SELECTED TOPICS ON PHOTON AND DI-PHOTON PRODUCTION

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LAPP-TH

Some topics on di-photon and photon production are presented. It includes some results on a complete next-to-leading di-photon production including fragmentation contribution and the validity of factorisation using isolated photon in e^+e^- collisions.

1 Di-photon production at hadronic collider

Double prompt photon production is interesting because it will be the main background to the Higgs search (for intermediate Higgs mass $M_H < 140$ GeV), so it has to be known very precisely.

1.1 fixed order calculation

It is well known that the photons can be produced directly in the partonic subprocess or emitted quasi-colinearly by a parton, itself scattered at high p_t by the hard subprocess. Di-photon production can be split in three parts:

- direct contribution where the two photons are emitted directly in the partonic subprocess;
- single fragmentation contribution where one of the two photons is produced via a fragmentation function of partons into photon, the other being emitted directly in the partonic subprocess;
- double fragmentation contribution where the two photons are produced via fragmentation functions of partons into photon.

In the direct contribution, the gluon-gluon fusion diagrams are also included. Indeed, due to the gluon flux, these diagrams although belonging to higher order are important.

These different contributions are not independent. More precisely, only the sum of the three contributions is physical. This is why when the photon is colinear to the parent quark, we get a singularity. This divergence is then absorbed at the scale M_f into the bare fragmentation function yielding an evolved one. So a M_f dependence appears both in the higher order of the direct contribution and in the leading order of the one fragmentation one. These dependences partially cancel out. The same cancellation occurs between the higher order of the single fragmentation and the leading order of the double fragmentation contribution.

The fragmentation contributions produce in fact events where the photons (one or two) are inside a jet. For this type of events, there is a large background due to π^0. To reduce this part an isolation criteria must be applied. Usually in hadronic collisions, a cone of size R is drawn around the photon and the transverse energy deposited in that cone is required to be less than E_{tc}.

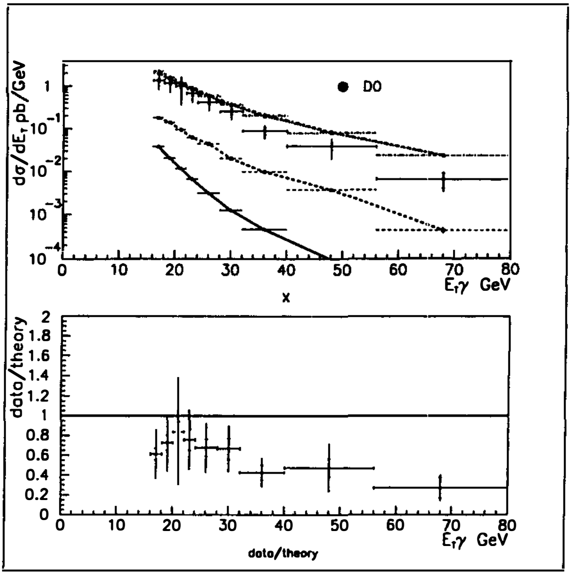

Figure 1: Di-photon differential cross section $d\sigma/dE_T$. Preliminary data points (statistical errors and systematics in quadrature) from the D0 collaboration [1]. The dash-dotted line is the full contribution, the dashed line the direct contribution, the dotted line the single fragmentation and the solid line the double fragmentation. The theoretical curves are from ref. [2].

1.2 summation of soft gluons

For some observables, summation of soft gluons is required because the fixed order calculation exploded at a border of the phase space: this is the case of $d\sigma/dq_t^2$ where q_t is the transverse momentum of the $\gamma\ \gamma$ pair. Indeed, if the partonic final state is composed only of the $\gamma\ \gamma$ pair, then this pair is produced at $q_t = 0$:

$$\frac{d\sigma}{dq_t^2} = \delta(q_t^2). \tag{1}$$

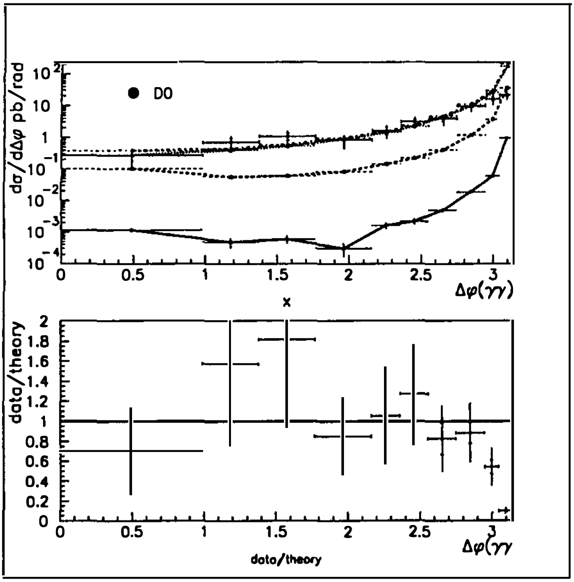

Figure 2: Di-photon differential cross section $d\sigma/d\phi\gamma\gamma$. Preliminary data points (statistical errors only) from the D0 collaboration [1]. The dash-dotted line is the full contribution, the dashed line the direct contribution, the dotted line the single fragmentation and the solid line the double fragmentation.

But, if the partonic final state is composed of the photon pair plus a parton p, then, due to energy-momentum conservation, $\vec{q_t} = -\vec{p}_{tp}$. That means that if the photon pair is produced at low q_t the parton p is forced to be soft or colinear to the initial state:

$$\frac{d\sigma}{dq_t^2} = A\,\delta(q_t^2) + B\left(\frac{1}{q_t^2}\right)_M + C\left(\frac{\ln(Q^2/q_t^2)}{q_t^2}\right)_M + D \tag{2}$$

where the plus distribution are defined:

$$\int_0^{M^2} dq_t^2\, f(q_t^2)\,\left(g(q_t^2)\right)_M = \int_0^{M^2} dq_t^2\,\left(f(q_t^2) - f(0)\right) g(q_t^2) \tag{3}$$

and Q^2 is the partonic center of mass energy $\hat{s}$. Now if $q_t^2 \ll Q^2$ ($q_t^2 \gg \Lambda^2$), two different scales appear in our problem and large logarithms $\ln(Q^2/q_t^2)$ can spoil the convergence of the perturbative serie. More precisely, we want to sum $\alpha_s^n \ln^m(Q^2/q_t^2)/q_t^2$ with $n-1 \leq m \leq n$. To achieve it, it is more convenient to work in the impact parameter space b which is the Fourier conjugate of q_t. At high Q^2, the result is[3]:

$$\begin{aligned}\frac{d\sigma_{ij\to\gamma\gamma}}{dq_t^2 dP_{t3} dy_3 dy_4} &= \int \frac{d\vec{b}}{(2\pi)^2}\, e^{-i\vec{q_t}.\vec{b}}\, R_{ij}(b, x_1, x_2)\, e^{S_{ij}^{pert}(b,Q^2)} \\ &\quad + Y_{ij}(P_{t3}, y_3, y_4)\end{aligned} \tag{4}$$

with:

$$S_{ij}^{pert}(b, Q^2) = -\int_{c_1/b^2}^{c_2 Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[A_{ij}(g(\bar{\mu}))\,\ln\left(\frac{c_2^2 Q^2}{\bar{\mu}^2}\right) + B_{ij}(g(\bar{\mu}))\right]. \tag{5}$$

The function R_{ij} contains the terms proportional to $\delta(q_t^2)$ in eq. (2), it can be written as:

$$\begin{aligned} R_{ij}(b,x_1,x_2) &= \sum_{k,l}\int_{x_1}^1 \frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}\, F_k^{H_1}\left(z_1,\frac{c_3^2}{b^2}\right)\, F_l^{H_2}\left(z_2,\frac{c_3^2}{b^2}\right) \\ & \left[C_{0,kl}\left(\frac{x_1}{z_1},\frac{x_2}{z_2},\frac{c_1}{c_2},g\left(\frac{c_3^2}{b^2}\right),c_3\right)\delta_{ik}\,\delta_{jl}\,\delta(1-z_1)\,\delta(1-z_2)\right. \\ & + C_{1,kl}\left(\frac{x_1}{z_1},\frac{x_2}{z_2},\frac{c_1}{c_2},g\left(\frac{c_3^2}{b^2}\right),c_3\right)\delta_{ik}\,\delta(1-z_1) \\ & \left. + C_{1,kl}\left(\frac{x_1}{z_1},\frac{x_2}{z_2},\frac{c_1}{c_2},g\left(\frac{c_3^2}{b^2}\right),c_3\right)\delta_{jl}\,\delta(1-z_2)\right], \end{aligned} \tag{6}$$

where c_1, c_2 and c_3 are some arbitrary scales. The functions A, B and C have the following perturbative development:

$$P(g(\bar{\mu})) = \sum_{n=n_0}\left(\frac{\alpha_s(\bar{\mu}^2)}{2\pi}\right)^n P^{(n)} \quad \begin{cases} n_0 = 1 \text{ if } P = A,\ B \\ n_0 = 0 \text{ if } P = C \end{cases} \tag{7}$$

Working at next-to-leading approximation, for the ij (i, $j = q,g$) initial state, the functions A, B and C are given by:

$$A_{ij}^{(1)} = \frac{1}{2}\left(a_{ii}(1) + a_{jj}(1)\right) \tag{8}$$

$$A_{qq}^{(2)} = C_F\left(\frac{67}{3} - \pi^2 - \frac{10}{9}N_F - 4\beta_0 \ln\left(\frac{c}{c1}\right)\right) \tag{9}$$

$$B_{ij}^{(1)} = -(b_{ii}+b_{jj}) - \frac{1}{2}\left(a_{ii}(1)+a_{jj}(1)\right)\ln\left(\frac{c_2^2c^2}{c_1^2}\right) \tag{10}$$

$$C_{0,ij}^{(0)} = K_{ij}\,|M|^2_{ij\to\gamma\gamma} \tag{11}$$

$$C_{1,ij}^{(0)} = K_{ij}\left[F(\hat{s},\hat{t},\hat{u}) + |M|^2_{ij\to\gamma\gamma}\left(B_{ij}^{(1)}\ln\left(\frac{c_2^2c^2}{c_1^2}\right) + \frac{1}{2}A_{ij}^{(1)}\ln^2\left(\frac{c_2^2c^2}{c_1^2}\right)\right)\right] \tag{12}$$

$$C_{1,ij}^{(1)} = K_{ij}\,|M|^2_{ij\to\gamma\gamma}\left(\frac{a_{ij}^{n-4}(z)}{(1-z)_+} + P_{ij}(z)\ln\left(\frac{c^2}{c_3^2}\right)\right). \tag{13}$$

where

$$K_{ij} = \frac{\alpha^2}{4C_iC_jS^2}\,\frac{2\pi P_t}{x_1x_2} \quad \begin{cases} C_q = N \\ C_g = N^2-1 \end{cases} \quad \text{and } c = 2e^{\gamma_E} \tag{14}$$

In addition, we define a and b such that every Altarelli-Parisi kernel can be written as:

$$\begin{aligned} P_{ij}^{(n)}(z) &= \frac{a_{ij}(z) - \epsilon\, a_{ij}^{n-4}(z)}{(1-z)_+} + b_{ij}\,\delta(1-z) & (15)\\ &= P_{ij} - \epsilon\frac{a_{ij}^{n-4}(z)}{(1-z)_+} & (16) \end{aligned}$$

F is the finite part of the virtual piece of the reaction $ij\to\gamma\gamma$ (note that this part is not known for the reaction $gg\to\gamma\gamma$).

But this is not the whole story because when b is large ($> 1/\Lambda$) the value of $\alpha_s(\bar{\mu})$ cannot be computed pertubatively. The value of the b integration is controlled by a saddle point at:

$$b_s = \frac{1}{\Lambda}\left(\frac{Q}{\Lambda}\right)^{-0.41}$$

This region is in fact important because for five flavours, $Q = 15$ GeV and $\Lambda = 0.15$ GeV, $b_s = 1$ GeV^{-1}. So we have to use the following trick [3]:

$$R(b,x_1,x_2)\, e^{S^{pert}(b,Q^2)} = R(b^*,x_1,x_2)\, e^{S^{pert}(b^*,Q^2)}\, \frac{R(b,x_1,x_2)\, e^{S^{pert}(b,Q^2)}}{R(b^*,x_1,x_2)\, e^{S^{pert}(b^*,Q^2)}} \tag{17}$$

where

$$b^* = \frac{b}{\sqrt{1+b^2/b_{max}^2}}. \tag{18}$$

So, no matter b is large, b^* is always small. The last term of (17) cannot be computed, it must be extracted from experiment.

So the cross section (4) is written:

$$\frac{d\sigma_{ij\to\gamma\gamma}}{dq_t^2 dP_{t3} dy_3 dy_4} = \int \frac{d\vec{b}}{(2\pi)^2}\, e^{-i\vec{q_t}.\vec{b}}\, R_{ij}(b,x_1,x_2)\, e^{S_{ij}^{pert}(b,Q^2)} e^{S_{ij}^{nonpert}(b,Q^2)} + Y_{ij}(P_{t3},y_3,y_4). \tag{19}$$

Now we can ask whether the non perturbative Sudakov form factor is universal. If we rewrite the last term of (17) as follow:

$$\frac{R(b,x_1,x_2)\, e^{S^{pert}(b,Q^2)}}{R(b^*,x_1,x_2)\, e^{S^{pert}(b^*,Q^2)}} = e^{S^{pert}(b,Q^2)-S^{pert}(b^*,Q^2)+\ln(R(b,x_1,x_2)/R(b^*,x_1,x_2))}, \tag{20}$$

all the Q^2 dependence comes from $S^{pert}(b,Q^2) - S^{pert}(b^*,Q^2)$. Since $S^{pert}(b,Q^2)$ is universal (it depends only on external legs) we conclude that the term which contains the Q^2 dependence is universal but in $R(b,x_1,x_2)$ there are terms which are specific of the partonic reaction and so the other terms are not universal. That means that these terms cannot be extracted from the study of the transverse momentum of the W and Z, we have then to measure it in di-photon production. Fixed target enegy experiments are very sensitive to the non perturbative Sudakov form factor. To illustrate that, we show in Fig. (3) the comparison between the WA70 data [4] and the theory using CSS summation formalism. Two types of parametrisation are used:

- Davies-Stirling-Webber parametrisation [5]:

$$e^{S^{non\,pert}(b,Q^2)} = e^{-b^2\,(g_2\,\ln(Q/(2Q_0))+g_1)} \tag{21}$$

  choosing $g_1 = 0.15$ GeV^2, $g_2 = 0.4$ GeV^2 and $Q_0 = 2$ GeV;

- Ladinsky-Yuan parametrisation [6]:

$$e^{S^{nonpert}(b,Q^2)} = e^{-b^2\,(g_2\,\ln(Q/(2Q_0))+g_1)-g_1\,g_3\,b\,\ln(100\,x_1\,x_2)} \tag{22}$$

  choosing $g_1 = 0.1$ GeV^2, $g_2 = 0.5$ GeV^2, $g_3 = -1.5$ GeV^{-1} and $Q_0 = 2$ GeV;

2 Factorisation and isolation criterium

Some problems seem to appear in the calculation of the one fragmentation contribution in the reaction $e^+ + e^- \to \gamma + X$ if an isolation criterium is put around the photon.

Berger, Guo and Qiu [7] found that the factorisation is broken: it remains a soft divergence at the end of the calculation. In fact the problem of that divergence is not relevant because it appears in one point of the phase space $x_\gamma = x_c = 1/(1+\epsilon_h) < 1$ and it is not weighted by any

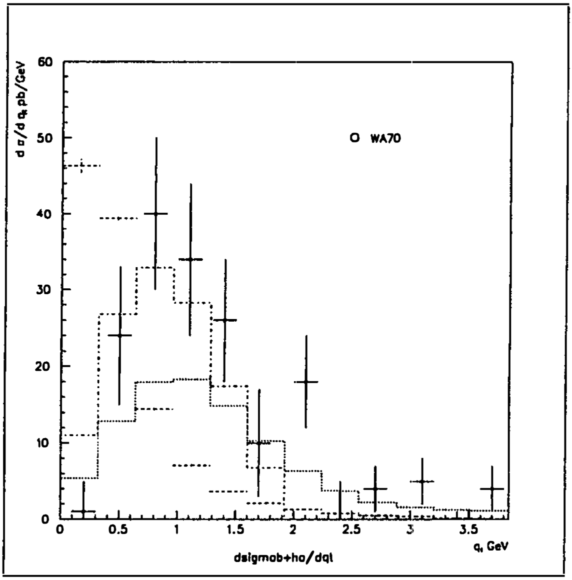

Figure 3: Di-photon differential cross section $d\sigma/dq_t$. Data points are from the WA70 collaboration [4]. The dashed histogram is the fixed order calculation, the dashed-dotted histogram is a resummed calculation using Ladinski-Yuan type of parametrisation, the dotted histogram is a resummed calculation using Davies-Stirling-Webber type of parametrisation for the non perturbative Sudakov form factor. Note that whatever the values of g_1 and g_2, the WA70 data cannot be described with the DSW parametrisation

delta distribution $\delta(x_\gamma - x_c)$ or so : this divergence is of zero measure. But some soft logarithm of $x_\gamma - x_c$ emerge and can spoil the convergence of the perturbative serie in this region. In a recent work Catani,Fontannaz and Pilon [8] have shown that in fact isolation criterium does not spoil factorisation, the long distance part can be reabsorbed into the same fragmentation functions as in the inclusive case. Once this singularities have been absorbed into fragmentation functions, the short distance cross section can still have a divergent behaviour at some points of the phase space when computed order by order in perturbation theory. These divergences are due to certain kinematical constraints that, limiting the fully inclusive character of the cross section, produce an imperfect compensation between real and virtual emission. Owing to its pertubative origin, this disease can be cured by summing the logarithmic divergences to all orders in perturbation theory.

References

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