrected charged-particle multiplicity distribution at  $\langle\sqrt{s}\rangle = 57 \text{ GeV}$  and predictions of the NLL (solid histogram) PS model and Lund LL (dotted curve) PS model.

### $\Lambda_{\overline{MS}}$ determination from three-jet ratio

Since a main source of three-jet events is  $q\bar{q}g$  production, the fraction of three-jet events,  $R_3$ , is sensitive to the value of  $\Lambda_{\overline{MS}}$  [14]. We define the jet by using the YCLUST jet-clustering algorithm [15].

Figure 3 shows how the jet multiplicity fractions vary with cut-off parameter,  $y_{cut}$ , together with results from the NLL PS generator for different values of  $\Lambda_{\overline{MS}}$ .  $\Lambda_{\overline{MS}}$  can be determined from  $R_3$  for different values of  $y_{cut}$ . We choose  $y_{cut}=0.08$  resulting  $\Lambda_{\overline{MS}} = 0.24 \pm 0.07 \text{ GeV}$ , since the ratio of the  $R_3$  at the parton level and that at the hadron level is almost constant above  $y_{cut}=0.08$ . Systematic uncertainties in determining  $\Lambda_{\overline{MS}}$  are estimated as follows: The fluctuations of  $\Lambda_{\overline{MS}}$  over the range  $0.04 \leq y_{cut} \leq 0.12$  indicate an uncertainty from the  $y_{cut}$  dependence of 0.034 GeV. The uncertainty arising from the  $Q_0$  dependence ( $1 \leq Q_0 \leq 3 \text{ GeV}$ ) and the  $a$  dependence ( $0.66 < a < 1.4$ ) are estimated to

be 0.050 GeV, 0.015 GeV, respectively. The uncertainty from  $\sigma_q$  ( $0.40 < \sigma_q < 0.43 \text{ GeV}$ ) is negligible. Adding all the above in quadrature, we obtain  $\Lambda_{\overline{MS}} = 0.24 \pm 0.07(\text{stat}) \pm 0.06(\text{sys}) \text{ GeV}$ .



**Fig. 3.** The acceptance-corrected multi-jet production rates measured by AMY and predictions of the NLL PS model for three different values of  $\Lambda_{\overline{MS}}$ .

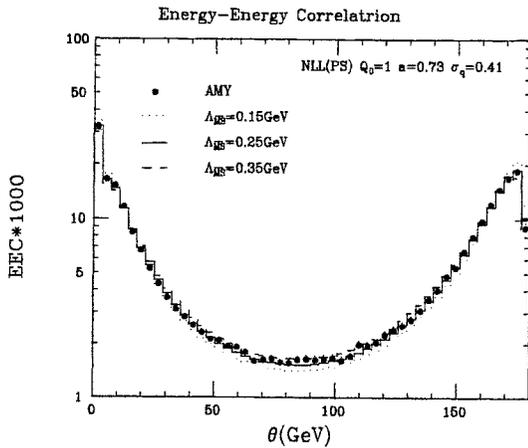
### $\Lambda_{\overline{MS}}$ determination from the $EEC$

The Energy-Energy Correlation ( $EEC$ ) [16] was introduced as a method to extract the strong coupling constant  $\alpha_s$ . The  $EEC$  is an energy-weighted angular correlation defined as

$$EEC(\theta) = \frac{d\Sigma}{d\theta}(\theta) = \frac{1}{N} \sum_{\text{events}} \sum_i \sum_j \frac{E_i E_j}{E_{vis}^2} \delta(\theta - \theta_{ij}),$$

where  $i$  and  $j$  run over all particles (charged and neutral) in the event,  $\theta_{ij}$  is the angle between particles  $i$  and  $j$ ,  $N$  is the total number of events, and  $E_{vis}$  is the total visible energy. While  $q\bar{q}$  events contribute to the  $EEC$  near  $\theta=0^\circ$  and  $180^\circ$ , events with hard gluon radiation contribute to the  $EEC$  in the intermediate angular region. Figure 4 shows the  $EEC$  for the corrected AMY data together with results from the NLL PS generator. We obtain  $\Lambda_{\overline{MS}}$  from the integrated value of  $EEC$  over the angular range  $\Theta \leq \theta \leq 180^\circ - \Theta$  for different values of  $\Theta$ . For smaller values of  $\Theta$ , fragmentation effects and the contribution from two jet events are expected to be large. We choose the relatively large value of  $\Theta=50^\circ$  to avoid uncertainties due to these effects, with the result  $\Lambda_{\overline{MS}}=0.30 \pm 0.02$ . The systematic uncertainty arising from the dependence on the choice of  $\Theta$  is estimated to be 0.039 GeV. The systematic uncertainties due to the  $Q_0$  dependence ( $1 \leq Q_0 \leq 3 \text{ GeV}$ ), the  $a$  dependence ( $0.66 < a < 1.4$ ) and the  $\sigma_q$  dependence ( $0.40 < \sigma_q < 0.43 \text{ GeV}$ ) are estimated to be 0.11 GeV,

0.075 GeV, and 0.011 GeV, respectively. Adding all of the above in quadrature, we obtain  $\Lambda_{\overline{MS}} = 0.30 \pm 0.02(\text{stat}) \pm 0.13(\text{sys})$  GeV.



**Fig. 4.** The acceptance-corrected AMY results for the *EEC* and predictions of the NLL PS model for three different values of  $\Lambda_{\overline{MS}}$ .

### Summary

After tuning the parameters to our previously reported distributions for event properties, the NLL Parton-Shower model agrees well with AMY data with the  $\chi^2$  comparable to that for LUND LL PS model. The tuned NLL PS model also reproduces the measured charged-particle multiplicities well.

The QCD scale parameter  $\Lambda_{\overline{MS}}$  is derived using the NLL PS model from the three methods:  $\Lambda_{\overline{MS}} = 0.31 \pm 0.02 \pm 0.11$  GeV (event shape tuning);  $0.24 \pm 0.07 \pm 0.06$  GeV ( $R_3$  at  $y_{cut} = 0.08$ );  $0.30 \pm 0.02 \pm 0.13$  GeV ( $\int EEC d\theta$  over  $50^\circ \leq \theta \leq 130^\circ$ ).

### Acknowledgements

We would like to thank Prof. K. Kato for helpful suggestions and discussions. We thank the TRISTAN staff for the excellent operation of the storage ring.

### References

1. R.K.Ellis, D.A.Ross, and A.E.Terrano, Nucl. Phys. **B178**, 421 (1981); F.Gutbrod, G.Kramer, and G.Schierholz, Z. Phys. **C21**, 235 (1984); A.Ali and F.Barreiro, Phys. Lett. **118B**, 155 (1982); T.D.Gottschalk and M.P.Shatz, Phys. Lett. **150B**, 451 (1985).

2. G.Altarelli and G.Parisi, Nucl. Phys. **B126**, 298 (1977); Yu.L. Dokshitzer, Sov. Phys. JETP **73**,1216 (1977); K.Konishi, A.Ukawa, and G. Veneziano, Phys. Lett. **78B**, 243 (1978); **80B**, 259 (1979); Nucl. Phys. **B157**, 45 (1979).
3. J.Kalinowski, K.Konishi, and T.R.Taylor, Nucl. Phys. **B181**, 221 (1981); J.Kalinowski, K.Konishi, P.N.Scharbach, and T.R.Taylor, Nucl. Phys. **B181**, 253 (1981).
4. J.F.Gunion and J.Kalinowski, Phys. Rev. **D29**, 1545 (1984); J.F. Gunion, J.Kalinowski and L.Szymanowski, Phys. Rev. **D32**, 2303 (1985).
5. K.Kato and T.Munehisa, Mod Phys. Lett. **A1**, 345 (1986); Phys. Rev. **D36**, 61 (1987); Phys. Rev. **D39**, 156 (1989).
6. B.Andersson *et al.*, Phys. Rep. **97**, 33 (1983).
7. T.Kamae, in: *XXIV International Conference on High Energy Physics*, eds. R. Kotthaus and J.H. Kuhn (Springer-Verlag,1988); K.Abe *et al.*(VENUS), Phys. Lett. **240B**, 232 (1990).
8. T.Kumita *et al.*(AMY), Phys. Rev. **D42**, 1339 (1990).
9. T.Sjöstrand and M.Bengtsson, Comput. Phys. Commun. **43**, 367 (1987).
10. T.Sjöstrand, Comput. Phys. Commun. **39** 347 (1986).
11. G.Marchesini and B.Webber, Nucl. Phys. **B238** 1 (1984); B. Webber, Nucl. Phys. **B238** 492 (1984); G. Marchesini and B. Webber, Nucl. Phys. **B310** 461 (1988).
12. Y.K.Li *et al.*(AMY), Phys. Rev. **D41**, 2675 (1990).
13. H.W.Zheng *et al.*(AMY), Phys. Rev. **D42**, 737 (1990).
14. J.Ellis, M.K.Gaillard, and G.G.Ross, Nucl. Phys. **B111**, 253 (1976); *ibid.* **B130**, 516 (1977).
15. W.Bartel *et al.*(JADE), Z. Phys. **C33**, 23 (1986).
16. C.L.Basham *et al.*Phys. Rev. **D17**, 2298 (1978).

## DISCUSSION

*Q. Haissinski (LAL, Orsay):* What is the scale that is being used for the value of  $\alpha_s(Q^2)$  in this NLL approach?

*A. Y. Sakai:* For each branching, one takes  $Q^2 = P_t^2$ . Therefore the scale is not unique. It decreases as the cascade develops.

*Q. M. Jacob (CERN):* The precision given on  $\Lambda_{\overline{MS}}$  seems very good. Does it include the uncertainties associated with the definition of the jets, i.e. recombination scheme dependence in the case of  $R_3$ ?

*A. Y. Sakai:* I understand that the answer was that it was not included.