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Addendum: Islands in multiverse models

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Abstract: We append a discussion on backreaction for the two-dimensional models that we considered to provide more details related to the technical issues mentioned in the companion erratum.

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In section 2.1, we considered vacuum solutions of Jackiw-Teitelboim (JT) gravity with positive, zero, and negative cosmological constant, and in section 2.2, we deformed these theories by coupling the background metric to a two-dimensional CFT with field content collectively denoted by ψ.

We can describe each of these three deformed theories in a unified way with the action

$$\begin{aligned} I[g_{\mu\nu},\phi,\psi] = \frac{\phi_0}{16\pi G_N}\int \mathrm{d}^2x\sqrt{-g}\mathcal{R} - \frac{1}{16\pi G_N}\int \mathrm{d}^2x\sqrt{-g}(\phi\mathcal{R}-V(\phi)) \\ + I_{\rm GHY}[g_{\mu\nu},\phi] + I_{CFT}[g_{\mu\nu},\psi], \end{aligned} \tag{1}$$

where $V(\phi)=2\phi$ results in a de Sitter (dS) solution, $V(\phi)=2$ results in a flat solution, and $V(\phi)=-2\phi$ results in an anti-de Sitter (AdS) solution. Varying this action with respect to the metric and the dilaton field ϕ produces the following equations of motion in the semiclassical limit:

$$\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\nabla^2\phi - \frac{1}{2}g_{\mu\nu}V(\phi) = -8\pi G_N\langle T_{\mu\nu}\rangle\,, \tag{2}$$

$$\mathcal{R} = V'(\phi)\,, \tag{3}$$

where $\langle T_{\mu\nu}\rangle$ is the expectation value of the covariant stress-energy tensor of the CFT. The vacuum solution corresponds to setting $\langle T_{\mu\nu}\rangle = 0$.

Upon including the coupling to the CFT, we showed that a contribution to $\langle T_{\mu\nu}\rangle$ coming from a trace anomaly can be removed by a suitable field redefinition, and we subsequently proceeded with the vacuum solution for ϕ in section 2.2 and beyond. However, we did not account for contributions from the Weyl anomaly and the Casimir energy when we examined an n-fold extension of dS_2. These contributions cancel when $n=1$.[1]

Begin with the line element defined in eqs. (2.22) and (2.23) and let $x^\pm = \sigma\pm\varphi$. As before, we can remove $\langle T_{+-}\rangle$ by a suitable redefinition of the constant ϕ_0 (as it arises from the conformal anomaly). The Weyl anomaly and Casimir energy [1] combine to give

$$\langle T_{\pm\pm}\rangle = \frac{c}{48\pi}\left(1-\frac{1}{n^2}\right), \tag{4}$$

where we recall that the spatial coordinate φ is $2\pi n$-periodic, and we have chosen the state in the z, $\bar{z}$ coordinates of (2.24) to be in vacuum. Therefore, the sourceless solutions for the dilaton will acquire a supplementary additive term due to the source on the right-hand-side of eq. (2) when $n>1$.

Our starting point was the case $\mathcal{R}=2$, i.e. dS_2^n. In this case, the solution of eq. (2) with $V(\phi)=2\phi$ and the source (4) is

$$\phi = \phi_r\frac{\cos\varphi}{\cos\sigma} - \frac{cG_N}{3}\left(1-\frac{1}{n^2}\right)(\sigma\tan\sigma+1), \tag{5}$$

cf. eq. (2.5). Notice, however, that when $\phi_r/G_N \gg c$, the vacuum contribution dominates over the additive correction, and so we can safely neglect the correction in this limit.

[1]We thank Edgar Shaghoulian for pointing this out to us.

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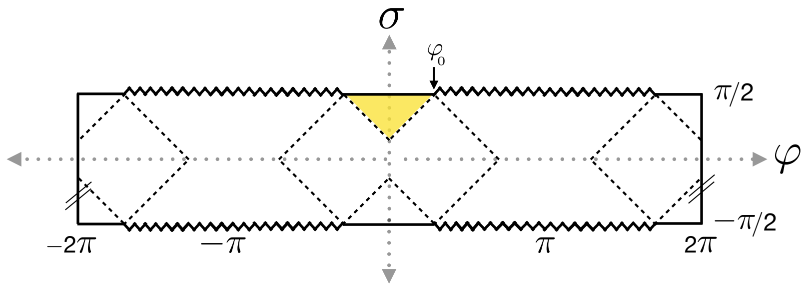

Figure 1. Penrose diagram for dS_2^n with $n = 2$. An expanding patch is shaded in yellow, and it intersects $\mathcal{I}^+$ on the interval $(-\varphi_0, \varphi_0)$ where $\varphi_0 < \pi/2$.

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In particular, we can still attempt to build up a JT multiverse using vacuum solutions as follows. Starting with pure dS_2^n with $\phi_r/G_N \gg c$, we drop the correction due to a CFT stress-energy source and work with the vacuum dS dilaton. If we want to include bubbles, the only choice is to patch in bubbles in which the dilaton obeys vacuum equations of motion; otherwise, the gluing would result in a dilaton that is not continuous across bubble interfaces. However, the CFT then cannot be in the Minkowski vacuum of (2.24), for which there would have to be nonzero $\langle T_{\pm\pm}\rangle$ sources in flat and AdS regions per eq. (4). Instead, the CFT is in some state such that $\langle T_{\pm\pm}\rangle$ vanishes everywhere. It is unclear whether such a CFT state is well defined and whether its entropy is close to that of the Minkowski vacuum of (2.24), so that eq. (2.27) continues to hold. But, with these caveats, the existing analysis goes forward.

Alternatively, if we do not neglect the source (4), we can solve for a backreacted dilaton on the manifolds that we specified in eqs. (2.22) and (2.23). Let us start with dS_2^n in the absence of any bubbles. Compared to the vacuum solution, the modified dilaton solution (5) causes the parts of $\mathcal{I}^+$ on which $\phi \to +\infty$ (i.e., the future boundary of the expanding patches) to shrink. By inspection, these are the parts of $\mathcal{I}^+$ for which

$$\cos\varphi > \frac{\pi c G_N}{6\phi_r}\left(1 - \frac{1}{n^2}\right) . \tag{6}$$

For such regions of $\mathcal{I}^+$ to exist, one must have that

$$\frac{\phi_r}{G_N} > \frac{\pi c}{6}\left(1 - \frac{1}{n^2}\right) , \tag{7}$$

which we assume here. This is illustrated in figure 1.

Our analysis of quantum extremal islands then proceeds essentially verbatim. We again consider a region R whose endpoints lie near the corners of the expanding patch centred about $\varphi = 0$, and we posit an island whose endpoints lie just beyond the patch's corners. Because the locations of the corners are now shifted relative to the vacuum case, instead of the ansatz (3.4), we write

$$\begin{aligned} \sigma_R &= \frac{\pi}{2} - \delta\sigma_R\,, & \sigma_I &= \frac{\pi}{2} - \delta\sigma_I\,, \\ \varphi_R &= \varphi_0 - \delta\varphi_R\,, & \varphi_I &= \varphi_0 + \delta\varphi_I, \end{aligned} \tag{8}$$

where we have defined

$$\varphi_0 = \arccos \frac{\pi c G_N}{6\phi_r}\left(1 - \frac{1}{n^2}\right). \tag{9}$$

Making these substitutions in (3.2), we get

$$\begin{aligned} S_{\text{gen}}((R\cup I)^c) \approx \frac{c}{3}\log\left[\frac{2n^2\left(\cos\left(\frac{\delta\sigma_I-\delta\sigma_R}{n}\right)-\cos\left(\frac{\delta\varphi_I+\delta\varphi_R}{n}\right)\right)}{\epsilon_{\text{rg}}\epsilon_{\text{uv}}\delta\sigma_I\delta\sigma_R}\right] \\ +2\phi_r\frac{\cos(\varphi_0+\delta\varphi_I)}{\delta\sigma_I} - \frac{\pi c}{12}\left(1-\frac{1}{n^2}\right)\frac{1}{\delta\sigma_I} + 2\phi_0\,, \end{aligned} \tag{10}$$

where we have set $4G_N = 1$. Next, if we expand $\cos(\varphi_0 + \delta\varphi_I)$ about φ_0 and assume that the sum $\delta\varphi_I + \delta\varphi_R$ and the difference $\delta\sigma_I - \delta\sigma_R$ are small, we arrive at

$$S_{\text{gen}}((R\cup I)^c) \approx \frac{c}{3}\log\left[\frac{(\delta\varphi_I+\delta\varphi_R)^2-(\delta\sigma_I-\delta\sigma_R)^2}{\epsilon_{\text{rg}}\epsilon_{\text{uv}}\delta\sigma_I\delta\sigma_R}\right] - 2\phi_r'\frac{\delta\varphi_I}{\delta\sigma_I} + 2\phi_0, \tag{11}$$

where we have defined

$$\phi_r' = \phi_r\sqrt{1-\left(\frac{\pi c(1-n^{-2})}{24\phi_r}\right)^2}. \tag{12}$$

This is identical in form to eq. (3.6), and so the rest of the analysis proceeds as before, but with $\phi_r \to \phi_r'$. While ϕ_r' now depends on n explicitly, any n-dependence only enters at $O((c/\phi_r)^2)$.

Next, we consider multiverse models in the presence of flat or AdS_2 bubbles, like the ones in section 3.2. The task is to show that the backreacted dilaton solutions continuously join up along bubble interfaces. Let us first examine the flat case. With a judicious choice of integration constants, the solution of eq. (2) with $V(\phi) = 2$ and the source (4) for ϕ in a flat bubble centred about $\varphi = 0$ reads (cf. eq. (2.12))

$$\phi = \phi_r - K_n + 2\frac{\cos\varphi - \cos\sigma}{\cos\varphi + \cos\sigma} - K_n\frac{\sigma\sin\sigma + \varphi\sin\varphi}{\cos\varphi+\cos\sigma}, \tag{13}$$

where we have defined

$$K_n = \frac{cG_N}{3}\left(1-\frac{1}{n^2}\right). \tag{14}$$

In particular, the flat solution (13) and the dS solution (5) coincide along $\sigma = |\varphi|$, and so they can be continuously joined together.

A consequence of the backreaction is that $\mathcal{I}^+$ in a flat bubble develops segments where $\phi \to -\infty$. By examining the behaviour of (13) as one approaches the lines $\sigma = \pi \pm \varphi$, one concludes that $\varphi < |\varphi_f|$ is the portion of $\mathcal{I}^+$ on which $\phi \to \infty$, where

$$\tan\varphi_f = \frac{4}{\pi K_n}. \tag{15}$$

This is illustrated in figure 2. Therefore, should islands still develop, we expect that the endpoints of a region R inside of a flat bubble should be placed just to the interior of $(\sigma, \varphi) = (\pi - \varphi_f, \pm\varphi_f)$. We were unable to locate extrema of $S_{\text{gen}}((R\cup I)^c)$ by placing

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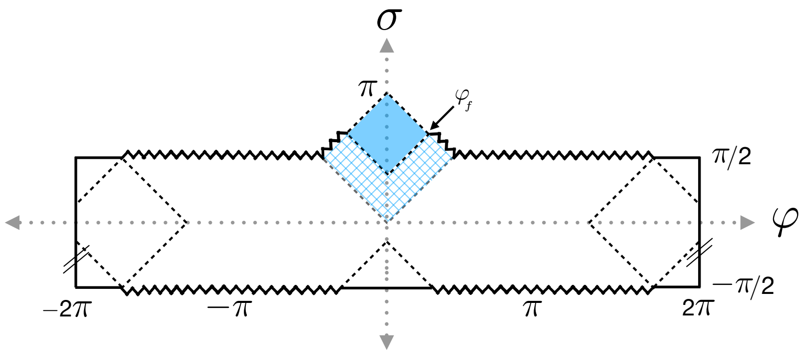

Figure 2. Penrose diagram for dS_2^n with $n = 2$ and a flat bubble centred at $\varphi = 0$. The domain of dependence of the part of $\mathcal{I}^+$ on which $\phi \to \infty$ is shaded in blue, but the metric is still flat in the blue hatched region.

(σ_I, φ_I) perturbatively to the past of $(\pi - \varphi_f, \varphi_f)$ and mirroring our earlier analysis, and so additional work would be needed to conclusively determine whether or not islands develop.

For an AdS_2 bubble, the solution of eq. (2) with $V(\phi) = -2\phi$ and the source (4) for ϕ reads (cf. eq. (2.16))

$$\phi = -\phi_r \frac{\cos\sigma}{\cos\tilde{\varphi}} - K_n(\tilde{\varphi}\tan\tilde{\varphi} + 1), \tag{16}$$

where $\tilde{\varphi} = 0$ corresponds to the centre of the bubble. Its value along the line $\sigma = \tilde{\varphi}$ for $0 < \tilde{\varphi} < \pi/2$ coincides with the value of the dS dilaton (5) along the line $\sigma = \pi + \varphi$ for a constant shift $\tilde{\varphi} = \varphi + \pi$. Similarly, the AdS dilaton's value along the line $\sigma = -\tilde{\varphi}$ for $-\pi/2 < \tilde{\varphi} < 0$ coincides with that of (5) along the line $\sigma = \pi - \varphi$ for a constant shift $\tilde{\varphi} = \varphi - \pi$. Therefore, with the appropriate coordinate translations, the backreacted dS and AdS dilaton profiles continuously join up along AdS_2 bubble walls.

In our previous analysis, we were only able to locate islands with endpoints in AdS_2 bubbles numerically in the regime $\phi_r/G_N \ll c$. Therefore, we cannot conclusively say whether or not islands develop when we take backreaction into account, which requires $\phi_r/G_N > c$ for $n > 1$ per eq. (7).

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