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On the Inaccessibility of Time Machines

Marija Tomašević

CPHT, CNRS, École polytechnique, Institut Polytechnique de Paris, 91120 Palaiseau, France; marija.tomasevic@polytechnique.edu

Abstract: We will explain why time machines, although allowed in General Relativity, cannot be accessed by observers once we include quantum effects. Moreover, we will show that traversable wormholes cannot be turned into time machines without invoking the effects of quantum gravity.

Keywords: time machines; quantum effects; AdS/CFT correspondence

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1. Introduction

Time machines, or closed timelike curves (CTCs), are infamously consistent with the laws of General Relativity. They are even abundant; behind every inner horizon of a rotating black hole, there seems to be a spacetime filled with CTCs [1]. Moreover, one can show that traversable wormholes, *generically,* tend to turn into time machines [2]. A wormhole connects two points in space but also in time, so it is not difficult to see how some additional tinkering with the wormhole can lead to closed timelike curves; one does this by sufficiently distorting the relative clock rates between the two mouths.

Therefore, time machines seem quite accessible to observers, albeit sometimes, one would need to jump into a black hole. Additionally, even though traversable wormholes of the 20th century are perceived to require the use of exotic matter and advanced civilizations, nowadays their construction is regarded as a physical possibility [3–11]. A key ingredient behind their construction is the use of quantum physics—specifically, the Casimir effect. However, just as quantum effects are crucial for the realization of traversable wormholes, they will be fatal for the realization of time machines.

Indeed, one can see the incompatibility of time machines and quantum physics already at the level of energy conditions. Einstein's equations allow any smooth Lorentzian manifold to be considered spacetime, so we need physical energy conditions to restrict the set of possible stress tensors that produce the spacetimes in question. The weakest energy condition suggested to be obeyed by quantum fields is known as the achronal average null energy condition (AANEC). In essence, the AANEC requires that the stress tensor along the fastest null geodesic must be positive on average. Additionally, in [12] it was shown that the AANEC is enough to forbid the creation of time machines[1].

As powerful as AANEC may be, it still does not explain what goes wrong if one tries to make a time machine out of a wormhole—it simply states that time machines cannot exist in semiclassical gravity. This can be seen as analogous to the arguments surrounding the black hole information paradox in AdS/CFT: the CFT is unitary; therefore, the paradox is lost. Such statements are clearly useful, but a physical realization is lacking in both examples, so we must try harder to understand the full picture.

The first step in this direction was already made by Hawking through a conjecture known as *the chronology protection principle* [13]. In essence, Hawking argues that we might try to make a wormhole into a time machine, but in doing so, we will encounter a singularity before we close the (timelike and/or lightlike, i.e., causal) curves. His argument is based on the expectation value of the renormalized stress tensor at a chronology horizon—the boundary between the regular and the CTC-filled part of the spacetime—which leads to a singularity as we try to make a causal geodesic into a closed one. In other words, it appears that any attempt to transform a wormhole into a time machine results in large vacuum polarization effects, which disrupt the internal structure of the wormhole, thereby preventing time machine formation. Note, also, that the chronology horizon is a Cauchy horizon; for inner horizons of rotating black holes, the chronology protection principle is then equivalent to the strong cosmic censorship conjecture [1].

The calculations of the stress tensor have been performed mostly for free fields and in spacetimes of low dimensions [13–19]. However, a linear free field, propagating in a fixed background with a chronology horizon, cannot be prevented from crossing to the side with CTCs: even though the field may be in a state that is singular on the horizon, we can still create localized wave-packets on top of that state, which can travel on geodesics across the horizon. Additionally, since the field is linear, these wave-packets would not be sensitive to the singularity in the background field state—interaction is needed for that. Therefore, making a statement about CTCs for interacting theories, preferably in all dimensions, would lead to strong support for the chronology protection principle.

Interacting quantum fields are notoriously difficult to handle, but this issue is ameliorated if the fields in question are holographic. The AdS/CFT correspondence maps a theory of (quantum) gravity in d dimensions to a strongly coupled field theory in $d-1$ dimensions. Additionally, in the case where the bulk is purely classical, an exact map is known, and one can completely reconstruct the bulk from boundary data. Since the bulk is simpler to understand, we will construct a time machine spacetime on the boundary, where strongly interacting fields propagate, and map these physics to a purely classical one in the bulk[2]. Therefore, in this work, we show how the chronology protection principle is upheld through the means of holography.

The first two sections of this work summarize [20,21]. Namely, we will see in Section 2 the basic structure of time machine spacetimes that we will use throughout, while in Section 3, we will see the bulk reconstruction from such a boundary time machine spacetime. We can obtain the bulk in two ways: through direct reconstruction or double Wick rotation of other known bulk metrics; we will review both methods. The last section, Section 4, is a work in progress with Roberto Emparan: we will show that time machines could be formed even for long wormholes but that the AANEC prevents the Cauchy horizon formation first. This implies that one does not need quantum gravity for the resolution of time machines, semiclassical backreaction must be sufficient enough. We finish, in Section 5, with a discussion and outlook.

2. Basic Structure of Time Machine Spacetimes

Let us begin by defining what we mean by a time machine [1,18].

Definition 1. *If a spacetime M contains a closed causal curve γ, then M contains a causality-violating time machine, and the curve γ traverses the time machine.*

Notice that the whole spacetime need not be a time machine. It is enough to have a "causality-violating region" in order to be referred to as a time machine.

In this case, we will have a boundary separating the causality-obeying region from the other one, so we have a notion of a Cauchy horizon, that is, a *causality horizon*:

Definition 2. *The future causality horizon of a spacetime M is the boundary of the causal future of the causality-violating region,*

$$H^+(J) \equiv \partial[J^+(J^0(M))], \tag{1}$$

where $J^0(M)$ is the set of all points in M which make up the causality-violating region, ∂ is a symbol for a boundary and J^+ indicates we take the future domain of dependence (causal future), that is,

$$J^+(p) \equiv \{q \in M | \exists \text{ a future-directed causal curve from } p \text{ to } q\}. \tag{2}$$

Intuitively, one can refer to the example of the Kerr black hole: the inner horizon in that case is also a causality-horizon, and all points beyond this horizon, towards the Kerr singularity, belong to the causality-violating region of this spacetime.

The first solution of Einstein's equations with CTCs has been found by Gödel in 1949 [22]. However, the simplest examples of time machine spacetimes are known as Misner–AdS spacetimes [23,24]. These spacetimes start from a completely regular initial Cauchy slice, which starts to tilt more as we progress toward the future and the past. An intuitive way to obtain them is to start with a regular $(1+1)$-dimensional cylinder with a time tilt,

$$ds^2 = -(d\tau + v d\psi)^2 + d\psi^2 \,, \tag{3}$$

where τ is the time coordinate, $\psi \sim \psi + L$ the spatial one, with L the radius of the cylinder, and $v = const$ represents the slope of the tilt. This spacetime is locally flat, but if we were to add time dependence to the tilt, we would obtain a different structure,

$$ds^2 = -(d\tau + a\tau\, d\psi)^2 + d\psi^2 \,, \tag{4}$$

where $a = const$; see Figure 1.

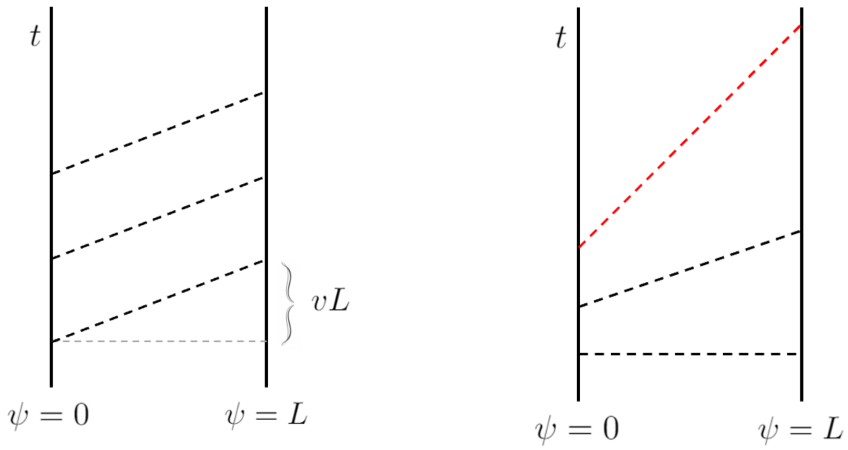

Figure 1. (**Left**) Tilted cylinder with $v = const$. The sides $\psi = 0$ and $\psi = L$ are identified along the dashed lines at the same time τ but at different times t. (**Right**) Misner–AdS_2 cylinder with linearly increasing $v = a\tau$. The red line indicates that lightlike-separated points are identified, so a closed null line is present. At later times, CTCs appear.

This spacetime is now locally AdS_2 with the radius $1/a$, as one can see through the coordinate change

$$t = \tau\, e^{a\psi} \,, \qquad x = \frac{1}{a} e^{a\psi} \,, \tag{5}$$

and since we have made the identifications $(\tau, \psi) \sim (\tau, \psi + L)$, this geometry is known as the Misner–AdS_2 spacetime. Writing out the Misner geometry fully,

$$ds^2 = -d\tau^2 - 2a\tau\, d\psi\, d\tau + (1 - a^2\tau^2) d\psi^2 \,, \tag{6}$$

makes it clear that $\partial/\partial\psi$ becomes timelike, and hence CTCs appear, when

$$|\tau| > \frac{1}{a} \,. \tag{7}$$

There are two regions with CTCs, one in the future and another in the past, separated from the regular part of the spacetime by respective chronology horizons $\tau = \pm 1/a$.

Extending this solution to higher dimensions is fairly straightforward. Misner–AdS$_{n+2}$ spacetime, with $n > 0$, then has the form

$$ds^2 = -(dt + td\psi)^2 + \sum_{i=1}^{n}(dy_i + y_i d\psi)^2 + d\psi^2 \,, \tag{8}$$

with t and y^i running along all of $\mathbb{R}$, and ψ periodically identified at fixed t and $\mathbf{y} = (y^1, \ldots, y^n)$ as $(t, \mathbf{y}, \psi) \sim (t, \mathbf{y}, \psi + \Delta)$. Since $g_{\psi\psi} = 1 - t^2 + \mathbf{y}^2$, we see that the ψ coordinate becomes timelike when $t^2 > 1 + \mathbf{y}^2$. The chronology horizons are set by $t = \pm\sqrt{1 + \mathbf{y}^2}$. Through a simple change of coordinates [21], we obtain

$$ds^2 = -\frac{d\tau^2}{1 - \tau^2} + (1 - \tau^2)d\phi^2 + \tau^2\Big(d\chi^2 + \sinh^2\chi\, d\Omega_{n-1}\Big) \,, \tag{9}$$

where the last factor is the hyperbolic space H_n. When we discuss the double Wick rotation method in the next section, we will mostly use this form of the metric.

Note that for $n = 1$, that is Misner–AdS$_3$, the metric looks the same as for a static Bañados–Teitelboim–Zanelli (BTZ) black hole [25]. In fact, it *contains* BTZ, as it also covers the region beyond the "singularity" $r = 0$ (in BTZ coordinates). In fact, the surface $r = 0$ is the chronology horizon for this time machine spacetime: one must jump inside the black hole in order to cross to the pathological part of the spacetime.

This observation can be generalized for all n: in [26,27], it was observed that the discrete quotient of AdS$_{n+2}$ by boost orbits gives a black hole that generalizes the BTZ solution. Misner–AdS$_{n+2}$ has these constant curvature black holes in the regular region, while the pathological part of the spacetime is beyond the $r = 0$ surface.

3. Holography of Time Machines

Now that we have our boundary time machine spacetime(s), we can obtain the associated bulk. In general, there are two ways one can do this. The first is the usual bulk reconstruction technique which uses a perturbative expansion around the boundary, developed in [28]. Unfortunately, this technique only leads to a complete bulk reconstruction in special cases: for two-dimensional boundary theories and bulk metrics that are conformally flat [29]. Therefore, we will use this method only for a two-dimensional time machine spacetime, as was done in [20].

The second method requires us to be a bit more clever. Namely, we can look at some of the known transformations, in which both the bulk and the boundary metric are known, as well as the boundary theory, and we can try to see if we can obtain Misner–AdS through some metric manipulation. Indeed, one can show that the double Wick rotation of certain metrics leads to the Misner–AdS metric on the boundary and to its associated higher-dimensional dual [21]. We will review both methods below. Once we obtain the bulk, we will analyze its symmetries and other properties in order to obtain the dual information about the time machine.

3.1. Two-Dimensional Time Machines

Let us start with the simplest example of a 2-dimensional boundary time machine. In order to reconstruct its associated bulk dual, we will follow the procedure outlined in [28]. In essence, in order to reconstruct the bulk, we require two pieces of information: the boundary metric and the boundary stress tensor of the conformal fields. We already know our boundary metric (4), which we can rewrite in the usual AdS$_2$ form as

$$ds^2 = -(\tau^2 - 1)d\phi^2 + \frac{d\tau^2}{\tau^2 - 1} \tag{10}$$

through the coordinate change $\psi = \phi - \ln\sqrt{|\tau^2 - 1|}$, or we can write in a null basis,

$$ds^2 = -\frac{4}{(x^+ - x^-)^2} dx^+ dx^- \tag{11}$$

through $x^\pm = (\tau \pm 1)e^\psi$, where we have rescaled $a = 1$. Recall that chronology horizons lie at $\tau = \pm 1$. How about the stress tensor?

Given that we are in a 2-dimensional spacetime with conformal fields, the form of the stress tensor can be fixed by symmetries and consistency relations. Namely, we have scaling invariance, so $x^\pm \to \lambda x^\pm$ should be a symmetry, and we also know that the stress tensor is conserved, $\nabla_\mu T^{\mu\nu} = 0$. For the metric (11), this implies that our stress tensor will have the form

$$T_{\mu\nu} dx^\mu dx^\nu = -\alpha \frac{c}{24\pi} \left(\left(\frac{dx^+}{x^+} \right)^2 + \left(\frac{dx^-}{x^-} \right)^2 \right) - \frac{c}{24\pi} g_{\mu\nu} dx^\mu dx^\nu, \tag{12}$$

where the last part is the anomalous trace term with the scalar curvature $R = -2$, and α is an arbitrary constant that fixes the state: depending on its sign and value, we can have Casimir energy or a thermal stress tensor, or it can give us an unexcited CFT; we will work with only one value of α, but all three cases have been analyzed in [20]. The constant c is the CFT central charge.

In (τ, ϕ) coordinates, the stress tensor becomes diagonal,

$$\langle T^\tau_\tau \rangle = -\left\langle T^\phi_\phi \right\rangle = \frac{c}{12\pi} \frac{\alpha}{1 - \tau^2}, \tag{13}$$

so these coordinates are adapted to frame comoving with the conformal fields. In this frame, when $\tau^2 < 1$ and ϕ is a spatial direction, the stress tensor is time-dependent but spatially homogeneous. When $\tau^2 > 1$ and ϕ is timelike, the stress-tensor is static but inhomogeneous. In both regions, the stress tensor diverges at the chronology horizons $\tau = \pm 1$. This is the divergence that will prevent excitations from crossing over to the CTC-filled part [19].

Now that we have our two main ingredients, we can reconstruct the bulk and see how this divergence is mapped. As mentioned above, the procedure we follow is the one introduced by de Haro et al. in [28], and the details are laid out in Appendix B of [20]. In (τ, ϕ) coordinates, where z is the holographic coordinate, we obtain

$$ds^2 = \frac{\ell^2}{z^2} \left[dz^2 - \left(1 + \frac{z^2}{4} + \frac{\alpha}{2} \frac{z^2}{1 - \tau^2} \right)^2 \frac{d\tau^2}{1 - \tau^2} + \left(1 + \frac{z^2}{4} - \frac{\alpha}{2} \frac{z^2}{1 - \tau^2} \right)^2 (1 - \tau^2) d\phi^2 \right]. \tag{14}$$

Recall that τ is timelike and ϕ spacelike when $\tau^2 < 1$ (this is the "normal" region), and they reverse roles when $\tau^2 > 1$ (this is the "pathological" region). Note that the metric is independent of ϕ, and therefore $k = \frac{\partial}{\partial \phi}$ is a Killing vector of the bulk. Its norm is given by

$$|k|^2 = g_{\phi\phi} = \frac{\ell^2}{z^2} \frac{1}{1 - \tau^2} \left((1 - \tau^2) \left(1 + \frac{z^2}{4} \right) - \frac{\alpha}{2} z^2 \right)^2, \tag{15}$$

which implies that k is spacelike for $|\tau| < 1$ and timelike for $|\tau| > 1$. The bulk then contains CTCs in precisely the same range of τ as in the boundary. However, when $|\tau| = 1$, the vector k is null only when $z = 0$, that is, on the boundary; at any finite distance z inside the bulk, k diverges when $|\tau| = 1$. We care about the null Killing vector since it indicates the location of our chronology (Cauchy) horizon. In other words, the chronological horizon can *only* be present at the boundary!

We can now proceed with the analysis of the bulk. Specifically, we will want to see what is the bulk dual of the divergent stress tensor. Bear in mind that a divergence in the holographic stress tensor only implies a divergence in the extrinsic curvature of certain hypersurfaces in the bulk, and this need not be a bulk curvature singularity. In fact, it cannot be a bulk curvature singularity if the bulk is locally a constant curvature AdS geometry, as it is in our case by construction. In addition, we saw that the chronology horizon actually lies on the boundary. So, if the bulk geometry is not singular, what can then prevent a quantum excitation of the field from crossing the chronology horizon?

To answer this question, we will first rewrite the metric in a more familiar form. Namely, for any value of α, it is locally equivalent to AdS_3 (by construction), so a suitable change of coordinates[3] will give us

$$ds^2 = \kappa\Big(r^2 + \kappa(1-2\alpha))dt^2 - r^2 d\phi^2\Big) + \frac{dr^2}{r^2 + \kappa(1-2\alpha)}\,, \tag{16}$$

where we have introduced a parameter

$$\kappa = \mathrm{sign}(ø^2 - 1) = \begin{cases} +1 & \text{upper patch (CTC region)}\,, \\ -1 & \text{lower patch (regular region)}\,. \end{cases} \tag{17}$$

As we mentioned before, we will work here only with one set of values of α, namely $\alpha > 1/2$; other values lead to similar results.

We can therefore introduce $r_0^2 = 2\alpha - 1 > 0$. Setting ourselves in the lower patch,

$$ds^2 = -(r^2 + r_0^2)dt^2 + \frac{dr^2}{r^2 + r_0^2} + r^2 d\phi^2, \tag{18}$$

we see that our CTC-free part of the spacetime is given simply by global AdS_3, with $-\infty < t < \infty$. In other words, the bulk is geodesically *complete*! What this means is that excitations of the CFT cannot exit the region $\tau^2 < 1$, and the chronology horizon of the boundary, $\tau^2 = 1$, is at $t = \pm\infty$ and is, therefore, never reached at any finite time in the bulk. An excitation along a null or timelike geodesic in the bulk will travel for an arbitrarily long proper time, or affine parameter, without ever arriving at the chronology horizon $\tau^2 = 1$.

From the bulk point of view, it is clear that there cannot be any excitation that can cross from the regular part to the pathological one—it would require said excitation to have infinite energy.

In practice, this geodesic completeness means that our lower patch of the bulk geometry is *disconnected* from the pathological part. In other words, we have a fragmentation of the bulk geometry so that it smoothly (in the bulk) disconnects the two patches; a sketch of the geometry is shown in Figure 2. If we study the upper patch (the pathological one), we would find that it has conical singularities, and is filled with CTCs, as is expected. However, since we care about physical excitations that start off in the regular patch, the pathological part of the spacetime is no longer of interest to us.

One might be wondering how the static bulk of global AdS_3 is compatible with the time dependence of the boundary geometry. After all, we are familiar with global AdS_3 geometry, and we know it does not lead to any time machine questions. However, note that the timelike Killing vector in the bulk does not generate time translations at the boundary since these are broken by the asymptotic boundary conditions. In other words, the choice of a singular conformal factor at $\tau^2 = 1$ allows the boundary geometry to cover the two patches that are disconnected through the bulk. The coordinate τ compactifies the infinite extent of the bulk time t into the finite interval $\tau^2 < 1$. Thus, although the lower-patch bulk is complete, its boundary geometry is not, and its extension to $\tau^2 > 1$ involves a different bulk spacetime, see Figure 2.

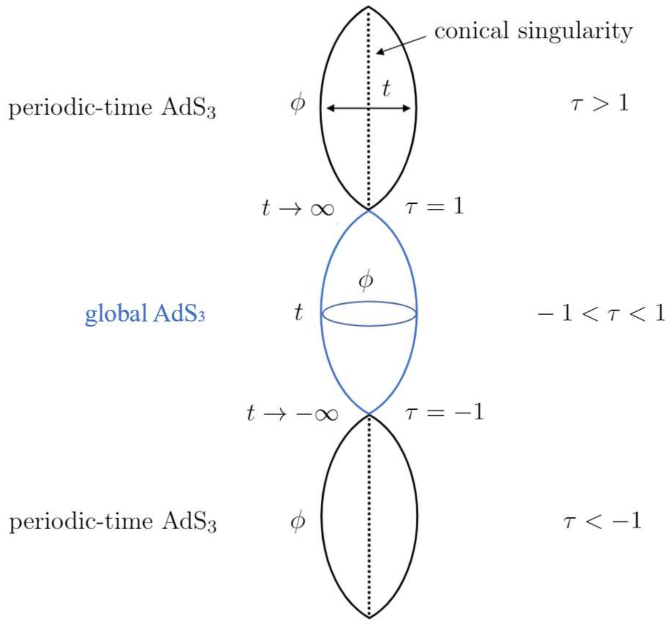

Figure 2. Sketch of the reconstructed bulk geometry; note that this is not a proper Penrose diagram. The boundary is the Misner–AdS$_2$ geometry with chronology horizons at $\tau = \pm 1$, but the bulk consists of three disconnected spacetimes. For $-1 < \tau < 1$, it is a geodesically complete, global AdS$_3$ spacetime, with time coordinate $t \in (-\infty, +\infty)$, and without CTCs. For $|\tau| > 1$, the bulk is an AdS$_3$ geometry, with CTCs (the time ϕ is periodically identified now), and with a conical singularity at the origin $r = 0$, since the "angular" coordinate t runs in $(-\infty, +\infty)$.

3.2. *n-Dimensional Time Machines*

As we will see, the main features of the reconstructed bulk will remain the same: the disconnection from the pathological part and the chronological horizon residing at the boundary. Nevertheless, it is instructive to see how we can obtain the bulk geometry in an arbitrary number of dimensions.

One caveat to keep in mind is that this construction will only work for Misner–AdS$_n$ spacetimes, but not other types of time machine spacetimes. Nevertheless, we expect the same qualitative behavior to occur whenever chronology horizons are present. This is due to the fact that the Misner geometry is analogous to the Rindler geometry: one obtains it in the limit close to a horizon, and whereas the Rindler limit zooms in on event horizons, the Misner limit does so for (some) chronology horizons[4].

Let us first outline the logic of our strategy in three dimensions $n = 1$. Since we saw in Section 2 that the BTZ black hole is a part of the Misner–AdS$_3$ spacetime, our goal would be to find the bulk dual to the extended BTZ geometry on the boundary (where by extended, we mean Misner–AdS$_3$). Luckily, such a construction has already been made in [30] for the standard BTZ geometry. The authors in [30] noticed that performing a double Wick rotation of a Schwarzschild–AdS$_4$ black hole in the bulk yields a geometry conformal to static BTZ on the boundary[5]. Let us write down the metric in the bulk,

$$ds^2 = -f(\rho)dT^2 + \frac{d\rho^2}{f(\rho)} + \rho^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{19}$$

where

$$f(\rho) = \rho^2 + 1 - \frac{\alpha}{\rho}, \tag{20}$$

with α the mass parameter of the bulk black hole, and we have set the cosmological constant to one. Let us transform it into a more convenient form by letting $\theta \to \arccos(1/r)$,

$$ds^2 = -f(\rho)dT^2 + \frac{d\rho^2}{f(\rho)} + \rho^2\left(\frac{dr^2}{r^2(r^2-1)} + \frac{r^2-1}{r^2}d\varphi^2\right). \tag{21}$$

Performing a double Wick rotation,

$$T \to i\phi, \qquad \varphi \to i\chi, \tag{22}$$

we obtain

$$ds^2 = \frac{d\rho^2}{f(\rho)} + \rho^2\left(\frac{dr^2}{r^2(r^2-1)} + \frac{f(\rho)}{\rho^2}d\phi^2 - \frac{r^2-1}{r^2}d\chi^2\right). \tag{23}$$

Finally, setting $r^2 = 1 - \tau^2$, we obtain

$$ds^2 = \frac{d\rho^2}{f(\rho)} + \frac{\rho^2}{1-\tau^2}\left(-\frac{d\tau^2}{1-\tau^2} + (1-\tau^2)\frac{f(\rho)}{\rho^2}d\phi^2 + \tau^2 d\chi^2\right). \tag{24}$$

In the limit $\rho \to \infty$, we obtain at the boundary the conformally rescaled BTZ geometry, written in terms of Misner–AdS_3 coordinates,

$$ds^2 = \frac{\rho^2}{1-\tau^2}\left(-\frac{d\tau^2}{1-\tau^2} + (1-\tau^2)d\phi^2 + \tau^2 d\chi^2\right). \tag{25}$$

In other words, we recover (9) with $n = 1$. To be more precise, we recovered the chronologically regular region where $\tau^2 < 1$; repeating the analysis for $\tau^2 > 1$ and being careful with additional minus signs, we obtain the pathological region of the time machine spacetime. We obtained the three-dimensional time machine on the boundary, but obtaining the higher-dimensional analogs is just as simple. One simply adds the remaining spheres, $d\chi^2 \to d\chi^2 + \sinh^2\chi d\Omega_{n-1}$, which go along for the ride. Putting everything together, including the regular and the pathological regions, we obtain a unified form as

$$ds^2 = \frac{\rho^2}{|\tau^2-1|}\left(\frac{d\tau^2}{\tau^2-1} - (\tau^2-1)\frac{f_\kappa(\rho)}{\rho^2}d\phi^2 + \tau^2\left(d\chi^2 + \sinh^2\chi\, d\Omega_{n-1}\right)\right) + \frac{d\rho^2}{f_\kappa(\rho)}, \tag{26}$$

with $\kappa = \pm 1$ defined as in (17), and

$$f_\kappa(\rho) = \rho^2 - \kappa + \frac{\kappa\alpha}{\rho^n}. \tag{27}$$

Note that now we have the boundary time machine geometry (and the bulk one), but no stress tensor on the boundary. In order to obtain the stress tensor, and show that it displays a similar divergence at the chronology horizons, we will use the method outlined in [31,32]. In essence, we will compute the Brown–York stress tensor, with added counterterms necessary for the finiteness of the stress tensor. One can obtain the d-dimensional stress tensor, and here we will outline the calculation for the three-dimensional case. We start with the metric for the Schwarzschild–AdS_4 solution, with $f(r) = r^2/\ell^2 + 1 - \alpha/r$, and we calculate the extrinsic data (the curvature and the scalar) of the boundary surface. The stress tensor is then given by[6]

$$T_{\mu\nu} = -\frac{1}{8\pi G}\left(K_{\mu\nu} - Kh_{\mu\nu} + \frac{2}{\ell}h_{\mu\nu} - \ell G_{\mu\nu}\right), \tag{28}$$

where $h_{\mu\nu}$ is the metric at the boundary, ℓ is associated to the three-dimensional cosmological constant, and $G_{\mu\nu}$ is the Einstein tensor associated to $h_{\mu\nu}$. The stress tensor we obtain is

$$T_\mu^\nu = \frac{1}{16\pi G}\frac{\ell\alpha}{r^3}\text{diag}(-2,1,1) \tag{29}$$

Double Wick rotation of this stress tensor simply means that $r^2 = 1-\tau^2$, with $\tau^2 < 1$. and we exchange one of the angles with time,

$$\left\langle T_\mu^\nu \right\rangle = \frac{1}{16\pi G}\frac{\ell\alpha}{(1-\tau^2)^3}\text{diag}(1,-2,1). \tag{30}$$

In the most general case, with varying $\kappa = \pm 1$ and renormalizing Newton's constant, we obtain

$$\left\langle T_\mu^\nu \right\rangle = \frac{1}{8\pi G}\frac{-\kappa\alpha}{|1-\tau^2|^{\frac{n+2}{2}}}\,\text{diag}(1,-(n+1),1,\ldots,1). \tag{31}$$

We see clearly that there is a divergence once again present at the chronology horizon, in the same fashion as before.

3.3. Adding Backreaction

Note that the stress tensor gives a divergence only if the boundary state has a non-vanishing α. However, there exist states in which α can go to zero, so one might wonder what happens to our argument for such a case.

While it is true that the stress tensor will then vanish, this will only happen when the CFT does not backreact: once we account for backreaction, the divergence at the horizon will return. Calculating this effect is no easy task, but with the help of double holography, we can obtain certain exact solutions which incorporate such a backreaction. In three dimensions, such a solution is known under the name of quantum BTZ black hole [33].

This black hole resides on a Karch–Randall brane [34], where gravity is dynamical and the CFT can backreact. Such a solution has been useful for many different purposes, including solidifying the claim of strong cosmic censorship in rotating BTZ [35], testing holographic complexity proposals [36], and even for constructions of quantum black holes in dS_3 and AF_3 spacetimes [37,38]. In this work, we can use the quantum BTZ to show that CFT backreaction leads to a divergent $r = 0$ surface.

In [21], we have written out the full metric and associated stress tensor. However, here we will only argue qualitatively why the chronology horizon is still impassable. Namely, this comes from the simple fact that the quantum BTZ is a section of a higher-dimensional black hole in AdS_4–in other words, the Schwarzschild–AdS_4 solution. Additionally, it is an equatorial section of this solution, so the (t, r) part of the 4D black hole is preserved. This implies that the black hole's inner structure is preserved as well, including the singularity structure at $r = 0$. In other words, the quantum BTZ black hole has a Schwarzschild-like, spacelike, curvature singularity at $r = 0$. Given that this is where our chronology horizon resides, it is clear nothing can cross through it, restoring chronology protection for all values of the CFT stress tensor.

4. Time in Traversable Wormholes

In this section, we will present a simple (but non-unique) recipe for creating a time machine out of a traversable wormhole, following Visser [18][7]. Indeed, this recipe consists of only three steps:

1. Acquire a traversable wormhole;
2. Induce a growing time-shift between the wormhole mouths;
3. Wait long enough.

We know how to construct solutions for traversable wormholes, so we can proceed to the next step[8]. However, before we do so, we have to emphasize that these steps in [18] revolve around traversable wormholes, which are *not* self-consistent. In other words, traversable models in the time before Gao, Jafferis, and Wall [3] have all consisted of fixed backgrounds and some "exotic" matter fields with negative energy, which would keep the wormhole from collapsing. Moreover, the role of achronicity was not yet fully understood, and so many of these models have short wormholes or even zero-length ones. Ultimately, we will see that this was the main reason why it was not realized that time machines cannot exist: a self-consistent model was not yet found, thereby allowing all kinds of non-consistent physics to seemingly occur. Nevertheless, for pedagogical purposes, we will argue how one would arrange for a wormhole time machine to form.

Step two tells us we should induce a time-shift between the mouths—what does this mean? In essence, this amounts to desynchronizing the clocks at each of the mouths. In other words, we can induce it by sending one of the mouths on a twin-paradox trip or, maybe, putting a massive object next to one of the mouths. The point is to rely on (special or general) relativistic time dilatation—going through the wormhole then connects you to different times. One can see the effect this will have if we let the time shift become large enough: the gravity of a massive object would slow time near one wormhole mouth so that a time difference between the ends of the wormhole would gradually accumulate.

That leads us to the final step three: wait long enough[9]. Enough here means until the time it takes from and to one of the mouths becomes shorter than the time shift one has induced.

Note that we can use the construction outlined in Section 2 to argue for the creation of CTCs in a wormhole. Namely, we first go to the near horizon region of a near-extremal black hole; then, our spacetime becomes of the type $AdS_2 \times X$, where X is some compact manifold. The Misner–AdS_2 geometry is then seen by a particle traveling in a circle through the wormhole[10]. When $v = const$, as in (3), we simply have one mouth boosted relative to the other—and no time machine results. However, if the mouths accelerate relative to each other (as in a twin-paradox trip, or when the mouths are at different gravitational potentials), then $v = a\tau$, as in (4), ends up creating a time machine [20]. Below, we briefly outline the physical intuition behind this construction.

Consider a wormhole where two mouths, A and B, are identified (one can put a tube between them, but for simplicity, we will first assume that the tube is short enough that its length can be neglected). Viewed from the outside, the two mouths are far apart and moving inertially, at rest relative to each other, so time runs at the same rate in both of them. At some moment, bring a black hole close to mouth B—if there is a tube joining the mouths, we assume that it remains approximately the same as before. The two mouths remain identified at all times, so an observer jumping into one mouth comes out of the other almost immediately (if there's a short tube joining them, jumping into one mouth and coming out of the other can be done in a very short time). On the other hand, when the observer compares the clocks in the two mouths through the external space, they will see that, due to the gravitational dilation caused by the nearby black hole, the clock near B slows down relative to that near A. At a sufficiently late time, this time difference will have accumulated to become larger than the light travel time between A and B through the external space. So, across the external space, mouth A is in the causal past of mouth B. It is then clear that there are closed causal curves.

Additionally, let us say that after this much time difference has accumulated, an observer leaves from near mouth A and travels across the external space to mouth B. When arriving there, she jumps into the wormhole and traverses it in almost no time to appear at A at a time earlier than she began the external trip: she has moved to her causal past on a closed causal curve.

Note that even if we have a tube between the mouths which is initially *longer* than the external distance, the effect will still be present[11], provided one waits long enough for mouth A to age sufficiently relative to mouth B and that the tube length remains approximately constant all the time, i.e., it looks like in order to prevent the formation of a time machine, the tube should grow longer and longer as time dilation accumulates.

Heuristic Argument Using the AANEC

Let us now argue why time machines cannot form, even in semiclassical General Relativity. The key point when discussing the formation of time machines is the realization of closed causal curves; one of the reasons why wormholes provide an apparently easy method to form such curves is due to their inherent structure, which allows for non-contractible cycles. However, the same inherent structure cannot be supported without obeying certain energy conditions, that is, the AANEC. This energy condition already restricted the set of all allowed (physical) wormholes; for instance, we cannot have a self-consistent solution with a wormhole whose length is shorter than the ambient space distance.

Can we use the AANEC then to restrict time machines as well? Yes, and in fact, it was shown in the original paper, that established the AANEC [12] that time machines are impossible under this energy restriction. Given that one can derive the AANEC from the fine-grained Generalized Second Law (GSL) [40], one can prove the absence of time machines using the GSL as well. However, what these papers [12,40] show is that the AANEC/GSL is incompatible with time machines; simply put, having one excludes the other. However, what they do not show is *when* does the AANEC/GSL restrict the time machine formation: at the moment of formation, or sooner? In this section, we will show that *the AANEC must act well before the time machine is formed, thereby indicating that no divergence of the stress tensor will be necessary to prevent the time machine formation, and low energy, semiclassical physics must be enough* .

In essence, the AANEC is applied to a null geodesic which represents the shortest path between wormhole mouths; this is why only long wormholes are allowed, since the AANEC then applies only to the exterior distance. However, in a setting where we are inducing a time shift and slowing down time at one of the mouths, we are shortening the effective length/time of the wormhole throat. Conversely, the ambient distance between the mouths becomes prolonged by the time shift. In other words, before making a time machine, we first need to make the wormhole the shorter path—which manifestly violates the AANEC in the throat.

Notice that we are making the wormhole shorter in the sense of how much time it takes to traverse it, and not by literally shortening it. Regardless, the AANEC applies to the geodesic, which takes the shortest time, so it becomes violated in the throat.

Let us expand a bit on this argument with some simple calculations. We will denote the proper time for exterior asymptotic observers by T and by t the proper time of observers at rest inside the wormhole[12]. The exterior distance will be denoted by D and the wormhole length by L. Now, we will need to discuss the mouths of the wormhole separately, and we will introduce appropriate parameters which will track our time-shifting for each mouth. In other words,

$$\text{Mouth 1: } t = e^{\phi_1} T, \qquad \text{Mouth 2: } t = e^{\phi_2} T, \tag{32}$$

where ϕ_i represents the gravitational potentials one induces when, for instance, bringing a massive body next to one of the mouths to induce a time shift. We will assume that $T = t = 0$ as initial conditions when all internal and external clocks are synchronized, and so $\phi_1 = \phi_2$ initially. Inducing the time shift then amounts to making one of the potentials bigger, say $\phi_2 > \phi_1$, which makes the second mouth "run faster", that is, the first mouth heavier. In order to make a time machine, we simply enter the heavier mouth and exit through the lighter one—this will allow us to "come back before we started" provided that ϕ_2 is large enough.

However, we will also gain energy in this process: the energy is not conserved, and our gravitational field is non-potential. In [2], it is explained how this feature of non-potentiality gives rise to time machines. There are various reasons behind this feature, but here we will be content with presenting one way: the difference in the potentials comes from identifying mouths that have different masses M_1 and M_2 and the same radius R—we change the mass of one mouth by surrounding it with a spherical mass shell. We will be using a simple model of Schwarzschild black holes for wormhole mouths. Then $e^{\phi_i} = 1 - M_i/R$, where small $\phi_i = -M_i/R$, and when $M_1 > M_2$ then $\phi_1 < \phi_2 < 0$. We will denote the difference between the potentials as

$$\Delta\phi = \phi_2 - \phi_1 > 0. \tag{33}$$

Now, in order to obtain some intuition of how time machine formation works, we will first analyze the case of a zero-length wormhole. In that case, when we enter the first mouth at some exterior time T, which in wormhole time parametrization is $t = e^{\phi_1}T$, we emerge at the second mouth also at time t, since $L = 0$, and this translates to some exterior time T' which is equal to $e^{-\phi_2}t$, that is $T' = e^{-\Delta\phi}T$. A time machine is formed when two conditions are satisfied

$$T' = e^{-\Delta\phi}T < T \tag{34}$$

but also

$$|\Delta T| < L + D = D. \tag{35}$$

We can approximate $1 - e^{-\Delta\phi} \sim \Delta\phi$, which then gives

$$T > T_c = \frac{D}{\Delta\phi}. \tag{36}$$

Therefore, we see that we make closed loops when the exterior time T to traverse the distance D becomes larger than the same distance, re-scaled by the time shift. In other words, the effective distance (in this case, only the exterior one) has shrunk due to the time shift.

Now we can take up a more serious example with a finite-length wormhole throat, for instance, as described by

$$ds^2 = -dt^2 + dr^2 + R^2 d\Omega^2, \tag{37}$$

where now it takes some time to cross the wormhole since it has finite length L. This wormhole, if long enough, can initially be a realistic, i.e., AANEC complying, traversable wormhole. We can denote this time as $\Delta t = t' - t$. Again, one enters the heavier mouth at time $T = e^{-\phi_1}t$, arrives at the other mouth at time $t' = t + L$, which corresponds to the exterior time to $T' = e^{-\phi_2}t' = e^{-\Delta\phi}T + e^{-\phi_2}L$. In order to close the loop, one travels back to the first mouth using the exterior path and arrives at mouth 1 at time T'', which is given by

$$T'' = D + T' = D + e^{-\Delta\phi}T + e^{-\phi_2}L. \tag{38}$$

As before, the time machine forms when $T'' < T$, that is,

$$e^{-\phi_2}L + D < \Delta\phi T, \tag{39}$$

or in other words, when we reach the critical point

$$T_c = \frac{e^{-\phi_2}L + D}{\Delta\phi}. \tag{40}$$

Notice that the wormhole length is allowed to be as long as we want: given any L, there exists a time T_c for which we form closed causal curves; we just have to wait a bit longer.

This proves wrong the common lore that "time machines only happen with short wormholes". In any case, we can simplify the expression by putting $\phi_2 \ll 1$, which gives

$$T_c = \frac{L + D}{\Delta\phi}. \tag{41}$$

Now that we see what the conditions to form a time machine are, we would like to see what conditions the AANEC imposes. In other words, we want to see when the exterior light ray arrives later than the interior one, thereby making the wormhole path a shorter one. The race starts at exterior time $T = e^{-\phi_1} t$, as before. However, now we send two light rays: one through the wormhole (inner light ray) and one through the exterior (outer light ray). We see that the light rays arrive at

$$T^{\text{out}} = D + T, \qquad T^{\text{inn}} = e^{-\phi_2} t' = e^{-\phi_2} L + e^{-\Delta\phi} T, \tag{42}$$

and so,

$$T^{\text{out}} > T^{\text{inn}} \tag{43}$$

when

$$T > \frac{e^{-\phi_2} L - D}{\Delta\phi}, \tag{44}$$

which for $\phi_2 << 1$ reduces to

$$T > T^{\text{achr}} = \frac{L - D}{\Delta\phi}, \tag{45}$$

while

$$T > T_c = \frac{L + D}{\Delta\phi}. \tag{46}$$

Since $T^{\text{achr}} < T_c$, we see that the AANEC becomes violated before we form a time machine! The only case where these two times become comparable is for very long throats, $L \gg 1$, and one might say that at that point, we need to revert back to the divergent stress tensor resolution. However, there are upper bounds on the length of the throat [5,41]. In essence, for very long throats, the temperature of the black holes becomes very small, leading us outside the regime of thermodynamics and allowing for fluctuations similar in size to excitations of our supporting field. In other words, quantum gravity fluctuations become relevant, and we cannot trust our solution any more. It seems then that quantum gravity does kick in for large L, one way or another. Nevertheless, for all intermediate stages, it is clear that the time machines must be ruled out on the basis of AANEC, at times sufficiently earlier than the time for the formation of closed causal curves, and when only physics well below the Planck scale is involved.

5. Discussion

We have shown two different perspectives on the problem of time machines, both leading to the conclusion that they are inaccessible to observers. The first perspective is a holographic one: we constructed a time machine on the boundary of AdS in arbitrary dimensions and looked at the associated bulk dual. The holographic map geometrizes the problem, translating the inaccessibility of time machines on the boundary to either geodesic completeness of the bulk, or to a curvature singularity at the chronology horizon. In both cases, it is clear that observers who start in the regular part of the spacetime cannot cross to the pathological part.

The second perspective uses only semiclassical General Relativity, with an energy condition imposed on the quantum fields. We have seen through a simple toy calculation that this condition becomes violated much before any chronology horizon is formed, implying that backreaction effects must kick in sooner than previously thought. Of course, this perspective necessitates that our time machine is one constructed through the wormhole protocol, outlined in Section 4, so we cannot apply the same conclusion to, for instance, Misner–AdS spacetimes. Regardless, the wormhole protocol is the most physical protocol one would use to construct a time machine.

There are a couple of lessons we can extract from these two perspectives. One lesson concerns the more general class of Cauchy horizons and their instability. We have seen that chronology horizons, which are a subclass of Cauchy horizons, lead to a divergent stress tensor of the quantum fields. However, this is not the first time quantum effects led to an unstable Cauchy horizon: the strong cosmic censorship is now widely believed to be upheld exactly because of this same reason [42]. Namely, if one computes the stress tensor of a conformally coupled massless scalar in the Reissner–Nordström geometry, one obtains a singularity at the inner horizon of this black hole[13]. One can then conjecture that *all Cauchy horizons will be unstable to quantum fluctuations*, thereby restoring predictability of General Relativity, with quantum effects.

However, note that trivial Cauchy horizons would not fall under this conjecture. By trivial, we mean Cauchy horizons obtained through the restriction to a region in a globally well-defined spacetime. Indeed, such horizons simply restrict the predictive power of an observer within the causal diamond, but they do not imply global predictivity is lost; the examples of globally lost predictivity include inner horizons of black holes and chronology horizons of the type discussed in this article.

Another lesson from the second perspective would be that semiclassical backreaction is sometimes enough to avoid the creation of curvature singularities (or maybe other types of scenarios that naively require the knowledge of quantum gravity). Namely, had the AANEC not been violated before the formation of the chronology horizon, we would have had a singularity in the traversable wormhole. We expect that the wormhole will self-correct once the backreaction effects are included. In other words, as we try to create a closed timelike curve, the wormhole tube would become longer and longer, never allowing for the closed curve to form[14].

Both of these lessons tell us that including quantum effects in gravitational physics can lead to an (unexpected) plethora of new results.

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Notes

1. In the same paper, the AANEC was used to forbid the realization of *short* traversable wormholes; all the known examples mentioned above are wormholes whose length is longer than the ambient space distance between the mouths.
2. One might worry that putting a CFT on a pathological spacetime might nullify the holographic map; after all, are pathologies on the boundary not mapped to pathologies in the bulk? This worry might be justified if our whole boundary spacetime was filled with CTCs (in other words, an eternal time machine). However, we will see that our models start with a completely regular spacetime which, at some point, develops a chronology horizon. This way, our initial value problem is well-defined, and one can proceed with the bulk reconstruction.
3. The exact transformation $(\tau, z) \to (t, r)$ is found in [20].
4. This is not true for Kerr inner horizons, as discussed in Section 5.
5. They also find the rotating BTZ solution on the boundary, but this will not be of interest to us here.
6. Note that the counterterms vary with respect to the number of spacetime dimensions.
7. For more details, see [39].

8 We emphasize that the experimental confirmation of their existence is still lacking.

9 In [18], this step is replaced by "bring the mouths close together". This version assumes that the second step simply induces a time-shift that changes the synchronization between the mouth clocks. As such, if the time-shift does not grow large enough, no time machine could have been created without making the ambient distance shorter than the time-shifted wormhole length.

10 Such a restriction is well justified when the particle is constrained to move along such lines, as is the case in the model, which uses magnetic field lines [5] or in the model with cosmic strings [7].

11 Disregarding backreaction.

12 If there are gravitational redshifts within the wormhole, then we simply need to know how the internal time hooks up to the exterior time at the two mouths.

13 The calculation done in [42] is believed to be correct also for the Kerr black hole, although a proper calculation has not been done yet. One could have thought that we could use our knowledge about Misner–AdS spacetimes to infer that the Kerr inner horizon is similarly singular. Namely, if in the near inner horizon limit of the Kerr black hole, one finds the solution of the kind Misner–$AdS_2 \times X$, where X is some compact manifold, then our problem would be solved—Misner–AdS_2 leads to a divergent stress tensor at the chronology horizon, which would confirm the strong cosmic censorship. However, this is not the case: as one takes the limit close to the inner horizon, one finds a Rindler geometry instead. This is to say that the chronology horizon is a completely regular surface; Misner–AdS_2 gives a quasi-regular surface only [21]. Nevertheless, as argued in [35], we expect higher-order quantum corrections to destabilize even these regular surfaces.

14 In progress [43].

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