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Regularization of electromagnetic field for self-force problem in de Sitter spacetime

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Abstract

The paper is concerned with the motion of a point electric charge in de Sitter spacetime. A point particle of mass m and charge q moving on a geodesic curve produces electromagnetic field that diverges at a particle's position. The field is determined by the electromagnetic Green's function by Higuchi and Lee (2008 *Phys. Rev.* D **78** 084031). The self-force contains both divergent and finite terms, and the latter are responsible for the radiation reaction. Our derivation of an effective equations of motion is based on conservation laws corresponding to the group of isometry of de Sitter space. The Nöther quantities consist of particle's individual characteristics and energy, momentum, and angular momentum carried by particle's electromagnetic field. Following the Detweiler–Whiting concept that a charge's motion should only be enforced by the regular component of its own field, we ignore the divergent terms in conservation laws. We assume that the divergencies are absorbed by particle's individual characteristics within the renormalization procedure. Finite radiative terms together with kinematic particle's characteristics constitute ten conserved quantities of closed particle plus field system. Their differential consequences yield the effective equations of motion of radiating charge in an external electromagnetic field and gravitation. Contributions to already renormalized particle's four-momentum and its inertial mass originated from electromagnetic field and background gravity are also derived from ten balance equations.

Keywords: de Sitter spacetime, Maxwell equations, renormalization, radiation reaction

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1. Introduction

Recent years have been rapidly growing interest in the problems of electromagnetic and gravitational self-force on a point-like particle moving in curved spacetime. It was due to the development of gravitational wave astronomy [1] and the need to produce accurate gravitational waveforms [2] for future space-based gravitational interferometer LISA; rapid advances in laser technology which produces necessity of correct description of motion of ultrarelativivstic electron in intense electromagnetic fields [3–7]; growing interest in the theory of radiation in spacetime dimensions other than four [8, 9]. The problems are intimately connected to each other.

Early studies of the electromagnetic self force is associated with Lorentz, Larmor, and Abraham. Lorentz [10] derived the self force by considering a finite size electron as a small continuously charged shell. Abraham [11, 12] generalized the self-force expression to be valid for relativistic charged particle. This is especially interesting since he did it before the advent of special relativity. Having decomposed the particle's electromagnetic field into singular and regular parts, designed from the retarded and the advanced Liénard-Wiechert fields, Dirac [13] produced equation of motion of a point charged particle in flat spacetime. This is well-known Lorentz-Abraham-Dirac (LAD) equation which describes the dynamics of a point charge acted upon an external force as well as its own electromagnetic field $\hat{F}$:

$$a^\mu = \frac{q}{m} F^\mu{}_\nu u^\nu + \tau_0 \left[\dot{a}^\mu - (a\cdot a) u^\mu\right]. \tag{1.1}$$

Here $u^\mu(s) = \mathrm{d}z^\mu(s)/\mathrm{d}s$ ($\mu = 0,1,2,3$) is a four-velocity in the position $z(s)$ of the charge world line parameterized by the proper time s, $a^\mu(s) = \mathrm{d}u^\mu(s)/\mathrm{d}s$ is its 4-acceleration, and $\dot{a}^\mu(s) = \mathrm{d}a^\mu(s)/\mathrm{d}s$ is the so-called 'jerk'. The self force is proportional to the small parameter $\tau_0 = 2q^2/(3m)$ of the dimension of time (6.2664×10^{-24} s for electron; in the dimension of length $\tau_0 = 1.8786\times10^{-15}$ m).

The backreaction terms dominate in dynamics of high energy particles moving in extremely strong electromagnetic fields. Such a situation occurs in astrophysics, e.g. relativistic jets of radiation and particles from super-massive black holes in the centers of some active galaxies. In series of papers [14–16] the behavior of plasma behavior in a neutron star's magnetosphere is studied. The consideration is based on the LAD equation describing particle's motion in flat spacetime. In 1960 DeWitt and Brehme [17] generalized LAD to curved spacetime, and their calculations was corrected by Hobbs [18] in 1968:

$$m\mathfrak{a}^\mu = f^\mu_{\rm ext} + q^2\left(\delta^\mu{}_\nu + u^\mu u_\nu\right)\left(\frac{2}{3}\frac{\mathrm{D}\mathfrak{a}^\nu}{\mathrm{d}s} + \frac{1}{3}R^\nu{}_\lambda u^\lambda\right) + f^\mu_{\rm tail}. \tag{1.2}$$

The covariant acceleration $\mathfrak{a}^\mu(s) = \mathrm{D}u^\mu(s)/\mathrm{d}s$ is the covariant derivative of four-velocity $u^\mu(s)$ along the world line; $R^\nu{}_\lambda$ is the Ricci tensor. The external force $f^\mu_{\rm ext} = g^{\mu\alpha}(qF_{\alpha\beta}u^\beta)$ is the covariant generalization of the Lorentz force. Expression in the right hand side of equation (1.2), i.e. the self-force, results from two types of particle's interactions with its own electromagnetic field. The first is a purely local and describes the radiation reaction depending on the current instant of time. The second is non-local one resulting from previously emitted radiation:

$$f^{\rm tail}_\mu = q^2 u^\nu(s)\int_{-\infty}^{s} \mathrm{d}s' \nabla_{[\mu} G^{\rm ret}_{\nu]\lambda'}\left(z(s), z(s')\right) u^{\lambda'}(s'). \tag{1.3}$$

Here $G^{\rm ret}_{\mu\nu'}$ is the retarded electromagnetic Green function in the curved spacetime [19, equation (1.33)], and ∇_μ is the covariant derivative in μth direction. The square brackets designate antisymmeterization of the indices. This interaction is due to the failure of Huygens principle: in curved spacetime electromagnetic waves propagate not just at the speed of light, but also at all speeds smaller than or equal to the speed of light. For this reason a massive particle 'fills' its own field, which acts on it just like the external one.

Having inspired by Dirac's seminal paper [13], Detweiler and Whiting [20] develop the regularization scheme based on the decomposition of Green's function in curved spacetime. The retarded field is splitted into singular and regular parts being the solutions to inhomogeneous and homogeneous Maxwell equations, respectively. Contrary to regular part, singular Green's function has no support inside the null cone and does not depend on the particle's past history. The regular Green's function had reasonable causal structure. It has no support on the future null cone. The function produces radiation field that detaches itself from the charge and leads an independent existence. The singular part of electromagnetic field is permanently attached to the charge and is carried along with it. It contributes to charge's inertia.

Having used this decomposition, Harte [21, 22] investigates the dynamics of extended charged bodies interacting with external fields as well as their own electromagnetic fields. A body moves in an arbitrary curved spacetime. Harte's scheme is based on the concept of approximate space-time symmetries. In the sufficiently small neighborhood of a point on particle's world line the momentarily comoving reference frame is introduced where the region looks nearly flat. Action principle is proclaimed to be invariant with respect to transformations from generalized Poincaré group [21]. Harte performs volume integrations of convolution of the stress energy tensor and generalized Killing fields [22]. Singular component of self field contributes into both the linear and angular momenta of charged body. In approximation of small charge distribution with slow internal dynamics the equation of center-of-mass motion is obtained. In specific case of structureless point charge the equation becomes the well-known DeWitt-Brehme-Hobbs equation (1.2).

It is obvious that the de Sitter group can be substituted for Poincaré group in Harte's scheme [21, 22]. In the standard theory of general relativity, de Sitter space is a highly symmetrical special vacuum solution. The isometry group of de Sitter space is the indefinite orthogonal group $SO(4,1)$ which contracts to Poincaré group for short distance kinematics. The choice of this slightly curved spacetime as the basic approximation of background gravity seems to be more adequate than the flat Minkowski space. The decomposition of the stress-energy tensor into singular and regular parts is necessary.

In the present paper, we study the self action problem of charged particle arbitrarily moving in de Sitter spacetime. We adopt the generalized Detweiler–Whiting axiom which states that the Lorentz force law involves the regular part of charge's self field only. We shall take the first step towards understanding how a gravity and a charge's electromagnetic potential energy change inertial properties of a charged particle, i.e. its individual momentum and its mass [22, equation (59)].

Topologically, de Sitter space is the direct product $\mathbb{R}\times\mathbb{S}^3$ where $\mathbb{R}$ is time axis and $\mathbb{S}^3$ is three-dimensional sphere [23]. The non-trivial topology makes electrodynamics very specific. A flux integral vanishes identically if there is no boundary surface in a compact manifold. The Gauss law says that the total outward flux of electromagnetic field through a closed surface is equal to the total charge of particles enclosed by this surface. So, the total charge in de Sitter space must be zero. These circumstances drastically change the method of usage of the retarded Green's function [24]. To reproduce the electromagnetic field, a set of initial data not only on a given point source is necessary, but also on a Cauchy surface consisting of points in the past of the field point. These circumstances make the self action problem highly nontrivial.

The question arises as how to deal with DeWitt-Brehme-Hobbs equation (1.2) in case of de Sitter metrics. One of our tasks is to figure it out.

Another task is to estimate the gravity correction for electron. We shall compare it with the small parameter τ_0 of LAD equation (1.1). Despite the fact of rapid development of experimental astrophysics, a man-maid machine only allows us to study the dynamics of radiating electron in very detail. The machine is multi-petawatt laser system [7]. Because of the third time derivative, LAD equation possesses pathological solutions [25–27], such as runaway solution (when acceleration grows exponentially with time) and preacceleration (when acceleration begins to increase prior the time at which the external force switches on). The reduction of order is necessary.

Landau and Lifshitz [28, § 76] replace the square of acceleration in the Larmor term by the negative scalar product $(u\cdot\dot{a})$ and substitute $m^{-1}\mathrm{d}f^{\mu}_{\mathrm{ext}}/\mathrm{d}\tau$ for the problematic derivative of particle's four-acceleration ('jerk'). The Landau–Lifshitz (LL) equation is of Newtonian class which avoids the nonphysical solutions of the LAD equation. LL approximation of LAD equation serves as equation of motion of radiating electron in an intense laser pulse in most experiments [7, 29, 30]. Validity of this approximation depends on the magnitude of applied force as well as type of external electromagnetic field [3–5, 7].

In [4] the series of the second order equations is investigated. They are obtained by successive iteration of order reduction procedure applied to LAD equation. The expansion parameter is $\tau_0\omega$ where $\tau_0 = 2q^2/(3m)$ and ω is a typical field frequency scale. Since the parameter is extremely small, the procedure can be truncated at any order of τ_0. The nth order approximation is denoted as LL_n. The LL equation represents only the first of an infinite series of approximations to LAD. The authors [4] show that the perturbation expansion generated by reduction of order has zero radius of convergence. It is because the coefficients in series expansions are alternate in sign (see [4, equations (6) and (7)]). An infinite order approximation LL_∞ is also evaluated. The properties of the iterative procedure demonstrates that effects of non-zero curvature of Universe will be important at some iteration.

The paper is organized as follows. In section 2 we sketch the topology of de Sitter space in context of CPT-symmetric Universe [31]. We draw a Penrose diagram illustrating a compactified de Sitter space. We study geodesic curves on the one-sheeted hyperboloid modeling the space. In section 3 we analyze the electromagnetic field produced by a point charge $+q$ placed at the so-called 'North pole', and a charge $-q$ at the 'South pole' of de Sitter space. The form of Penrose diagram of CPT-symmetric de Sitter space allows us to apply the retarded Green's function in usual way. A Cauchy surface is not necessary. In section 4 we develop the regularization procedure based on the global conservation laws associated with ten linearly independent Killing vectors of de Sitter space. We derive the equation of motion of a point-like charge acted upon an external force as well as its own electromagnetic field and background gravity. In section 5, we summarize the main ideas and results.

2. Free particle in de Sitter space

De Sitter space dS_4 is a vacuum solution of the Einstein field equations with cosmological constant $\Lambda = 3H^2$. The Hubble constant H defines the Ricci scalar $\mathcal{R} = 12H^2 = 4\Lambda$. We define de Sitter space as a submanifold of flat spacetime $\mathbb{M}_5$ of one higher dimension spanned by Minkowski rectangular coordinates y:

$$\eta_{MN}y^My^N = \frac{1}{H^2}. \tag{2.1}$$

We choose the mostly plus metric tensor $\hat{\eta} = \mathrm{diag}(-1,1,1,1,1)$. Capital Latin letters denote indices running from 0 to 4. Following [31], we accept hypothesis of CPT-symmetric Universe: top half of hyperboloid (2.1) pictures our Universe, while the bottom half of $\mathbb{H}_4$ describes anti-Universe—a cosmos with antiparticles, interactions and opposite direction of time arrow.

The metric of de Sitter spacetime dS_4 is defined via restriction of five-dimensional line element $\mathrm{d}s^2 = \eta_{MN}\mathrm{d}y^M\mathrm{d}y^N$ to the surface of one sheet hyperboloid (2.1). Conformally Einstein coordinates [32, § 2.21.2] cover the whole spacetime. In the CPT-symmetric Universe the time arrow is pointing away from Big Bang in both directions [31]. To adequately describe this circumstance we change the evolution variable [32, equation (2.21.9)] $\eta = \tau + \pi/2$. The de Sitter metric [32, equation (2.21.8)] takes the form [24, equation (4.1)]

$$\mathrm{d}s^2 = \frac{1}{H^2\cos^2\tau}\left(-\mathrm{d}\tau^2 + \mathrm{d}\chi^2 + \sin^2\chi\mathrm{d}\omega^2\right), \tag{2.2}$$

where $\mathrm{d}\omega^2 = \mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\phi^2$ is the spherical surface element. Coordinates of a point $y \in \mathbb{H}_4$ is specified as

$$\begin{aligned} y^0 &= \frac{1}{H}\tan\tau, \\ y^1 &= \frac{1}{H}\frac{\sin\chi}{\cos\tau}\sin\vartheta\cos\phi, \\ y^2 &= \frac{1}{H}\frac{\sin\chi}{\cos\tau}\sin\vartheta\sin\phi, \\ y^3 &= \frac{1}{H}\frac{\sin\chi}{\cos\tau}\cos\vartheta, \\ y^4 &= \frac{1}{H}\frac{\cos\chi}{\cos\tau}, \end{aligned} \tag{2.3}$$

with the range of parameters given by $\tau \in (-\pi/2, \pi/2)$, $\chi \in [-\pi, \pi]$, polar angle $\vartheta \in [0, \pi]$, and azimuthal angle $\phi \in [0, 2\pi)$. The one-sheeted hyperboloid (2.1) is visualized as the circular cylinder of height π and unit radius: $\mathcal{C} = \{(\chi, \tau) \in \mathbb{R}^2 | \tau \in (-\pi/2, \pi/2), \chi \in [-\pi, \pi]\}$. Future $\mathcal{I}_+$ and past $\mathcal{I}_-$ infinities are identified with edges $\tau = +\pi/2$ and $\tau = -\pi/2$, respectively. The vertical edges of the strip, $\mathcal{V}_- = \{\tau \in (-\pi/2, \pi/2), \chi = -\pi\}$ and $\mathcal{V}_+ = \{\tau \in (-\pi/2, \pi/2), \chi = +\pi\}$, are glued. To each point on this cylindrical surface corresponds two-dimensional sphere with radius $R = \frac{1}{H}\frac{\sin\chi}{\cos\tau}$ spanned by polar and azimuthal angles. If $\chi = 0$ and $\chi = \pm\pi$ the radius vanishes. The spheres shrink to points, named as [24] 'North pole' and 'South pole', respectively.

As de Sitter space is the slightly curved Minkowski spacetime, it is reasonable to use the *stereographic coordinates* being 'a projection the de Sitter hypersurface into target Minkowski spacetime' [33–35]:

$$y^\alpha = \Omega(x)x^\alpha, \tag{2.4}$$

$$y^4 = \pm\frac{1}{H}\Omega(x)\left(1 - \frac{H^2}{4}x^2\right), \tag{2.5}$$

where $x^2 = (x \cdot x)$, and

$$\Omega(x) = \left(1 + \frac{H^2}{4}x^2\right)^{-1}. \tag{2.6}$$

Small Greek letters denote indices running from 0 to 3. The 'dot' designates the scalar product in flat spacetime of four dimensions: $(x \cdot x) = \eta_{\mu\nu}x^\mu x^\nu$. Fourth coordinate (2.5) can be presented as $y^4 = \pm\frac{1}{H}(2\Omega - 1)$.

The de Sitter metric is the induced metric from the standard five-dimensional flat metric on dS_4. Inserting equation (2.4) in the line element $\mathrm{d}s^2 = \eta_{MN}\mathrm{d}y^M\mathrm{d}y^N$ we restrict it to the surface of hyperboloid $\mathbb{H}_4$:

$$\begin{aligned} \mathrm{d}s^2 &= \eta_{MN}\mathrm{d}y^M\mathrm{d}y^N\big|_{\mathbb{H}_4} \\ &= \Omega^2(x)\eta_{\alpha\beta}\mathrm{d}x^\alpha\mathrm{d}x^\beta. \end{aligned} \tag{2.7}$$

The metric tensor is

$$g_{\alpha\beta}(x) = \Omega^2(x)\eta_{\alpha\beta}. \tag{2.8}$$

The squared function (2.6) is the common spacetime dependent conformal factor. We denote $\hat{\eta} = \mathrm{diag}(-1,1,1,1)$ the metric tensor of Minkowski space.

Let's find the transformation from stereographic coordinates to conformally Einstein coordinates. Two charts are necessary to cover the surface of one-sheeted hyperboloid (2.1):

- (a) $\chi \in [-\pi/2, \pi/2]$: $y^4 = +\frac{1}{H}(2\Omega - 1)$.
  Substituting the right-hand side of equation (2.3) for y^4, we derive the conformal factor

$$\Omega(x) = \frac{\cos\tau + \cos\chi}{2\cos\tau}, \tag{2.9}$$

  and coordinate transformation

$$\begin{aligned} x^0 &= \frac{2}{H}\frac{\sin\tau}{\cos\tau + \cos\chi}, \\ x^1 &= \frac{2}{H}\frac{\sin\chi}{\cos\tau + \cos\chi}\sin\vartheta\cos\phi, \\ x^2 &= \frac{2}{H}\frac{\sin\chi}{\cos\tau + \cos\chi}\sin\vartheta\sin\phi, \\ x^3 &= \frac{2}{H}\frac{\sin\chi}{\cos\tau + \cos\chi}\cos\vartheta. \end{aligned} \tag{2.10}$$

  Putting $\tau = 0$ and $\chi = 0$ we obtain the Big Bang [31] point where $x^\mu = 0$.
- (b) $\chi \in [-\pi, -\pi/2] \cup [\pi/2, \pi]$: $y^4 = -\frac{1}{H}(2\Omega - 1)$.
  In this case the conformal factor becomes

$$\Omega(x) = \frac{\cos\tau - \cos\chi}{2\cos\tau}. \tag{2.11}$$

  The coordinate transformation takes the form

$$\begin{aligned} x^0 &= \frac{2}{H}\frac{\sin\tau}{\cos\tau - \cos\chi}, \\ x^1 &= \frac{2}{H}\frac{\sin\chi}{\cos\tau - \cos\chi}\sin\vartheta\cos\phi, \\ x^2 &= \frac{2}{H}\frac{\sin\chi}{\cos\tau - \cos\chi}\sin\vartheta\sin\phi, \\ x^3 &= \frac{2}{H}\frac{\sin\chi}{\cos\tau - \cos\chi}\cos\vartheta. \end{aligned} \tag{2.12}$$

  If $\tau = 0$ and $\chi = \pm\pi$ we obtain the Big Bounce [31] point where $x^\mu = 0$.

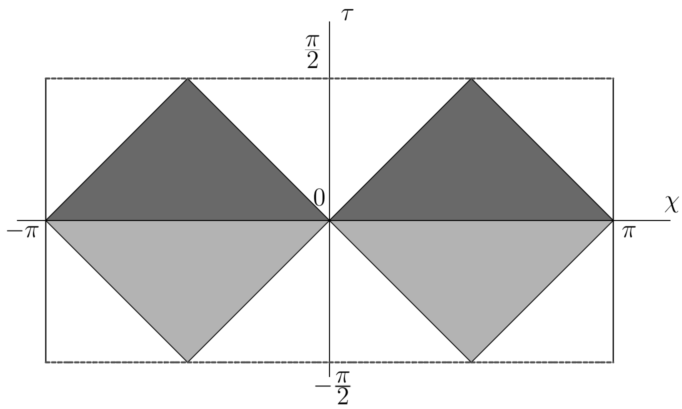

Figure 1. Sketch of compactified de Sitter space. Coordinates (χ,τ) cover one-sheeted hyperboloid (2.1); spherical angles (ϑ,ϕ) are suppressed. Bottom edge $\tau=-\pi/2$ corresponds to the past infinity $\mathcal{I}^-$ while the top one $\tau=+\pi/2$ represents future infinity $\mathcal{I}^+$. Zero level $\tau=0$ depicts Big Bang $(\chi=0)$ and Big Bounce $(\chi=\pm\pi)$. We draw causal structure for two specific points, $\chi=0$ and $\chi=\pm\pi$. Null cones separate time-like and space-like domains. The latter are shaded in grey. Vertical 'edges' $\chi=\pi$ and $\chi=-\pi$ are identified.

Both the Big Bang and the Big Bounce are the fixed points of de Sitter group transformations.

Having analyzed the chart (b), we see that the conformal factor (2.11) vanishes if and only if $\chi=\pm\tau$. Both the rays, $\chi-\tau=0$ and $\chi+\tau=0$, begin at the coordinate origin and end at infinities, either past $\mathcal{I}^-(\tau=-\pi/2)$ or future $\mathcal{I}^+(\tau=+\pi/2)$. In the Penrose diagram pictured in figure 1 they are represented by the straight lines with $\pm45^\circ$ slope. They are out of the domain $\mathcal{D}_b=\{\tau\in(-\pi/2,\pi/2),\chi\in[-\pi,-\pi/2]\cup[\pi/2,\pi]\}$. Therefore, the coordinate transformation (2.12) is justified. The rays lie within the domain $\mathcal{D}_a=\{\tau\in(-\pi/2,\pi/2),\chi\in[-\pi/2,\pi/2]\}$. At their points the conformal factor (2.9) is equal to 1. According to equation (2.6), this immediately gives $(x\cdot x)=0$. The past directed light rays end at past infinity $\mathcal{I}^-$, while future-directed ones end at future infinity $\mathcal{I}^+$.

Analogously, when the chart (a) is considered, the conformal factor (2.9) does not vanish within the domain $\mathcal{D}_a$ and coordinate transformation (2.12) is correctly defined. The segments $\chi\pm\tau=\pm\pi$ belong to the domain $\mathcal{D}_b$. At these points the conformal factor (2.11) is equal to 1. According to equation (2.6), this implies $(x\cdot x)=0$. The light rays begin at point $(\tau=0,\chi=\pm\pi)$ and end at infinities, either past or future. They are pictured in figure 1.

Causal structure of de Sitter space is induced by that in 'host' Minkowski spacetime of five dimensions [23, § II.7]. The interval between two points, x and x', in $\mathrm{d}S_4$ is nothing but the projection of interval in five-dimensional flat spacetime on the hyperboloid (2.1):

$$\begin{aligned}\rho(x,x') &= \eta_{MN}(y^M-y'^M)(y^N-y'^N)|_{\mathbb{H}_4}\\ &= \Omega(x)\Omega(x')\eta_{\mu\nu}(x^\mu-x'^\mu)(x^\nu-x'^\nu).\end{aligned} \tag{2.13}$$

The interval is proportional to the standard one of special relativity. Thus two events, x and x', in dS_4 are future connected if $(x-x')^2\leqslant 0$ and $x^0-x'^0>0$. The events are space-like separated if $(x-x')^2>0$. When $(x-x')^2=0$, the points lie on the light cone.

Starting with the covariant metric tensor (2.8), we present the basic objects of de Sitter metric in the simple and clear form. Putting it in general expression [32, equation (1.3.3)] for Christoffel symbols of the 2nd kind we obtain

$$\Gamma^{\mu}{}_{\alpha\beta} = \frac{H^2}{2}\Omega(x)\left(-\delta^{\mu}{}_{\alpha}x_{\beta} - \delta^{\mu}{}_{\beta}x_{\alpha} + \eta_{\alpha\beta}x^{\mu}\right). \tag{2.14}$$

Standard calculations yield Riemann tensor [32, equation (1.3.6)]

$$R_{\nu\alpha\rho\beta} = -H^2\Omega^4(x)\left(\eta_{\alpha\rho}\eta_{\nu\beta} - \eta_{\alpha\beta}\eta_{\nu\rho}\right), \tag{2.15}$$

Ricci tensor [32, equation (1.3.9)]

$$R_{\alpha\beta} = 3H^2\Omega^2(x)\eta_{\alpha\beta}, \tag{2.16}$$

and Ricci scalar [32, equation (1.3.10)]

$$\mathcal{R} = 12H^2. \tag{2.17}$$

It is easy to show that the tensors satisfy the source free Einstein field equation $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$ where Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu} = -3H^2\Omega^2(x)\eta_{\mu\nu}$ and cosmological constant $\Lambda = 3H^2$.

2.1. Geodesics in de Sitter space

The full geodesic equation is

$$\frac{\mathrm{d}^2x^{\mu}}{\mathrm{d}s^2} + \Gamma^{\mu}{}_{\alpha\beta}\frac{\mathrm{d}x^{\alpha}}{\mathrm{d}s}\frac{\mathrm{d}x^{\beta}}{\mathrm{d}s} = 0, \tag{2.18}$$

where s is the proper time defined by the relation

$$g_{\alpha\beta}\frac{\mathrm{d}x^{\alpha}}{\mathrm{d}s}\frac{\mathrm{d}x^{\beta}}{\mathrm{d}s} = \mp 1. \tag{2.19}$$

Here sign '$-$' stands for time-like geodesics while sign '$+$' defines space-like ones. Inserting the Christoffel symbols (2.14) we obtain the equation on geodesics in de Sitter space

$$\ddot{x}^{\mu} + \frac{1}{2}H^2\Omega(x)\left[-2(x\cdot\dot{x})\dot{x}^{\mu} + (\dot{x}\cdot\dot{x})x^{\mu}\right] = 0. \tag{2.20}$$

The 'dot' denotes the scalar product in flat spacetime of four dimensions, e.g. $(x\cdot\dot{x}) = \eta_{\alpha\beta}x^{\alpha}\dot{x}^{\beta}$. According to equations (2.8) and (2.19), the squared four-velocity $(\dot{x}\cdot\dot{x})$ takes value either $-\Omega^{-2}$ for time-like geodesics or $+\Omega^{-2}$ for space-like ones.

We overmultiply the equation on $-(H^2/2)\Omega^2(x)x_{\mu}$. After some algebra we derive the non-homogeneous second-order differential equation

$$\ddot{\Omega} + \kappa H^2\left(\Omega - \frac{1}{2}\right) = 0, \tag{2.21}$$

where parameter $\kappa = \Omega^2(x)(\dot{x}\cdot\dot{x})$ can takes three values:

$$\kappa = \begin{cases} -1 & , \quad \text{time-like} \quad \text{geodesics} \\ +1 & , \quad \text{space-like} \quad \text{geodesics} \\ 0 & , \quad \text{null} \quad \text{geodesics} \end{cases}. \tag{2.22}$$

Rewriting the projection relation (2.5), we express the conformal factor Ω in terms of 4th coordinate of the 'host' Minkowski space of five dimensions:

$$\Omega = \frac{1}{2}\left(1 \pm Hy^4\right). \tag{2.23}$$

Inserting this in equation (2.21) we obtain homogeneous second-order differential equation on the fourth space coordinate

$$\ddot{y}^4 + \kappa H^2 y^4 = 0. \tag{2.24}$$

We deal with the manifestly covariant theory. It is reasonable to assume that the coordinates (2.4) satisfy analogous equation, namely

$$\ddot{y}^\mu + \kappa H^2 y^\mu = 0. \tag{2.25}$$

Let us prove that it is equivalent to geodesic equation (2.20). Substituting $\Omega(x)x^\mu$ for y^μ we derive the equation

$$\frac{\mathrm{d}^2}{\mathrm{d}s^2}(\Omega x^\mu) + \kappa H^2 \Omega x^\mu = 0. \tag{2.26}$$

Inserting the differential $\dot{\Omega} = -(H^2/2)\Omega^2(x\cdot\dot{x})$ of conformal factor (2.6), taking into account equation (2.21), and replacing the parameter κ by $\Omega^2(x)(\dot{x}\cdot\dot{x})$, we obtain the geodesic equation (2.20).

2.1.1. Time-like geodesics. The solution to equation (2.21) where $\kappa = -1$ is parameterized by hyperbolic functions

$$\Omega = \frac{1}{2} + B_1 \cosh(Hs) + B_2 \sinh(Hs), \tag{2.27}$$

where B_1 and B_2 are constants of integration. As the equations (2.21) and (2.24) are equivalent to each other, they are associated with initial values of the fourth coordinate of $\mathbb{M}_5$, namely $B_1 = (H/2)y^4(0)$ and $B_2 = (1/2)\dot{y}^4(0)$.

The solution to equation (2.25) is as follows

$$y^\alpha(s) = y^\alpha(0)\cosh(Hs) + \frac{1}{H}\dot{y}^\alpha(0)\sinh(Hs). \tag{2.28}$$

Taking into account the coordinate transformation (2.4), we obtain the coordinate functions of the time-like geodesic curve $\zeta \subset \mathrm{d}S_4$

$$x^\alpha(s) = \frac{A^\alpha \cosh(Hs) + B^\alpha \sinh(Hs)}{\frac{1}{2} + B_1 \cosh(Hs) + B_2 \sinh(Hs)}. \tag{2.29}$$

Four-vectors A and B are connected with initial values of coordinates of $\mathbb{M}_5$:

$$A^\alpha = y^\alpha(0), \qquad B^\alpha = \frac{1}{H}\dot{y}^\alpha(0). \tag{2.30}$$

Initial values satisfy the constraint (2.1) and its differential consequence:

$$\eta_{MN} y^M(0) y^N(0) = \frac{1}{H^2}, \tag{2.31}$$

$$\eta_{MN} y^M(0) \dot{y}^N(0) = 0. \tag{2.32}$$

These yield the following relations on vectors A and B involved in equation (2.29):

$$(A\cdot A) = \frac{1}{H^2}\left(1 - 4B_1^2\right), \tag{2.33}$$

$$(A\cdot B) = -\frac{4}{H^2}B_1 B_2. \tag{2.34}$$

It is easy to show that the proper time condition $\Omega^2(x)(\dot{x}\cdot\dot{x}) = -1$ is equivalent to the constraint

$$\eta_{MN}\dot{y}^M\dot{y}^N = -1. \tag{2.35}$$

The initial five-velocity satisfies this relation too. It produces the relation

$$(B\cdot B) = \frac{1}{H^2}\left(-1-4B_2^2\right). \tag{2.36}$$

Recall that the 'dot' denotes the scalar product in flat spacetime of four dimensions, e.g. $(A\cdot A) = \eta_{\alpha\beta}A^\alpha A^\beta$. Direct calculations show that the coordinate functions (2.29) satisfy the geodesic equation (2.20).

Inserting coordinate functions of points $x(s)\in\zeta$ and $x(s')\in\zeta$ in equation (2.13) we derive, that they are separated by the squared interval

$$\rho(x,x') = \frac{2}{H^2}\left[1-\cosh\left(H(s-s')\right)\right]. \tag{2.37}$$

We expand the argument of hyperbolic cosine in powers of small parameter H. Restricting ourselves to the first nontrivial term, we obtain $\rho(x,x') = -(s-s')^2$. As would be expected, the approximated interval reduces to the difference of proper instants s and s' depicting two points on time-like geodesic line in flat Minkowski space of four dimensions.

2.1.2. Space-like geodesics. Putting $\kappa = +1$ in equation (2.24) we obtain the equation on simple harmonic oscillations. The solution is defined by trigonometric functions. Taking into account equation (2.23), we find the conformal factor

$$\Omega(s) = \frac{1}{2} + B_1\cos(Hs) + B_2\sin(Hs), \tag{2.38}$$

where constants $B_1 = (H/2)y^4(0)$ and $B_2 = (1/2)\dot{y}^4(0)$.

Putting $\kappa = +1$ equation (2.25) we see that the coordinates y^α also satisfy the oscillatory equations. Since $y^\alpha = \Omega(x)x^\alpha$, the space-like geodesic curve ξ is spanned by coordinate functions

$$x^\alpha(s) = \frac{A^\alpha\cos(Hs) + B^\alpha\sin(Hs)}{\frac{1}{2} + B_1\cos(Hs) + B_2\sin(Hs)}, \tag{2.39}$$

where four-vectors A and B are related to the initial data (2.30). The constraints (2.31) and (2.32) produce the relations (2.33) and (2.34), respectively.

The proper time condition $\Omega^2(x)(\dot{x}\cdot\dot{x}) = +1$ is equivalent to the constraint

$$\eta_{MN}\dot{y}^M\dot{y}^N = 1. \tag{2.40}$$

It produces the relation

$$(B\cdot B) = \frac{1}{H^2}\left(1-4B_2^2\right), \tag{2.41}$$

(cf equation (2.36)).

Putting coordinate functions (2.39) of points $x(s)\in\xi$ and $x(s')\in\xi$ in equation (2.13) we derive that the points are separated by the squared the interval

$$\rho(x,x') = \frac{2}{H^2}\left[1-\cos\left(H(s-s')\right)\right]. \tag{2.42}$$

Expanding the argument of cosine in powers of H we obtain $\rho(x,x') = (s-s')^2$ after passing to $H\to 0$ limit. The approximated interval reduces to the difference of proper instants s and s' depicting two points on space-like geodesic line in flat Minkowski space of four dimensions.

2.1.3. Null geodesics. In the domain $\mathcal{D}_a$ the locus of points satisfying $(x \cdot x) = 0$ are specified by the relations $\chi = \pm\tau$ where time $\tau \in [0, \pi/2)$. Inserting this in equation (2.10) we find out the coordinates of points of null cones in the standard form

$$x^\mu = k^\mu s, \tag{2.43}$$

where $k^\mu = (1, \pm n^i)$, $n^i = \sin\vartheta\cos\phi, \sin\vartheta\sin\phi, \cos\vartheta$, is null four-vector. The vertex of future light cone is placed at the coordinate origin $(\chi, \tau) = (0, 0)$ (North pole). Proper time $s = \frac{1}{H}\tan\tau$ plays role of affine parameter: $s \in [0, +\infty)$.

Expression (2.43) is true of the domain $\mathcal{D}_b$. So, for $(\chi, \tau) = (\pm\pi, 0)$ (South pole) the past cone is specified by $\chi = \pm(\tau + \pi)$ where time $\tau \in (-\pi/2, 0]$. Inserting this in equation (2.12) we obtain equation (2.43). Affine parameter $s \in (-\infty, 0]$.

Taking into account equation (2.13), we see that points on null cones with vertex at point x' can be parameterized as $x^\mu = x'^\mu + rk^\mu$ where r is affine parameter and $(k \cdot k) = 0$. This circumstance allows us to introduce the retarded coordinates in de Sitter space (see appendix C).

3. Electric charge in de Sitter space

The space-like geodesics in de Sitter space are expressed in terms of trigonometric functions which repeat their values at regular intervals. This reflects the circumstance that dS_4 is topologically $\mathbb{R} \times \mathbb{S}^3$ where $\mathbb{R}$ is time axis. A flux integral vanishes identically if there is no boundary surface in a compact manifold. In [24] the electromagnetic field produced by a point charge $+q$ placed at the so-called 'North pole', and a charge $-q$ at the 'South pole' has been calculated. Both the particles are at rest.

Let us consider time-like geodesics of two opposite charges in CPT-symmetric Universe [31]. The set of parameters $B_1 = 1/2$, $B_2 = 0$, $A^\mu = 0$, $B^0 = 1/H$, and $B^i = 0$ satisfy equations (2.33), (2.34) and (2.36). Putting these in equation (2.29), we obtain the time-like geodesic curve of static charge:

$$\begin{aligned} x^0(s) &= \frac{2}{H}\frac{\sinh(Hs)}{1 + \cosh(Hs)}, \\ x^i(s) &= 0. \end{aligned} \tag{3.1}$$

Let us consider the positive charge situated at 'North pole'. We suppose that it moves forward in time. The static charge 'lives' in the open subset $U = \{(\chi, \tau) \in \mathbb{R}^2 | \chi \in [-\pi/2, \pi/2], \tau \in [0, \pi/2)\}$. Putting $\chi = 0$ in equation (2.10), we obtain

$$x^0(\tau) = \frac{2}{H}\frac{\sin\tau}{\cos\tau + 1}, \tag{3.2}$$

where $\tau \in [0, +\pi/2)$. Having compared with equation (3.1), we derive the relations between evolution parameters

$$\cosh(Hs) = \frac{1}{\cos\tau}, \qquad \sinh(Hs) = \tan\tau. \tag{3.3}$$

To clarify the situation we present the zeroth component of geodesic curve in the form

$$\begin{aligned} x^0(s) &= \frac{2}{H}\sqrt{\frac{\cosh(Hs) - 1}{\cosh(Hs) + 1}} \\ &= \frac{2}{H}\sqrt{\frac{1 - \cos\tau}{1 + \cos\tau}}. \end{aligned} \tag{3.4}$$

Since $\tau \in [0, +\pi/2)$, the coordinate function raises from 0 to $2/H$. Passing to $H \to 0$ we obtain $\lim_{H\to 0} x^0(s) = +s$. If $Hs \to \infty$ then zeroth coordinate $x^0(s)$ approaches to Horizon $2/H$.

We suppose that the negative charge placed at the 'South pole' moves backward in time. Putting $\chi = \pi$ in equation (2.12), we obtain the coordinate function (3.2) where $\tau \in (-\pi/2, 0]$. Proper time s and evolution parameter τ are related by equation (3.3). Zeroth component of geodesic curve takes the form

$$x^0(s) = -\frac{2}{H}\sqrt{\frac{\cosh(Hs)-1}{\cosh(Hs)+1}} = -\frac{2}{H}\sqrt{\frac{1-\cos\tau}{1+\cos\tau}}. \tag{3.5}$$

The coordinate function changes within the interval $(-2/H, 0]$. Passing to $H \to 0$ we obtain $\lim_{H\to 0} x^0(s) = -s$. If $Hs \to -\infty$ then zeroth coordinate $x^0(s)$ approaches to Horizon $-2/H$ in anti-Universe.

Gauss law is satisfied because to each charged particle in our Universe corresponds antiparticle in its mirror counterpart consisting of antimatter. Penrose diagram of CPT-symmetric Universe is pictured in figure 1. It is quite different from that in [24, figure 1(b)]. The world line of a static charge placed at North pole originates from Big Bang moment $\tau = 0$, not from remote past $\tau = -\pi/2$. As a consequence, the charge moves inside the causality region, not at its edge (cf figure 1 and [24, figure 1(b)]). For this reason, the electromagnetic potential is related to Green's function in usual way in CPT-symmetric de Sitter space.

De Sitter space is slightly curved spacetime in the absence of matter or energy. This residual curvature implies a positive cosmological constant $\Lambda = 3H^2$ determined by observation. The inverse Hubble length $(7.2817 \pm 0.0584) \times 10^{-27}\mathrm{m}^{-1}$ corresponds to the Hubble constant $H = 67.36 \pm 0.54\ (\mathrm{Km\,s}^{-1})\,\mathrm{Mps}^{-1}$ [36, equation (14)]. Let us imagine a proton that was born at the moment of Big Bang, i.e. $T_U = 13.7$ billion light years ago. Dimensionless argument of hyperbolic cosine is $HT_U \approx 0.9438$. The zeroth coordinate $x^0(T_U) \approx 12.7661$ billion light years. If the dynamics of a charged particle in the processing chamber of a man-made machine is studied, the Big Bang looks as an event at the remote past. Another important point, the Horizon $2/H \approx 29.03$ billion light years, is the very distant future.

Do we have calculate time and distance from the Big Bang in de Sitter space? The answer is negative. De Sitter space is the quotient of the pseudo-orthogonal group $SO(4,1)$ and the Lorentz group $\mathcal{L}$, namely $dS_4 = SO(4,1)/\mathcal{L}$ [33]. Transformations from ten-parameter group $SO(4,1)$ do not change the interval (2.13). The group manifold is a bundle with de Sitter space as base space and the Lorentz group $\mathcal{L}$ as fiber. Similarly to Minkowski spacetime, we can choose the coordinate origin at an arbitrarily point and direct coordinate axes as we like a.s. For example, see [37] where well-known Twin Paradox in de Sitter Space is studied. Transitions between inertial frames is governed by the Lorentz group.

The first charge 'lives' in our Universe while the second one exists in its mirror counterpart consisting of antimatter. These worlds are not causally connected. With a great degree of accuracy we can put the interval $s \in (-\infty, +\infty)$ instead of the time interval from Big Bang to Horizon in our Universe.

Let a point-like charge q moves along a world line $\zeta \subset U$ parameterized by four functions $z^\alpha(s)$ of proper time parameter s. The charged particle produces current

$$j^\nu(x) = q\int_{-\infty}^{+\infty} \mathrm{d}s\, u^\nu(s)\frac{\delta^4(x - z(s))}{\sqrt{-g(x)}}. \tag{3.6}$$

where $u^\nu(s) = \mathrm{d}z^\nu(s)/\mathrm{d}s$ is the four-velocity. We take into account that in curved spacetime the δ-function definition involves the determinant of the metric tensor $\sqrt{-g(x)}$ (see [19, equation (13.2)]). Corresponding electromagnetic potential is

$$A_\mu^{\mathrm{ret}}(x) = 4\pi \int_U \mathrm{d}^4x' \sqrt{-g(x')} G^{\mathrm{ret}}_{\mu\nu'}(x,x') j^{\nu'}(x'), \tag{3.7}$$

where $G^{\mathrm{ret}}_{\mu\nu'}(x,x')$ is the retarded Green's function [24]. DeWitt-Brehme-Hobbs equation (1.2) is valid in de Sitter space. The equation adapted for de Sitter metric (2.8), Christoffel symbols (2.14), and Ricci tensor (2.16) does not distinguish between short-range and long-range radiations. To find corresponding expressions we apply Harte's scheme [22] to the self action problem of charged particle arbitrarily moving in de Sitter spacetime.

3.1. Maxwell equations in a curved spacetime

At each point of a four-dimensional Riemannian manifold (M,g), a given current density $\hat{j}$ generates the electromagnetic field which satisfies the Maxwell equations

$$\mathrm{d}\hat{F} = 0, \quad \text{(a)} \qquad \mathrm{d}^\star\hat{F} = 4\pi{}^\star\hat{j}, \quad \text{(b)} \tag{3.8}$$

where 4π is the area of unit three-dimensional sphere. For a given coordinate chart (U,x) on M, the field is determined by the Faraday 2-form $\hat{F} = \frac{1}{2}F_{\alpha\beta}\,\mathrm{d}x^\alpha \wedge \mathrm{d}x^\beta$ where the differentials of a set of coordinates serve as basic 1-forms. To define the Hodge dual 2-form ${}^\star\hat{F} = \hat{\omega}(\mathbf{F})$ we construct two-vector $\mathbf{F} = \frac{1}{2}F^{\mu\nu}\,\mathbf{e}_\mu \wedge \mathbf{e}_\nu$ with components

$$F^{\mu\nu} = g^{\mu\alpha} F_{\alpha\beta}\, g^{\beta\nu}, \tag{3.9}$$

and then find the dual of $\mathbf{F}$ with respect the volume 4-form $\hat{\omega}$. Tensor $g^{\mu\alpha}$ is the reciprocal of the metric tensor $g_{\alpha\beta}$ which induces this 4-form

$$\hat{\omega} = \sqrt{-g(x)}\,\mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3. \tag{3.10}$$

We denote $g(x)$ the determinant $\det|g_{\alpha\beta}|$ of the metric tensor. So, the components of the Hodge dual 2-form ${}^\star\hat{F} = \frac{1}{2}{}^\star F_{\gamma\delta}\,\mathrm{d}x^\gamma \wedge \mathrm{d}x^\delta$ heavily depends on metrics:

$$\begin{aligned} {}^\star F_{\gamma\delta} &= \frac{1}{2}\sqrt{-g}\,\epsilon_{\gamma\delta\mu\nu} F^{\mu\nu} \\ &= \frac{1}{2}\sqrt{-g}\,\epsilon_{\gamma\delta\mu\nu} g^{\mu\alpha} F_{\alpha\beta} g^{\beta\nu}. \end{aligned} \tag{3.11}$$

Levi–Civita symbol $\epsilon_{\gamma\delta\mu\nu}$ is defined by 0 if any two labels are the same, $+1$ if γ,δ,μ,ν is an even permutation of $0,1,2,3$, and -1 if the set of labels is an odd permutation of these numbers. The volume form (3.10) is oriented as $\epsilon_{0123} = +1$.

Hodge dual 3-form ${}^\star\hat{j} = \hat{\omega}(\mathbf{j})$ of a four-vector current $\mathbf{j} = j^\nu \mathbf{e}_\nu$ is

$$\begin{aligned} {}^\star\hat{j} = \sqrt{-g}\,\big(& j^0\,\mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \\ & - j^1\,\mathrm{d}x^0 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \\ & + j^2\,\mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^3 \\ & - j^3\,\mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2\big)\,. \end{aligned} \tag{3.12}$$

Routine calculations yield the second pair (3.8(b)) of Maxwell's equations in curved spacetime:

$$\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}\,g^{\mu\alpha}F_{\alpha\beta}\,g^{\beta\nu}\right) = -4\pi\sqrt{-g}j^\nu. \tag{3.13}$$

Despite the use of partial derivatives, these equations are covariant under arbitrary curvilinear coordinate transformations. Thus if one replaces the partial derivatives with covariant derivatives, the extra terms thereby introduced would cancel out [38].

Let us consider the first pair (3.8(a)) of Maxwell's equations in curved spacetime. We suppose that the Faraday 2-form is exact on coordinate chart $(U,x) \subset M$: $\hat{F} = \mathrm{d}\hat{A}$. The electromagnetic field is a covariant antisymmetric tensor of degree 2 which can be defined in terms of the 1-form electromagnetic potential by

$$\begin{aligned} F_{\alpha\beta} &= \frac{\partial A_\beta}{\partial x^\alpha} - \frac{\partial A_\alpha}{\partial x^\beta} \equiv \partial_\alpha A_\beta - \partial_\beta A_\alpha \\ &= \nabla_\alpha A_\beta - \nabla_\beta A_\alpha \equiv \nabla_{[\alpha}A_{\beta]}, \end{aligned} \tag{3.14}$$

where ∇_α denotes the covariant derivative. Inserting the second line of equation (3.14) into equation (3.13) and adopting the covariant Lorentz gauge

$$\nabla_\alpha A^\alpha = 0. \tag{3.15}$$

one arrives at the well-known wave equation

$$\Box A^\alpha - R^\alpha{}_\beta A^\beta = -4\pi j^\alpha, \tag{3.16}$$

for the electromagnetic vector potential in a curved spacetime. Here $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$ is the covariant wave operator and $R^\alpha{}_\beta$ is the Ricci tensor.

The Green's function technique is applied to solve the wave equation [19, 20, 24]. The retarded electromagnetic potential is given by equation (3.7) where $G^{\rm ret}_{\mu\nu'}(x,x')$ is the retarded Green's function of the equation (3.16).

3.2. Electrodynamics in de Sitter space

Stereographic coordinates (2.4)–(2.6) result the *conformally flat* metric tensor (2.8). Electrodynamics in a conformally flat spacetime of four dimensions possesses remarkable properties [39, S. II] which simplify the procedure of solving of wave equation (3.16).

Inserting the square root of negative determinant $\sqrt{-g} = \Omega^4(x)$ and the reciprocal of de Sitter metric tensor $g^{\mu\alpha} = \Omega^{-2}(x)\eta^{\mu\alpha}$ into Maxwell's equation (3.13) we obtain

$$\frac{\partial}{\partial x^\mu}\left(\eta^{\mu\alpha}F_{\alpha\beta}\eta^{\beta\nu}\right) = -4\pi\Omega^4(x)j^\mu. \tag{3.17}$$

It is of great importance that the conformal factors disappear in the left hand side of this equation. This circumstance motivates us to substitute $\partial_\alpha A_\beta - \partial_\beta A_\alpha$ for $F_{\alpha\beta}$ (see the first line of equation (3.14)). We arrive at

$$\eta^{\mu\alpha}\frac{\partial^2}{\partial x^\mu \partial x^\alpha}\left(A_\beta \eta^{\beta\nu}\right) - \eta^{\beta\nu}\frac{\partial}{\partial x^\beta}\frac{\partial\left(\eta^{\mu\alpha}A_\alpha\right)}{\partial x^\mu} = -4\pi\Omega^4(x) j^\nu. \tag{3.18}$$

To simplify the equation we adopt here the non-covariant Lorentz gauge

$$\frac{\partial\left(\eta^{\mu\alpha}A_\alpha\right)}{\partial x^\mu} = 0. \tag{3.19}$$

The potential satisfying this condition differs from that satisfying the covariant Lorentz gauge (3.15). Thus we use the notation A_μ. Finally we obtain the wave equation

$$\Box A_\beta \eta^{\beta\nu} = -4\pi\Omega^4(x) j^\nu, \tag{3.20}$$

where $\Box = \eta^{\mu\alpha}\partial_\mu\partial_\alpha$ is the standard d'Alembert operator in flat spacetime.

It is of great importance that the left hand side of this equation is just the same as in Minkowski space. We apply the Green's function method (3.7)

$$A_\beta(x)\eta^{\beta\nu} = 4\pi\int_U \mathrm{d}^4x'\Omega^4(x')G(x-x')j^\nu(x'), \tag{3.21}$$

where we take into account that $\sqrt{-g(x')} = \Omega^4(x')$. The function $G(x-x')$ gives a response to δ-shape excitation

$$\Box G(x-x') = -\delta^4(x-x') \tag{3.22}$$

where $\Box$ is the standard d'Alembert operator in Minkowski space.

There are two solutions: (causal) retarded Green's function and its advanced (acausal) counterpart. We restrict ourselves to the well-known retarded solution to equation (3.22)

$$G^{\mathrm{ret}}(x-x') = \frac{1}{4\pi}\frac{\delta(x^0 - x'^0 - |\mathbf{x}-\mathbf{x}'|)}{|\mathbf{x}-\mathbf{x}'|}, \tag{3.23}$$

where $\mathbf{x} = \{x^1, x^2, x^3\}$. The retarded Green's function has support only on the past light cone of the field point x. We insert the function (3.23) and the δ-shape current (3.6) in equation (3.21). After some algebra we obtain the retarded potential in de Sitter space

$$A_\beta(x)\eta^{\beta\nu} = q\frac{u^\nu(s^{\mathrm{ret}}(x))}{r(x)}. \tag{3.24}$$

The potential is inversely proportional to the standard retarded distance in Minkowski space

$$r(x) = -\eta_{\alpha\beta}(x^\alpha - z^\alpha(s^{\mathrm{ret}}))u^\beta(s^{\mathrm{ret}}), \tag{3.25}$$

i.e. to the scalar product on null linking vector $K^\alpha = x^\alpha - z^\alpha(s^{\mathrm{ret}}(x))$ and particle's four-velocity, taken with opposite sign.

For further convenience Greek indices are raised and lowered with the Minkowski metric tensor $\hat{\eta} = \mathrm{diag}(-1,1,1,1)$ in the present paper. With this in mind we rewrite the potential (3.24) in the form

$$A_\beta(x) = q\frac{u_\beta(s^{\mathrm{ret}}(x))}{r(x)}. \tag{3.26}$$

The denotation is not mathematically correct. Indeed, in de Sitter space we apply the covariant metric tensor $g_{\alpha\beta} = \Omega^2(x)\eta_{\alpha\beta}$ in the operation of lowering indices. However, usage $\eta_{\alpha\beta}$ seems very natural because de Sitter metric tensor is proportional to Minkowski metric tensor.

In appendix A we demonstrate that the potential (3.26) in the non-covariant gauge (3.19) is gauge equivalent to the Higuchi-Lee potential (A.17) in the covariant Lorentz gauge. Recall that in terms of stereographic coordinates the line element (2.8) of de Sitter Universe satisfies the conformally flat space condition. Hobbs [40] proved that in spacetime with conformal metric the 'tail' term (1.3) vanishes (see also [41]). In appendix B we show that the integral (1.3) defining the nonlocal tail term of the radiation reaction force vanishes in de Sitter space.

In this specific case the scheme developed by Detweiler and Whiting is based on the symmetric Green's function [20, equation (6)] where tail function $v(x,z)$ vanishes:

$$G^{\rm sym}(x,z) = \frac{1}{8\pi}\delta\left(T^2 - \mathbf{R}^2\right). \tag{3.27}$$

We denote $T = x^0 - z^0$ and $R^i = x^i - z^i$. The inhomogeneous field is the one-half sum of the retarded and the advanced solutions, while the homogeneous one becomes the one-half difference of these fields.

3.2.1. Electromagnetic field in de Sitter space. The retarded electromagnetic field in de Sitter space $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ can be derived from the potential (3.26) by using the differentiation rule

$$\frac{\partial s^{\rm ret}(x)}{\partial x^\alpha} = -k_\alpha, \tag{3.28}$$

where $k_\alpha = K_\alpha / r$ is the normalized linking null vector $K_\alpha = x_\alpha - z_\alpha[s^{\rm ret}(x)]$. The adjective 'normalized' means the property $(k\cdot u) = -1$. It allows us to differentiate a function of x which contains an implicit reference to the retarded time s. In particular

$$\frac{\partial r(x)}{\partial x^\alpha} = -u_\alpha + [-(u\cdot u) + r(a\cdot k)]\, k_\alpha, \tag{3.29}$$

where $(u\cdot u) = \eta_{\mu\nu}u^\mu u^\nu$ and $(a\cdot k) = \eta_{\mu\nu}a^\mu k^\nu$. It is worth noting that for time-like geodesics $(u\cdot u) = -\Omega^{-2}(z)$ (see normalization relation (2.19)).

The rule can be derived as follows. Let us consider the interval (2.13) where x be a field point while x' is the source point $z(s)\in\zeta$ at which the world line $\zeta \subset {\rm dS}_4$ intersects the past light cone of x. The points are linked by null geodesic: $\rho(x,z(s)) = \Omega(x)\Omega(z(s))(x - z(s))^2 = 0$. We assume that neither field point nor source point lie on the surface where either $\Omega(x)$ or $\Omega(z(s))$ vanishes. Therefore, in de Sitter space the null geodesics lie on the surface of Minkowski light cone $(x - z(s))^2 = 0$.

Suppose that the field point x is shifted to the new point $x + \delta x$. This infinitesimal displacement yields corresponding change in the retarded time $s + \delta s$. New points, $x + \delta x$ and $z(s + \delta s)$, are related by null geodesic: $\rho(x + \delta x, z(s + \delta s)) = 0$. Expanding this in series of δx and δs, restricting ourselves to the first order in these small parameters, and using $(x - z(s))^2 = 0$ we obtain

$$K_\alpha \delta x^\alpha + r\delta s = 0, \tag{3.30}$$

where $K_\alpha = x_\alpha - z_\alpha(s)$ is the linking null vector and r is the retarded distance (3.25). This immediately gives the differentiation rule (3.28).

These allows us to derive the components of retarded electromagnetic field 2-form. After some algebra we obtain

$$F^{\rm ret}_{\alpha\beta}(x) = \frac{q}{r^2}\Omega^{-2}\left(u_\alpha k_\beta - u_\beta k_\alpha\right) + \frac{q}{r}\left[a_\alpha k_\beta - a_\beta k_\alpha + (a\cdot k)\left(u_\alpha k_\beta - u_\beta k_\alpha\right)\right]. \tag{3.31}$$

All the particle's kinematic characteristics are referred to the retarded time $s^{\rm ret}(x)$ associated with point x where the field strengths are measured. The centered dot designates the scalar product of two four-vectors produced by metric tensor of flat spacetime.

4. Radiation reaction force

For a point charge arbitrarily moving in flat spacetime the Harte's procedure [22] yields the flows of energy, momentum, and angular momentum through thin tube around particle's world line. At first the calculation of linear momentum flow has been performed by Teitelboim in his pioneering paper [42] on the derivation of the LAD equation (1.1) within the frame of retarded causality. The components of momentum four-vector carried by electromagnetic field of a point-like charge are

$$p^\alpha_{\rm em} = \int_{\sigma(t)} {\rm d}\sigma_\mu T^{\mu\alpha}, \tag{4.1}$$

where ${\rm d}\sigma_\mu$ is the vectorial surface element on a space-like surface $\sigma(t)$. In [42, figure 1] the surface $\sigma(t)$ is the section of future light cone and covariant hyperplane being orthogonal to particle's four-velocity. Poisson [43, figure 3] calculates flows through thin world tube $r =$ const enclosing particle's world line. In [44] the hyperplane $x^0 =$ const is used.

The stress-energy tensor

$$T^{\alpha\beta}(x) = \frac{1}{4\pi}\eta^{\alpha\mu}\eta^{\beta\nu}\left[F_{\mu\gamma}\eta^{\gamma\delta}F_{\nu\delta} - \frac{1}{4}\eta_{\mu\nu}F_{\sigma\gamma}\eta^{\gamma\delta}F_{\rho\delta}\eta^{\sigma\rho}\right], \tag{4.2}$$

is composed from the retarded Liénard-Wiechert field strengths (see equation (3.31) where $\Omega = 1$.) It admits a natural decomposition into singular (bound) and regular (emitted) components:

$$T^{\alpha\beta}(x) = \underbrace{T^{\alpha\beta}_{(-4)}(x) + T^{\alpha\beta}_{(-3)}(x)}_{\rm bound} + \underbrace{T^{\alpha\beta}_{(-2)}(x)}_{\rm radiative}. \tag{4.3}$$

Each term has been labeled according to its dependence on the distance r. So, the subscript (-2) means that the radiative term is inversely proportional to the squared distance.

Volume integration of the radiative part $\hat{T}_{\rm rad} = \hat{T}_{(-2)}$ of the stress-energy tensor (4.2) gives the Larmor relativistic rate of power of electromagnetic waves that detach themselves from the charge and lead an independent existence. Integration of the bound part $\hat{T}_{\rm bnd} = \hat{T}_{(-4)} + \hat{T}_{(-3)}$ yields singular terms inversely proportional to the distance. They constitute the bound part of electromagnetic radiation which is permanently attached to the charge and is carried along with it. It contributes to charge's inertia.

A point-like charge is the limiting case of an extended body. Teitelboim's decomposition is in line of the Harte's scheme. In order to calculate the electromagnetic field contribution in charge's inertia, the bound component $T^{\alpha\beta}_{\rm bnd}$ should be inserted in between the round brackets in [22, equation (35)]. The term $\hat{T}_{(-4)}$ is composed from the velocity field [42, equation (2.3a)] which is inversely proportional to the squared distance. While the term $\hat{T}_{(-3)}$ contains also the acceleration field [42, equation (2.3b)] scaled as $1/r$. The singular field F^S involved in [22, equation (35)] can not be extracted from the retarded Liénard-Wiechert field. According to [22], F^S originates from new symmetric Green's function [20, equation (16)!]. In case of flat spacetime it is one-half of the sum of the retarded and advanced field strengths. The stress-energy tensor composed from F^S is not equal to $\hat{T}_{\rm bnd}$. Regular field F^R becomes one-half difference of the retarded and advanced field strengths. The stress-energy tensor composed from F^R is not equal to $\hat{T}_{\rm rad}$. Their sum has no of true physical sense.

Similarly, the torque of the stress-energy tensor which defines the angular momentum tensor

$$M^{\alpha\beta}_{\rm em} = \int_{\sigma(t)} {\rm d}\sigma_\mu \left(x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha}\right), \tag{4.4}$$

can be decomposed. The bound and radiative parts of integrand in equation (4.4) are as follows [44]:

$$M_{\text{rad}}^{\mu\alpha\nu} = z^\mu(s)T_{\text{rad}}^{\alpha\nu} - z^\nu(s)T_{\text{rad}}^{\alpha\mu} \tag{4.5}$$

$$+ (x^\mu - z^\mu(s))\,T_{(-3)}^{\alpha\nu} - (x^\mu - z^\nu(s))\,T_{(-3)}^{\alpha\mu},$$

$$M_{\text{bnd}}^{\mu\alpha\nu} = z^\mu(s)T_{\text{bnd}}^{\alpha\nu} - z^\nu(s)T_{\text{bnd}}^{\alpha\mu} \tag{4.6}$$

$$+ (x^\mu - z^\mu(s))\,T_{(-4)}^{\alpha\nu} - (x^\mu - z^\nu(s))\,T_{(-4)}^{\alpha\mu},$$

where s is the retarded instant and $\hat{T}_{(-3)}$, $\hat{T}_{(-4)}$ denote the parts of $\hat{T}_{\text{bnd}}$ which are scaled as r^{-3} and r^{-4}, respectively.

In [44] the LAD equation (1.1) is derived via analysis of ten conserved quantities corresponding to Poincaré invariance of a closed system consisting of a point-like charge and its electromagnetic field:

$$P^\alpha = p_{\text{part}}^\alpha + p_{\text{em}}^\alpha, \quad M^{\alpha\beta} = M_{\text{part}}^{\alpha\beta} + M_{\text{em}}^{\alpha\beta}. \tag{4.7}$$

It is natural to assume that the divergences are absorbed by particle's individual characteristics. They are proclaimed to be finite and measurable. The radiative terms which leave the charge and lead to an independent existence are valuable for conservation laws:

$$P^\alpha = p_{\text{part}}^\alpha + p_{\text{rad}}^\alpha, \quad M^{\alpha\beta} = M_{\text{part}}^{\alpha\beta} + M_{\text{rad}}^{\alpha\beta}, \tag{4.8}$$

(cf equation (4.7)). The system is then placed in an external field. Changes in particle's mechanical 4-momentum p_{part}^α and momentum p_{rad}^α carried by particle's electromagnetic field should be balanced by external force: $\dot{p}_{\text{part}}^\alpha + \dot{p}_{\text{rad}}^\alpha = f_{\text{ext}}^\alpha$. Similarly, changes in angular momenta should be balanced by torque of the external force: $\dot{M}_{\text{part}}^{\alpha\beta} + \dot{M}_{\text{rad}}^{\alpha\beta} = z^\alpha f_{\text{ext}}^\beta - z^\beta f_{\text{ext}}^\alpha$. Careful analysis of ten algebraic equations yields the expression for particle's mechanical four-momentum

$$p_{\text{part}}^\mu = mu^\mu - \frac{2q^2}{3}a^\mu, \tag{4.9}$$

and its time derivative

$$\frac{\mathrm{d}p_{\text{part}}^\mu}{\mathrm{d}s} + \frac{2q^2}{3}(a\cdot a)\,u^\mu = f_{\text{ext}}^\mu. \tag{4.10}$$

The second term in equation (4.9) describes the electromagnetic field's contribution to particle's inertia. Substituting the left hand side of equation (4.9) for p_{part}^μ we obtain the LAD equation (1.1).

4.1. Radiation reaction force in de Sitter space

The dynamics of a free massive charge in the de Sitter space is governing by the Lagrangian

$$L(z,\dot{z}) = -m\sqrt{-g_{\alpha\beta}(z)\dot{z}^\alpha\dot{z}^\beta}, \tag{4.11}$$

where $g_{\alpha\beta}(z) = \Omega^2(z)\eta_{\alpha\beta}$. Here overdot denotes differentiation with respect to any time parameter τ. Variation of action based on this function yields the equation of motion

$$\frac{\mathrm{d}p_\alpha}{\mathrm{d}\tau} + \frac{1}{2}H^2\Omega(z)\left(p\cdot\frac{\mathrm{d}z}{\mathrm{d}\tau}\right)z_\alpha = 0, \tag{4.12}$$

where p_α is particle's four-momentum

$$p_\alpha(\tau) = m\Omega\frac{\dot{z}_\alpha}{\sqrt{-(\dot{z}\cdot\dot{z})}}, \tag{4.13}$$

and $(\dot{z}\cdot\dot{z}) \equiv \eta_{\alpha\beta}\dot{z}^\alpha\dot{z}^\beta$. The equation (4.12) can be rewritten as manifestly covariant one: $\dot{z}^\mu\nabla_\mu(p_\alpha) = 0$. In coordinate treatment we have

$$\frac{\mathrm{d}p_\alpha}{\mathrm{d}\tau} - \Gamma^\gamma{}_{\alpha\beta}p_\gamma\frac{\mathrm{d}z^\beta}{\mathrm{d}\tau} = 0, \tag{4.14}$$

where $\Gamma^\gamma{}_{\alpha\beta}$ are Christoffel symbols of the 2nd kind (2.14) (see also equation (9) in [34]). The equation, completely rewritten using proper time s satisfying the condition $g_{\alpha\beta}(z)\dot{z}^\alpha\dot{z}^\beta = -1$, takes the form

$$m\Omega^2(z)\left[\frac{\mathrm{d}^2z_\alpha}{\mathrm{d}s^2} + \frac{1}{2}H^2\Omega(z)\left[-2(z\cdot\dot{z})\dot{z}_\alpha + (\dot{z}\cdot\dot{z})z_\alpha\right]\right] = 0. \tag{4.15}$$

Overdot designates differentiation with respect to proper time s. Raising of indices by means of the reciprocal metric tensor $g^{\mu\alpha}(z) = \Omega^{-2}(z)\eta^{\mu\alpha}$ we obtain the geodesic equation (2.20). The equation is solved in section 2.1 where the time-like, space-like, and null geodesics are found.

The equation of motion of test charge acted upon an external electromagnetic field $\hat{F}^{\rm ext}$ is

$$\frac{\mathrm{d}p_\alpha}{\mathrm{d}\tau} - \Gamma^\gamma{}_{\alpha\beta}p_\gamma\frac{\mathrm{d}z^\beta}{\mathrm{d}\tau} = qF^{\rm ext}_{\alpha\beta}\frac{\mathrm{d}z^\beta}{\mathrm{d}\tau}. \tag{4.16}$$

The field enforces it to deviate from geodesic motion (cf equation (4.14)). In terms of kinematic quantities $u(s) = \mathrm{d}z(s)/\mathrm{d}s$ and $a(s) = \mathrm{d}u(s)/\mathrm{d}s$ the equation of motion of test charge in de Sitter space takes the form

$$m\left\{a^\mu + \frac{1}{2}H^2\Omega(z)\left[-2(z\cdot u)u^\mu + (u\cdot u)z^\mu\right]\right\} = f^\mu_{\rm ext}, \tag{4.17}$$

where the external force $f^\mu_{\rm ext} = g^{\mu\alpha}(qF^{\rm ext}_{\alpha\beta}u^\beta)$. All tensors are evaluated at $z(s)$, the current position of the particle on the world line. The equation governs the evolution of a test charged particle subject only to background gravity and external electromagnetism. Our next task is to calculate the self-force due to particle's own field.

We base our consideration on the balance of conserved quantities which, according to Nöther theorem, correspond to the symmetry of the problem. In [33–35, 45] the idea of de Sitter invariant special relativity has been considered in very details. The group of isometry $SO(4,1)$ of de Sitter space involves the Lorentz group SO(3,1) as the subgroup. The space rotations describes isotropy of space while the boosts indicates the equivalence of inertial frames of reference. The set of generators of corresponding infinitesimal transformations [33, equation (14)]

$$L_{\alpha\beta} = x_\alpha\frac{\partial}{\partial x^\beta} - x_\beta\frac{\partial}{\partial x^\alpha}. \tag{4.18}$$

consists of the usual space rotations $J_k = \varepsilon_k{}^{ij}L_{ij}$ and boosts $K_i = L_{i0}$.

The combination of usual space translations and proper conformal transformations constitute four non-commutative generators which are responsible for homogeneity. The generators [33, equation (15)]

$$\Pi_\alpha = \left[1 - \frac{H^2}{4}(x\cdot x)\right]\frac{\partial}{\partial x^\alpha} + \frac{H^2}{2}x_\alpha x^\beta\frac{\partial}{\partial x^\beta}, \tag{4.19}$$

reduce to usual space-time translations in vanishing cosmological limit $H\to 0$.

According to the Nöther theorem, the group of isometry $SO(4,1)$of de Sitter spacetime produces ten conserved quantities:

- de Sitter momentum [45, equations (27) and (28)]

$$\pi^\alpha_{\rm part} = \Omega^2(z)\left[\left(2-\Omega^{-1}(z)\right)\delta^\alpha{}_\beta + \frac{1}{2}H^2 z^\alpha z_\beta\right] p^\beta_{\rm part}; \tag{4.20}$$

- angular momentum [45, equation (29)]

$$\lambda^{\alpha\beta}_{\rm part} = \Omega^2(z)\left(z^\alpha p^\beta_{\rm part} - z^\beta p^\alpha_{\rm part}\right). \tag{4.21}$$

The tangent momentum four-vector p^β is related to its cotangent counterpart (4.13) by means of the operation of rising indices. In the proper time parametrization $p_\alpha = m\Omega^2(z)\dot z_\alpha$ and, therefore, $p^\beta = g^{\beta\alpha}(z)p_\alpha = m\dot z^\beta$.

When considering an isolated system of a point-like charge and its electromagnetic field, the conserved momentum is

$$\Pi^\alpha = \pi^\alpha_{\rm part} + \pi^\alpha_{\rm em}, \tag{4.22}$$

where the second term is the de Sitter momentum carried by the particle's electromagnetic field (3.31):

$$\pi^\alpha_{\rm em} = \int_{-\infty}^{s} {\rm d}s' \int_0^\pi {\rm d}\vartheta \sin\vartheta \int_0^{2\pi} {\rm d}\varphi\, \Omega^4(x)\Omega^2(z) r^2 \tag{4.23}$$
$$\times\, \Omega^2(x)\left[\left(2-\Omega^{-1}(x)\right) T^{\alpha\beta}_{\rm dS} r_\beta + \frac{1}{2}H^2 x^\alpha\left(x_\gamma T^{\gamma\beta}_{\rm dS} r_\beta\right)\right].$$

By r_β we denote the coordinate partial derivative (3.29) of the retarded distance (3.25). The outward-directed surface element (C.7) is derived in appendix C. Similarly, the total conserved angular momentum is

$$\Lambda^{\alpha\beta} = \lambda^{\alpha\beta}_{\rm part} + \lambda^{\alpha\beta}_{\rm em}, \tag{4.24}$$

with electromagnetic field's contribution

$$\lambda^{\alpha\beta}_{\rm em} = \int_{-\infty}^{s} {\rm d}s' \int_0^\pi {\rm d}\vartheta \sin\vartheta \int_0^{2\pi} {\rm d}\varphi\, \Omega^4(x)\Omega^2(z) r^2 \Omega^2(x)\left(x^\alpha T^{\beta\gamma}_{\rm dS} r_\gamma - x^\beta T^{\alpha\gamma}_{\rm dS} r_\gamma\right). \tag{4.25}$$

The stress-energy tensor of de Sitter space is

$$T^{\alpha\beta}_{\rm dS}(x) = \frac{1}{4\pi} g^{\alpha\mu} g^{\beta\nu}\left[F_{\mu\gamma} g^{\gamma\delta} F_{\nu\delta} - \frac{1}{4} g_{\mu\nu} F_{\sigma\gamma} g^{\gamma\delta} F_{\rho\delta} g^{\sigma\rho}\right]$$
$$= \frac{1}{4\pi}\Omega^{-6}(x)\, T^{\alpha\beta}(x), \tag{4.26}$$

where $T^{\alpha\beta}(x)$ is the stress-energy tensor (4.2) of Minkowski space. The retarded electromagnetic field $\hat F$ is given by equation (3.31).

In this section we derive the radiation reaction force from ten balance equations (4.22) and (4.24). Changes in total de Sitter momentum Π^α and angular momentum $\Lambda^{\alpha\beta}$ should be balanced by the external Lorentz force and its torque:

$$\dot{\pi}^{\alpha}_{\rm part} + \dot{\pi}^{\alpha}_{\rm em} = \Omega^2(z)\left[\left(2-\Omega^{-1}(z)\right)f^{\alpha}_{\rm ext} \quad + \frac{1}{2}H^2(f_{\rm ext}\cdot z)z^{\alpha}\right]; \tag{4.27}$$

$$\dot{\lambda}^{\alpha\beta}_{\rm part} + \dot{\lambda}^{\alpha\beta}_{\rm em} = \Omega^2(z)\left(z^{\alpha}f^{\beta}_{\rm ext} - z^{\beta}f^{\alpha}_{\rm ext}\right). \tag{4.28}$$

As the factor $\Omega^{-6}(x)$ in the electromagnetic stress-energy tensor (4.26) cancels the factor $\Omega^6(x)$ in field's contributions to conserved quantities (4.23) and (4.25), the time derivatives of this quantities take the form

$$\dot{\pi}^{\alpha}_{\rm em} = \frac{1}{4\pi}\int_0^{\pi} {\rm d}\vartheta\sin\vartheta\int_0^{2\pi}{\rm d}\varphi\,\Omega^2(z)r^2\left[\left(2-\Omega^{-1}(x)\right)T^{\alpha\beta}r_{\beta} + \frac{1}{2}H^2x^{\alpha}\left(x_{\gamma}T^{\gamma\beta}r_{\beta}\right)\right], \tag{4.29}$$

$$\dot{\lambda}^{\alpha\beta}_{\rm em} = \frac{1}{4\pi}\int_0^{\pi} {\rm d}\vartheta\sin\vartheta\int_0^{2\pi}{\rm d}\varphi\,\Omega^2(z)r^2\left(x^{\alpha}T^{\beta\gamma}r_{\gamma} - x^{\beta}T^{\alpha\gamma}r_{\gamma}\right). \tag{4.30}$$

Angular integration can be handled via relations (D.8) presented in appendix D. Resulting expressions depend on kinematic quantities referred to the particle's current position $z(s)$ on the world line. They consist of (short-range) divergent terms and distance-free radiative terms which leave the point source and lead to independent existence. In the pioneering spirit of Detweiler-Whiting [20] and Harte [22], it is natural to assume that the divergences are absorbed by particle's individual characteristics. Then they are proclaimed to be finite and measurable. The radiative terms which leave the charge and lead to an independent existence are valuable for balances of conservation laws only. Having performed the solid angle integration, we obtain

$$\begin{aligned}\dot{\pi}^{\alpha}_{\rm rad} = q^2\Bigg\{&\Omega^2\left[\left(2-\Omega^{-1}\right)\delta^{\alpha}{}_{\beta} + \frac{1}{2}H^2z^{\alpha}z_{\beta}\right]\frac{2}{3}\Omega^2\left[(a\cdot a)+\frac{1}{4}H^4(z\cdot u)^2\right]u^{\beta}\\ &-\frac{1}{4}H^2u^{\alpha}+\frac{1}{3}H^2\Omega^2\left[(z\cdot a)u^{\alpha}-(z\cdot u)a^{\alpha}\right]\Bigg\};\end{aligned} \tag{4.31}$$

$$\dot{\lambda}^{\alpha\beta}_{\rm rad} = \frac{2}{3}q^2\left\{\Omega^4\left[(a\cdot a)+\frac{1}{4}H^4(z\cdot u)^2\right]\left(z^{\alpha}u^{\beta}-z^{\beta}u^{\alpha}\right)+\Omega^2\left(u^{\alpha}a^{\beta}-u^{\beta}a^{\alpha}\right)\right\}. \tag{4.32}$$

We substitute they for electromagnetic field contributions $\dot{\pi}^{\alpha}_{\rm em}$ and $\dot{\lambda}^{\alpha\beta}_{\rm em}$ in the balance equations (4.27) and (4.28), respectively. We obtain the system of ten linear algebraic equations on eight components $\pi^{\alpha}_{\rm part}$ and $\dot{\pi}^{\alpha}_{\rm part}$. After some algebra we derive the following equation on $\pi^{\alpha}_{\rm part}$:

$$\begin{aligned}&\frac{1}{2}H^2\omega^2(z)(z\cdot u)\left(z^{\alpha}\pi^{\beta}_{\rm part}-z^{\beta}\pi^{\alpha}_{\rm part}\right)+\omega(z)\left(u^{\alpha}\pi^{\beta}_{\rm part}-u^{\beta}\pi^{\alpha}_{\rm part}\right)\\ &\quad-\omega(z)\left(z^{\alpha}\dot{\pi}^{\beta}_{\rm rad}-z^{\beta}\dot{\pi}^{\alpha}_{\rm rad}\right)+\dot{\lambda}^{\alpha\beta}_{\rm rad}=0,\end{aligned} \tag{4.33}$$

where $\omega(z) = (2-\Omega^{-1})^{-1}$. The solution is

$$\pi^{\alpha}_{\rm part} = \Omega^2(z)\left[\left(2-\Omega^{-1}\right)\delta^{\alpha}{}_{\beta}+\frac{1}{2}H^2z^{\alpha}z_{\beta}\right]\left(M(s)u^{\beta}-\frac{2q^2}{3}a^{\beta}+\frac{q^2}{4}H^2\Omega^{-1}(z)z^{\beta}\right), \tag{4.34}$$

where $M(s)$ is a scalar function which can be interpreted as inertial mass. It will be defined below. Assuming this we take into account that in curved spacetime 'm is not usually conserved, but rather varies according to' [22, equation (59)].

Having compared expressions (4.20) and (4.34), we establish the particle's mechanical momentum:

$$p^{\beta}_{\rm part} = M(s)u^{\beta}-\frac{2q^2}{3}a^{\beta}+\frac{q^2}{4}H^2\Omega^{-1}(z)z^{\beta}. \tag{4.35}$$

This is the generalization of mechanical momentum (4.9) arising in flat spacetime. The momentum of 'dressed' charge consists of the Schott term and the contribution due to curvilinear background. The terms describe the reversible form of emission and absorbtion of field energy-momentum, which never gets far from a charge.

Inserting equations (4.31) and (4.34) into the momentum balance equation (4.27) we obtain the auxiliary equation which involves the self-action terms:

$$M\left\{a^\beta - \frac{H^2}{2}\Omega\left[\,2(z\cdot u)u^\beta + \Omega^{-2}z^\beta\right]\right\} + \dot{M}u^\beta - \frac{2q^2}{3}\dot{a}^\beta + \frac{2q^2}{3}H^2\Omega(z)(z\cdot u)a^\beta$$
$$+\frac{2q^2}{3}\Omega^2\left[(a\cdot a) + \frac{1}{4}H^4(z\cdot u)^2\right]u^\beta + \frac{q^2}{6}H^4(z\cdot u)z^\beta = f^\beta_{\rm ext}. \tag{4.36}$$

Thus can be treated as generalization of the geodesic equation (2.20). Deviation of electric charge's motion from geodesic is caused by action of both the external electric field and its own field.

Let us derive the differential equation on scalar function $M(s)$. We overmultiply equation (4.36) on the cotangent four-velocity vector u_β. Since the external force $f^{\rm ext}_\beta = qF^{\rm ext}_{\beta\mu}u^\mu$, the contraction $f^{\rm ext}_\beta u^\beta$ vanishes. Taking into account the normalization condition $(u\cdot u) = -\Omega^{-2}$ and its differential consequences

$$(u\cdot a) = -\frac{H^2}{2}\Omega^{-1}(z\cdot u),$$
$$(u\cdot\dot{a}) = -(a\cdot a) - \frac{H^2}{2}\Omega^{-1}\left[-\Omega^{-2} + (z\cdot a)\right] - \frac{H^4}{4}(z\cdot u)^2, \tag{4.37}$$

we obtain the following differential equation

$$\frac{{\rm d}M}{{\rm d}s} = -\frac{q^2}{6}H^4\Omega^2(z\cdot u)^2 + \frac{q^2}{3}H^2\Omega\left[(u\cdot u) + (z\cdot a)\right].$$

The solution is

$$M(s) = m + \frac{q^2}{3}H^2\Omega(z)(z\cdot u), \tag{4.38}$$

where m is constant of integration which can be interpreted as (already renormalized) rest mass of a charge. Therefore, the curvilinear background contributes also in the inertial mass of charged particle.

Putting this function and its time derivative into auxiliary equation (4.36) we obtain

$$m\left\{a^\beta - \frac{H^2}{2}\Omega(z)\left[\,2(z\cdot u)u^\beta + \Omega^{-2}(z)z^\beta\right]\right\} - \frac{2q^2}{3}\dot{a}^\beta + q^2H^2\Omega(z)(z\cdot u)a^\beta$$
$$+\frac{2q^2}{3}\Omega^2\left\{(a\cdot a) + \frac{1}{2}H^2\Omega^{-1}\left[-\Omega^{-2} + (z\cdot a)\right] - \frac{1}{2}H^4(z\cdot u)^2\right\}u^\beta = f^\beta_{\rm ext}. \tag{4.39}$$

after canceling of like terms. This is the Dewitt-Brehme-Hobbs equation (1.2) adapted for de Sitter metric (2.8), Christoffel symbols (2.14), and Ricci tensor (2.16). Keeping in mind that Greek indices are raised and lowered with the Minkowski metric tensor, we write the projection operator in equation (1.2) as $\delta^\mu{}_\nu + \Omega^2(z)u^\mu u_\nu$. We take into account relations (4.37). The tail (1.3) vanishes (see appendix B).

5. Conclusions

We derive the electromagnetic self force in de Sitter space via analysis of Nöther conserved quantities corresponding to the group of isometry $SO(4,1)$. The derivation involves the volume integration of electromagnetic field's stress–energy tensor which has a singularity on a particle trajectory. The consideration is based on the splitting of electromagnetic field's stress-energy tensor and its torque into bound and radiative parts which contribute to the self force differently [42]. Volume integration of bound parts yields divergent terms which are permanently 'attached' to the charge and are carried along with it. They contribute in particle's mechanical momentum (4.35). In addition to the usual Schott term, the mixed term arises because of interaction of electromagnetic field and gravity. Volume integration of radiative parts produces long range electromagnetic radiation which dissipates in space. The amount of radiated energy, momentum, and angular momentum depends on all previous evolution of a source. The changes in energy-momentum and angular momentum carried by the electromagnetic field should be balanced by corresponding changes of particle's 4-momentum and angular momentum, respectively. Radiation reaction force is the consequence of these balance equations.

The intriguing phenomenon is discovered: a moving electric charge in expanding Universe changes its rest mass because of interaction of particle's electromagnetic field and gravitation (see equation (4.38)). This was previously established for a scalar charge [46, 47]. A similar effect occurs in flat $(2+1)$ Minkowski spacetime [48, equation (3.27)]. The mass change does not lead to dismission of energy conservation.

The Green's function of a point-like charge in de Sitter space derived by Higuchi and Lee [24] consists of terms being proportional to Dirac δ-function and Heaviside θ-function of Singe world function. The retarded electromagnetic field should spread not only along a light-cone surface, but also within its interior. Both the self-action and interaction between charged particles should be nonlocal in time, i.e. should depend on a whole particle history [19]. Nevertheless, we demonstrate that in de Sitter space a time-nonlocal term of the electromagnetic potential of point charge originated from Higuchi and Lee Green's function can be turned into the local expression by gauging out its nonlocal part. Moreover, this expression does not contribute to the self force. The self-action comes entirely from the basic local term of the electromagnetic potential and has form (3.26). It is a minimal generalization of the flat-space Lorentz-Abraham-Dirac expression to the de Sitter geometry.

As for the possible experiment with electron in multi-petawatt laser system, the order of gravity corrections is tiny: $\tau_0 H^2 \approx 10^{-67} m^{-1}$. They are important for the fifth iteration of reduced LAD equation (1.1).

In the recent paper [6] the LL equation for a charged particle acted upon an external scalar field is investigated. The problem is reformulated in terms of curved spacetime with constant curvature. Coupling the scalar field potential $\Phi(x)$ with the particle's rest mass m, the authors form the position dependent mass $\mathcal{M} = m + \Phi(x)$ which determines the conformal factor in the conformally flat space metric tensor. The dynamics of a point-like charge in the scalar field $\Phi(x)$ is reduced to the geodesic motion of a free particle in curved spacetime.

The special attention is paid [6] to the Nöther quantities arising from the symmetry of the problem with respect to the transformations from Poincaré group. The authors restrict themselves to analysis of evolution of the energy and momentum which correspond to the time and space translations, respectively. The other isometries, i.e. space rotations and boosts, are not considered. Subtle balance between bound Schott term and long-range Larmor contribution to the total self-force expression has been studied in very details. The authors conclude that the scalar self-force 'is not frictional, even in the ultra-relativistic limit, and can even be anti-damping. This leads to behavior that would not be expected from radiation reaction alone, such as a particle in a time-independent field gaining energy.'

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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Appendix A. Higuchi-Lee electromagnetic potential

In [24] Higuchi and Lee found the electromagnetic vector potential in de Sitter space dS_4 via constructing the bivector Green's function of the wave equation (3.16). The retarded Green's function [24, equations (4.13)–(4.15)] is the combination of terms proportional to Dirac delta-function and Heaviside step function:

$$G^{\rm ret}_{\mu\nu'}(x,x') = G^{\delta}_{\mu\nu'}(x,x')\delta(1-Z) + G^{\theta}_{\mu\nu'}(x,x')\theta(Z-1). \tag{A.1}$$

The arguments of generalized functions involve the scalar function [24, equation (4.4)] of base point x' and field point x

$$Z(x,x') = \frac{1}{2}\left[1+\cos[H\mu(x,x')]\right]. \tag{A.2}$$

Here $\mu(x,x')$ is the distance between x' and x along the geodesic connecting these points (see [49, equation (1.3)]). We express it in terms of Synge's world function [19, equation (3.1)]. This two-point scalar function represents one-half of the squared geodesic distance between its arguments: $\mu(x,x') = \sqrt{2\sigma(x,x')}$. If the spacelike geodesic is parameterized by the proper time parameter s, the distance between points $x(s)$ and $x(s')$ becomes $\mu(x,x') = s-s'$.

In case of timelike arrangement the value of $\mu(x,x')$ becomes purely imaginary, and we have

$$Z(x,x') = \frac{1}{2}\left[1+\cosh(H\sqrt{-2\sigma(x,x')})\right]. \tag{A.3}$$

In section 2.1 we study the geometry of de Sitter spacetime. We solve geodesic equation (2.20) and obtain explicit expressions for timelike and spacelike geodesics. De Sitter spacetime interval (2.13) is also calculated. Comparing equations (2.37) and (2.42) with expressions (A.2) and (A.3), we rewrite the two-point function $Z(x,x')$ in terms of interval (2.13):

$$Z(x,x') = 1 - \frac{H^2}{4}\Omega(x)\Omega(x')\eta_{\mu\nu}(x^\mu - x'^\mu)(x^\nu - x'^\nu). \tag{A.4}$$

The function is invariant under the action of de Sitter isometry group $SO(4.1)$. Expressions (A.2)–(A.4) demonstrate that $Z(x,x')=1$ if and only if $\sigma(x,x')=0$ or, equivalently, $x=x'$.

The coefficient functions in the Green's function (A.1) are as follows

$$G^{\delta}_{\mu\nu'}(x,x') = \frac{H^2}{8\pi}\bar{g}_{\mu\nu'}(x,x'), \tag{A.5}$$

$$G^{\theta}_{\mu\nu'}(x,x') = -\frac{H^2}{12\pi}\left[\left(\frac{1}{Z}+\frac{1}{2Z^2}\right)\bar{g}_{\mu\nu'}(x,x') - \frac{1}{H^2}\frac{1}{Z^3}\partial_\mu Z\partial_{\nu'}Z\right]. \tag{A.6}$$

They involve the parallel propagator

$$\bar{g}_{\mu\nu'}(x,x') = \frac{2}{H^2}\left(\partial_\mu\partial_{\nu'}Z - \frac{1}{Z}\partial_\mu Z\partial_{\nu'}Z\right). \tag{A.7}$$

This is modified expression [24, equation (4.6)] where $2Z-1$ is substituted for $\cos[H\mu(x,x')]$. Recall that these functions are related by equation (A.2).

Inserting the parallel propagator (A.7) in equation (A.6), we present the coefficient function of the θ-term as the following second-order derivative:

$$\begin{aligned} G^{\theta}_{\mu\nu'}(x,x') &= -\frac{H^2}{12\pi}\frac{1}{H^2}\left[\left(\frac{2}{Z}+\frac{1}{Z^2}\right)\partial_\mu\partial_{\nu'}Z - \left(\frac{2}{Z^2}+\frac{2}{Z^3}\right)\partial_\mu Z\partial_{\nu'}Z\right] \\ &= \partial_\mu\partial_{\nu'}\frac{1}{6\pi}\left(\frac{1}{2Z}-\ln Z\right) \\ &\equiv \partial_\mu\partial_{\nu'}\Phi(x,x'). \end{aligned} \tag{A.8}$$

A.1. Potential of a point-like charge in the covariant Lorentz gauge

Inserting the Higuchi-Lee Green's function (A.1) in the electromagnetic potential (3.7) generated by the current (3.6) of a point-like charge we obtain

$$A_\mu = 4\pi q\int \mathrm{d}\tau\, G^{\rm ret}_{\mu\nu'}(x,z(\tau))\dot{z}^{\nu'}(\tau) \equiv A^\delta_\mu + A^\theta_\mu, \tag{A.9}$$

where $\dot{z} \equiv \mathrm{d}z(\tau)/\mathrm{d}\tau$, and

$$A^\delta_\mu = q\frac{H^2}{2}\int \mathrm{d}\tau\,\theta(x,z(\tau))\bar{g}_{\mu\nu'}(x,z(\tau))\dot{z}^{\nu'}\delta(Z(x,z(\tau))-1), \tag{A.10}$$

$$\begin{aligned} A^\theta_\mu &= 4\pi q\int \mathrm{d}\tau\,\theta(x,z(\tau))\theta(Z-1)\partial_\mu\dot{\Phi}(x,z(\tau)) \\ &= -4\pi q\int \mathrm{d}\tau\,\theta(x,z(\tau))\delta(Z-1)\dot{Z}\partial_\mu\Phi(x,z(\tau)) + \partial_\mu\Lambda^{(1)}, \end{aligned} \tag{A.11}$$

$$\Lambda^{(1)} = 4\pi q\int \mathrm{d}\tau\,\theta(x,z(\tau))\theta(Z-1)\dot{\Phi}(x,z(\tau)). \tag{A.12}$$

Unit step function $\theta(x,z(\tau))$ 'cuts' the retarded domain where zeroth coordinate x^0 of the field point is larger that this $z^0(\tau)$ of the source point. The integration is performed over $\tau \in]-\infty,\infty[$ by default, and the evolution parameter τ in this section is meant arbitrary since all the expressions (A.9)–(A.12) are reparametrization-invariant. After rearranging terms in (A.9) via extracting the full derivative we obtain:

$$A_\mu = \tilde{A}_\mu + \partial_\mu\Lambda^{(1)} \tag{A.13}$$

where

$$\begin{aligned} \tilde{A}_\mu &= q\int \mathrm{d}\tau\,\theta(x,z)\left\{\partial_\mu\dot{Z} - \left[\frac{1}{Z}-\frac{1}{Z^2}\right]\frac{\dot{Z}\partial_\mu Z}{3}\right\}\delta(Z-1) \\ &= q\int \mathrm{d}\tau\,\theta(x,z(\tau))\delta(Z-1)\partial_\mu\dot{Z}. \end{aligned} \tag{A.14}$$

The second term in curly brackets vanishes at point $Z=1$ specified by $\delta(Z-1)$. Thus the potential (A.9) is local, up to the gauge term $\partial_\mu\Lambda^{(1)}$. It is of worth noting that $\tilde{A}_\mu$ has no a subluminal tail in despite it possesses a contribution from A^θ_μ.

A.2. Gauge equivalence of vector potentials

According to equation (A.4), the interval (2.13) is the argument of δ-function in the expression (A.14) for the potential $\tilde{A}_\mu$. We are interesting in the retarded root $s^{\rm ret}(x)$ of vanishing interval equation $\rho(x,z(s))=0$. Having performed the integration in (A.14) over s where $Z[x,z(s)]$ is given by equation (A.4) we obtain

$$\begin{aligned}\tilde{A}_\mu(x) &= q\left[\frac{u_\mu(s^{\rm ret})}{r} - \frac{\partial}{\partial x^\mu}\ln\left(1+\frac{H^2}{4}(z^{\rm ret}\cdot z^{\rm ret})\right) + \frac{\partial}{\partial x^\mu}\ln\left(1+\frac{H^2}{4}(x\cdot x)\right)\right]\\ &= q\frac{u_\mu(s^{\rm ret})}{r} + \partial_\mu\Lambda^{(2)}(x),\end{aligned} \tag{A.15}$$

where $z^{\rm ret}\equiv z[s^{\rm ret}(x)]$ and

$$\Lambda^{(2)}(x) = q\ln\frac{\Omega(z^{\rm ret})}{\Omega(x)}. \tag{A.16}$$

Thus, the potential $\tilde{A}_\mu$ is gauge-equivalent to the potential (3.26). As follows from (A.13), the same is true for Higuchi-Lee potential (A.9):

$$A_\mu = q\frac{u_\mu(s^{\rm ret})}{r} + \partial_\mu(\Lambda^{(1)}+\Lambda^{(2)}). \tag{A.17}$$

Appendix B. Estimation of the radiation reaction tail

Let us consider the integral (1.3) defining the nonlocal tail term of the radiation reaction force. The retarded Green function (A.1) depends on two points, $z(s)$ and $z(s')$, on the same timelike world line. The Synge's function $\sigma(z(s),z(s'))=0$ if and only if $z(s)=z(s')$. Therefore, the only singularity arising upon integration is at $s=s'$ when $\sigma=0$ and $Z=1$. According to the regularization prescription of [19, pg. 19], the upper limit of the integral (1.3) 'is cut short at $s'=s^-:=s-0^+$ to avoid the singular behavior of the retarded Green's function at coincidence'. For this reason the δ-term of the Green function (A.1) does not contribute to the integral (1.3).

Thus the tail term (1.3) takes the form:

$$f^{\rm tail}_\mu = 4\pi q^2 u^\nu(s)\int_{-\infty}^{s^-}{\rm d}s'\{\partial_{[\mu}G^\theta_{\nu]\lambda'}\theta(Z-1)\}u^{\lambda'}(s'). \tag{B.1}$$

The square brackets mean antisymmeterization in indices μ and ν. The coefficient function $G^\theta_{\mu\nu'}(x,x')$ is the mixed partial derivative (A.8). The expression in curly braces becomes

$$\begin{aligned}\partial_{[\mu}G^\theta_{\nu]\lambda'}\theta(Z-1) &= \partial_{[\mu}(\partial_{\nu]}\partial_{\lambda'}\Phi)\theta(Z-1)\\ &= (\partial_{[\mu}\partial_{\nu]}\partial_{\lambda'}\Phi)\theta(Z-1) + (\partial_{[\mu}Z)(\partial_{\nu]}\partial'_\lambda\Phi)\delta(Z-1).\end{aligned}$$

Since $\partial_{[\mu}\partial_{\nu]}\equiv 0$, the 2-nd δ-term survives only. It does not contribute to the integral (B.1) because of the regularization prescription. Thus $f^{\rm tail}_\mu=0$.

Appendix C. Retarded coordinates in dS_4

The retarded field (3.31) depends on the state of the charge's motion at the instant at which its world line intersects the past light cone of x. Thus, the past light cone defines a natural mapping between the field-point x and a specific point $z(s)$ on the world line. The point $x^\alpha(s,r,\vartheta,\varphi)$ in de Sitter space is linked to the point $z^\alpha(s)$ on the charge's world line by a null geodesic of affine-parameter length r:

$$x^\alpha(s,r,\vartheta,\varphi) = z^\alpha(s) + rk^\alpha(s,\vartheta,\varphi). \tag{C.1}$$

Angular variables define the direction of null vector k^α while the retarded instant s designates its tail:

$$k^\alpha(s,\vartheta,\varphi) = \Omega[z(s)]\Lambda^\alpha{}_{\alpha'}(s) n^{\alpha'}(\vartheta,\varphi). \tag{C.2}$$

The Lorentz matrix

$$\Lambda^\alpha{}_{\alpha'}(s) = \left[\begin{array}{c|c} \Omega u^0 & \Omega u_{k'} \\ \hline \Omega u^i & \delta^i{}_{k'} + \Omega^2 \dfrac{u^i u_{k'}}{\Omega u^0 + 1} \end{array}\right] \tag{C.3}$$

defines the transformation to momentarily comoving frame where particle's four-velocity becomes $u^{\alpha'}(s) = (\Omega^{-1}[z(s)], 0, 0, 0)$. We build the orthogonal to four-velocity hyperplane spanned by three spatial axes. In this non-inertial frame the null vector has the following components

$$n^{\alpha'}(\vartheta,\varphi) = (1, \cos\varphi\sin\vartheta, \sin\varphi\sin\vartheta, \cos\vartheta), \tag{C.4}$$

where azimuthal angle φ and polar angle ϑ will refer to this choice of axes. Expression (C.1) defines the transformation from Cartesian coordinates $x^\alpha = (x^0, x^1, x^2, x^3)$ to the curvilinear coordinates $r^\beta = (s, r, \vartheta, \varphi)$.

The volume element in de Sitter space is $\mathrm{d}V = \mathrm{d}^4x\,\Omega^4(x)$. We now turn to the metric tensor as it viewed in the retarded coordinates r^β. Basic vectors $\mathbf{e}_\beta = (\partial x^\alpha/\partial r^\beta)\mathbf{e}_\alpha$ are as follows

$$\begin{aligned}
\mathbf{e}_s &= \left(u^\alpha + r\frac{\partial k^\alpha}{\partial s}\right)\mathbf{e}_\alpha, \\
\mathbf{e}_r &= k^\alpha \mathbf{e}_\alpha, \\
\mathbf{e}_\vartheta &= r\Omega(z(s))\Lambda^\alpha{}_{\alpha'}(s) n_\vartheta^{\alpha'}(\vartheta,\varphi), \\
\mathbf{e}_\varphi &= r\Omega(z(s))\Lambda^\alpha{}_{\alpha'}(s) n_\varphi^{\alpha'}(\vartheta,\varphi),
\end{aligned}$$

where $n_\vartheta^{\alpha'} = (0, \cos\varphi\cos\vartheta, \sin\varphi\cos\vartheta, -\sin\vartheta)$ and $n_\varphi^{\alpha'} = (0, -\sin\varphi\sin\vartheta, \cos\varphi\sin\vartheta, 0)$. Desired metric tensor is composed from their scalar products

$$\hat{G} = \begin{bmatrix} g_{ss} & -1 & g_{s\vartheta} & g_{s\varphi} \\ -1 & 0 & 0 & 0 \\ g_{\vartheta s} & 0 & r^2\Omega^2 & 0 \\ g_{\varphi s} & 0 & 0 & r^2\Omega^2\sin^2\vartheta \end{bmatrix}. \tag{C.5}$$

We use the standard relation $\eta_{\alpha\beta}\Lambda^\alpha{}_{\alpha'}\Lambda^\beta{}_{\beta'} = \eta_{\alpha'\beta'}$. Combining $\mathrm{d}V = \mathrm{d}^4x\Omega^4(x)$ and $\det(\hat{G}) = -r^4\Omega^4(z)\sin^2\vartheta$, we derive the volume element in terms of retarded coordinates:

$$\mathrm{d}V = \mathrm{d}s\,\mathrm{d}r\,\mathrm{d}\vartheta\,\mathrm{d}\varphi\,\Omega^4(x)\Omega^2(z) r^2\sin\vartheta. \tag{C.6}$$

Another result we shall need is the outward-directed surface element of a cylinder $r = const$ in de Sitter space

$$\Sigma_\alpha = \Omega^4(x)\Omega^2(z)\frac{\partial r}{\partial x^\alpha} r^2 \sin\vartheta \, \mathrm{d}s\mathrm{d}\vartheta\mathrm{d}\varphi, \tag{C.7}$$

where coordinate derivative of the retarded distance is given by equation (3.29).

Appendix D. Solid angle integration of de Sitter momentum (4.29) and angular momentum (4.30)

Let us introduce kinematic variables

$$U^\mu = \Omega u^\mu, \tag{D.1}$$

$$K^\mu = \Omega^{-1} k^\mu, \tag{D.2}$$

$$R \ = \Omega r, \tag{D.3}$$

and new evolution parameter $\mathrm{d}\tau = \Omega^{-1}\mathrm{d}s$. Conformal factor Ω is referred to $z(s)$.

These yield new four-acceleration

$$\begin{aligned} A^\mu &= \frac{\mathrm{d}U^\mu}{\mathrm{d}\tau} \\ &= \Omega^2 \left[a^\mu - \frac{1}{2}H^2\Omega(z\cdot u)u^\mu \right]. \end{aligned} \tag{D.4}$$

Inserting these into equation (3.31) we rewrite the retarded electromagnetic field in terms of new variables

$$F^{\mathrm{ret}}_{\alpha\beta}(x) = \frac{q}{R^2}\left(U_\alpha K_\beta - U_\beta K_\alpha\right) + \frac{q}{R}\left[A_\alpha K_\beta - A_\beta K_\alpha + (A\cdot K)\left(U_\alpha K_\beta - U_\beta K_\alpha\right)\right]. \tag{D.5}$$

Putting equations (D.1) and (D.2) in the expressions (C.2) and (C.3) defining null vector we obtain

$$K^\alpha(s,\vartheta,\varphi) = \Lambda^\alpha{}_{\alpha'}(s) n^{\alpha'}(\vartheta,\varphi), \tag{D.6}$$

where the Lorentz matrix takes usual form

$$\Lambda^\alpha{}_{\alpha'}(s) = \left[\begin{array}{c|c} U^0 & U_{k'} \\ \hline U^i & \delta^i{}_{k'} + \dfrac{U^i U_{k'}}{U^0+1} \end{array}\right]. \tag{D.7}$$

Our next aim is to derive the relations defining the solid angle integration of polynomials of components of null vector K defined by equation (D.6). The angular integration can be handled via the relations which involve spatial components of four-vector (C.4)

$$\begin{aligned} \frac{1}{4\pi}\int_0^\pi \mathrm{d}\vartheta\, \sin\vartheta \int_0^{2\pi} \mathrm{d}\varphi\, n^i &= 0, \\ \frac{1}{4\pi}\int_0^\pi \mathrm{d}\vartheta\, \sin\vartheta \int_0^{2\pi} \mathrm{d}\varphi\, n^i n^j &= \frac{1}{3}\delta^{ij}, \\ \frac{1}{4\pi}\int_0^\pi \mathrm{d}\vartheta\, \sin\vartheta \int_0^{2\pi} \mathrm{d}\varphi\, n^i n^j n^k &= 0. \end{aligned}$$

Using the relation $\eta_{\alpha\beta}\Lambda^{\alpha}{}_{\alpha'}\Lambda^{\beta}{}_{\beta'} = \eta_{\alpha'\beta'}$ and taking into consideration that $\Lambda^{\alpha}{}_{0'} = U^{\alpha}$, we derive the following relations

$$
\begin{aligned}
&\frac{1}{4\pi}\int_0^{\pi} \mathrm{d}\vartheta \,\sin\vartheta \int_0^{2\pi} \mathrm{d}\varphi\, K^{\alpha} = U^{\alpha},\\
&\frac{1}{4\pi}\int_0^{\pi} \mathrm{d}\vartheta \,\sin\vartheta \int_0^{2\pi} \mathrm{d}\varphi\, K^{\alpha}K^{\beta} = \frac{1}{3}\eta^{\alpha\beta} + \frac{4}{3}U^{\alpha}U^{\beta},\\
&\frac{1}{4\pi}\int_0^{\pi} \mathrm{d}\vartheta \,\sin\vartheta \int_0^{2\pi} \mathrm{d}\varphi\, K^{\alpha}K^{\beta}K^{\gamma} = 2U^{\alpha}U^{\beta}U^{\gamma}\\
&\qquad + \frac{1}{3}\left(U^{\alpha}\eta^{\beta\gamma} + U^{\beta}\eta^{\alpha\gamma} + U^{\gamma}\eta^{\alpha\beta}\right).
\end{aligned} \tag{D.8}
$$

In terms of new variables (D.1)–(D.4) the stress-energy tensor (4.2) decomposed into the bound and radiative components (4.3) the form

$$
\begin{aligned}
T^{\alpha\beta}_{(-4)} &= \frac{q^2}{R^4}\left(U^{\alpha}K^{\beta} + K^{\alpha}U^{\beta} - K^{\alpha}K^{\beta} + \frac{1}{2}\eta^{\alpha\beta}\right),\\
T^{\alpha\beta}_{(-3)} &= \frac{q^2}{R^3}\left[A^{\alpha}K^{\beta} + K^{\alpha}A^{\beta} + (A\cdot K)\left(U^{\alpha}K^{\beta} + K^{\alpha}U^{\beta} - 2K^{\alpha}K^{\beta}\right)\right],\\
T^{\alpha\beta}_{(-3)} &= \frac{q^2}{R^2}\left[(A\cdot A) - (A\cdot K)^2\right]K^{\alpha}K^{\beta}.
\end{aligned} \tag{D.9}
$$

Inserting equations, (D.2) and (D.3) in equation (C.1), we obtain the relation $x^{\alpha} = z^{\alpha}(s) + RK^{\alpha}$, Taking this into account we express the projectors involving in equations (4.29) and (4.30) in terms of new variables:

$$
\begin{aligned}
2 - \Omega^{-1}(x) &= 2 - \Omega^{-1}(z) - R\frac{H^2}{2}(z\cdot K),\\
x^{\alpha}x_{\beta} &= z^{\alpha}z_{\beta} + R\left(z^{\alpha}K_{\beta} + K^{\alpha}z_{\beta}\right) + R^2K^{\alpha}K_{\beta}.
\end{aligned} \tag{D.10}
$$

So, we have the mathematical tools for the solid angle integration of the proper time derivatives of Noether quantities (4.29) and (4.30). The electromagnetic field's momentum (4.29) consists of the distant-dependent bound term

$$
\frac{\mathrm{d}\pi^{\alpha}_{\mathrm{bnd}}}{\mathrm{d}s} = \frac{q^2}{\Omega R}\left[(2-\Omega^{-1}(z))\delta^{\alpha}{}_{\beta} + \frac{H^2}{2}z^{\alpha}z_{\beta}\right]\left[\frac{1}{2}A^{\beta} + \frac{H^2}{4}\Omega(z\cdot U)U^{\beta} - \frac{H^2}{4}\Omega z^{\beta}\right], \tag{D.11}
$$

and the distant-free radiative part

$$
\begin{aligned}
\frac{\mathrm{d}\pi^{\alpha}_{\mathrm{rad}}}{\mathrm{d}s} = \frac{q^2}{\Omega}\Bigg\{&\left[(2-\Omega^{-1}(z))\delta^{\alpha}{}_{\beta} + \frac{H^2}{2}z^{\alpha}z_{\beta}\right]\frac{2}{3}(A\cdot A)U^{\beta}\\
&+ \frac{H^2}{3}\left[(z\cdot A)U^{\alpha} - (z\cdot U)A^{\alpha}\right] - \frac{H^2}{4}U^{\alpha}\Bigg\}.
\end{aligned} \tag{D.12}
$$

Restoring the original variables, we obtain equation (4.31). So far as the bound part (D.11), we arrive at the expression

$$
\frac{\mathrm{d}\pi^{\alpha}_{\mathrm{bnd}}}{\mathrm{d}s} = \frac{q^2}{r}\left[(2-\Omega^{-1}(z))\delta^{\alpha}{}_{\beta} + \frac{H^2}{2}z^{\alpha}z_{\beta}\right]\left[\frac{1}{2}a^{\beta} - \frac{H^2}{4}\Omega^{-1}z^{\beta}\right]. \tag{D.13}
$$

Recall that we assume that $r = \text{const}$. There is the total time derivative in the right hand side of equation. Having integrated it, we finally obtain

$$\pi^{\alpha}_{\text{bnd}} = \frac{q^2}{r}\left[(2-\Omega^{-1}(z))\delta^{\alpha}{}_{\beta} + \frac{H^2}{2}z^{\alpha}z_{\beta}\right]\frac{1}{2}u^{\beta}. \tag{D.14}$$

The Coulomb-lire singularity should be coupled with particle's individual momentum (4.20):

$$\pi^{\alpha}_{\text{part}} + \pi^{\alpha}_{\text{bnd}} = \Omega^2(z)\left[\left(2-\Omega^{-1}(z)\right)\delta^{\alpha}{}_{\beta} + \frac{1}{2}H^2 z^{\alpha}z_{\beta}\right]\left(p^{\beta}_{\text{part}} + \frac{q^2}{2r}\Omega^{-2}u^{\beta}\right). \tag{D.15}$$

The singularity is absorbed by particle's momentum within the regularization procedure.

The angular momentum (4.30) carried by electromagnetic field also consists of two terms, singular and regular:

$$\frac{\mathrm{d}\lambda^{\alpha\beta}_{\text{bnd}}}{\mathrm{d}s} = \frac{q^2}{2\Omega R}\left\{z^{\alpha}\left[A^{\beta} + \frac{H^2}{2}\Omega(z\cdot U)U^{\beta}\right] - z^{\beta}\left[A^{\alpha} + \frac{H^2}{2}\Omega(z\cdot U)U^{\alpha}\right]\right\}; \tag{D.16}$$

$$\frac{\mathrm{d}\lambda^{\alpha\beta}_{\text{rad}}}{\mathrm{d}s} = \frac{2q^2}{3\Omega}\left[(A\cdot A)\left(z^{\alpha}U^{\beta} - z^{\beta}U^{\alpha}\right) + U^{\alpha}A^{\beta} - U^{\beta}A^{\alpha}\right]. \tag{D.17}$$

Being rewritten in terms of original variables the proper time derivative of radiative part takes the form equation (4.32). The bound part is simply

$$\frac{\mathrm{d}\lambda^{\alpha\beta}_{\text{bnd}}}{\mathrm{d}s} = \frac{q^2}{2r}\left(z^{\alpha}a^{\beta} - z^{\beta}a^{\alpha}\right). \tag{D.18}$$

After integration we obtain

$$\lambda^{\alpha\beta}_{\text{bnd}} = \frac{q^2}{2r}\left(z^{\alpha}u^{\beta} - z^{\beta}u^{\alpha}\right). \tag{D.19}$$

Coupled this with the particle's individual angular momentum (4.21), we arrive at

$$\lambda^{\alpha\beta}_{\text{part}} + \lambda^{\alpha\beta}_{\text{bnd}} = \Omega^2(z)\left[z^{\alpha}\left(p^{\beta}_{\text{part}} + \frac{q^2}{2r}\Omega^{-2}u^{\beta}\right) - z^{\beta}\left(p^{\alpha}_{\text{part}} + \frac{q^2}{2r}\Omega^{-2}u^{\alpha}\right)\right]. \tag{D.20}$$

In flat spacetime one usually assume that the parameter m involving in particle's action term is the unphysical bare mass. It absorbs inevitable infinity within the regularization procedure and becomes the observable rest mass of the particle. The bare mass parameter is in Lagrangian (4.11) governing the dynamics of a free massive charge in the de Sitter space. In section 4.1, the expression (4.35) for particle's momentum is derived where the contribution due to its own electromagnetic field is taken into account. The time-dependent mass function (4.38) contains the observable mass m which absorbs the Coulomb-like divergency which arises in bound momentum (D.14) and angular momentum (D.19).

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References

[1] Abbott B P *et al* 2019 GWTC-1: a gravitational-wave transient catalog of compact binary mergers observed by LIGO and Virgo during the first and second observing runs *Phys. Rev.* X **9** 031040
[2] Oltean M, Epp R J, Sopuerta C F, Spallicci A D A M and Mann R B 2020 Motion of localized sources in general relativity: gravitational self-force from quasilocal conservation laws *Phys. Rev.* D **101** 064060
[3] Price W, Formanek M and Rafelski J 2022 Radiation reaction and limiting acceleration *Phys. Rev.* D **105** 016024
[4] Ekman R, Heinzl T and Ilderton A 2021 Reduction of order, resummation and radiation reaction *Phys. Rev.* D **104** 036002
[5] Yaghjian A D 2021 Lorentz-Abraham-Dirac and Landau-Lifshitz equations of motion and the solution to a relativistic electron in a counterpropagating laser beam *Phys. Rev. Accel. Beams* **24** 114002
[6] Noble A, Burton D A, Docherty L and Jaroszynski D A 2021 Self-force on a charged particle in an external scalar field *New J. Phys.* **23** 115007
[7] Blackburn T G 2020 Radiation reaction in electron-beam interactions with high-intensity lasers *Rev. Mod. Plasma Phys.* **4** 5
[8] Andriot D and Gómez G L 2017 Signatures of extra dimensions in gravitational waves *J. Cosmol. Astrophys. Phys.* JCAP06(2017)048 Andriot D and Gómez G L 2019 *J. Cosmol. Astrophys. Phys.* JCAP05(2019)E01 (erratum)
[9] Gaĺtsov D V and Khlopunov M 2020 Synchrotron radiation in odd dimensions *Phys. Rev.* D **101** 084054
[10] Lorentz H A 2010 *The Theory of Electrons and its Applications to the Phenomena of Light and Radiant Heat* (Nabu Press)Lorentz H A 1953 *Teoriya Electronov* (Gosudarstvennoye Izdatelstvo Tekhniko-teoreticheskoy literatury) (in Russian)
[11] Abraham M 1905 *Theorie Der Elektrizität* vol 1 (Springer)
[12] Abraham M 1949 *The Classical Theory of Electricity and Magnetism* (Blackie & Son Ltd)
[13] Dirac P A M 1938 Classical theory of radiating electrons *Proc. R. Soc.* A **167** 148–69
[14] Rylov Yu A 1989 The algebraical structure of the electromagnetic tensor and description of charged particles moving in the strong electromagnetic field *J. Math. Phys.* **30** 521–36
[15] Rylov Yu A 1988 Self-consistent model of the global structure of axially-symmetric pulsar magnetosphere in massless approximation *Astrophys. Space Sci.* **143** 269–300
[16] Rylov Yu A 1989 The global structure of the pulsar magnetospheres *Astrophys. Space Sci.* **158** 297–333
[17] Dewitt B S and Brehme R W 1960 Radiation damping in a gravitational field *Ann. Phys.* **9** 220–59
[18] Hobbs J M 1968 A vierbein formalism of radiation damping *Ann. Phys.* **47** 141–65
[19] Poisson E, Pound A and Vega I 2011 The motion of point particles in curved spacetime *Living Rev. Relativ.* **14** 7
[20] Detweiler S and Whiting B F 2003 Self-force via a Green's function decomposition *Phys. Rev.* D **67** 024025
[21] Harte A I 2008 Approximate spacetime symmetries and conservation laws *Class. Quantum Grav.* **25** 205008
[22] Harte A I 2009 Electromagnetic self-forces and generalized Killing fields *Class. Quantum Grav.* **26** 155015
[23] Enayati M, Gazeau J-P, Pejhan H and Wang A 2022 The de Sitter group and its representations: a window on the notion of de Sitterian elementary systems (arXiv:2201.11457v3 [math-ph])
[24] Higuchi A and Lee Y C 2008 How to use retarded Green's functions in de Sitter spacetime *Phys. Rev.* D **78** 084031
[25] Rohrlich F 1990 *Classical Charged Particles* 2nd edn (Addison-Wesley)
[26] Parrott S 1987 *Relativistic Electrodynamics and Differential Geometry* (Springer)
[27] Villarroel D 2002 Preacceleration in classical electrodynamics *Phys. Rev.* E **66** 046624
[28] Landau L D and Lifshitz E M 1962 *The Classical Theory of Fields* 2nd edn (Pergamon)
[29] Hadad Y, Labun L, Rafelski J, Elkina N, Klier C and Ruhl H 2010 Effects of radiation reaction in relativistic laser acceleration *Phys. Rev.* D **82** 096012
[30] Keitel C H, Szymanowski C, Knight P L and Maquet A 1998 Radiative reaction in ultra-intense laser-atom interaction *J. Phys. B: At. Mol. Opt. Phys.* **31** L75–L83

[31] Boyle L, Finn K and Turok N 2022 The Big Bang, CPT and neutrino dark matter *Ann. Phys., NY* **438** 168767
[32] Müller T and Grave F 2010 Catalogue of spacetimes (arXiv:0904.4184v3 [gr-qc])
[33] Aldrovandi R, Beltrán Almeida J P and Pereira J G 2007 de Sitter special relativity *Class. Quantum Grav.* **24** 1385–404
[34] Aldrovandi R, Beltrán Almeida J P and Pereira J G 2006 A singular conformal spacetime *J. Geom. Phys.* **56** 1042–56
[35] Aldrovandi R, Beltrán Almeida J P and Pereira J G 2004 Cosmological term and fundamental physics *Int. J. Mod. Phys.* D **13** 2241–8
[36] Aghanim N *et al* (Planck Collaboration) 2020 Planck 2018 results. VI. Cosmological parameters *Astron. Astrophys.* **641** 67
[37] Boblest S, Müller T and Wunner G 2010 Twin paradox in de sitter spacetime (arXiv:1009.3427v1 [gr-qc])
[38] Red'kov V M, Tokarevskaya N G, Bychkouskaya E M and Spix G J 2009 Maxwell equations in Riemannian space-time, geometry effect on constitutive relations in media *Nonlinear Phenom. Complex Syst.* **12** 232–50
[39] Bičák J and Krtouš P 2001 Accelerated sources in de Sitter spacetime and the insufficiency of retarded fields *Phys. Rev.* D **64** 124020
[40] Hobbs J M 1968 Radiation damping in conformally fat universes *Ann. Phys., NY* **47** 166–72
[41] Roberts M D 1989 The motion of a charged particle in a spacetime with a conformal metric *Class. Quantum Grav.* **6** 419–23
[42] Teitelboim C 1970 Splitting of the Maxwell tensor: radiation reaction without advanced fields *Phys. Rev.* D **1** 1572–82
[43] Poisson E 1999 An introduction to the Lorentz-Dirac equation (arXiv:gr-qc/9912045)
[44] Yaremko Y 2003 On the regularization procedure in classical electrodynamics *J. Phys. A: Math. Gen.* **36** 5149–56
[45] Aldrovandi R, Beltrán Almeida J P, Mayor C S O and Perejra J G 2007 de Sitter relativity and quantum physics *AIP Conf. Proc.* **962** 175
[46] Burko L M, Harte A I and Poisson E 2002 Mass loss by a scalar charge in an expanding Universe *Phys. Rev.* D **65** 124006
[47] Haas R and Poisson E 2005 Mass change and motion of a scalar charge in cosmological spacetimes *Class. Quantum Grav.* **22** S739–52
[48] Yaremko Y 2007 Radiation reaction in 2+1 electrodynamics *J. Math. Phys.* **48** 092901
[49] Allen B and Jacobson T 1986 Vector two-point functions in maximally symmetric spaces *Commun. Math. Phys.* **103** 669–92