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STATUS OF THE CKM MATRIX

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Abstract

I briefly review recent progress in the the determination of the CKM matrix.

1 Introduction

The only source of flavor and CP violation in the SM is the CKM matrix, but most models of new physics naturally involve new sources of flavor and CP violation. The precise verification of the CKM mechanism is therefore central in the search for new physics and represents the modern equivalent of the tests of the universality of the charged currents. CKM studies are made difficult by the ubiquitous presence of strong interactions. In most cases, theoretical errors have become the dominant source of uncertainty: we are learning slowly but steadily how to minimize them. Significant recent progress in this direction is due to a synergy with experiment [1]. The selection of topics presented below is incomplete, but I hope it reflects the main directions of progress in the field.

The CKM matrix has a highly hierarchical structure, that is best exposed in the Wolfenstein parameterization,

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\varrho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4), \tag{1}$$

where $\lambda \approx 0.22$ is the sine of the Cabibbo angle. There are only four independent parameters: λ, A, ρ and η.

2 The Cabibbo angle

We see from (1) that, up to higher orders in λ, the upper left 2×2 submatrix is nothing but the Cabibbo matrix. Indeed, because of the smallness of $|V_{ub}| \approx 0.004$, the unitarity of the first row of the CKM matrix can be verified by a comparison of λ extracted from V_{ud} and V_{us}. Of course, λ can also be extracted from the second row, using DIS and W decay data, but with much lower precision [2].

The most precise determination of $|V_{ud}|$ comes from superallowed Fermi transitions (SFT), i.e. $0^+ \to 0^+$ nuclear β decays. Nine different such decays give consistent results and the error of the final value, $|V_{ud}| = 0.9740(5)$ [3] or $\lambda \equiv |V_{us}| = 0.2265(22)$, is dominated by the theoretical uncertainty in radiative corrections and nuclear effects. Neutron β decay provides a valuable alternative and starts being competitive, $\delta V_{ud} \sim 0.0015$, with further improvements expected at PERKEO. Theoretically, however, the cleanest channel is $\pi^+ \to \pi^0 e\nu$, which is penalized by a 10^{-8} BR. The present uncertainty based on preliminary PIBETA results, $\delta V_{ud} \sim 0.006$, is still far from being competitive, but the goal of PIBETA is to reduce it by a factor 3.

So far, the extraction of $|V_{us}|$ has been dominated by old data on semileptonic $K \to \pi l \nu$ decays (K_{l3}). For several years, K_{l3} data have preferred a value of λ lower than that coming from SFT, leading to a $\sim 2.3\sigma$ violation of unitarity. Last year, however, the BNL experiment E865 has published a new K^+ result implying a much higher λ than the old ones, in good agreement with unitarity. A new, thorough analysis of K_L semileptonic decays by the KTeV Collaboration [4], as well as new K_{Se3} and K_L results by KLOE [5] and

K_L, K^+ data from NA48 have confirmed the E865 result, improving significantly the experimental accuracy. The new results' average is $\lambda = 0.2259(22)$. The dominant source of error here is the theoretical error in the determination of the form factor at zero momentum $f_+(0)$. The form factor can be chirally expanded

$$f_+(0) = 1 + f_2 + f_4 + ... \tag{2}$$

where f_n are SU(3) breaking correction of $\mathcal{O}(M^n_{K,\pi}/(4\pi f_\pi)^n)$. While f_2, thanks to the Ademollo-Gatto theorem, can be precisely calculated, the real challenge is the estimate of f_4. It has recently be computed for the first time in quenched lattice QCD [6]. This exploratory analysis agrees with the reference quark model value by Leutwyler and Roos, and can be hopefully improved in several ways. It has also recently been realized that f_4 can be constrained by data on the slope and curvature of the form factor [7], but that requires higher experimental accuracy, an interesting challenge for present experiments. A 0.5% determination of $|V_{us}|$ in the next few years is conceivable.

The apparent violation of unitarity and the unclear experimental situation for K_{l3} of the last years have stimulated fresh ideas and a revisitation of older ones. A first example is the extraction of $|V_{us}|$ from hadronic τ decays [1, 8]. This requires a precise value of the strange quark mass, that can be obtained from lattice QCD or from sum rules. The value of λ obtained in [8] is compatible with unitarity and the present uncertainty, $\delta V_{us} \sim 0.035$, is dominated by the experimental errors on the τ BRs, expected to decrease significantly with B-factories data. A second possibility is to use hyperon decays [9], fitting the ratio of axial over vector current from data. While the experimental error on $|V_{us}|$ is close to 1%, SU(3) breaking effects require a dedicated lattice study (the convergence of the chiral expansion is slower) and have not yet been included. A third recent proposal [3] is to extract $|V_{us}/V_{ud}|$ from the experimental ratio

$$\frac{\Gamma(K \to \mu\bar{\nu}_\mu(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}_\mu(\gamma))} = \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{f_K^2 m_K (1 - m_\mu^2/m_K^2)^2}{f_\pi^2 m_\pi (1 - m_\mu^2/m_\pi^2)^2} R_{rc} \tag{3}$$

using the radiative corrections factor $R_{rc} = 0.9930(35)$ and the new, partially unquenched lattice result $f_K/f_\pi = 1.210(4)(13)$ by the MILC collaboration [10]. The resulting $\lambda = 0.2221(27)$ has an uncertainty dominated by the lattice and, in principle, great potential for improvement. On the other hand, unquenched calculations have not yet reached maturity and the MILC error estimate is presently debated.

3 V_{cb}

The parameter A can be best determined from V_{cb}, see (1). The exclusive determination of $|V_{cb}|$ uses the extrapolation of the $B \to D^* l\nu$ rate to the kinematic endpoint where the D^* is produced at rest (zero-recoil). In this limit, the form factor $F(1)$ is known, up to corrections suppressed by at least two powers of $m_{c,b}$ that have to be computed, e.g. on the lattice. Since one needs to estimate only the $\mathcal{O}(10\%)$ correction to the heavy quark limit, an interesting accuracy can be reached even with present methods. In fact, current lattice QCD and sum rule results are both consistent with $F(1) = 0.91 \pm 0.04$ [1]. The overall uncertainty is therefore close to 5%: $|V_{cb}^{excl}| = 41.5(1.0)_{ex}(1.8)_{th} \times 10^{-3}$, but the two most precise experimental results, by Babar and Cleo, differ by almost 3σ [11, 12]. Semileptonic decays to D mesons give consistent but less precise results. Progress is expected especially from unquenched lattice calculations.

While the non-perturbative unknowns in the exclusive determination of $|V_{cb}|$ have to be calculated, those entering the inclusive semileptonic decays, $B \to X_c l\nu$, can be measured in a self-consistent way. Indeed, the inclusive decay rate depends only on the hadronic structure of the decaying B meson, but the sensitivity is actually suppressed by two powers of Λ_{QCD}/m_b, as the highly energetic decay products are (generally) unable to probe the long wavelengths characteristic of the B meson. The differential rate for $B \to X_c l\nu$ can therefore be expressed as a double expansion in α_s and Λ_{QCD}/m_b (Heavy Quark Expansion), whose leading term is nothing but the parton model result. However, the HQE results for the spectra can be compared to experiment only after smearing over a range of energies $\gg \Lambda_{QCD}$ and away from the endpoints. This is evident in the case of the hadronic mass spectrum, is dominated by resonance peaks that have no counterpart in the HQE: the HQE results have no *local* meaning.

The moments (weighted integrals) of the lepton energy and hadronic mass spectra, as well as the photon spectrum in radiative decays, are therefore employed, often with a lower cut on the charged lepton energy. Their HQE is analogous to that of the integrated rate,

$$\Gamma_{cl\nu} = \frac{G_F^2 m_b^5 \eta^{ew}}{192\pi^3} |V_{cb}|^2 z(r) \left[1 + a_1(r)\frac{\mu_\pi^2}{m_b^2} + a_2(r)\frac{\mu_G^2}{m_b^2} + b_1(r)\frac{\rho_D^3}{m_b^3} + b_2(r)\frac{\rho_{LS}^3}{m_b^3}\right], \tag{4}$$

where $r = (m_c/m_b)^2$, the Wilson coefficients a_i, b_i are series in α_s, and power corrections up to $1/m_b^3$ have been kept. Theoretical predictions are therefore given in terms of α_s, of properly defined *quark* masses $m_{c,b}$ and of the B meson matrix elements of four *local* operators, $\mu^2_{\pi,G}$, $\rho^3_{D,LS}$. Because they depend on the various parameters in different ways, the moments serve a double purpose: they allow to constrain the non-perturbative parameters and they test the overall consistency of the HQE framework. Effects that cannot be described by the HQE (and so violate *parton-hadron duality*) and higher order power corrections can be severely constrained.

In this sense, the new Babar analysis [13], based on [14], represents a real step forward, both in completeness and accuracy. It shows a remarkable consistency of a variety of leptonic and hadronic moments, leading to an excellent fit, values of the quark masses in agreement with lattice and spectral sum rule determinations, important bounds on the other non-perturbative parameters in agreement with other independent constraints, and $|V_{cb}^{incl}| = 41.4(0.4)_{ex}(0.4)_{hqe}(0.6)_{th} \times 10^{-3}$. The main results have been recently confirmed [15]. Semileptonic and radiative moments from Belle, Cleo, Delphi, and CDF can be included as well, without deteriorating the quality of the fit. A 1% determination of $|V_{cb}|$ might be possible, but requires some theoretical effort.

4 The unitarity triangle

As illustrated in Fig. 1, various measurements constrain differently $\bar{\rho} = \rho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$. The triangle in the $(\bar{\rho}, \bar{\eta})$ plane with vertices in (0,0), (1,0), and $(\bar{\rho}, \bar{\eta})$ represents the unitarity relation $\sum_i V_{ib}^* V_{id} = 0$ and is usually called *unitarity triangle*.

The ratio $|V_{ub}/V_{cb}|$ measures the left side of the unitarity triangle, identifying a circle in the $(\bar{\rho}, \bar{\eta})$ plane. The determination of $|V_{ub}|$ from $b \to u$ semileptonic decays parallels that of $|V_{cb}|$, but the exclusive determination ($B \to \pi l \nu, B \to \rho l \nu$, etc.) is penalized by the absence of a heavy quark normalization for the form factors at a certain kinematical point, while the inclusive determination is affected by the kinematic cuts necessary to isolate $b \to u$ transitions from the dominant $b \to c$ background. Moreover, if theoretical precision is lower, so is statistics, by two orders of magnitude. In the exclusive case, lattice QCD and light cone sum rules complement each other, but as the first unquenched calculations appear the accuracy does not exceed

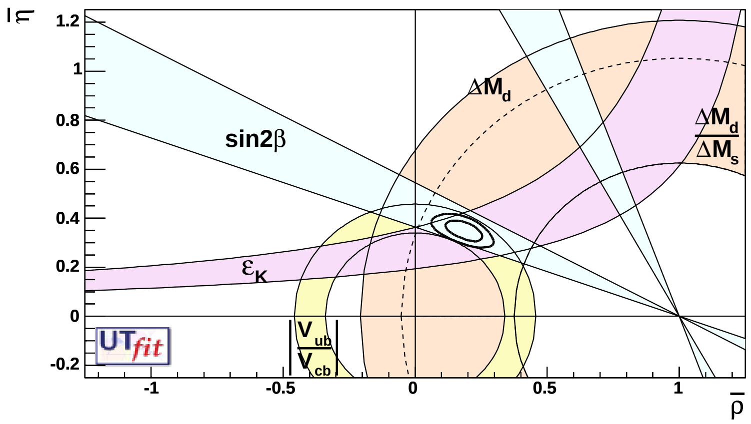

Figure 1: *Determination of the Unitarity Triangle using various constraints.*

15-20%, with central values for $|V_{ub}|$ around 0.0035. In the inclusive case, the cuts destroy the convergence of the HQE and introduce a sensitivity to local b-quark wave function properties like the Fermi motion, not suppressed by powers of $1/m_b$. Different strategies have been proposed (cuts on the hadronic invariant mass $M_X < M_D$, on the electron energy, on the q^2 of the lepton pair, and combinations thereof), each of them with peculiar experimental and theoretical systematics [16]. Recently, an intense theoretical activity has concerned the optimization of the cuts, subleading non-perturbative effects, the resummation of Sudakov logs, the role of the radiative decay spectrum in constraining the shape function, etc. As witnessed by the latest HFAG average of inclusive determinations, $|V_{ub}| = 4.70(44)\ 10^{-3}$, the present error is close to 10% and dominated, again, by theory. Improvements will come from high statistics experimental data, in particular from a precise determination of the radiative spectrum, from a careful application of the constraints on the shape function coming from spectral moments, and from the $b \to u$ differential rate itself. Eventually, the variety of complementary approaches that have been developed will be extremely useful.

The other interesting side of the unitarity triangle is proportional to $|V_{td}/V_{cb}|$, which can be accessed only via loop induced FCNC transitions, more sensitive to new physics. The useful observables are ε_K, ΔM_d, and $\Delta M_s/\Delta M_d$, from K^0, B_d^0, and B_s^0 mixing. Their theoretical interpretation depends crucially on input from lattice QCD, whose accuracy generally does not exceed 10-15% accuracy at present. B physics lattice simulations are multiscale, and present lattices can resolve neither the b quark (too heavy if one wants to minimize discretization errors), nor the light quarks: various extrapolations are therefore needed. In addition, most calculations are performed without dynamical sea quarks (*quenched QCD*). Although error bars have not shrunk much, there has been significant progress in the last few years and more will come. The next frontier are unquenched simulations, that might reduce the lattice error by a factor three but are still in their infancy. It is easy to realize the dramatic impact this could have in Fig. 1. A measurement of ΔM_s at Tevatron would also have an important impact, even if it agrees with the SM. Alternative and promising routes to access V_{td} are the rare decays $K \to \pi\nu\bar{\nu}$ and $B \to \rho\gamma$.

Finally, various CP asymmetries measure directly some of the angles of the unitarity triangle. The measurement of $\sin 2\beta$ from the CP asymmetry in $B \to J/\Psi K_S$, in particular, has become a clean and very precise input (see Fig. 1). The measurement of the other angles is more difficult and is affected by various theoretical systematics, but is becoming the focus of the B-factories [17].

Global fits to the unitarity triangle give $\bar{\rho} = 0.172(47)$ and $\bar{\eta} = 0.348(28)$ [18] or $\bar{\rho} = 0.189(78)$ and $\bar{\eta} = 0.358(44)$ [19], according to the two main methodologies on the market. They mostly differ in the treatment of theoretical errors, but have been shown to be practically equivalent at the 95% CL [1]. The agreement between the various constraints is impressive. For instance, one can compare the direct and indirect determinations of $\sin 2\beta$, $0.707^{+0.043}_{-0.053}$ and 0.739 ± 0.048, respectively. The prediction for the angle γ is $62^\circ \pm 7^\circ$, while Belle analysis gives $81(19)(13)(11)^\circ$. The expected value for ΔM_s is $18.3(1.6)\,ps^{-1}$, to be compared with the direct lower bound $\Delta M_s > 14.5\,ps^{-1}$: in the absence of new physics Tevatron should be able to measure it soon.

In summary, the CKM mechanism describes successfully a host of data. Present errors are dominantly theoretical: lattice QCD still represents the best hope, but theory control can be very often improved by new data, a lesson never to forget.

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