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TEUTA # KINEMATICAL ANALYSIS PROGRAM FOR COLLIDING BEAM EVENTS

by

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ABSTRACT

TEUTA is a FORTRAN IV program that does kinematical analysis of events with correlated momenta, such as events without a visible vertex. Incoming, outgoing, and target particles are all treated as four-component vectors (m_i, $1/p_i$, λ_i, φ_i) thus allowing TEUTA to be used for colliding beam, unknown target perticle, and disintegrating particle experiments with no program changes.

TEUTA is written in subroutine form where everything is kept as simple as possible so that the user need supply only the peripheral routines necessary to blend TEUTA into his system.

The additional peripheral programs described in this report were developed at SLAC for streamer chamber data analysis.

To use TEUTA one must first supply data describing an event in the form of two adjoint four-vector arrays, P_i^m and I_i, and full correlation error matrix, G_{ij}. The P_i^m array contains m_i, $1/p_i$, λ_i, φ_i, and the I_i array contains integers that tell if the P_i^m data is fixed, measured, or unknown. Then one may supply any number of hypotheses consisting of lists of fixed masses preceded by plus or minus signs to depict incoming and outgoing particles. TEUTA calculates new "best values," P_i^s, such that energy and momentum are conserved and the expression $(P_i^m - P_i^s) G_{ij}^{-1} (P_j^m - P_j^s)$ is minimized.

I. GENERAL DESCRIPTION

The FORTRAN IV program TEUTA is written primarily for the analysis of the data produced in a streamer chamber (SC). At the present stage of development, where SC is used as a detector in a scattering experiment, the following characteristics are of interest for data analysis: The primary vertex is invisible, the target material is easily changeable, and there are no significant coulomb or nuclear multiple scattering effects. All other characteristics are identical to those in bubble chamber analysis. For the details about SC operations see Ref. 1, and for geometric analysis of the events see Ref. 2. In our procedure we adopt the method of least squares fit with correlations, successfully used in the Berkeley SQAW and CERN GRIND[3] programs. Our treatment differs in that we use a full correlation matrix and treat all particles, incoming and outgoing, on the same footing. As a result, unknown quantities (m, $1/p$, λ, φ) for the target particle as well as any other particle can be found, and colliding beam experiments are readily analyzed. The full correlation matrix allows us to analyze an invisible vertex, where one cannot omit the correlations between the tracks.

Non ignarus mali, we intended to supply kinematical analysis programs for the group which does not have computer group service.

A. Input Parameters

TEUTA requires the following input parameters:

$1/p$	inverse momentum (in same units as the masses)
λ	deep angle at vertex (in radians)
φ	azimuthal angle at vertex (in radians)
GI	full correlation error-matrix between $1/p$, λ, φ of all "tracks."

Table I gives the block of measured quantities.

In TEUTA we distinguish three types of input parameters:

(1) Fixed, a quantity which is fixed at the beginning of fitting, without error, and remains unchanged by the fitting process. (Masses are mostly fixed but any other quantity may also be fixed.)

(2) Measured, a quantity which is changed by a least squares fitting process. (Masses may not be measured quantities.)

(3) Unknown, a quantity which is initially unknown and is changed by the fitting process. This quantity is not weighted by error. TEUTA finds a best value for the unknown quantity and its corresponding errors. (Any quantity may be unknown, but for one vertex nor more than three may be used.)

For each hypothesis which one would like to try, TEUTA constructs from input parameters the vectors for each particle:

$$VNF_i \equiv (m_i , 1/p_i , \lambda_i , \varphi_i)$$

and also the adjoined vector:

$$IVNF_i \equiv (I1_i , I2_i , I3_i , I4_i)$$

where I1, I2, etc. are the numbers 1, 2 or 3 if the corresponding quantity in VNF is fixed, measured, or unknown.

TEUTA is designed for one-vertex fits. However, one can easily construct a quasi-multivertex fit by fitting one vertex at a time, using the unknown quantities of the first vertex as fixed quantities in the solution of the second vertex, etc.

We did not consider multivertex fits, not only because of our desire to keep TEUTA simple, but also because invisible vertices and visible vertices are qualitatively different, and it would not have been good to treat them in the same manner. Further, the very big full correlation matrices encountered in a multivertex fit could have caused numerical and spatial difficulties.

B. Description of Least Squares Fit

This is a brief summary of the notation and the method of least squares used in TEUTA. TEUTA tries to find "best values" for a set of correlated quantities measured to some precision. The "best values" are those which satisfy the constraints of conservation of the total four-momenta components and, furthermore, which give a "minimum" value to the sum of weighted differences between measured and "best values." The number of degrees of freedom is equal to the number of constraints (four, minus the number of unknown quantities). This problem is well known and extensive literature as well as proofs and limitations can be found in Ref. 4. For the notations used in TEUTA, see Table I.

VNF (I,J) is the array of starting values, when:

I = 1, 2, 3, ... NP number of particles (incoming and outgoing)

J = 1 for mass

= 2 for 1/p

= 3 for λ

= 4 for φ

VNF may contain fixed, measured, or unknown quantities.

CR2 (I, J) is the array of old corrections to VNF

CR (I, J) is the array of new corrections for the next iteration, such that after each iteration better values are obtained:

$$V(I, J) = VNF(I, J) + CR(I, J)$$

Vector FCON(N), N = 1, 4 has the components of a total four-vector and it is a function only of CR(I, J).

$$FCON(1) = \sum_{I=1}^{NP} \frac{A(I)}{V(I, 2)} \cos\left(V(I, 3)\right) \cos\left(V(I, 4)\right)$$

$$FCON(2) = \sum_{I=1}^{NP} \frac{A(I)}{V(I, 2)} \cos\left(V(I, 3)\right) \sin\left(V(I, 4)\right)$$

$$FCON(3) = \sum_{I=1}^{NP} \frac{A(I)}{V(I, 2)} \sin\left(V(I, 3)\right)$$

$$FCON(4) = \sum_{I=1}^{NP} A(I) \sqrt{\left(\frac{1}{V(I, 2)}\right)^2 + V(I, 1)^2}$$

A(I) is -1. or +1. for incoming and outgoing particles, respectively.

GI(M,N) is the full error matrix (see Table III). In the fitting procedure we use part of GI(M,N), $GI(K_1, K_2)$. K always indicates the measurable quantities; L always indicates the unknown quantities.

To write programmed formulas we shall introduce the following notation:

C_K are the correction components of CR(I, J) to measured quantities

C^*_L are the correction components of CR(I,J) to unknown quantities ($L \leq 3$)

$\bar{C}_K$ are approximate values of C_K

$\bar{C}^*_L$ are approximate values of C^*_L (for the very first approximation we use $\bar{C}_K = 0$ and $\bar{C}^*_L = 0$)

F_λ are the components of FCON(N) vector $\lambda = 1, 4$

$$F_{\lambda K} \equiv \frac{\partial F_\lambda}{\partial C_K}$$

$$F^*_{\lambda L} \equiv \frac{\partial F_\lambda}{\partial C^*_L}$$

We define the following quantities:

$$\chi^2 = C_{K_1} \, GI^{-1}_{K_1, K_2} \, C_{K_2} \qquad (1)$$

$$\bar{\chi}^2 = \chi^2 + \alpha_\lambda F_\lambda(C_K, C^*_L)$$

$\bar{\chi}^2$ is only a function of C_K, C^*_L and α_λ. We try to find the minimum of Eq. (1) under the conditions $\alpha_\lambda F_\lambda = 0$. The conditions for the minimum of Eq. (1) are:

$$\frac{\partial \bar{\chi}^2}{\partial C_K} = 2 C_{K_1} \, GI^{-1}_{K_1, K} + 2 \alpha_\lambda F_{\lambda, K} = 0 \qquad (2)$$

$$\frac{\partial \bar{\chi}^2}{\partial C^*_L} = 2 \alpha_\lambda F^*_{\lambda L} = 0 \qquad (3)$$

$$\frac{\partial \bar{\chi}^2}{\partial \alpha_\lambda} = 2 F_\lambda = 0 \qquad (4)$$

Because the functions $F_{\lambda,K}$, $F^*_{\lambda,L}$, F_λ are not linear in C_K and C^*_L, solution of the system is not straightforward. To solve this sysetm, we

linearize the F_λ functions and by iteration try to find the best solution for C_K, C_L^* and α_λ. We perform a Taylor expansion for each component F_λ around the "first" approximation $\bar{C}_K$ and $\bar{C}_L^*$:

$$F_\lambda(C_K, C_L^*) = F_\lambda(\bar{C}_K, \bar{C}_L^*) + F_{\lambda K}(\bar{C}_K, \bar{C}_L^*) \cdot \left[C_K - \bar{C}_K\right] +$$

$$+ F_{\lambda L}^*(\bar{C}_K, \bar{C}_L^*) \cdot \left[C_L^* - \bar{C}_L^*\right] + \text{NEGLECT.} \qquad (5)$$

The expansion Eq. (5) and Eqs. (2), (3) and (4) define the iteration procedure, where $\bar{C}$ and $\bar{C}^*$ are "first" iterations, C and C* "second" iterations, and so on.

For the very first iteration $\bar{C}_K = 0$, $\bar{C}_L^* = 0$. This means that we use as the first approximation just the measured quantities and, for the unknown quantities, the values calculated from constraint equations. From Eqs. (5) and (4) we obtain

$$F_{\lambda K} C_K + F_{\lambda L} C_L^* - R_\lambda = 0$$

where

$$R_\lambda = F_{\lambda K} \bar{C}_K + F_{\lambda L} \bar{C}_L^* - F_\lambda(\bar{C}, \bar{C}^*)$$

From Eq. (2) we see that

$$C_K = - GI_{KK_2} F_{K_2 \lambda} \alpha_\lambda \qquad (6)$$

Defining a (4×4) matrix, $H_{\lambda\lambda'} = F_{\lambda K} G_{KK_2} F_{K_2 \lambda}$, we get the linear system of equations:

$$- H_{\lambda\lambda'} \alpha_{\lambda'} + F_{\lambda L}^* C_L^* = R_\lambda$$

$$F_{L\lambda}^* \alpha_\lambda = O_L \text{ (zeros)} \qquad (7)$$

Equation (7) represents a linear system of $(\lambda+L)$ equations and $(\lambda+L)$ unknown; (α_λ, C_L^*). Solving Eq. (7), we can get from Eq. (6) the rest of the C_K unknowns,

and our iteration loop is closed. We stop the iteration when the new four-vectors

$$V(I,J) = VNF(I,J) + CR(I,J)$$

satisfy conservation of energy and momentum to a specified degree of accuracy and $\bar{\chi}^2$ has approached its minimum to a specified degree of accuracy. During the iteration some guiding tests, with cut-step remedies (see SQAW, Ref. 2) are performed.

In testing $\bar{\chi}^2$ as well for the final probability for the given hypothesis, we calculate χ^2 from the expression $\chi^2 = -\alpha_\lambda R_\lambda$ so that we do not need the inverse of GI.

C. Error Calculation

To calculate the error associated with the new (V) fitted quantities, we need just to transform the GI matrix from the VNF system to the V system. Using the assumption that we can linearize F_λ, the transformation matrix from the VNF system to the V system is easy to obtain:

$$\frac{\partial(VNF + CR)}{\partial(VNF)} \tag{8}$$

For the proof of this statement see Ref. 5. We put only the relation which is used in TEUTA.

From Eq. (7) one gets the $(\lambda+L) \times (\lambda+L)$ matrix of the system:

$$\begin{vmatrix} -H_{\lambda\lambda} & F^*_{\lambda L} \\ F^*_{L\lambda} & O_{LL} \end{vmatrix} \tag{9}$$

If we invert the matrix (9), we get a new inverse $(\lambda+L) \times (\lambda+L)$, which we call

$$\begin{vmatrix} W_{\lambda\lambda} & V_{\lambda L} \\ V_{L\lambda} & X_{LL} \end{vmatrix} \tag{10}$$

(N. B. Only for Eq. (10) did we use complete inversion of the matrix; in Eq. (7) only a partial inversion was used.)

Then we can write:

$$C^*_L = V_{L\lambda} R_\lambda$$

$$\alpha_\lambda = W_{\lambda\lambda'} R_{\lambda'} \qquad (11)$$

$$C_K = GL_{KK_2} F_{K_2\lambda} \alpha_\lambda$$

and get the derivatives of the new "coordinates" (VNF + CR) with respect to the old one (VNF). This gives us the transformations (8). Keeping in mind the expansion condition (5), we have

$$\frac{\partial R_\lambda}{\partial VF_K} = - F_{\lambda K}$$

and we can easily write the complete transformation matrix

$$A_{K_1K_2} = E_{K_1K_2} + GI_{K_1K} F_{K\lambda} W_{\lambda\lambda'} F_{\lambda'K_2}$$

$$E_{K_1K_2} = E_{KK} \quad \text{is the unit matrix.}$$

We treat measured and unknown quantities as independent variables so that we get for $\frac{\partial(VNF^* + C^*)}{\partial VNF}$:

$$D_{LK} = - V_{L\lambda} F_{\lambda K} \ .$$

Finally, our new error matrices are:

$$\overline{GI}_{KK'} = A_{KK_1} GI_{K_1K_2} A_{K_2K'} \quad \text{(measured variables)}$$

$$\overline{GI}^*_{LL'} = D_{LK_1} GI_{K_1K_2} D_{K_2L'} \quad \text{(unknown variables)}$$

$$\overline{K}_{KL} = A_{KK_1} GI_{K_1K_2} D_{K_2L} \quad \text{(correlation between measured and unknown).}$$

II. HOW TO USE TEUTA

TEUTA is a group of subroutines designed to calculate "best values" for the description of an event. TEUTA is not complete by itself, but must be used in conjunction with peripheral routines that provide the initial event descriptions, that supply and control hypotheses of the events, and that monitor the results. Information is communicated between TEUTA and the peripheral routines through the /TEUTA 1/ common block and through subroutine parameters.

All of the users' routines that communicate with TEUTA should have the following common statement:

```
* * *
COMMON/TEUTA 1/NTRAKS, VNF(10,4), IVNF(10,4), GI(30,30),
IPDEBG,V(10,4),GT(30,30),ERMASS(3,33),CHISQ,PROBL,
NDEG,CON,IFLEK(15),GAM,ELE,PIC,PIZ,KAC,KAZ,
PRO,NEU,LAM,SIP,SIN,SIZ,KSZ,KSN,OMN,UNK,KIN,KOUT, INTAP,IOUTAP, NPARTS
REAL KAC,KAZ,NEU,LAM,KSZ,KSN
```

A. Description of the Event

The user must set the tape unit numbers for his system. This is done by setting

KIN	the standard input unit
KOUT	the standard output unit
INTAP	the data input tape
IOUTAP	the data output tape

For each event the user must fill in the description variables in the /TEUTA 1/ common block. The description variables are:

NTRAKS: the total number of "tracks" in the event. The target is considered an incoming track.

VNF(I,J): $1/p$, λ, and φ for each track in the event:

VNF(I,1) will be filled in by TEUTA.

VNF(I,2) $1/p$, the inverse momentum in $(\text{GeV/c})^{-1}$ of the Ith track.

If $1/p \geq 10^{5}$ TEUTA sets $p = 0$

If $1/p \leq 10^{-5}$ TEUTA treats p as unknown

VNF(I,3) λ, the dip angle, in radians, of the Ith track.

VNF(I,4) φ, the azimuthal angle, in radians, of the Ith track.

IVNF(I,J): the adjoint matrix for VNF that tells if the adjoining piece of data if fixed, measured, or unknown.

If IVNF(I,J) = 1, the data is fixed, i.e., its value is held constant throughout the fitting process.

If IVNF(I,J) = 2, the data is measured, i.e., its value will be adjusted to a "best value" by the fitting process.

If IVNF(I,J) = 3, the data is unknown, i.e., it will be given an initial value by solution of constraints using the fixed and measured data. It will then be adjusted to a "best value" by the fitting process.

Only the IVNF(I,2), IVNF(I,3) and IVNF(I,4) elements should be set. IVNF(I,1) will be set by TEUTA, through calling the hypothesis.

GI(I,J): the full correlation error matrix. It is a symmetrical matrix whose rows and columns correspond to:

$$
\begin{aligned}
\text{I or J} &= 1 : \Delta\, 1/p_1 \\
&= 2 : \Delta\lambda_1 \\
&= 3 : \Delta\varphi_1 \\
&= 4 : \Delta\, 1/p_2 \\
&= 5 : \Delta\lambda_2 \\
&= 6 : \Delta\varphi_2 \\
&= 7 : \Delta\, 1/p_3 \\
&\;\;\vdots
\end{aligned}
$$

i.e.,

$$
\begin{aligned}
GI(1,1) &= \Delta\, 1/p_1\, \Delta\, 1/p_1 \\
GI(1,2) &= \Delta\, 1/p_1\, \Delta\lambda_1 \\
GI(1,3) &= \Delta\, 1/p_1\, \Delta\varphi_1 \\
GI(1,4) &= \Delta\, 1/p_1\, \Delta\, 1/p_2 \\
GI(1.5) &= \Delta\, 1/p_1\, \Delta\lambda_2 \\
GI(1,6) &= \Delta\, 1/p_1\, \Delta\varphi_2 \\
&\;\;\vdots \\
GI(2,1) &= \Delta\lambda_1\, \Delta\, 1/p_1 \\
GI(2,2) &= \Delta\lambda_1\, \Delta\lambda_1 \\
&\;\;\vdots
\end{aligned}
$$

After the user fills in these description variables, he must call the SETCOM routine in TEUTA. This routine catalogs all the event data and returns control to the users' program. This is done by the following statement:

***

CALL SETCOM

This completes the description of the event.

B. Specification of Hypotheses

The user may test any number of hypotheses for one event. A hypothesis is tested by calling the HYP subroutine:

***

CALL HYP(NTYP, NPART, P1, P2, ...)

where

NTYP is the type of hypothesis. If:

NTYP = 1 the situation is normal and the listed masses, P1, P2, ... , are tried as the masses for the first, second,... tracks.

NTYP = 2 an unseen "track" in the event is hypothesized and another track is automatically added to the end of VNF. It is assumed that the last mass (P_{NPART}) listed corresponds to the added "track." The 1/p, λ, and φ for the added track will be unknown and TEUTA will fill in the appropriate 3's in IVNF.

NPART: the total number of "tracks" for the hypothesis. This includes the extra "track" if a type 2 hypothesis is used.

P1: the mass, in GeV, of the first track. This mass will be preceded by a minus sign if the track is an incoming track. A positive mass will indicate an outgoing track. The mnemonics in /TEUTA 1/ may be used here. TEUTA will set this mass into VNF(1,1) and set a " 1" in IVNF(1,1) unless the mnemonic UNK is used, in which case it will set a 3 in IVNF(1,1).

P2: the mass, in GeV, of the second track.

etc.

For the users' convenience /TEUTA 1/ contains mnemonics for the masses of many commonly used particles. The mnemonics are preset by TEUTA and are:

Mnemonic	Mass (GeV)	Particle
GAM	0.	Gamma ray
ELE	$5.11006\ 10^{-4}$	Electron
PIC	0.139577	Pion (positive or negative)
PIZ	0.134974	Pion zero
KAC	0.49378	Kaon (positive or negative)
KAZ	0.49782	Kaon zero
PRO	0.938256	Proton
NEU	0.939550	Neutron
LAM	1.11563	Lambda
SIP	1.18953	Sigma positive
SIN	1.19733	Sigma negative
SIZ	1.1922	Sigma zero
KSZ	1.3147	Xi zero
KSN	1.3212	Xi negative
OMN	1.674	Omega minus
UNK	- - -	Unknown. This indicates to Teuta that this mass is unknown and is to be solved for by TEUTA.

Given the event data and a hypothesis, TEUTA will calculate a set of "best values" for the event.

C. Monitoring the Results

For each hypothesis TEUTA will communicate results to the user through the /TEUTA 1/ common block.

Debugging Information:

IPDEBG is a debug print parameter. If:

IPDEBG = 0 before a hypothesis call, no debugging information will be printed.

IPDEBG = 1 before a hypothesis call, debugging information will be printed.

Format: The quantity name and values will be printed for each variable.

Status Information:

The IFLEK(15) vector describes the status of a hypothesis. The structure of the IFLEK vector is:

IFLEK (1) Error Code (see list)

(2) Number of iterations

(3) Total number of times the SOLVE (subroutine to solve linear system of equation) subroutine failed to converge in the FIT routine

(4) Number of consecutive times the SOLVE subroutine failed to converge in the FIT routine

(5) The last iteration that the SOLVE subroutine failed to converge in the FIT routine

(6) Total number of times the constraints have increased from the previous iteration

(7) Number of consecutive times that the constraints have increased from one iteration to the next

(8) The last iteration that the constraints increased from the previous iteration

(9) Total number of times that chi squared increased from one iteration to the next

(10) Number of consecutive times that chi squared increased from one iteration to the next

(11) The last iteration that chi squared increased from the previous iteration

(12) Total number of times that one of the momenta became negative

(13) Number of consecutive times that one of the momenta became negative

(14) The last iteration that one of the momenta became negative.

(15) Not used

D. List of Error Codes

-01 A singularity occurred in the SOLVE subroutine during the matrix inversion to calculate the new error matrix. Still the fitted values are printed, but error could be wrong.

00 No error (O.K.).

01 The number of consecutive iterations with a negative momentum is $\geq$ NCNP (now set 2).

02 The number of consecutive iterations in which the constraints increased is $\geq$ NCCS (now set 2).

03 The number of tracks is ≤ 2 or > 10.

04 An unknown φ or λ occurred in the data in a track other than the added track in a type 2 hypothesis.

05 The number of consecutive iterations in which chi squared increased is $\geq$ NCCI (now set 2).

06 The first parameter, NTYP, of the HYP subroutine is not a 1 or 2.

07 A discrepancy exists in the number of data tracks and the number of masses in the hypothesis.

08 The square root of the sum of the squares of the constraints for the first iteration is > TSPCT $*\sum P_i$ (now set $0.1 * \sum P_i$).

09 An unknown momentum is numerically undefined, i.e., it is not possible to calculate from given constraints.

10 There are more than 3 unknowns in the input data.

11 There are more than ITSF iterations (now set 10)

12 There is more than 1 unknown mass in the input data.

13 A singularity occurred in the SOLVE subroutine during the fitting process.

14 The number of consecutive iterations in which the SOLVE subroutine failed to converge is $\geq$ NCNSS (now set 2).

15 The number of negative momenta on one iteration exceeded one.

16 For one iteration chi squared was < TSCHIL (0.0) or was > TSCHIU (1000.0).

We incorporated the facility to do (NS) steps, so-called free steps, where no testing for error is done.

Numerical Information:

If IFLEK (1) = 00 or -01, numerical results will be returned:

V(I,J) : the "best values" for the event. V is in the same form as VNF, i.e., V(I,1) = m_i , V(I,2) = $1/p_i$, V(I,3) = λ_i, and V(I,4) = ϕ_i.

GT(I,J) : the full correlation error matrix as transformed from the VNF system to the V system. GT is in the same form as GI.

ERMASS(I,J) : the correlation error matrix in the V system of any unknown masses in the hypothesis.

ERMASS(1,1) = $\Delta m_1 \Delta m_1$

ERMASS(1,2) = $\Delta m_1 \Delta m_2$

ERMASS(1,3) = $\Delta m_1 \Delta m_3$

ERMASS(1,4) = $\Delta m_1 \Delta 1/p_1$

ERMASS(1,5) = $\Delta m_1 \Delta \lambda_1$

.
.
.

ERMASS(2,1) = $\Delta m_2 \Delta m_1$

ERMASS(2,2) = $\Delta m_2 \Delta m_2$

.
.
.

ERMASS(2,6) = $\Delta m_2 \Delta \varphi_1$

.
.
.

m_1, m_2, m_3 refer here to the first, second and third UNK masses located in the hypothesis.

CHISQ : Chi squared

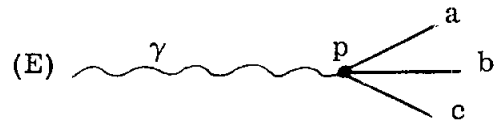
PROBL : the probability for a given hypothesis, calculated from CHISQ and NDEG

NDEG : the number of degrees of freedom

CON : the square root of the sum of the squares of the constraints

Example:

Find the "best values" for the γ-p inelastic scattering event:



where a, b, c could be proton, 2π or 2K, etc., (E) is for TEUTA 5-track event.

Track I = 1 γ

= 2 Target (with $p=\lambda=\varphi=o$)

= 3 a

= 4 b

= 5 c

In the event (E) we suppose the following:

Fixed quantities are: all masses, and 1/p, λ and φ for the target proton .

Measured quantities are all but the incident momentum of γ.

Unknown is 1/p for incident γ .

Step 1: Read the data: [At the present

Read unit for the cards is KIN = 5

Read unit for the tape INTAP = 10

Write unit for printer KOUT = 6

Write unit for tape IOUTAP = 12]

- - - - - -

NTRAKS = 5

VNF (I, J) (See Table I - Appendix)

I \ J	1 (m_I)	2 ($1/p_I$)	3 (λ_I)	4 (φ_I)
1	———	———	0.0	0.0
2	———	1.0×10^5	0.0	0.0
3	———	$\frac{1.0}{0.27693}$	0.25135	-1.32868
4	———	$\frac{1.0}{7.61232}$	0.03828	0.02529
5	———	$\frac{1.0}{4.84303}$	-0.07175	0.01607

IVNF = (I, J) (See Table II - Appendix)

I \ J	1	2	3	4
1	———	3	2	2
2	———	1	1	1
3	———	2	2	2
4	———	2	2	2
5	———	2	2	2

(The sign ——— means it will be filled by TEUTA)

GI(I, J) (See Table III - Appendix)

I \ J	7	8	9	10	11	12	13	14	15
7.	0.1320×10^{-2}	-0.2268×10^{-4}	0.2606×10^{-3}	-0.1693×10^{-4}	-0.1368×10^{-4}	-0.2135×10^{-5}	0.8787×10^{-5}	-0.5564×10^{-5}	-0.6087×10^{-5}
8.	-0.2268×10^{-4}	0.1108×10^{-4}	-0.2070×10^{-5}	0.2168×10^{-5}	0.6279×10^{-5}	0.5686×10^{-6}	-0.1808×10^{-5}	0.5107×10^{-5}	0.5677×10^{-6}
9.	0.2606×10^{-3}	-0.2070×10^{-5}	0.5554×10^{-4}	-0.1799×10^{-4}	-0.1090×10^{-5}	-0.4197×10^{-5}	0.1337×10^{-4}	0.3628×10^{-6}	-0.4797×10^{-5}
10.	-0.1693×10^{-4}	0.2168×10^{-5}	-0.1799×10^{-4}	0.3742×10^{-3}	-0.1859×10^{-5}	0.7615×10^{-4}	-0.1173×10^{-3}	-0.1720×10^{-5}	0.3555×10^{-4}
11.	-0.1368×10^{-4}	0.6279×10^{-5}	-0.1090×10^{-5}	-0.1859×10^{-5}	0.1368×10^{-4}	-0.4644×10^{-6}	0.1340×10^{-5}	0.4394×10^{-5}	-0.3875×10^{-6}
12.	-0.2135×10^{-5}	0.5686×10^{-6}	-0.4197×10^{-5}	0.7615×10^{-4}	-0.4644×10^{-6}	0.1664×10^{-4}	-0.3064×10^{-4}	-0.4355×10^{-6}	0.9280×10^{-5}
13.	0.8787×10^{-5}	-0.1808×10^{-5}	0.1337×10^{-4}	-0.1173×10^{-3}	0.1340×10^{-5}	-0.3064×10^{-4}	0.1805×10^{-3}	0.1288×10^{-5}	-0.4611×10^{-4}
14.	-0.5564×10^{-5}	0.5107×10^{-5}	0.3628×10^{-6}	-0.1720×10^{-5}	0.4394×10^{-5}	-0.4355×10^{-6}	0.1288×10^{-5}	0.7830×10^{-5}	-0.3924×10^{-6}
15.	-0.6087×10^{-5}	0.5677×10^{-6}	-0.4797×10^{-5}	0.3555×10^{-4}	-0.3875×10^{-6}	0.9280×10^{-5}	-0.4611×10^{-4}	-0.3924×10^{-6}	0.1243×10^{-4}

All elements not shown ($J \leq 15$, $I \leq 15$) are zero except for $J = 2$, $I = 2$ and $J = 3$, $I = 3$ which are 0.1000×10^{-5}.

Step 2 :

CALL SETCOM

Step 3 :

Try first hypothesis: $\gamma + p \longrightarrow p + \pi^+ + \pi^-$

CALL HYP(1, 5, -GAM, -PRO, PRO, PIC, PIC)

Step 4 :

The results will be communicated to the user through the /TEUTA 1/ common block. In the SLAC system we can print the results, the hypothesis, and/or the debugging information.

To print the hypothesis:

CALL PHYP

To print the results:

CALL WRITE

To print the debug information:

Put IPDEBG = 1 before a calculation to be printed.

Printed Output:

CALL PHYP

PARTICLE		MASS	P	LAMBDA	PHI
1	-GAM	0.00000	12.49856*	0.	0.
2	-PRO	0.93826	0.	0.	0.
3	PRO	0.93826	0.27693	0.25135	-1.32868
4	PIC	0.13958	7.61232	0.03828	0.02529
5	PIC	0.13958	4.84303	-0.07175	0.01607

PARTICLE: the track number and the mnemonic for the particle name. A minus indicates an incoming particle.

MASS: mass in GeV

P: momentum in GeV/c

LAMBDA: λ in radians

PHI: φ in radians

*An asterisk beside a number indicates that the variable was initially unknown.

CALL WRITE

EVENT NUM 112 HYP 1 CHI SQ = 0.30345007E 00 DEG = 3 PROB = 0.95937824E 00 CONSTR = 0.68297354E-05

REEL 0 SEQ 0 GEOFIT = 0. X0 = 0.0 Y0 = 0.0 Z0 = 0.0

0 2 0 0 -1 0 0 -1 0 0 -1 0 0 -0 DX0 = 0.0 DY0 = 0.0 DZ0 = 0.0

		F-MASS	U-MASS	D-MASS	F-P	U-P	D-P	F-LAM	U-LAM	D-LAM	F-PHI	U-PHI	D-PHI
1	-GAM	0.00000	0.00000	0.	12.2787	12.4968*	0.7180	0.01	0.	0.06	0.02	0.	0.05
2	-PRO	0.93826	0.93826	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	PRO	0.93826	0.93826	0.	0.2772	0.2769	0.0014	14.37	14.40	0.17	-76.17	-76.13	0.20
4	PIC	0.13958	0.13958	0.	7.4473	7.6123	0.6877	2.14	2.19	0.16	1.46	1.45	0.14
5	PIC	0.13958	0.13958	0.	4.7879	4.8430	0.1433	-4.15	-4.11	0.14	0.89	0.92	0.08

EVENT NUM : event number

HYP : hypothesis number

CHI SQ : chi squared

DEG : number of degrees of freedom

PROB : probability

CONSTR : square root of the sum of the squares of the constraints

REEL : reel number

SEQ : sequence number

GEOFIT : geometrical fit indication

X0 , Y0, Z0 : coordinates of the vertex

DX0 , DY0 , DZ0 : error on the coordinates

Row of integers: IFLEK (1) ... IFLEK (14)

The first three lines will be printed for every call of the WRITE subroutine.

If IFLEK (1) = 00 or -01, the fit will also be printed.

Track Number	track number
Particle Name	mnemonic with a preceding minus if it is incoming
F-MASS	fitted mass in GeV
U-MASS	unfitted mass in GeV
D-MASS	mass error in GeV
F-P	fitted momentum in GeV/c
U-P	unfitted momentum in GeV/c
D-P	momentum error in GeV/c
F-LAM	fitted λ in degrees
U-LAM	unfitted λ in degrees
D-LAM	λ error in degrees
F-PHI	fitted φ in degrees
U-PHI	unfitted φ in degrees
D-PHI	φ error in degrees

* An asterisk beside a number indicates that it was initially unknown.

Computing time 1.2 seconds (7090)

E. Limitations

The maximum number of tracks that may be used in an event is ten. This includes the added track in a type 2 hypothesis.

The maximum number of unknowns allowed is three. The only combinations of unknowns allowed are:

(1) The 1/p, λ, φ of the added track of a type 2 hypothesis

(2) One unknown mass

(3) One, two or three unknown 1/p's

(4) One unknown mass and one or two unknown 1/p's

It is clear that one can change just the UNKNOWN routine and calculate other possible unknown variables. Except from the constraints equation, one can get the first guess from somewhere else (rough estimate). The comments in UNKNOWN explain the operation, and it is possible just to write a new UNKNOWN routine if one wants to use other sets of unknown quantities. In the case of UNKNOWN momentum or mass equal to zero, a singularity occurs in SOLVE.

TEUTA at present stage (10-particles) needs 22,047 decimal words. The minimum requirement (4-particles) is 17,689 decimal words.

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APPENDIX

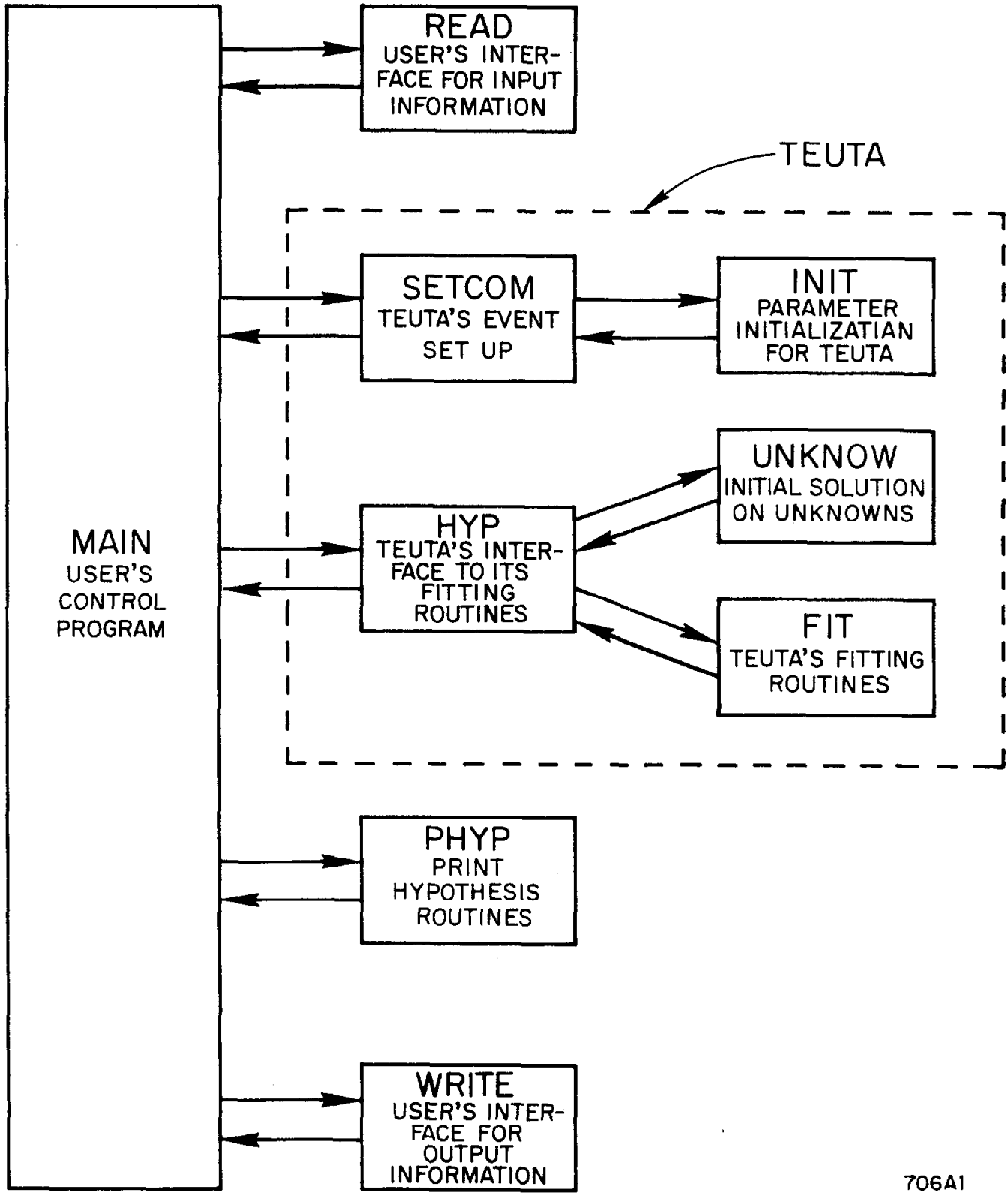
READ
USER'S INTER-FACE FOR INPUT INFORMATION
TEUTA
SETCOM
TEUTA'S EVENT SET UP
INIT
PARAMETER INITIALIZATIAN FOR TEUTA
UNKNOW
INITIAL SOLUTION ON UNKNOWNS
MAIN
USER'S CONTROL PROGRAM
HYP
TEUTA'S INTER-FACE TO ITS FITTING ROUTINES
FIT
TEUTA'S FITTING ROUTINES
PHYP
PRINT HYPOTHESIS ROUTINES
WRITE
USER'S INTER-FACE FOR OUTPUT INFORMATION
706A1

TABLE I

Structure of VNF

One row for each particle

m_1	$1/p_1$	λ_1	φ_1
⋮	⋮	⋮	⋮
m_n	$1/p_n$	λ_n	φ_n

If $1/p_i \geq 10^5$ $\quad p_i = 0$

TABLE II

Structure of IVNF

One row for each particle

$i_{1,1}$	$i_{1,2}$	$i_{1,3}$	$i_{1,4}$
⋮	⋮	⋮	⋮
$i_{n,1}$	$i_{n,2}$	$i_{n,3}$	$i_{n,4}$

If $1/p_n \leq 10^{-5}$ $\quad i_{n,2} = 3$

The IVNF array is in exact correspondence with the VNF array and each $i_{n,m}$ describes the corresponding piece of data in VNF.

$i_{n,m} = 1$ for fixed
$i_{n,m} = 2$ for measured
$i_{n,m} = 3$ for unknown

TABLE III

Structure of GI

NTRACKS = Total number of tracks

N / M	$1/p_1$	λ_1	φ_1	$1/p_2$		φ_n
$1/p_1$	$\Delta 1/p_1 \Delta 1/p_1$	$\Delta 1/p_1 \Delta\lambda_1$	$\Delta 1/p_1 \Delta\varphi_1$	$\Delta 1/p_1 \Delta 1/p_2$	. . .	$\Delta 1/p_1 \Delta\varphi_n$
λ_1	$\Delta\lambda_1 \quad \Delta 1/p_1$	$\Delta\lambda_1 \quad \Delta\lambda_1$	⋮	⋮		⋮
φ_1	$\Delta\varphi_1 \quad \Delta 1/p_1$	⋮				
$1/p_2$	$\Delta 1/p_2 \Delta 1/p_1$					
	⋮					
φ_n	$\Delta\varphi_n \quad \Delta 1/p_1$					

Structure of GT

Same as GI only they are error of the fitted values.

Structure of V

Same as VNF only it is for fitted values.