MUON CAPTURE IN COMPLEX NUCLEI

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The muon capture interaction in a theory with basic V and A couplings is described by four effective couplings[1]

g_V = renormalized vector coupling

g_M = weak magnetism coupling

g_A = renormalized axial-vector coupling

g_P = induced pseudoscalar coupling

The values of these coupling constants for muon capture on the assumption of UFI (Universal Fermi Interaction with conserved vector current) are usually given as[**]

$$g_V = .97\, g_V^{(\beta)} = .98 \times \frac{10^{-5}}{M_p^2} \tag{1a}$$

$$g_M = 3.7\, g_V \tag{1b}$$

$$g_A = g_A^{(\beta)} \simeq -1.2\, g_V^{(\beta)} \tag{1c}$$

$$g_P = 8\, g_A \tag{1d}$$

where the superscript β refers to the values in β-decay. The equations for g_A and g_P depend upon qualitative features of dispersion relations arguments, essentially the ignoring of the contributions from integrals starting at $S' = (3m_\pi)^2$, which is large compared to the value of $q^2 \simeq m_\mu^2$ relevant for muon capture. (Here $q = p_\nu - p_\mu$). The dependence on q^2 of the couplings is assumed to be unimportant for muon capture processes except possibly for the pseudoscalar coupling which is more correctly written as

$$g_P \simeq 7.5\, g_A \frac{m_\pi^2 + m_\mu^2}{m_\pi^2 + q^2} \tag{2}$$

If the predictions of Eqs. (1) should not be verified, a variety of possible reasons might be considered:

(1) Equations (1c) and (1d) are incorrect because of failure of the qualitative dispersion relation arguments mentioned above. (Deviations of 20% in g_P or of 1 to 2% in g_A would not be surprising, but such small effects are not important with respect to present experiments.)

(2) Equations (1) are correct but there exist additional couplings which make small contributions to β-decay but are important for μ-capture. Within the framework of vector leptonic currents these would be the effective scalar and weak pseudomagnetism[2], which could arise from strangeness-conserving currents with the "wrong" behavior under G.

(3) The weak interactions of electrons and muons are not the same.

In spite of many papers it seems to me to be premature to attempt to determine from experiment the quantitative values of the four (or six) effective couplings. We therefore restrict ourselves here to a comparison between experimental data and theoretical predictions based on UFI. Ideally only the data from muon capture by hydrogen should be used; unfortunately, these experiments at present yield essentially only one result and they are not likely in the future to yield more than two (capture from the two hyperfine states). Therefore data from muon capture in complex nuclei must be used.

In Table I we list the experiments (with emphasis on recent ones) that seem to provide the most significant quantitative comparison with theory.

1. *Capture in hydrogen*. Both experiments appear to give results lower than theory by 2 to 3 times the quoted error, which means by about 20% of the theoretical value. There are two sources of theoretical uncertainty (1) the values of $g_V^{(\beta)}$ and $g_A^{(\beta)}$, (2) molecular properties. If we fit the measured ft-value of the neutron within quoted errors, then the first uncertainty is no more than 5%. I am not aware, however, of any reliable estimates of the second uncertainty and so it is not clear to me whether the discrepancy between experiment and theory is really significant.

2. *Comparisons between mu-capture and β-decay*. Certain nuclear properties may cancel out when the muon capture rate is calculated directly from the ft-value of a corresponding β transition. This is done in the next two entries in Table 1; thus the ft-value of $H^3 \rightarrow He^3 + e^- + \bar{\nu}$ is used to calculate the rate of $\mu^- + He^3 \rightarrow H^3 + \nu$. The experimental result for this capture rate is now one of the most precise experimental numbers in muon capture physics, while the theoretical number may well be the least uncertain. The agreement between experiment and UFI theory for the case of He^3 represents the most significant confirmation of the theory now available.

The He^3 experiment measures the combination of Fermi and Gamow-Teller coupling constants $G_F^2 + 3G_G^2$, wheras the next experiment (the classical Godfrey experiment) depends only on G_G^2, since it is a pure Gamow-Teller transition. The agreement between theory and experiment is excellent; however, the uncertainties are such that it is impossible to extract the Fermi coupling constant G_F^2 by combining this with the He^3 result. A similar determination of the Gamow-Teller coupling has recently been made[7] at Columbia by a study of the capture rate from the ground state (0+) of O^{16} to the ground state (2-) of N^{16}.

3. *Total capture rates*. The greatest amount of data on muon capture gives total capture rates by complex nuclei. These may be calculated[9] using the closure approximation, which results in two major sources of

uncertainty: (1) the appropriate value for $\bar{\nu}$, the "mean neutrino momentum" and (2) the appropriate nuclear model for the capturing nucleus' ground state. In this approximation the total capture rates are proportional to $(G_F^2 + 3G_G^2)$, the same combination measured in the He^3 experiment, plus a relativistic contribution of about 15%. The uncertainties are least for the case of He^4, where the value of $\bar{\nu}$ can be estimated from observed triton recoils and the necessary nuclear parameter can be derived from electron scattering experiments. Theory and experiment agree but neither has the precision of the He^3 case. The relativistic contribution (about 20% for He^4) is pretty well buried in the errors, although including it improves the agreement.

Total capture rates in O^{16}, F^{19}, Ne^{20}, Cl^{37}, and Ca^{40} fit UFI when shell model wave functions are used for the nucleus provided $\bar{\nu}$ is arbitrarily set at about 75 Mev[11]. This value of $\bar{\nu}$ is considerably lower than would be obtained if the shell model were used for the excited states and corresponds to a greater nuclear excitation for the case of Ca than is indicated by recent experiments on the neutron spectrum[12] or the γ-ray spectrum in radiative capture[13]. Recent careful measurements[14] of total capture rates in heavy elements indicate a precision fit to the Primakoff formula with UFI between $Z = 48$ and $Z = 83$ provided $\bar{\nu}$ is set at the constant value of about 75 Mev. The theoretical uncertainties in attempting to derive coupling constants from such a fit are known to be large[15].

4. Experiments sensitive to the induced pseudoscalar. The next three experiments have in common a sensitivity to different combinations of coupling constants than the experiments discussed before; in particular, they are rather sensitive to the value of the induced pseudoscalar coupling. It is possible to consider a modification of Eq. (2) of the form

$$g_P = 7.5\, g_A \frac{m_\pi^2 + m_\mu^2}{m_\pi^2 + q^2} + \Delta g_P \qquad (2a)$$

where Δg_P is considered as independent of q^2 since it is not associated with the one-pion pole.

The first experiment measures the capture rate from the ground state (0^+) of O^{16} to the first excited state (0^-) and third excited state (1^-) of N^{16}. The ratio of these rates measured by Devons at Columbia is compared to the theoretical value (based on Elliott and Flowers wave functions for N^{16}) calculated by Rood[16]. In view of the sensitivity of the theoretical calculation both to the configuration admixtures and the radial wave functions, the agreement between theory and experiment must be considered satisfactory. Better agreement[⊠] is obtained with a value of $\Delta g_P \simeq 4\, g_A$.

The next experiment measures the total rate of the radiative capture in calcium. In fact, only γ-rays with energies greater than 60 Mev are measured and the theory is used to convert this into a total rate. The large effect of the pseudoscalar is in part due to the possibility of approaching closer to the one-pion pole if the γ-ray is emitted by the nucleon current. In view of the large theoretical uncertainty associated with the complex nucleus effects, theory and experiment can not be considered as in significant disagreement. Better agreement is obtained with $\Delta g_P \simeq 10 g_A$.

The last experiment is a measurement of the asymmetry of neutrons with energies greater than 20 Mev from the capture of polarized muons in calcium. While the complex nucleus effects are not easy to calculate, they would be expected to be small for high-energy neutrons and to decrease rather than increase the asymmetry. The theoretical result includes no complex nucleus effects but is decreased in magnitude by the estimated relativistic corrections, which are larger for high-energy neutrons. The theoretical magnitude can be increased to a little over .4 by letting $\Delta g_P \sim 25\ g_A$. This decreases butdoes not eliminate the disagreement.

Conclusions

1. In general the agreement between UFI and experiment seems to be satisfactory. The precision of the agreement is no better than 20% with the exception of the one experiment of capture in He^3. Therefore there exist many alternatives to UFI which fit the data as well or better.

2. The most striking anomalous result is the large neutron asymmetry found at Dubna. This experiment should be repeated by other groups and by different methods.

3. In view of the difficulty in interpreting results from complex nuclei additional precision in the determination of the coupling constants is most likely to be obtained from (a) further experiments on capture in hydrogen and theoretical study of the associated molecular problems, (b) experiments on muon capture in deuterium, and (c) further theoretical analysis of the capture in He^3 aided by additional electron-scattering experiments on He^3 and H^3.

TABLE I

Comparison of experiment and theory

EXPERIMENT	RESULT	THEORETICAL UFI
Total: Liquid hydrogen	CERN[3] 450 ± 50 sec^{-1}	585 ± ?
Total: $p\mu p$	Columbia[4] 467 ± 40	570 ± ?
$\mu^- + He^3 \rightarrow H^3 + \nu$	Dubna[8] 1410 ± 140 Berkeley[5] 1520 ± 50 Carnegie Tech[19] 1440 ±90	1530 ± 150
$\mu^- + C^{12} \rightarrow B^{12} + \nu$	Carnegie Tech[6] 6750^{+300}_{-750}	6900 ± 1500
Total: He^4	Northwestern[10] 368 ± 47	270 ± 80
$\mu^- + O^{16} \rightarrow N^{16*} + \nu \; \frac{0^-}{1^-}$	Columbia[7] .38 ± .07	55 ± ? (Ref. 16)
$\frac{\mu^- + Ca \rightarrow \gamma + \nu + \text{all}}{\mu^- + Ca \rightarrow \nu + \text{all}}$	CERN[13] 3.05 ± 0.35 x 10^{-4}	2.3 x 10^{-4} ± ? (Ref. 17)
$\mu^- + Ca \rightarrow n$ asymmetry	Dubna[18] -1.0 ± 0.15	-.20 ± ?

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FOOTNOTES

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†† In the theoretical calculations the number 1.2 in Eq. (1c) is put equal to 1.19 to fit the ft-value of the neutron and 1.21 to fit the ft-value of the triton. The number 8 in Eq. (1d) should most likely be closer to 7; however, it is left at the conventional value of 8.

⊠ I am informed (K. Crowe, private communication) that a recent measurement at Berkeley gives a value for this ratio approximately twice that found at Columbia. This result, if confirmed, would agree equally well with theory; however, the agreement might become worse if a positive Δg_P were assumed.