

Baryon asymmetry from primordial black holes

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We propose a new scenario of the baryogenesis from primordial black holes (PBH). Assuming the presence of microscopic baryon (or lepton) number violation, and the presence of an effective CP-violating operator such as $\partial_\alpha F(\mathcal{R}_{...})J^\alpha$, where $F(\mathcal{R}_{...})$ is a scalar function of the Riemann tensor and J^α is a baryonic (leptonic) current, the time evolution of an evaporating black hole generates baryonic (leptonic) chemical potential at the horizon; consequently PBH emanates asymmetric Hawking radiation between baryons (leptons) and antibaryons (leptons). Though the operator is higher-dimensional and largely suppressed by a high mass scale M_* , we show that a sufficient amount of asymmetry can be generated for a wide range of parameters of the PBH mass M_{PBH} , its abundance Ω_{PBH} , and the scale M_* .

Subject Index B70, B73, E72, E76

1. Introduction

The standard model of particle physics is completed by the discovery of the Higgs boson, and is surprisingly consistent with the experimental data up to the 1 TeV scale. However, there still remain several unsolved questions, e.g., what is the dark matter in the universe? and why are baryons more abundant than antibaryons?

In order to answer these questions, gravitational effects might play important roles. One of the interesting possibilities for gravitational effects will be primordial black holes (PBH) (Refs. [1,2]), which may be created in the early universe. PBHs could be formulated in the early universe by various processes such as large density fluctuations by inflation (Ref. [3]), preheating (Ref. [4]), in particular the tachyonic preheating (Ref. [5]), or bubble collisions (Ref. [6]) associated with first-order phase transitions in the universe (Ref. [7]).¹ A PBH evaporates by Hawking radiation until the present time if its mass M is lighter than $M = 10^{15}$ g. Consequently the abundance of the PBHs around $M = 10^{15}$ g is strongly constrained by observations of the cosmic gamma ray (Refs. [10,11]). If the mass is between 10^9 g and 10^{13} g, the Hawking radiation from the PBHs affects the big bang nucleosynthesis (BBN) and the abundance in this mass region is also strongly constrained (see, e.g., Ref. [12]). A PBH with larger mass also plays various important roles in cosmology. It may contribute to the dark matter in the universe for $M \gtrsim 10^{15}$ g although its abundance is severely constrained (Refs. [13,14]). PBHs may also explain the origin of BHs with mass $M = \mathcal{O}(10^{30})$ g

¹ See also Ref. [8] for discussion on PBH formation within the recently proposed framework of graviton condensates (Ref. [9]).

(Refs. [15,16]), whose binary mergers are observed in the recent detections of gravitational waves by LIGO (Ref. [17]).

On the other hand, PBHs with smaller mass $M < 10^8$ g will play a different role. One of the important roles of lighter PBHs will be to give a stage for generating baryon asymmetry. Hawking (Ref. [18]), Carr (Ref. [19]), and Barrow (Ref. [20]) proposed a scenario of baryogenesis in which grand unified theory-scale particles/right-handed neutrinos are created by Hawking radiation and then decay in a C- and CP-violating manner (see also Refs. [21–24]). Recently Hook proposed a different scenario for baryogenesis by using asymmetric Hawking radiation due to a dynamically generated baryonic chemical potential at the horizon (Ref. [25]). There the CP-violating interaction of the baryonic (or leptonic) current J^α and the scalar curvature \mathcal{R} ,

$$\frac{1}{M_*^2} \partial_\alpha \mathcal{R} J^\alpha, \quad (1)$$

is assumed, and the time evolution of the universe is used to generate the chemical potential $\mu = \dot{\mathcal{R}}/M_*^2$ for baryons. The same interaction is used in gravitational baryogenesis (Ref. [26]). Indeed, the mechanism (Ref. [25]) essentially utilizes the idea of spontaneous baryogenesis (Refs. [27,28]) and gravitational baryogenesis (Ref. [26]) scenarios.

In this paper, we propose a new mechanism for the baryogenesis from evaporating PBHs. The mechanism is similar to that of Ref. [25], but instead of using the time evolution of the universe, we make use of the time evolution of the mass of the PBH itself for generating the baryonic chemical potential.² This leads to a big difference between our mechanism and Ref. [25]. Since the scalar curvature around the PBH in a vacuum is vanishing, we need to use higher-dimensional operators such as

$$\frac{1}{M_*^4} \partial_\alpha (\mathcal{R}_{\mu\nu\rho\sigma} \mathcal{R}^{\mu\nu\rho\sigma}) J^\alpha, \quad (2)$$

where $\mathcal{R}_{\mu\nu\rho\sigma}$ is the Riemann tensor.³ The operator is largely suppressed by the scale M_* and seems to be negligible but we show that it is sufficient to generate the desired asymmetry. This is due to the fact that the dynamically generated chemical potential μ , as well as the temperature of the Hawking radiation T_H , is time-dependent through the mass of the BH. Indeed μ/T_H is increasing as the PBH evaporates and becomes larger than 1 at the late stage of the evaporation. Consequently the asymmetry in the Hawking radiation becomes maximal after the PBH mass becomes smaller than a critical mass. The critical mass is determined by the scale M_* .

² A similar idea is proposed in Ref. [29]. Note that by chemical potential, we here mean asymmetry of propagations between particles and antiparticles due to the interaction with the background geometry. If particles enter thermal equilibrium, the distributions become asymmetric. In the case of Hawking radiation, the radiation from black holes becomes asymmetric as if there is a chemical potential. Hence, in the present paper we call the term μJ^0 a chemical potential term.

³ Our investigation does not depend much on the specific form of the higher-dimensional operators. We can instead use the Gauss–Bonne type

$$\partial_\mu (\mathcal{R}^2 + \mathcal{R}^{\mu\nu\rho\sigma} \mathcal{R}_{\mu\nu\rho\sigma} - 4\mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu}) J^\mu,$$

which can be transformed to the scalaron picture (Ref. [30]), and is safe from the point of view of ghost modes. The first and the third terms in the Gauss–Bonne term vanish around the Schwarzschild BH, so it gives the same effective CP-violating operator as the one we introduced.

Once the chemical potential is generated at the horizon, Hawking radiation produces lepton/baryon asymmetry. Since the sphaleron process (Refs. [31–33]) violates $B + L$, it is necessary to generate $B - L$ if the typical Hawking temperature is higher than 100 GeV. We thus assume violation of the $B - L$ number in the underlying microscopic theories such as interactions with right-handed neutrinos or some effects related to the quantum gravity (Ref. [34]). CP symmetry is broken by the effective operator (2). The time dependence of the PBH mass due to Hawking radiation induces time-dependent and position-dependent chemical potential, which is apparently a nonequilibrium process. In this way, Sakharov's three conditions (Ref. [35]) are satisfied in the present model.⁴

The paper is organized as follows. In the next section, we explain the basic mechanism of the scenario, and estimate the order of the asymmetry. We show that in some region of the parameter space of the PBH mass M_{PBH} and the scale M_* , the desired asymmetry $n_B/s \simeq 8.7 \times 10^{-11}$ (Ref. [36]) can be generated. A possible origin of the higher-dimensional operator (2) is given in Sect. 3, and we estimate the order of the scale M_* . In Sect. 4 we show that washout of the generated lepton number outside the horizon does not occur for the typical interaction discussed in Sect. 3. Section 5 is devoted to a summary and discussions. In Appendixes A and B, we discuss Hawking radiation with the chemical potential. In Appendix C, we give an analytical approximation of the function $g_n(X)$ used in Sect. 2.

2. Baryo- (lepto-) genesis at the BH horizon

2.1. CP-violating interactions

The scenario of gravitational baryogenesis (Ref. [26]) assumes the CP-violating interaction (1) where M_* is the scale of the underlying theory that generates such an interaction. In an expanding universe, the time derivative of the scalar curvature $\dot{\mathcal{R}}$ is nonzero and the interaction generates a chemical potential⁵ $\mu = \dot{\mathcal{R}}/M_*^2$. If a B -violating interaction is present⁶ and the system is in thermal equilibrium, the distribution becomes asymmetric between baryons and antibaryons. Then once the temperature drops below the freezing-out temperature of the B -violating interaction, the asymmetry remains in the later universe. The scenario is applied to the evaporating BH by the Hawking radiation (Ref. [25]). The term μJ^0 is similarly generated by the evolution of the universe and the Hawking radiation becomes asymmetric, but the freezing-out scenario is different. Since the thermal radiation from black holes is created, not by the thermal process of B -violating interaction, but by a genuine quantum process, the condition of the thermal equilibrium and the freezing-out in (Ref. [26]) does not need to be introduced in the analysis of (Ref. [25]).

In this section, we generalize the idea of the gravitational baryogenesis from a PBH (Ref. [25]) by taking the direct effect of the decay of the PBH mass $M(t)$. Since the scalar curvature \mathcal{R} vanishes outside the BH in vacuum,⁷ we consider an operator such as in Eq. (2). More generally, we can

⁴ It is often stated that Sakharov's three conditions are not necessary in spontaneous baryogenesis. In the present scenario, since the CP parity of $F(\mathcal{R} \dots) \propto \mathcal{R}_{\mu\nu\lambda\sigma}^2$ is even, the higher-dimensional operator breaks C and CP symmetries.

⁵ As explained in footnote 2, the energy spectrum becomes asymmetric between particles and antiparticles. Then, as long as the typical time scale of the interaction is smaller than that of the expansion of the universe, particles enter in thermal equilibrium and μJ^0 term can be interpreted as a chemical potential.

⁶ In the present Sect. 2.1, for simplicity, we use B (aryon) to represent the current J^μ . It can be either baryons or leptons but a necessary condition is that it has nonvanishing $B - L$ charge.

⁷ In Refs. [25,26], the radiation-dominated universe with the trace anomaly of the energy-momentum tensor is studied so as to make \mathcal{R} nonvanishing.

consider a class of higher-dimensional operators,⁸

$$\frac{a_n}{M_*^{4n}} \partial_\alpha (\mathcal{R}_{\mu\nu\rho\sigma} \mathcal{R}^{\mu\nu\rho\sigma})^n J^\alpha, \quad n \geq 2. \quad (3)$$

It can be further generalized to

$$\partial_\alpha F(\mathcal{R}_{...}) J^\alpha \quad (4)$$

where $F(\mathcal{R}_{...})$ is any scalar function made of the curvature tensors. For the square of the Riemann curvature of the Schwarzschild BH given by⁹

$$\mathcal{R}_{\mu\nu\rho\sigma} \mathcal{R}^{\mu\nu\rho\sigma} = \frac{3M^2}{4\pi^2 M_P^4 r^6}, \quad (5)$$

a nonvanishing chemical potential $\mu = a_n \partial_0 (\mathcal{R}_{\mu\nu\rho\sigma} \mathcal{R}^{\mu\nu\rho\sigma})^n / M_*^{4n}$ is generated if the BH mass M is decaying. Here we have introduced the reduced Planck scale,

$$M_P := (8\pi G)^{-1/2} = 2.43 \times 10^{18} \text{ GeV} = 4.3 \times 10^{-6} \text{ g} = (2.7 \times 10^{-43} \text{ s})^{-1}, \quad (6)$$

where G is the Newton constant. Note that the chemical potential is dependent on time through $M(t)$. It also changes with distance r from the BH. Since the Hawking radiation is generated by the Bogoliubov transformation between the vacua of quantum fields near the horizon and at far-infinity from the BH, the chemical potential near the horizon is relevant to generation of the asymmetry of the Hawking radiation. The propagations of baryon and antibaryon become different in the vicinity of the horizon, which shift the energy between them. Accordingly the generated asymmetry is proportional to the chemical potential evaluated at the horizon $r = r_H$. Here we note that even if we instead evaluate the chemical potential at, e.g., $r \simeq 2r_H$, it does not change our conclusion very much (see the second paragraph of Sect. 5).

2.2. Basic properties of evaporating BH

We summarize some basic facts about an evaporating BH. For simplicity¹⁰ we consider the Schwarzschild black hole with the metric,

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \frac{1}{\left(1 - \frac{2GM}{r}\right)} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2, \quad (7)$$

where M is the mass of the black hole. The radius of the horizon and the Hawking temperature are given by

$$r_H = 2GM = \frac{M}{4\pi M_P^2},$$

⁸ In addition to the CP-violating interaction, we implicitly assume the existence of a B -violating operator since otherwise this term vanishes by performing an integration by parts. See Sect. 3 and Appendix A for a discussion of the physical meaning of this operator.

⁹ See, e.g., Ref. [37].

¹⁰ In general, a BH can have charge and angular momentum, but these would be quickly lost before most of the BH mass disappears (Refs. [38–40]).

$$T_H = \frac{M_P^2}{M}. \quad (8)$$

Through the Hawking radiation, particles are emitted (Ref. [41]) from the BH with the rate

$$\begin{aligned} \frac{dN}{d\omega dt} &= \frac{1}{2\pi} \frac{\Gamma}{\exp(\omega/T_H) \pm 1}, \\ \frac{dE}{d\omega dt} &= \frac{1}{2\pi} \frac{\Gamma\omega}{\exp(\omega/T_H) \pm 1}, \end{aligned} \quad (9)$$

where N, E are the number and energy of emitted particles, ω is the frequency, and Γ is the absorption probability (or gray-body factor), which is caused by gravitational scatterings of emitted particles outside the horizon. The absorption probability depends on particle species, especially on the spin of the emitted particles. At low frequency, $\omega \rightarrow 0$, the absorption cross sections $\sigma := \pi\omega^{-2}\Gamma$ of spin 0 and 1/2 particles are constant while those of spin 1 and 2 particles are proportional to the square and fourth power of frequency, respectively:

$$\sigma \rightarrow \begin{cases} \text{const.} & \text{for spin 0 and 1/2,} \\ \omega^2 & \text{for spin 1,} \\ \omega^4 & \text{for spin 2.} \end{cases} \quad (10)$$

As a result, most of the energy emitted from the PBH is carried by scalars and fermions (Ref. [42]).

We emphasize that the spectrum of the Hawking radiation is (almost) thermal, not because thermal plasma at temperature T_H is realized due to sufficiently fast interactions between emitted particles, but simply because the quantum vacuum at the horizon behaves as if it is in the thermal equilibrium for an observer at far-infinity. In fact, even extremely weakly coupled particles (such as gravitons), that can be thermalized only at $T_H \gtrsim M_P$, are emitted according to the Hawking thermal spectrum.

Once we take into account the Hawking radiation, the spacetime is no longer stationary, and the metric (7) is no longer appropriate to describe the evaporating BH. Since the mass in Eq. (7) is the Arnowitt-Deser-Misner mass, which includes the energy of the emitted radiation, it cannot correctly describe the mass of a decaying BH itself. The simplest alternative is the outgoing Vaidya metric (Ref. [43]), which is a solution of the Einstein equation describing outgoing null dust:

$$ds^2 = \left(1 - \frac{2GM(u)}{r}\right) du^2 + 2du dr - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2, \quad (11)$$

where $u = t - r_*$, $r_* = r + 2M \log |(r - 2M)/2M|$. The apparent horizon is located at $r = r_H = 2GM(u)$. The corresponding energy-momentum tensor is given by

$$T_{\mu\nu} = -\frac{dM}{du} \frac{1}{4\pi r^2} l_\mu l_\nu, \quad l_\mu = \partial_\mu u, \quad (12)$$

which describes dust with energy density $\rho = (-dM/du)/(4\pi r^2)$ moving with a four-velocity l^μ . The mass $M(u)$ represents the Bondi mass, which is nothing but the mass of the BH itself. In the following, we consider the time-evolution of the Bondi mass $\partial_u M(u)$.

Because of the (almost) thermal Hawking radiation, the black hole loses its energy following

$$\frac{dM}{du} \simeq -(8\pi)^2 \frac{M_P^4}{M^2} \alpha, \quad (13)$$

where α is a numerical coefficient (Ref. [42]) that can be determined by taking the effects of the absorption cross section σ . As we discussed above, the dominant contribution to α in the standard model comes from fermions. In Ref. [42], it is shown that the contribution to α from ν_e and ν_μ is 1.575×10^{-4} . Then, summing all the fermionic degrees of freedom in the standard model, we obtain¹¹

$$\begin{aligned}\alpha &= 1.575 \times 10^{-4} \times \frac{3}{2} \times (1 + 2 + 4 \times 3) \\ &= 3.5 \times 10^{-3}.\end{aligned}\quad (14)$$

Solving Eq. (13), the time dependence of M is given by

$$M(u) = M_{\text{PBH}} \left(1 - \frac{u - u_{\text{ini}}}{\tau}\right)^{1/3}, \quad (15)$$

and the lifetime of the black hole τ becomes

$$\tau = \frac{M_{\text{PBH}}^3}{M_{\text{P}}^4} \frac{1}{3(8\pi)^2 \alpha}. \quad (16)$$

Here we take $M = M_{\text{PBH}}$ at the initial time $u = u_{\text{ini}}$. From Eq. (15), we can see that the PBH completely evaporates until today if M_{PBH} is smaller than $10^{20} M_{\text{P}} \sim 10^{15}$ g.

2.3. Dynamically generated chemical potential

Since the square of the Riemann tensor outside a PBH is

$$\mathcal{R}_{\mu\nu\rho\sigma} \mathcal{R}^{\mu\nu\rho\sigma} = \frac{12r_{\text{H}}^2}{r^6} = \frac{3M(u)^2}{4\pi^2 M_{\text{P}}^4 r^6}, \quad (17)$$

we have a chemical potential μ , if the CP-violating interaction (2) is present:

$$\mu = \frac{3}{2\pi^2} \frac{M}{M_{\text{P}}^4 M_*^4 r^6} \frac{dM}{du} \simeq -96\alpha \frac{1}{MM_*^4 r^6}. \quad (18)$$

By taking $r = r_{\text{H}}$, the chemical potential evaluated at the horizon becomes

$$\mu|_{r=r_{\text{H}}} = -\frac{3}{2}(8\pi)^6 \alpha M_{\text{P}} \left(\frac{M_{\text{P}}}{M}\right)^7 \left(\frac{M_{\text{P}}}{M_*}\right)^4. \quad (19)$$

Then the ratio of $\mu|_{r_{\text{H}}}$ to the Hawking temperature $T_{\text{H}} = M_{\text{P}}^2/M$ is given by

$$\frac{\mu|_{r_{\text{H}}}}{T_{\text{H}}} = -\frac{3}{2}(8\pi)^6 \alpha \left(\frac{M_{\text{P}}}{M}\right)^6 \left(\frac{M_{\text{P}}}{M_*}\right)^4 = -\left(\frac{M_{\text{cr}}}{M}\right)^6, \quad (20)$$

where we have defined the critical mass M_{cr} by

$$M_{\text{cr}} = 8\pi M_{\text{P}} \left(\frac{3\alpha}{2}\right)^{1/6} \left(\frac{M_{\text{P}}}{M_*}\right)^{2/3} \sim 10 \times \left(\frac{M_{\text{P}}}{M_*}\right)^{2/3} M_{\text{P}}. \quad (21)$$

¹¹ Here we assume that the value of α is the same for all the fermionic degrees of freedom, and the coefficients in the parentheses are 1 for the $SU(2)_L$ singlet lepton, 2 for the doublet lepton, and 4×3 for up and down quarks with color degrees of freedom. The coefficient $3/2$ is a transformation factor from the 2 generation calculation to 3.

Note that the (absolute value of the) ratio is increasing as the BH mass M decreases. For $M < M_{\text{cr}}$, the ratio exceeds 1 and the asymmetry of the radiation becomes maximal. This indicates that if the initial mass of the PBH is smaller than the critical mass, only baryons are emitted.¹²

Parametrizing the scale M_* as $M_* = 10^y M_{\text{P}}$ and the initial mass of the PBH as $M_{\text{PBH}} = 10^y M_{\text{P}}$, the condition of $M_{\text{PBH}} = M_{\text{cr}}$ becomes

$$y = 1 - \frac{2}{3}x. \quad (22)$$

We will see later that this relation plays an important role in generating the asymmetry.

2.4. Leptogenesis from a PBH

Now let us explicitly calculate the asymmetry produced by the evaporation of *one* PBH. Since we have in mind a model in which the CP-violating interaction is induced by interactions with right-handed neutrinos, we hereafter suppose that the chemical potential induced at the horizon is the leptonic one. Then, as we see that the temperature of the universe at the epoch of evaporation is much higher than the electroweak scale, sphaleron processes transmute the generated leptons into baryons.

First note that the averaged energy per each emitted massless particle is

$$\langle E \rangle = \frac{n_{\text{other}}}{n_{\text{tot}}} \langle E_{\text{other}} \rangle + \frac{n_L}{n_{\text{tot}}} \langle E_L \rangle + \frac{n_{\bar{L}}}{n_{\text{tot}}} \langle E_{\bar{L}} \rangle, \quad (23)$$

where the subscripts L (\bar{L}) and “other” represent the leptons (antileptons) and the other emitted particles (including only scalars and fermions) in the standard model respectively. The number density n_i for each species is given by

$$\begin{aligned} n_{\text{other}} &= \frac{g_{\text{other}}}{(2\pi)^3} \int d^3k (\exp(k/T_{\text{H}}) + 1)^{-1} = g_{\text{other}} \frac{3\zeta(3)}{4\pi^2} T_{\text{H}}^3, \\ n_L &= \frac{g_L}{(2\pi)^3} \int d^3k (\exp((k + \mu)/T_{\text{H}}) + 1)^{-1} = -\frac{g_L}{\pi^2} T_{\text{H}}^3 \text{Li}_3(-\exp(-\mu/T_{\text{H}})), \\ n_{\bar{L}} &= -\frac{g_{\bar{L}}}{\pi^2} T_{\text{H}}^3 \text{Li}_3(-\exp(\mu/T_{\text{H}})), \end{aligned} \quad (24)$$

where g_i is the internal degrees of freedom. In the case of the standard model, $g_L = g_{\bar{L}} = 9$ and $g_{\text{other}} = 76$; $n_{\text{tot}} = n_{\text{other}} + n_L + n_{\bar{L}}$ is the total number. The polylogarithmic function $\text{Li}_a(z)$ is defined by $\text{Li}_a(z) := \sum_{k=1}^{\infty} z^k/k^a$. In the present convention, $n_L > n_{\bar{L}}$ since $\mu < 0$. Of course, it is reversed if the sign of the coefficient of the CP-violating operator is reversed. The averaged energy for each species is $\langle E_i \rangle$ and the explicit expressions are

$$\begin{aligned} \langle E_{\text{other}} \rangle &= \frac{\int d^3k k (\exp(k/T_{\text{H}}) + 1)^{-1}}{\int d^3k (\exp(k/T_{\text{H}}) + 1)^{-1}} = \frac{7\pi^4}{180\zeta(3)} T_{\text{H}}, \\ \langle E_L \rangle &= \frac{\int d^3k k (\exp((k + \mu)/T_{\text{H}}) + 1)^{-1}}{\int d^3k (\exp((k + \mu)/T_{\text{H}}) + 1)^{-1}} = 3T_{\text{H}} \frac{\text{Li}_4(-\exp(-\mu/T_{\text{H}}))}{\text{Li}_3(-\exp(-\mu/T_{\text{H}}))}, \\ \langle E_{\bar{L}} \rangle &= 3T_{\text{H}} \frac{\text{Li}_4(-\exp(\mu/T_{\text{H}}))}{\text{Li}_3(-\exp(\mu/T_{\text{H}}))}. \end{aligned} \quad (25)$$

¹² Of course, particles without the baryon number are also emitted.

Notice that $\langle E_L \rangle$ behaves as $\langle E_L \rangle \sim T_H$ for $|\mu| < T_H$, and $\langle E_L \rangle \sim |\mu|$ for $T_H < |\mu|$.

By using these formulas, the lepton number asymmetry can be estimated as

$$\delta N_L = \int_{M_{\min}}^{M_{\text{PBH}}} \frac{dM}{\langle E \rangle} \frac{n_L + n_{\bar{L}}}{n_{\text{tot}}} \frac{n_L - n_{\bar{L}}}{n_L + n_{\bar{L}}}. \quad (26)$$

Here we have introduced the lower cutoff M_{\min} for the mass of the PBH, under which the typical energy scale of the Hawking radiation becomes higher than M_* and the present analysis becomes questionable. It is determined either by the condition $T_H = M_*$ or by $|\mu| = M_*$, and given by

$$M_{\min} = \max \left(\frac{M_{\text{P}}^2}{M_*}, \left(\frac{M_{\text{cr}}^6 M_{\text{P}}^2}{M_*} \right)^{1/7} \right). \quad (27)$$

For $M < M_{\min}$, either T_H or $|\mu|$ is larger than the scale M_* , and the present investigations are no longer valid.¹³ Then changing the integration variable in Eq. (26) from M to $X = (M/M_{\text{cr}})^2$, δN_L becomes

$$\delta N_L = \frac{1}{2} \left(\frac{M_{\text{cr}}}{M_{\text{P}}} \right)^2 f(X_{\min}, X_0). \quad (28)$$

The function $f(X_{\min}, X_0)$ is defined by an integral $f(X_{\min}, X_0) = \int_{X_{\min}}^{X_0} dX g(X)$, where the integrand is given by

$$g(X) = \frac{\frac{g_L}{\pi^2} (-\text{Li}_3(-\exp(-1/X^3)) + \text{Li}_3(-\exp(1/X^3)))}{\frac{7\pi^2 g_{\text{other}}}{240} + \frac{3g_L}{\pi^2} (-\text{Li}_4(-\exp(-1/X^3)) - \text{Li}_4(-\exp(1/X^3)))},$$

and

$$X_0 = \left(\frac{M_{\text{PBH}}}{M_{\text{cr}}} \right)^2, \quad X_{\min} = \max \left(\left(\frac{M_{\text{P}}^2}{M_{\text{cr}} M_*} \right)^2, \left(\frac{M_{\text{P}}^2}{M_{\text{cr}} M_*} \right)^{2/7} \right). \quad (29)$$

Note that, from Eq. (21), X_{\min} is given by

$$X_{\min} = \begin{cases} \left(\frac{M_{\text{P}}^2}{M_{\text{cr}} M_*} \right)^2 \geq 1 & \text{for } M_* < \frac{M_{\text{P}}}{256\pi^3 \sqrt{6\alpha}} \sim 10^{-3} M_{\text{P}}, \\ \left(\frac{M_{\text{P}}^2}{M_{\text{cr}} M_*} \right)^{2/7} \leq 1 & \text{for } M_* > \frac{M_{\text{P}}}{256\pi^3 \sqrt{6\alpha}}. \end{cases} \quad (30)$$

The function $g(X)$ is depicted numerically in Fig. 1. One can see that it is peaked around $X = \mathcal{O}(1)$, which indicates that the dominant lepton number asymmetry is produced when the BH mass is comparable with the critical mass. The damping behavior of $g(X)$ for large values of X means that

¹³ It does not necessarily mean that the asymmetry is not produced for $M < M_{\min}$, but for the validity of the analysis, we exclude the region from the integral. Since the dominant asymmetry is produced near (or a bit less than) $M \sim M_{\text{cr}}$, the produced asymmetry is not affected by the introduction of the cutoff unless $\mu = T_H$ at the critical mass M_{cr} is smaller than M_* . In the case of $M_{\min} < M_{\text{cr}}$, our analysis might provide a conservative value of the asymmetry.

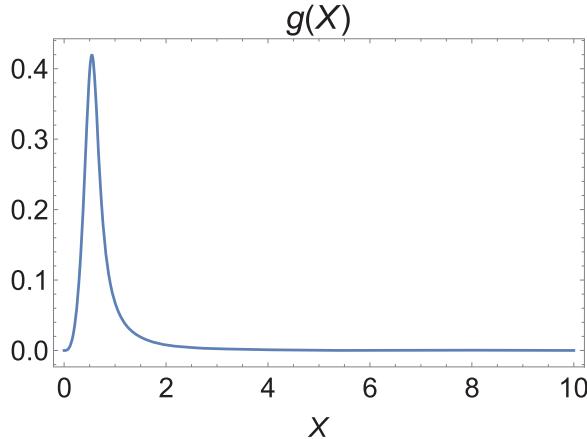


Fig. 1. The function $g(X)$ is plotted with the SM values of $g_L = g_{\bar{L}} = 9$ and $g_{\text{other}} = 76$. Since the generated lepton number is proportional to the integral of $g(X)$, the asymmetry is efficiently produced around $X \sim 0.5$, namely $M \sim 0.7M_{\text{cr}}$. For larger values of $X = (M/M_{\text{cr}})^2$, the chemical potential $|\mu|$ becomes negligibly small. For smaller values, the asymmetry is maximal but the number of emitted particles is reduced due to large $|\mu|$.

when the BH mass is larger than M_{cr} , the radiation is almost symmetric and never contributes to the lepton number. On the other hand, the damping in small values of X implies that the chemical potential of leptons is too large, and the number of emitted leptons is significantly suppressed. See also Appendix C for the behavior and analytical approximations of $f(X_{\min}, X_0)$ and $g(X)$.

2.5. Lepton asymmetry in the universe

In order to discuss the lepton asymmetry in the universe, we briefly discuss the cosmological history of the light PBHs. When the density perturbation becomes as large as the order one $\delta\rho/\rho \sim 1$, the PBH can be formed. The mass of the PBH is determined by the energy within the Hubble horizon, namely,

$$M_{\text{PBH}} \simeq 4\pi\gamma M_{\text{P}}^2 H_{\text{ini}}^{-1}, \quad (31)$$

where γ is a numerical factor depending on the details of the gravitational collapse. Since it is usually considered to be $\gamma \lesssim 0.2$, we take $\gamma = 0.2$ for simplicity. Here H_{ini} is the Hubble parameter at the time of the PBH formation. After it is formed, it emanates the Hawking radiation and when the Hubble parameter becomes the inverse of the PBH lifetime,

$$H_{\text{eva}} \simeq \frac{M_{\text{P}}^4}{M_{\text{PBH}}^3} 3(8\pi)^2 \alpha, \quad (32)$$

the PBH evaporates completely. Thus the ratio of the Hubble parameters is given by

$$\frac{H_{\text{eva}}}{H_{\text{ini}}} = 48\pi\alpha\gamma^{-1} \left(\frac{M_{\text{P}}}{M_{\text{PBH}}} \right)^2. \quad (33)$$

If the universe continues to be in the radiation-dominated phase, the Hubble parameter is related to the scale factor of the universe as $H \propto a^{-2}$. Then the ratio of the scale factors a during the

evaporation is given by

$$\left(\frac{a_{\text{eva}}}{a_{\text{ini}}}\right) = \left(\frac{\gamma}{48\pi\alpha}\right)^{1/2} \left(\frac{M_{\text{PBH}}}{M_{\text{P}}}\right). \quad (34)$$

Since the energy density of the universe changes as

$$\rho(t) = \rho_{\text{rad}}(t_i) \left(\frac{a(t_i)}{a(t)}\right)^4 + \rho_{\text{PBH}}(t_i) \left(\frac{a(t_i)}{a(t)}\right)^3, \quad (35)$$

the ratio of the energy density of PBHs ρ_{PBH} to the total energy density of the universe ρ_{rad} increases as the universe expands. Of course, the evaporation transfers the energy from the PBH to the radiation component and the actual evolution is more complicated (Ref. [44]). It is not further discussed in the present paper.

The temperature of radiation just after the PBH evaporation T_{eva} can be estimated from

$$H_{\text{eva}} \simeq \left(\frac{\pi^2 g_*}{90}\right)^{1/2} \frac{T_{\text{eva}}^2}{M_{\text{P}}}. \quad (36)$$

Here g_* is the effective degrees of freedom. Then we have

$$T_{\text{eva}} \sim 1.1 \times 10^{11} \text{ GeV} \left(\frac{\alpha}{3.5 \times 10^{-3}}\right)^{1/2} \left(\frac{106.75}{g_*}\right)^{1/4} \left(\frac{10^5 M_{\text{P}}}{M_{\text{PBH}}}\right)^{3/2}. \quad (37)$$

Notice that the typical value of T_{eva} is much higher than the electroweak scale for $M_{\text{PBH}} \lesssim 10^{11} M_{\text{P}} \sim 10^5$ g, and the lepton asymmetry produced by the PBH evaporation can be converted into the baryon asymmetry by the sphaleron process.

We also note that the Hubble parameter H_{ini} must be smaller than the Hubble parameter during inflation $H_{\text{inf}} \sim 10^{14} \sqrt{r/0.1}$ GeV, where r is the tensor to scalar ratio. Thus we have the lower bound on the PBH,

$$M_{\text{PBH}} \gtrsim \frac{4\pi\gamma M_{\text{P}}^2}{H_{\text{inf}}} \sim 6 \times 10^4 \left(\frac{0.1}{r}\right)^{1/2} M_{\text{P}}. \quad (38)$$

Only PBHs satisfying the condition can be created in our universe.

Having the above cosmological history in mind, we can estimate the lepton asymmetry after the PBH evaporation,

$$\frac{n_L}{s} = \frac{n_{\text{PBH}}}{s} \delta N_L = \Omega_{\text{PBH}} \frac{\rho_{\text{tot}}}{s} \frac{\delta N_L}{M_{\text{PBH}}}, \quad (39)$$

where $\Omega_{\text{PBH}} = \rho_{\text{PBH}}/\rho_{\text{tot}}$ is the ratio of the energy density of the PBHs to the total energy density at the epoch of evaporation, which includes the radiation from PBHs. Assuming domination of the radiation after the evaporation, and using $\rho_{\text{tot}}/s = 3T_{\text{eva}}/4$, Eqs. (28) and (37), we have

$$\begin{aligned} \frac{n_L}{s} &\simeq 8.7 \times 10^{-9} \left(\frac{106.75}{g_*}\right)^{1/4} \left(\frac{\alpha}{3.5 \times 10^{-3}}\right)^{5/6} \\ &\times \Omega_{\text{PBH}} f(X_{\min}, X_0) \left(\frac{10^5 M_{\text{P}}}{M_{\text{PBH}}}\right)^{5/2} \left(\frac{10^{-2} M_{\text{P}}}{M_*}\right)^{4/3}, \end{aligned} \quad (40)$$

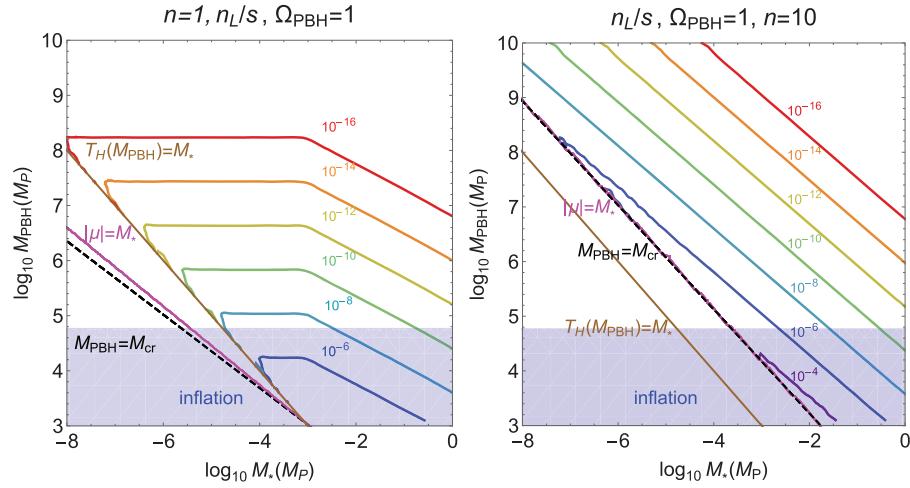


Fig. 2. The contour plot of the lepton asymmetry produced by the evaporation of the PBH. The observed baryon asymmetry is $n_B/s \simeq 8.7 \times 10^{-11}$. The left-hand panel shows the result (Eq. (40)) for $\Omega_{\text{PBH}} = 1$. The dotted line expresses the value of M_{cr} as a function of M_* , and the brown (and magenta) line represents the condition of $T_H = M_*$ (and $|\mu| = M_*$), below which the typical energy scale becomes higher than the scale M_* and the present calculation is no longer valid. The plot is drawn using the formulas in Appendix C. Below $M_* < 10^{-3}M_P$, the produced asymmetry becomes flat and independent of M_* . It is due to our prescription of cutting off the X integral at X_{min} (see the first paragraph of Appendix C and Eq. (30)), and the asymmetry in this region may be interpreted as a conservative estimation, as noted in footnote 13. A PBH with mass lower than $M_{\text{PBH}} \sim 10^5 M_P$ is not created in our universe as discussed in Eq. (38). The graph shows that the baryogenesis from the PBH works as far as $\Omega_{\text{PBH}} > 10^{-2}$. The right-hand panel shows the lepton asymmetry in the case of a generalized CP-violating operator for $n = 10$ discussed in Sect. 2.6. We use Eq. (49) with $a_n = 1/n$ and $\Omega_{\text{PBH}} = 1$. For $n \sim 10$, more asymmetry is efficiently generated compared to the $n = 1$ case in the left-hand panel, and the density ratio of PBH can be as low as $\Omega_{\text{PBH}} = 10^{-6}$. Below the dashed line, the asymmetry is suppressed because the function $g_n(X)$ significantly decreases for $X \lesssim 1$. The magenta line ($|\mu| = M_*$) almost coincides with the dashed line. Unlike the left-hand panel, the produced asymmetry does not become flat because, within the region of M_* in the graph, $X_{\text{min}} < 1$ is always satisfied and the integral is independent of the lower cutoff X_{min} .

which is conserved until now under the assumption that there is no other entropy production. From this rough estimation, we can see that the observed amount of asymmetry, $n_L/s \sim 10^{-10}$, can be successfully produced unless $f(X_{\text{min}}, X_0)$ is too small.

In the left-hand panel of Fig. 2, we plot the lepton asymmetry generated in the presence of the CP-violating interaction (2) for $\Omega_{\text{PBH}} = 1$ as a function of M_* .¹⁴ In order to generate the observed baryon asymmetry $n_B/s \simeq 8.7 \times 10^{-11}$, the mass of the PBH (the vertical axis) and the scale M_* suppressing the interaction must be on the line with $\sim 10^{-10}$. If the density ratio of PBHs is less than 1, i.e., $\Omega_{\text{PBH}} = 10^{-s}$ with $s > 0$, the parameters (M_{PBH}, M_*) must be on the line with a larger value $\sim 10^{s-10}$. There are three lines in the figure. On the dotted line (the lowest line), the initial mass M_{PBH} of PBH is equal to the critical mass M_{cr} . The other two lines represent $|\mu| = M_*$ and $T_H = M_*$. The region below these lines is beyond the reach of this paper since the typical energy scale is larger than M_* .

From Eq. (30), we see that the value $M_* \sim 10^{-3}M_P$ corresponds to $X_{\text{min}} \sim 1$, and the M_* dependence of asymmetry becomes different between the right-hand region with $X_{\text{min}} \lesssim 1$ and the

¹⁴ The definition of M_* should be understood as a renormalized one by the effect discussed in the second paragraph of Sect. 5.

left-hand region with $X_{\min} \gtrsim 1$. For $X_{\min} \lesssim 1$, according to Fig. 1, f (an integral of g) becomes almost independent of X_{\min} . Since the asymmetry is most dominantly generated around the dotted line ($X = 1$) and the critical mass increases as M_* decreases (see Eq. (21)), the asymmetry also increases when M_{PBH} is fixed. On the other hand, for $X_{\min} \gtrsim 1$, f strongly depends on the lower cutoff X_{\min} . It is shown in the first paragraph of Appendix C that the produced asymmetry becomes independent of M_* for $X_{\min} \gtrsim 1$. This is the reason why the generated asymmetry becomes flat as a function of M_* below $M_* \sim 10^{-3} M_P$.

The shaded region is not allowed because below $M_{\text{PBH}} \sim 10^5$ g, a PBH is not created in our universe (Eq. (38)). Therefore, Fig. 2 shows that the baryogenesis from the PBH works as far as $\Omega_{\text{PBH}} > 10^{-2}$ for the simplest CP-violating operator of dimension 8. It is based on our conservative assumption¹⁵ that the lepton asymmetry produced in the region of $T_H > M_*$ is not counted.

2.6. More general CP-violating interactions

So far we have studied the CP-violating interaction of Eq. (2). We extend it to more general higher-dimensional operators introduced in Eq. (3).¹⁶ The calculation is the same as the simplest case discussed so far. The chemical potential is dynamically generated at the horizon and the ratio to the Hawking temperature is given by

$$\left. \frac{\mu}{T_H} \right|_{r=r_H} = - \left(\frac{M_{\text{cr}}}{M} \right)^{4n+2} \quad (41)$$

where the critical mass is given by

$$M_{\text{cr}} =: \sqrt{C_n} M_P, \\ C_n = (na_n)^{1/(2n+1)} (128\alpha\pi^2)^{1/(2n+1)} \left(32\sqrt{3}\pi^2 \right)^{2n/(2n+1)} \left(\frac{M_P}{M_*} \right)^{4n/(2n+1)}. \quad (42)$$

Notice that, compared to the $n = 1$ case in Eq. (21), the exponent of (M_P/M_*) in $\sqrt{C_n}$ is larger and accordingly the critical mass becomes larger. It is good for producing a larger asymmetry since the asymmetry is produced when the BH mass is around the critical mass. Parametrizing M_* as $M_* = 10^x M_P$ and $M_{\text{PBH}} = 10^y M_P$, the condition of $M_{\text{PBH}} = M_{\text{cr}}$ for $n \rightarrow \infty$ with $a_n = 1/n$ becomes

$$y = 1.37 - x. \quad (43)$$

On the other hand, as shown in Eq. (41), the chemical potential becomes too large when M becomes larger than M_{cr} . It is bad for asymmetry generation because it enhances the typical energy $\langle E \rangle \sim \mu$ of emitted particles and consequently reduces the number of particles $dM/\langle E \rangle$ while the BH decreases its mass by dM . These two effects compete as n becomes large.

¹⁵ The assumption is partially based on the fact that the typical energy of the Hawking radiation is given by $T_H (> |\mu|)$ and thus the number of emanated particles is drastically reduced for very high T_H .

¹⁶ A truncation of higher-order terms is assumed in this subsection. In order to estimate the asymmetry starting from the ultraviolet theory such as the model in Sect. 3, it is necessary to compute the full propagator in a curved background without relying on the derivative expansion. We want to come back to this problem in future publications.

By introducing a new integration variable $X = (M/M_{\text{cr}})^2$ as before, we obtain the lepton number emitted from a single PBH as

$$\delta N_L = \frac{1}{2} \left(\frac{M_{\text{cr}}}{M_{\text{P}}} \right)^2 f_n(X_{\text{min}}, X_0), \quad (44)$$

where f_n is an integral $f_n(X_{\text{min}}, X_0) = \int_{X_{\text{min}}}^{X_0} dX g_n(X)$ over the PBH mass X . Here the function g_n is given by

$$g_n(X) = \frac{\frac{g_L}{\pi^2} (-\text{Li}_3(-\exp(-1/X^{2n+1})) + \text{Li}_3(-\exp(1/X^{2n+1})))}{\frac{7\pi^2}{240} g_{\text{other}} + \frac{3g_L}{\pi^2} (-\text{Li}_4(-\exp(-1/X^{2n+1})) - \text{Li}_4(-\exp(1/X^{2n+1})))} \quad (45)$$

and the lower bound of the integral is given by

$$X_{\text{min}} = \max \left(\left(\frac{M_{\text{P}}^2}{M_{\text{cr}} M_*} \right)^2, \left(\frac{M_{\text{P}}^2}{M_{\text{cr}} M_*} \right)^{2/(4n+3)} \right). \quad (46)$$

By using Eq. (42), it is found that

$$X_{\text{min}} = \begin{cases} \left(\frac{M_{\text{P}}^2}{M_{\text{cr}} M_*} \right)^2 \geq 1 & \text{for } M_* < M_{*\text{eq},n} := (na_n 128\alpha\pi^2)^{-1/2} (32\sqrt{3}\pi^2)^{-n} M_{\text{P}}, \\ \left(\frac{M_{\text{P}}^2}{M_{\text{cr}} M_*} \right)^{2/(4n+3)} \leq 1 & \text{for } M_* > M_{*\text{eq},n}, \end{cases} \quad (47)$$

and, for $n = 10$, $M_{*\text{eq},n=10} \sim 10^{-28} M_{\text{P}}$. Thus, in the region of $M_* > 10^{-8} M_{\text{P}}$, $X_{\text{min}} < 1$ is always satisfied. The function $f_n(X_{\text{min}}, X_0)$ behaves similarly to the previously defined function $f(X_{\text{min}}, X_0) = f_1(X_{\text{min}}, X_0)$, but, as shown in Fig. C1, the function g_n (the integrand of f_n) is more sharply peaked around $X_0 \sim 1$. In the large- n limit,

$$\delta N_L = \frac{1}{2} (na_n)^{1/2n} (32\sqrt{3}\pi^2) \left(\frac{M_{\text{P}}}{M_*} \right)^2 f_n(X_{\text{min}}, X_0). \quad (48)$$

Hence, for large n , the lepton asymmetry is found to be

$$\begin{aligned} \frac{n_L}{s} &\simeq 9.3 \times 10^{-7} \left(\frac{106.75}{g_*} \right)^{1/4} \left(\frac{\alpha}{3.5 \times 10^{-3}} \right)^{1/2} \\ &\times \Omega_{\text{PBH}} f_n(X_{\text{min}}, X_0) \left(\frac{10^5 M_{\text{P}}}{M_{\text{PBH}}} \right)^{5/2} \left(\frac{10^{-2} M_{\text{P}}}{M_*} \right)^2 (na_n)^{1/2n}. \end{aligned} \quad (49)$$

According to the behavior of a_n , the asymptotic behavior of $(na_n)^{1/2n}$ is given as

$$(na_n)^{1/(2n)} \rightarrow \begin{cases} 1 & \text{for } a_n \sim \frac{1}{n}, \\ \left(\frac{e}{n} \right)^{1/2} & \text{for } a_n \sim \frac{1}{n!} \sim n^{-n} e^n. \end{cases} \quad (50)$$

As discussed in Appendix C, the function f_n is suppressed by n as $f_n \sim 1/n$, and the asymmetry vanishes in the $n \rightarrow \infty$ limit. It is due to the rapid increase of the chemical potential μ for a larger n at $M < M_{\text{cr}}$. However, it is interesting to note that, for moderate values of n , the higher-dimensional

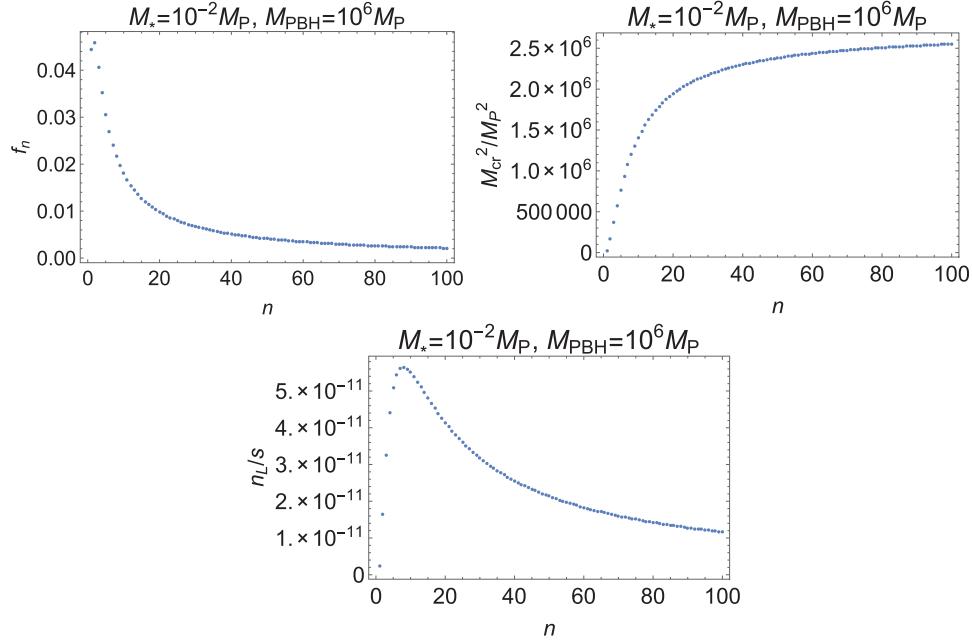


Fig. 3. Plots of f_n , M_{cr}^2/M_p^2 , and $n_L/s = f_n \times M_{\text{cr}}^2/(2M_p^2)$ as functions of n . Here we take $M_* = 10^{-2}M_p$, $M_{\text{PBH}} = 10^6M_p$. Although f_n is a decreasing function of n for $n \geq 2$, M_{cr} is monotonically increasing and thus n_L/s has a peak around $n = 10\text{--}20$.

operators are more efficient in producing a larger amount of asymmetry thanks to the increase in M_{cr} . In order to see the competition between these two effects of taking large n , we plot n dependences of the function f_n , the critical mass M_{cr}^2/M_p^2 and the lepton asymmetry $n_L/s = f_n \times M_{\text{cr}}^2/(2M_p^2)$ in Fig. 3. It can be seen that f_n behaves as $1/n$ for large n , as expected. However, M_{cr} is a monotonically increasing function of n . As a result, n_L/s has a peak around $n = 10\text{--}20$.

We now explicitly estimate the produced asymmetry created by the higher-dimensional interactions, assuming $(na_n)^{1/2n} \sim 1$. In the right-hand panel of Fig. 2, we plot the produced asymmetry n_L/s in Eq. (49) for $\Omega_{\text{PBH}} = 1$ and $n = 10$. We can see that the observed asymmetry, $n_B/s \simeq 8.7 \times 10^{-11}$, is realized for a wide range of parameters as far as $\Omega_{\text{PBH}} > 10^{-6}$.

We note that, if the mass of the PBH is less than 10^8 g (Ref. [12]), there are no observational constraints. Thus the mass region in which the present baryogenesis scenario works is all allowed. It is also interesting to note that the generalized higher-dimensional interactions suppressed by a power of M_* can generate larger baryon asymmetry. This is due to the fact that the critical mass M_{cr} , around which the highest lepton number is generated, becomes larger for larger n . But it also indicates that the present calculation of using the derivative expansions needs improvement. Actually, when the BH mass is reduced to the critical mass M_{cr} , the scale of the curvature tensor becomes comparable to M_* , and the Compton wave length of the underlying particles that induce the CP-violating coupling to the background gravity becomes comparable to the horizon radius of the BH. It is important to study the asymmetric Hawking radiation without resorting to the derivative expansions adopted in the present paper. We hope to come back in future investigations.

3. Possible origin of CP-violating operators

We discuss a possible origin of the CP-violating higher-dimensional operators through interactions with right-handed neutrinos. Note that we have not succeeded to obtain such interactions by explicit

calculation. Moreover, the mechanism in the previous section is not restricted to the right handed neutrino model which we discuss in this section.¹⁷

In order to generate these operators, the underlying physics needs to violate CP symmetry and the lepton number. One of the simplest possibilities is the Majorana mass terms of right-handed neutrinos. As usual, imaginary phases of the mass matrix break the CP symmetry. The Lagrangian we consider is

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + y_N \bar{L} N_R \tilde{\Phi} + \frac{M_N}{2} \bar{N}_R^c N_{Ri} + \text{h.c.}, \quad (51)$$

where y_N is the neutrino Yukawa coupling, and M_N is the mass of right-handed neutrinos. Recently, McDonald and Shore (Refs. [45,46]) explicitly calculated loop diagrams containing the above interactions coupled with the gravity in the external lines. They took the conformally flat metric, $g_{\mu\nu} = (1 + h)\eta_{\mu\nu}$ where the field h is treated as a background field, and then calculated the two-loop Feynman diagrams in Fig. 4 to obtain the higher-dimensional operator used in the gravitational leptogenesis,

$$\frac{1}{M_{\text{MS}}^2} \partial_\mu \mathcal{R} j^\mu. \quad (52)$$

Since right-handed neutrinos are integrated out, the mass scale of the higher-dimensional interaction is essentially given by¹⁸

$$\frac{1}{M_{\text{MS}}^2} \sim \frac{1}{(16\pi^2)^2} \frac{\text{Im}((y_N y_N^\dagger)^2)}{M_N^2}. \quad (53)$$

The higher-dimensional operators utilized in the present paper will also arise by calculating similar diagrams with more gravitons in the external lines.¹⁹ To prove it, we need to consider a more generic form of the metric and further involved calculations are necessary, so we leave it for future investigations. From dimensional and diagrammatic arguments, it can be expected that the mass scale M_* is given by

$$\frac{1}{M_*^4} \sim \frac{1}{(16\pi^2)^2} \frac{\text{Im}((y_N y_N^\dagger)^2)}{M_N^4}. \quad (54)$$

Let us now give a rough estimation for the scale M_* using this formula. The seesaw mechanism indicates that the typical order of M_N is given by $M_N \sim (y_N v)^2 / m_v \sim y_N^2 10^{16} \text{ GeV}$, where $v \sim 100 \text{ GeV}$ and $m_v \sim 10^{-3} \text{ eV}$. The scale M_* is proportional to M_N but also dependent on the Yukawa coupling y_N and the CP-violating parameter $\epsilon := \text{Im}((y_N y_N^\dagger)^2) / |(y_N y_N^\dagger)^2|$. If we take the Yukawa coupling as $y_N \sim 1$, the smallness of ϵ makes the scale M_* larger than 10^{16} GeV . On the other

¹⁷ If right-handed neutrinos are responsible for the CP-violating higher-dimensional interactions, then ordinary leptogenesis can also occur and we need to take it into account in calculating the total abundance of baryon asymmetry. In the present paper, we consider only the abundance generated through the PBHs.

¹⁸ These authors further claimed the possible appearance of an enhancement factor depending on the mass hierarchy of right-handed neutrinos. We do not discuss such effects here.

¹⁹ It is not evident what types of higher-dimensional operators can be induced, but their specific forms are not very important in our discussions, as commented in footnote 3.

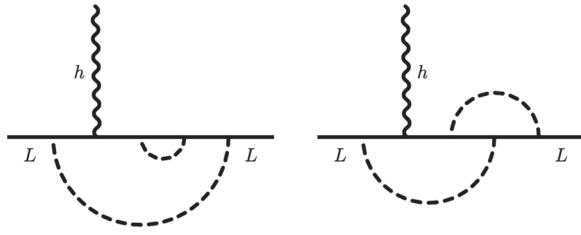


Fig. 4. The diagrams inducing the effective operator Eq. (52) by integrating out right-handed neutrinos. The quantities L and h are the lepton doublet and external gravitational field, respectively.

hand, if we assume that y_N is as small as, e.g., $y_N = 10^{-5}$ and the CP-violating parameter is of order 1, the scale M_* can be lowered to 10^{11} GeV. Note that from recent arguments on the hierarchy problem of the electroweak scale, it is preferable to take the scale of right-handed neutrinos relatively lower. Hence the scale M_* may be expected to take a value in the region, $10^{-7} M_P < M_* < M_P$.

We also emphasize that, other than right-handed neutrinos, if CP is broken in the UV theory including gravity, we can naturally expect the appearance of the operators like Eq. (3). These are left for future studies.

4. Effect of washout outside the BH

If the CP-violating interaction such as Eq. (3) is present, decay of the PBH mass by Hawking radiation breaks time-reversal symmetry and generates chemical potential at the horizon. Then the radiation becomes asymmetric. As already mentioned, the distribution looks thermal but it does not mean that the lepton-number-violating interaction is in thermal equilibrium. Hawking radiation is emitted because of the quantum mechanical effect, and reflects the fact that the quantum vacuum near the horizon is different from the Minkowski vacuum. In this respect, the mechanism of baryogenesis is very different from the gravitational baryogenesis (Ref. [26]), in which decoupling of the baryon-number-violating interaction is necessary to fix the final amount of asymmetry.

It is, however, necessary to check whether the generated asymmetry is washed out by the baryon-(or lepton-) number-violating interactions outside the black hole horizon. As discussed in the previous section, we assumed that the microscopic process violates the lepton number through the interaction with right-handed neutrinos. By integrating them out, we have the following dimension-5 operator:

$$S_L = \int d^4x \left(\frac{1}{\Lambda} LLHH \right). \quad (55)$$

After the Higgs acquires the vev $\langle H \rangle = v \simeq 246$ GeV, it gives the mass to the neutrinos. Hence Λ is determined to be

$$\Lambda \sim \frac{v^2}{m_\nu} \sim 2.5 \times 10^{-2} M_P, \quad (56)$$

where m_ν is the neutrino mass and we put $m_\nu = 10^{-3}$ eV.

The cross section is estimated as $\sigma \sim 1/\Lambda^2$ at the low-energy scale. The particle number density is given by $n \sim T_H^3$ near the horizon where the particles are created. Then the scattering rate of the

lepton-number-violating interaction at the horizon is given by

$$\Gamma_L^0 = \sigma n = T_H^3 / \Lambda^2 \sim 1.6 \times 10^{-12} \left(\frac{10^5 M_P}{M} \right)^3 M_P. \quad (57)$$

But the rate is much slower than the typical time scale of the created particle to move away from the BH, namely $\Gamma_L^0 \ll r_H^{-1} \sim (M_P/M) M_P$ for the BH mass $M \gtrsim 10^5 M_P$. Hence the particle density is quickly diluted by the factor $(r_H/r)^2$, where r is the distance from the BH, and the interaction rate is reduced to $\Gamma_L = \Gamma_L^0 (r_H/r)^2$. Since the Hubble parameter of the universe at the epoch of evaporation in Eq. (32) is estimated to be $H_{\text{eva}} \sim 10^{-14} (10^5 M_P/M)^3 M_P$, the dilution factor $(r_H/r)^2$ instantly makes Γ_L much smaller than H_{eva} . Hence the interaction is not in chemical equilibrium at the time of evaporation with the Hubble H_{eva} .

Next we check the condition of washout when the particles are at higher energy than M_N . The infinite blue shift near the BH horizon enhances the energy of the particle by the factor $(1 - r_H/r)^{-1/2}$. Then, near the horizon, the cross section of lepton-number-violating interaction is replaced by its high energy counterpart $\sigma \sim y_N^2/s \sim y_N^2 (1 - r_H/r)/T_H^2$. Taking the dilution factor $(r_H/r)^2$ into account, we have

$$\Gamma_L \sim y_N^2 T_H \frac{r_H^2}{r^2} \left(1 - \frac{r_H}{r} \right). \quad (58)$$

The maximal value of Γ_L is obtained by

$$\Gamma_{\text{max}} \sim 0.15 y_N^2 T_H = 0.15 \frac{y_N^2}{4\pi r_H} \quad (59)$$

at the position $r \sim 1.5 r_H$. Again this is much smaller than r_H^{-1} by a factor $10^{-2} y_N^2$, and the emitted particles move away quickly from the BH, so the scattering rate is reduced to $\Gamma_L \sim (r_H/r)^2 \Gamma_{\text{max}}$. Compared with the Hubble H_{eva} in Eq. (32), when the particle moves to $r \sim 10^3 r_H$, $\Gamma_L < H_{\text{eva}}$ is satisfied. Hence, if $y_N < 0.1$ the time that the particle moves to that position is sufficiently short for the lepton-number-violating interaction to occur, and the generated lepton number is never washed out.

5. Summary and discussions

In this paper, we have proposed a new scenario of baryogenesis from evaporating PBHs. The key element is the CP-violating operator (2), which generates the splitting of the energy spectrum between particles and antiparticles if the BH is decaying. The mechanism is similar in spirit to gravitational baryogenesis (Ref. [26]) or the mechanism by Hook (Ref. [25]), who applied gravitational baryogenesis to PBHs, but an essential difference is that we make use of the time evolution of the BH itself (Ref. [29]), not the cosmological evolution, to generate the chemical potential. Because of this, the ratio of the chemical potential to the Hawking temperature μ/T_H becomes a function of the decaying mass of evaporating BHs and, when the mass becomes less than the critical mass, the ratio $|\mu/T_H|$ exceeds 1. After this epoch, maximal asymmetry can be generated. Due to such an efficiency of generation mechanism, even though the CP-violating operator is largely suppressed by a high-energy scale M_* , we show that a sufficient amount of baryon asymmetry $n_B/s \simeq 8.7 \times 10^{-11}$ can be obtained in a wide range of parameter space if the density ratio of PBHs at the epoch of evaporation is $\Omega_{\text{PBH}} > 10^{-6}$. If the scenario really explains the baryon asymmetry of the universe, it

constrains model buildings beyond the standard model because the PBH can radiate heavy particles that may decay later, e.g., during the BBN. Thus the present scenario favors simpler model building, such as Ref. [47].

The estimation of the generated asymmetry and the requirement for Ω_{PBH} in the present paper will change if we take other effects into account. In our analysis we evaluated the chemical potential at the horizon $r = r_{\text{H}}$. There is, however, a discussion (Refs. [48,49]) that the Hawking radiation originates in the larger region $r = cr_{\text{H}} > r_{\text{H}}$. According to Ref. [49], $c = 3\sqrt{3}/2 \sim 2.6$ is given for high-frequency modes. But the effect can be always absorbed in the definition of the scale M_* . Indeed if we instead evaluate μ at $r = cr_{\text{H}}$, the chemical potential is reduced by c^{-6n} and the critical mass M_{cr} is reduced by the factor $c^{-6n/(4n+2)}$. From the definition of the critical mass, it is equivalent to increasing the effective M_* by $c^{6n/(4n+2)}$. For $n = 1$ and $n = 10$ with $c = 3\sqrt{3}/2$, the numerical factors are 2.6 and 3.9 respectively.

In the analysis we used the adiabatic approximation, namely we have implicitly assumed that the time scale characterizing the Hawking radiation is shorter than that of the change of mass of the PBH. Here we confirm the validity of this assumption. The typical time scale of Hawking radiation is estimated from the uncertainty relation between time and energy,

$$\Delta t \sim \frac{1}{\Delta E} \sim \frac{1}{T_{\text{H}}}. \quad (60)$$

The assumption of the adiabaticity is justified if the change of the PBH mass is negligibly small during Δt . Using Eq. (13), we have

$$\begin{aligned} \left| \frac{dM}{dt} \Delta t \right| &\simeq \frac{\pi N}{480} \frac{M_{\text{P}}^4}{M^2 T_{\text{H}}} \\ &= \frac{\pi N}{480} \left(\frac{M_{\text{P}}}{M} \right)^2 M, \end{aligned} \quad (61)$$

which is much smaller than M for $M \gg M_{\text{P}}$. Therefore, we can safely treat the system adiabatically.

If PBHs are responsible for the baryon asymmetry in the universe, they can also be responsible for gravitational waves. PBHs emit gravitational waves either by Hawking radiation or by a formation of PBH binaries, but the Hawking radiation would provide the strongest signal (Ref. [50]). In Ref. [50], they estimated the peak frequency $f^{(\text{peak})}$ and the peak amplitude $h_0^2 \Omega_{\text{GW}}$ as

$$f^{(\text{peak})} = 4 \times 10^{12} \text{ Hz} \left(\frac{M_{\text{PBH}}}{10^5 M_{\text{P}}} \right)^{1/2}, \quad h_0^2 \Omega_{\text{GW}} \sim 10^{-7}, \quad (62)$$

if PBH dominates the universe. Note that the peak amplitude does not depend on M_{PBH} . In Fig. 5, the peak frequency of the gravitational waves from the PBH is plotted as a function of its mass.

It is amusing to consider a cosmological history in which PBHs dominate in the early universe. But the expected gravitational waves seem to be difficult to observe since their frequencies are too high for near-future experiments.

The final comment is on the formation mechanism of the PBHs. There are various possibilities to create PBHs in the early universe as summarized in the introduction. The bubble collisions (Ref. [6]) associated with the first-order phase transition in the universe (Ref. [7]) are becoming more interesting recently, since the discovery of the Higgs boson, and strong constraints on the TeV-scale physics have stimulated reconsideration of our view of cosmological history. In particular, the revival of radiative symmetry breaking via the Coleman Weinberg mechanism (Refs. [51–53]) suggests that

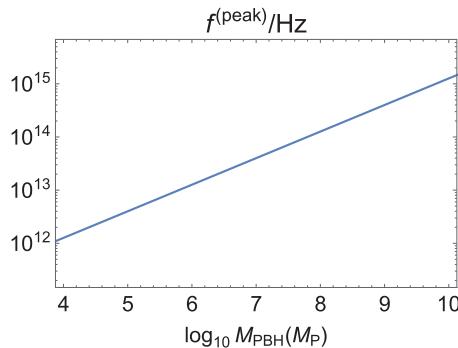


Fig. 5. The peak frequency of gravitational waves from Hawking radiation from a PBH as a function of M_{PBH} .

the phase transition will be a strong first-order type. In such models, bubble collisions of the true vacua can generate strong gravitational waves (Ref. [54]), topological objects such as monopoles (Ref. [55]), or PBHs (Ref. [6]). It is interesting to pursue further cosmological consequences of the strong first-order phase transitions, in relation to particle physics models beyond the SM.

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Appendix A. Is chemical potential physical?

If the (lepton number) current is conserved, an operator of the type in Eq. (4) seems to vanish by integrating by parts or by rotating the phase of leptons as ψ as $\psi \rightarrow \exp(iF(\mathcal{R}_{...}))\psi$. It implies that the lepton number violation is necessary to justify the microscopic origin of such CP-violating effective operators. Indeed McDonald and Shore (Ref. [46]) obtained such a term by integrating the right-handed neutrinos whose interactions violate the CP and the lepton number conservation microscopically.

But it is still paradoxical why we are not allowed to do such phase rotation to remove the effect of lepton asymmetry. First note that if the interaction is replaced by μJ^0 where $\mu = \dot{F}$ is assumed to be a constant, the phase rotation becomes $\psi \rightarrow \exp(i\mu t)\psi$. This is nothing but the shift of energy $\omega \rightarrow \omega - \mu$ for leptons and $\omega \rightarrow \omega + \mu$ for antileptons. Since the vacuum state is defined so as to fill all negative energy levels, the phase rotations simply change the definitions of the vacuum. Thus such phase rotation should be correlated with the definition of the vacuum state, and with the definition of the lepton number of the vacuum.

In the case we are discussing, the current is coupled to the total derivative $\partial_\alpha F(\mathcal{R})$ and $F(\mathcal{R})$ is a smooth function in spacetime. In particular, it vanishes at $r = \infty$ and takes a nonvanishing value at the BH horizon $r = r_H$. Therefore, if we assume that $F(\mathcal{R})$ is a slowly changing function in

time and can be expanded as $F(t, r) = F_0(r) + \mu(r)t + \dots$, the phase rotation changes the energy levels of leptons $\omega \rightarrow \omega - \mu(r)$ as a function of the position r . In this sense, the situation is similar to the discussion of chiral anomaly as the spectral flow in the Hamiltonian formulation. Now the question is, how we can define the vacuum of quantum field at $r = \infty$ and $r = r_H$ separately. And this is nothing but the issue of Hawking radiation. At $r = \infty$, $F(\mathcal{R})$ vanishes and there is no ambiguity in defining the vacuum. At $r = r_H$, we first define the appropriate vacuum state (such as the Unruh vacuum) so that an infalling observer does not encounter any divergences, and then calculate the effective action. Once the effective operator is induced as performed in Refs. [45,46], then we can no longer shift the energy level (see also Refs. [56,57].) This is the reason why we should not rotate the basis after we calculate the effective interaction, and the CP-violating operator and the resulting chemical potential derived from Eq. (51) have a physical meaning.²⁰ It will be interesting to obtain the asymmetry in spectrum of the Hawking radiation without resorting to the calculation of the effective interaction, i.e., by explicitly calculating the lepton wave function in the eikonal approximation with the coupling to right-handed neutrinos included. We want to come back to this problem in future. (See also Appendix B.)

Appendix B. Hawking radiation with a chemical potential

Here we briefly explain why the chemical potential modifies the spectrum of Hawking radiation. We start from the following action in the Schwarzschild black hole geometry:

$$\begin{aligned} S &= \int \sqrt{-g} d^4x \left(\bar{\psi} i \not{\partial} \psi + \frac{C}{M_*^4} g^{\mu\nu} \partial_\mu (\mathcal{R}_{\alpha\beta\gamma\delta} \mathcal{R}^{\alpha\beta\gamma\delta}) \bar{\psi} \not{\partial} \psi \right) \\ &= \int \sqrt{-g} d^4x \bar{\psi} \left(i \partial_\mu - \frac{C}{M_*^4} \partial_\mu (\mathcal{R}_{\alpha\beta\gamma\delta} \mathcal{R}^{\alpha\beta\gamma\delta}) \right) \gamma^\mu \psi. \end{aligned} \quad (\text{B.1})$$

The action has the same form as

$$\int \sqrt{-g} d^4x \bar{\psi} (i \partial_\mu - e A_\mu) \gamma^\mu \psi, \quad (\text{B.2})$$

by identifying $C \partial_\mu (\mathcal{R}_{\alpha\beta\gamma\delta} \mathcal{R}^{\alpha\beta\gamma\delta}) / M_*^4$ with the gauge potential A_μ . Thus the coupling to the background gravity is nothing but the pure gauge configuration (but it cannot be gauged away as discussed in the previous appendix). It is now instructive to briefly sketch the derivation of Hawking radiation in the case of the charged black hole following Refs. [58,59]. At the outer event horizon, the action of the charged fermion becomes

$$S \simeq \int dt dr_* (\bar{\psi} (i \partial_t - e A_t) \gamma^t \psi - \bar{\psi} i \partial_{r_*} \gamma^{r_*} \psi), \quad A_t = -\frac{eQ}{r}. \quad (\text{B.3})$$

Here we omit the contribution from angular components for simplicity, and Q is the charge of the black hole. In order to obtain the outgoing flow of the energy at the future infinity, we usually impose the ingoing boundary condition for the current at the horizon. But since the above scalar potential diverges, $A_U \propto A_t/U$ at the horizon $U = 0$ in the Kruskal coordinate, we need to take the gauge in which $A_t = 0$ at the horizon (see discussions, e.g., Refs. [58,59]).

$$A'_t = A_t + \partial_t \Lambda, \quad \Lambda = \frac{eQ}{r_H} t. \quad (\text{B.4})$$

²⁰ From a different point of view, Eq. (2) would be meaningful since this operator contains running coupling γ_N . Because this coupling depends on scale, the operator cannot be absorbed by fermion rotation in a covariant way.

This shifts the energy level of the charged particles and the spectrum of the Hawking radiation is obtained by replacing ω by $\omega - \mu = eQ/r_H$. Thus eQ/r_H indeed plays the role of the chemical potential in the thermal radiation.

The same procedure is applicable to the current setup. In our case, the “gauge transformation” to regularize the action at the horizon in the Kruskal coordinate is given by

$$\psi \rightarrow \exp \left(i \frac{C}{M_*^4} \partial_t (\mathcal{R}_{\mu\nu\rho\sigma} \mathcal{R}^{\mu\nu\rho\sigma}) \Big|_{r=r_H} t \right) \psi. \quad (\text{B.5})$$

Thus

$$\mu = \frac{C}{M_*^4} \partial_t (\mathcal{R}_{\alpha\beta\gamma\delta} \mathcal{R}^{\alpha\beta\gamma\delta}) \Big|_{r=r_H} \quad (\text{B.6})$$

becomes the chemical potential to describe the thermal radiation from the PBH.

Appendix C. Analytical approximation

In this appendix, we present an analytical approximation of the function $g_n(X)$ that is defined in Eq. (45) and appears in the calculation of the asymmetry. We investigate the large- n behavior of the integral. For large X , it can be expanded with respect to $1/X$ as

$$g_n(X) \simeq \frac{40g_L}{7((2g_L + g_{\text{other}})\pi^2)} \frac{1}{X^{2n+1}} + \frac{40g_L(46g_L - 7g_{\text{other}})}{49(2g_L + g_{\text{other}})^2\pi^4} \left(\frac{1}{X^{2n+1}} \right)^3 + \dots \quad (\text{C.1})$$

For $X \rightarrow \infty$, the first term gives a good approximation for $g_n(X)$ and we have an approximated formula for the integral:

$$f_n(X_{\min}, X_{\max}) = \int_{X_{\min}}^{X_{\max}} dX g_n(X) \simeq -\frac{20}{7\pi^2} \frac{g_L}{2g_L + \frac{1}{3}g_{\text{other}}} \frac{1}{n} \frac{1}{X_{\min}^{2n}}, \quad (\text{C.2})$$

for $1 \leq X_{\min} \leq X_{\max}$. Then, from Eq. (44), we have $\delta N_L \propto M_{\text{cr}}^2 X_{\min}^{-2n}$. In this region ($X > 1$), $T_H > \mu$ and X_{\min} is given by $X_{\min} = (M_p^2/M_{\text{cr}} M_*)^2$. By using the result for the critical mass M_{cr} in Eq. (42), it turns out that δN_L is independent of the mass scale M_* . Since the entropy s is determined by the life time of the PBH, namely M_{PBH} , and independent of M_* , n_L/s also becomes

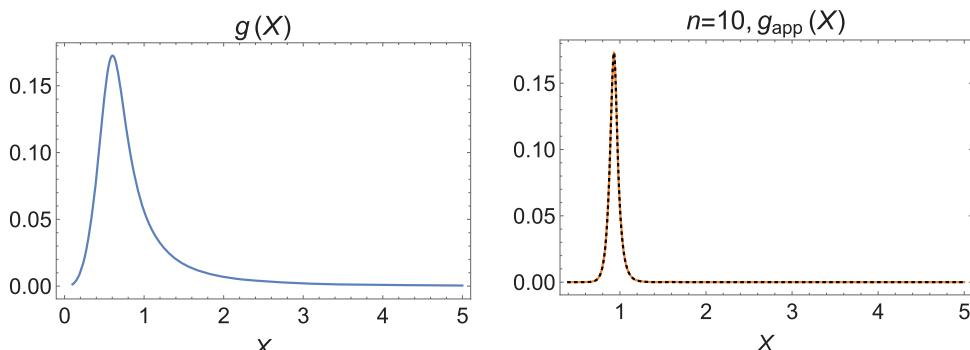


Fig. C1. The solid orange and dashed black lines correspond to $g(X)$ and $g_{\text{app}}(X)$, respectively. One can see that $g_{\text{app}}(X)$ is a good approximation for all the region of X , and the integrand has a strong peak around $X \sim 1$. Note that the position of the peak for $n = 10$ is closer to $X = 1$ than for $n = 1$.

independent of M_* . The region $1 < X_{\min}$ is given by the region where the line of $M_{\text{PBH}} = M_{\text{cr}}$ is below the line of $T_{\text{H}}(M_{\text{PBH}}) = M_*$, and corresponds to the region with $M_* \lesssim 10^{-3}M_{\text{P}}$ in the left-hand panel of Fig. 2. This is the reason why the generated asymmetry is constant as a function of M_* for $M_* \lesssim 10^{-3}M_{\text{P}}$. But we notice that the behavior is given by our prescription to cut the integral at X_{\min} where the typical energy of the Hawking radiation becomes higher than M_* and the present investigations become questionable. Though it is beyond our approximation to estimate δN_L for $X \lesssim X_{\min}$, a greater asymmetry may be produced in this region. In this sense, the estimation in this paper would give a conservative value for lepton asymmetry.

On the other hand, in the region $X \sim 0$, we have

$$\begin{aligned} g_{\text{app}}(X) &= -\frac{40g_L X^{2n+1} \left(1 + \pi^2 (X^{2n+1})^2\right)}{30g_L + 60g_L \pi^2 (X^{2n+1})^2 + 7(2g_L + g_{\text{other}})\pi^4 (X^{2n+1})^4}, \\ &= -\frac{4}{3} \frac{CX^{2n+1} \left(1 + \pi^2 (X^{2n+1})^2\right)}{C + 2C\pi^2 (X^{2n+1})^2 + (X^{2n+1})^4}, \end{aligned} \quad (\text{C.3})$$

where $C := 30g_L/(7\pi^4(2g_L + g_{\text{other}}))$. Surprisingly, as shown in Fig. C1, this expression gives a good approximation even for $X > 1$, as well as for $X \sim 0$. Since Eq. (C.3) is a rational function of X , its integral can be performed analytically, and we have

$$\begin{aligned} f_{\text{app}}(X_{\min}, X_0) := \int_{X_{\min}}^{X_0} g_{\text{app}}(X) &= -\frac{1}{3} \frac{\sqrt{C}}{(C\pi^4 - 1)^{1/2}} \left[\frac{1}{n+1} \pi^2 \right. \\ &\quad \left\{ -H_n(X_{\min}, X_{\min}) G_{n-}(X_0) F_{--}(X_0) X_0^{6n+4} \right. \\ &\quad + H_n(X_0, X_0) G_{n-}(X_{\min}) F_{--}(X_{\min}) X_{\min}^{6n+4} \\ &\quad + H_n(X_{\min}, X_{\min}) G_{n+}(X_0) F_{-+}(X_0) X_0^{6n+4} \\ &\quad \left. - H_n(X_0, X_0) G_{n+}(X_{\min}) F_{-+}(X_{\min}) X_{\min}^{6n+4} \right\} \\ &\quad + \frac{1}{n} \left\{ H_n(X_{\min}, X_{\min}) G_{n-}(X_0) F_{+-}(X_0) X_0^{2n+2} \right. \\ &\quad - H_n(X_0, X_0) G_{n-}(X_{\min}) F_{+-}(X_{\min}) X_{\min}^{2n+2} \\ &\quad - H_n(X_{\min}, X_{\min}) G_{n+}(X_0) F_{++}(X_0) X_0^{2n+2} \\ &\quad \left. + H_n(X_0, X_0) G_{n+}(X_{\min}) F_{++}(X_{\min}) X_{\min}^{2n+2} \right\} \Big] \\ &\quad \times \left[(X_0 X_{\min})^{4n+2} (2C\pi^2 + X_0^{4n+2}) (2C\pi^2 + X_{\min}^{4n+2}) \right. \\ &\quad \left. + C(H_n(X_0, X_0) + H_n(X_{\min}, X_{\min}) - C) \right]^{-1}. \end{aligned} \quad (\text{C.4})$$

Here we have defined

$$F_{\pm\pm}(Y) := {}_2F_1 \left(1, 1; \frac{2n + (1/2)}{2n + 1} \pm \frac{1}{2}; \frac{1}{\left(\pi^2 \pm \frac{(C\pi^4 - 1)^{1/2}}{\sqrt{C}} \right) Y^{4n+2} + 1} \right),$$

$$G_{n\pm}(Y) := Y^{4n+2} \pm \sqrt{C} (C\pi^4 - 1)^{1/2} + C\pi^2, \\ H_n(Y, Z) := 2C\pi^2 Y^{4n+2} + Z^{8n+4} + C, \quad (C.5)$$

where ${}_2F_1$ is the hypergeometric function. In the numerical calculation for drawing the figures, we used the expression of Eq. (C.4).

Finally let us examine the large- n behavior of $f_{\text{app}}(X_0, X_{\min})$. We concentrate on the region $X_0 > 1$ for simplicity. The behavior of X_{\min} is given by

$$X_{\min} \rightarrow 1 - \frac{1}{8n} (10 \log 2 + \log 3 + 4 \log \pi), \quad (C.6)$$

and thus

$$X_{\min}^{2n+2} \rightarrow \frac{1}{4\pi \times 3^{1/4} \sqrt{2}} := D \simeq 0.043. \quad (C.7)$$

Then the hypergeometric function is approximated for large n as

$$F_{\pm\pm}(Y) \rightarrow {}_2F_1 \left(1, 1; 1 \pm \frac{1}{2}; \frac{1}{\left(\pi^2 \pm \frac{(C\pi^4 - 1)^{1/2}}{\sqrt{C}} \right) Y^{4n+2} + 1} \right) \\ \rightarrow \begin{cases} 1 & \text{for } Y = X_0, \\ {}_2F_1 \left(1, 1; 1 \pm \frac{1}{2}; \frac{1}{\left(\pi^2 \pm \frac{(C\pi^4 - 1)^{1/2}}{\sqrt{C}} \right) D^{-2} + 1} \right) & \text{for } Y = X_{\min}. \end{cases} \quad (C.8)$$

Note that

$${}_2F_1 \left(1, 1; \frac{3}{2}; x \right) = \frac{\arcsin(\sqrt{x})}{(x - x^2)^{1/2}}, \quad {}_2F_1 \left(1, 1; \frac{1}{2}; x \right) = \frac{1}{1 - x} + \frac{\sqrt{x} \arcsin(\sqrt{x})}{(1 - x)^{3/2}}. \quad (C.9)$$

Then, by picking up the leading power of X_0 , f_{app} becomes

$$f_{\text{app}} \rightarrow -\frac{1}{3} \frac{1}{n} \left(\frac{C}{C\pi^4 - 1} \right)^{1/2} \\ \times X_0^{8n+4} \left[\pi^2 D^3 (G_{n-}(X_{\min}) F_{n--}(X_{\min}) - G_{n+}(X_{\min}) F_{n-+}(X_{\min})) (X_{\min}) \right. \\ \left. + D (-G_{n-}(X_{\min}) F_{n+-}(X_{\min}) + G_{n+}(X_{\min}) F_{n++}(X_{\min})) \right] \\ \times \frac{1}{X_0^{8n+4} [D^2 (2C\pi^2 + D^2) + C]} \\ = \frac{1}{n} \left(\frac{C}{C\pi^4 - 1} \right)^{1/2} \frac{1}{[D^2 (2C\pi^2 + D^2) + C]} \\ \times \left[\pi^2 D^3 (G_{n-}(X_{\min}) F_{n--}(X_{\min}) - G_{n+}(X_{\min}) F_{n-+}(X_{\min})) (X_{\min}) \right. \\ \left. + D (-G_{n-}(X_{\min}) F_{n+-}(X_{\min}) + G_{n+}(X_{\min}) F_{n++}(X_{\min})) \right], \quad (C.10)$$

from which it is concluded that $f_{\text{app}} \sim 1/n$, and vanishes for $n \rightarrow \infty$.

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