MUON TRACKING STUDIES IN A SKEW PARAMETRIC RESONANCE IONIZATION COOLING CHANNEL

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Abstract

Skew Parametric-resonance Ionization Cooling (SPIC) is an extension of the Parametric-resonance Ionization Cooling (PIC) framework that has previously been explored as the final 6D cooling stage of a high-luminosity muon collider. The addition of skew quadrupoles to the PIC magnetic focusing channel induces coupled dynamic behavior of the beam that is radially periodic. The periodicity of the radial motion allows for the avoidance of unwanted resonances in the horizontal and vertical transverse planes, while still providing periodic locations at which ionization cooling components can be implemented. A first practical implementation of the magnetic field components required in the SPIC channel is modeled in MADX. Dynamic features of the coupled correlated optics with and without induced parametric resonance are presented and discussed.

INTRODUCTION

The limit on the minimum achievable emittance in muon ionization cooling comes from the equilibrium between the cooling process and multiple Coulomb scattering in the absorber material. The concept of Parametric-resonance Ionization Cooling (PIC) is to push this limit by an order of magnitude in each transverse dimension by focusing the muon beam very strongly in both planes at thin absorber plates. This creates a large angular spread of the beam at the absorber locations, which is then cooled to its equilibrium value resulting in greatly reduced transverse emittances. Achieving adequately strong focusing using conventional magnetic optics would require unrealistically strong magnetic fields. Instead, PIC relies on a resonant process to provide the necessary focusing. A parametric resonance is induced in a cooling channel, causing focusing of the beam with the period of the channel's free oscillations. To attain simultaneous focusing in both planes at regular locations, the horizontal and vertical betatron oscillation periods must be commensurate with each other and with the channel's period. A magnetic channel possessing such optical properties, called a Twin helix channel, has been successfully developed and simulated [1].

Another important condition necessary for implementation of PIC is compensation of the beam smear from one focal point to another, to a degree where it is small compared to the focused beam size. Since the angular spread at the focal point is on the order of 100 mrad rms while the beam size is a fraction of a mm, this can be quite challenging. To mitigate this problem, the Twin helix channel was designed using continuous helical fields eliminating fringe-field effects. Significant progress has been made on compensation of aberrations using helical multipole fields [1]. However, multipole fields in combination with correlated optics introduce another serious problem, namely, non-linear resonances causing loss of dynamical stability.

To illustrate this problem, consider the Hamiltonian term of a continuous harmonically-varying octupole field $H_{oct} = n_{oct}\,(6x^2y^2 - x^4 - y^4)/4$ where $n_{oct} \sim \cos(2\pi mz/L)$ is the normalized octupole strength, m is an integer, z is the longitudinal coordinate, L is the channel period length, $x \sim \cos(2\pi\nu_x z/L)$ and $y \sim \cos(2\pi\nu_y z/L)$ are the horizontal and vertical transverse betatron coordinates, respectively, and ν_x and ν_y are the horizontal and vertical betatron tunes, respectively. Multiple octupole harmonics are needed in a cooling channel to compensate spherical aberrations. However, as can be clearly seen from the Hamiltonian, with our choice of betatron tunes of $\nu_x = 0.25$ and $\nu_y = 0.5$, any octupole harmonic m causes resonances in both planes. Dispersion further complicates the resonance structure. Selecting different betatron tunes does not help; as long as the betatron periods are integer multiples of the channel period as required by PIC, multipole fields will tend to cause non-linear resonance. This makes it difficult to find a set of multipoles sufficient for aberration compensation that does not cause beam instabilities.

To overcome this problem, we developed the concept of Skew PIC (SPIC). We introduce coupling in a cooling channel in such a way that the point to point focusing needed for PIC is preserved but the canonical betatron tunes are shifted from their resonant values, i.e. the canonical phase advances in the two planes are shifted from $m\pi$ values. A simple way to think of it is that the beam is azimuthally rotated between consecutive focal points. This moves the dispersion and betatron motion away from non-linear resonances. It also offers a number of other benefits: (a) it allows for control of the dispersion size for chromatic compensation; (b) it reduces the dimensionality of the aberration compensation problem to just the radial dimension and therefore reduces the number of required compensating multipoles; (c) it equates the parametric resonance rates in the two planes, and therefore only one resonance harmonic is needed; (d) it equates the two cooling decrements in the two transverse dimensions. In this paper, we present a design of a SPIC channel and first results of dynamics studies in this channel.

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SKEW PIC IMPLEMENTATION IN MADX

A cooling channel compatible with SPIC is initially implemented using a lumped elements approach in MADX [2] for ease of simulation and to provide a direct analog to the numerical simulations studied [3]. A step-like alternating curvature function K(s) is implemented using thin dipoles of equal strength but opposite curvature. A straight quadrupole extends over the length of the channel. A step-like alternating coupling function g(s) with relative wavelength $\lambda_g=2\lambda_K$ is implemented using zero-length skew quadrupoles inserted between each thin dipole. Figure 1 demonstrates the phasing and periodicity of the curvature and coupling functions implemented in the SPIC channel. The horizontal and vertical tunes are set to 0.5 and 1.0, respectively, by optimizing the dipole strength and straight quadrupole strength in the absence of coupling. Coupling is then introduced, and the dipole and straight quadrupole strengths are readjusted to reach the SPIC solution. The nonzero coupling shifts the tunes away from their initial integer and half-integer values. The SPIC solution is obtained by constraining particular values in the transfer matrix from one periodic absorber position to the next. Figure 2 shows the reference orbit geometry for one period of a SPIC-compatible channel.

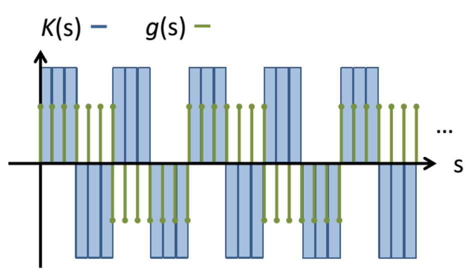


Figure 1: Schematic demonstrating periodicity and phasing of curvature and coupling functions.

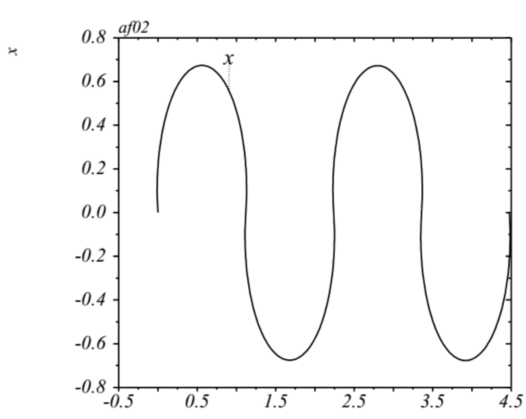


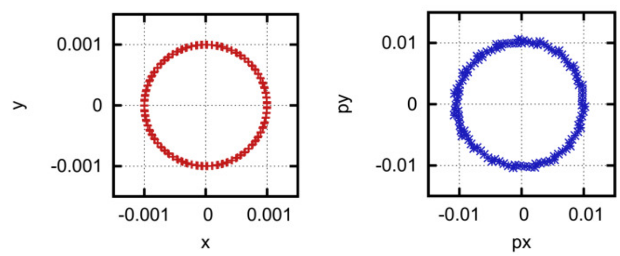
Figure 2: Reference orbit geometry of a SPIC channel.

Single particles are tracked through the SPIC channel to verify the dynamic features of the coupled correlated optics induced in the SPIC channel.

SKEW PIC DYNAMIC FEATURES

Periodic Rotational Behavior

The correlated optics of the previously studied PIC solution naturally defines periodic positions for the absorbers used for ionization cooling due to the periodic nature of the x and y transverse motion. The introduction of coupling in the SPIC channel results in x and y transverse motion that is no longer periodic in each respective plane; the transverse motion instead becomes radially periodic. At these periodic positions, the particle rotates in x-y phase space as shown in Figure 3. Because the SPIC solution enforces little to no coupling between particle coordinates and angles, the angular component also rotates in p_x-p_y phase space, also shown in Figure 3.

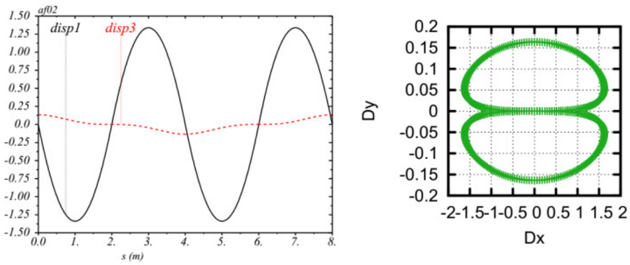


(a) (b)

Figure 3: (a) Particle rotation in x-y phase space at periodic points in SPIC channel. (b) Particle rotation in p_x-p_y phase space at same periodic points in SPIC channel. Note that these plots were generated using two separate particles.

Periodic Alternating Dispersion

The introduction of coupling with skew quadrupoles into the PIC channel induces a vertical dispersion component that oscillates with a period equal to the period of the coupling function. The magnitude of the vertical dispersion function is proportional to the coupling strength. SPIC requires small dispersion at the periodic absorber locations to minimize the energy straggle within the absorber, and thus the relative phase shift between the curvature function K(s) and the coupling function g(s) is adjusted to meet these conditions. We note that the vertical dispersion function generally has maxima at the intended absorber positions where the horizontal dispersion function has negative-going zero crossings, and thus the condition of small dispersion at the absorber positions can be met with small coupling strength. Figure 4 illustrates the horizontal and vertical dispersion functions in the SPIC channel.



(a) (b)

Figure 4: (a) Horizontal (black) and vertical (red) dispersion functions in one periodic cell of SPIC channel. (b) Horizontal and vertical dispersion components parameterized in s. The vertical dispersion has maxima at zeros of the horizontal dispersion.

INDUCING PARAMETRIC RESONANCE

Parametric resonant behavior was induced in the SPIC channel with quadrupoles of sinusoidally-varying strength. As in [1], the resonance focal point was made to coincide with the periodic absorber position by adjusting the period and phase of the parametric resonance quadrupoles. We note here that while two separate sets of quadrupoles were necessary to induce a half-integer parametric resonance in each of the transverse planes of the PIC channel, only one set is required for the SPIC channel due to the coupling between the planes. These quadrupoles were implemented in the SPIC channel by adding a normal quadrupole component of appropriate strength to each of the existing zero-length skew quadrupoles. Figure 5 demonstrates the phasing and periodicity of the parametric resonance inducing quadrupoles.

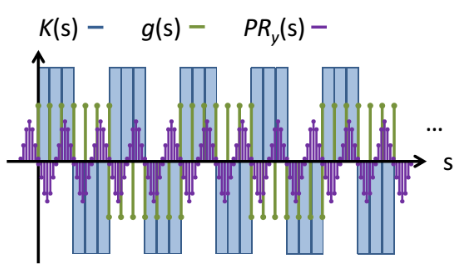
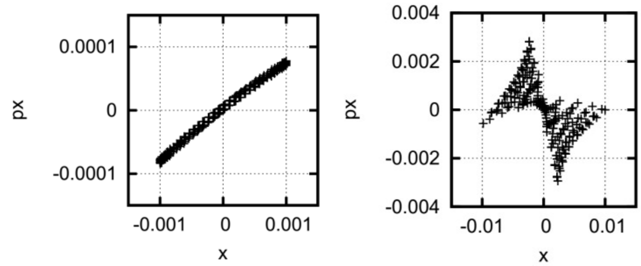


Figure 5: Schematic demonstrating periodicity and phasing of parametric resonance inducing quadrupoles with respect to curvature and coupling functions.

A distinctive feature of the parametric resonance is the periodic transformation of the transverse phase space motion from elliptical orbits to hyperbolic motion. The particle amplitudes are damped over successive periods; the corresponding angular growth eventually leads to instability and loss in the absence of the intended ionization cooling. This hyperbolic transformation is demonstrated in Figure 6.



(a) (b)

Figure 6: (a) Elliptical x-p_x phase space at periodic locations in SPIC channel. (b) Hyperbolic x-p_x phase space at same periodic locations in SPIC channel due to parametric resonance.

The amplitude damping/angular growth rate varies with the strength of the induced parametric resonance. A strong parametric resonance is ideal for fast damping and fast ionization cooling, but the motion can become unstable quickly and must be controlled.

SUMMARY

A first pass at implementation of a channel compatible with Skew Parametric-resonance Ionization Cooling (SPIC) has been performed using a lumped elements approach in MADX. The coupled correlated optics of the channel results in particle motion that is radially periodic at positions intended for ionization cooling. Particle tracking simulations have demonstrated this dynamic behavior as well as the expected phase space transformation in the presence of a parametric resonance. These initial results are promising and form the basis for further SPIC simulation studies.

ACKNOWLEDGMENTS

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