EPJ Web of Conferences **301**, 02001 (2024) https://doi.org/10.1051/epjconf/202430102001
XLV Symposium on Nuclear Physics 2024

β and γ vibrations in the IBM

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Abstract. The properties of β- and γ-bands are analyzed in the framework of the IBM. It is shown that the general features of the observed excitation energies and the transition probabilities to the ground state band can be explained simply in terms of the intrinsic states of the IBM.

1 Introduction

The identification and interpretation of vibrational excitations in deformed nuclei still provides an enormous challenge to nuclear structure physics. The quadrupole degree of freedom leads to two fundamental modes of vibration, called β with $K^P = 0^+$ and γ with $K^P = 2^+$ [1]. Multiphonon excitations include $\beta\beta$ with $K^P = 0^+$, $\beta\gamma$ with $K^P = 2^+$ and $\gamma\gamma$ with $K^P = 0^+$, 4^+. The observed $K^P = 2^+_1$ bands in deformed nuclei display a fairly regular behavior in their decay pattern to the ground state band, are reasonably well understood and can be interpreted a genuine γ vibration. On the contrary the observed low-lying $K^P = 0^+_2$ bands exhibit a wide variety in their decay probabilities to the ground state band which continues to raise a lot of discussion about the nature of these bands [2]. In recent years, many low-lying $K^P = 0^+$ have been discovered in deformed nuclei, as many as 16 in ^{154}Gd, 15 in ^{168}Er and 12 in ^{162}Dy [3]. In the majority of deformed nuclei, the excitation energy of the γ band is below that of the first excited $K^P = 0^+$ band, and the $g \to \gamma$ transitions dominate the $g \to 0^+_2$ transitions.

It is the purpose of this contribution to analyze the properties of β- and γ-bands in a schematic study of the Interacting Boson Model. It will be shown that the intrinsic IBM automatically leads to the above mentioned features about the vibrational energies and quadrupole transitions.

2 Intrinsic IBM

We consider the schematic IBM Hamiltonian expressed in terms of the d-boson energy and the quadrupole-quadrupole interaction

$$\begin{aligned} H &= (1-\xi)\hat{n}_d - \frac{\xi}{4(N-1)} Q(\chi)\cdot Q(\chi) , \\ \hat{n}_d &= \sqrt{5}\left(d^\dagger \tilde{d}\right)^{(0)} , \\ Q(\chi) &= \left(s^\dagger \tilde{d} + d^\dagger \tilde{s} + \chi\, d^\dagger \tilde{d}\right)^{(2)} , \end{aligned} \tag{1}$$

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and study its properties as a funcion of the coefficients ξ and χ. In general, the solutions can be obtained numerically by diagonalizing the Hamiltonian. In this contribution, we employ the large N limit in which approximate solutions can be obtained in closed form and which may help to achieve a better understanding of the basic features of the model.

The large N (or classical) limit of the IBM Hamiltonian can be studied by introducing a coherent (or intrinsic) state which has the form of a boson condensate, a superposition of monopole and quadrupole bosons [4, 5]

$$\begin{aligned} |g\rangle &= \frac{1}{\sqrt{N!}}\left(b_c^\dagger\right)|0\rangle\,, \\ b_c^\dagger &= \frac{1}{\sqrt{1+\beta^2}}\left[s^\dagger + \beta\left(d_0^\dagger \cos\gamma + \frac{1}{\sqrt{2}}(d_2^\dagger + d_{-2}^\dagger)\sin\gamma\right)\right]\,. \end{aligned} \tag{2}$$

Here β and γ denote the deformation parameters in the IBM. The associated energy surface is obtained as the expectation value of the (normal ordered) Hamiltonian in the intrinsic state

$$\frac{1}{N}E(\beta,\gamma) = \frac{1}{N}\langle g| : H : |g\rangle = (1-\xi)\frac{\beta^2}{1+\beta^2} - \xi\frac{\frac{2}{7}\chi^2\beta^4 - 4\sqrt{\frac{2}{7}}\chi\beta^3\cos 3\gamma + 4\beta^2}{4(1+\beta^2)^2}\,. \tag{3}$$

The equilibrium shape is defined as the minimum in the energy surface and is characterized by β_0, γ_0. Quantum phase transitions can be studied by varying the control parameter ξ over the range $0 \leq \xi \leq 1$ [4].

For $\chi = 0$, the Hamiltonian exhibits a second-order phase transition between the $U(5)$ and $SO(6)$ limits separated by the critical point $\xi_c = \frac{1}{2}$. For $0 \leq \xi < \xi_c$ the equilibrium shape is spherical ($\beta_0 = 0$) and for $\xi_c < \xi \leq 1$ it is deformed ($\beta_0 > 0$).

For $\chi = \mp\sqrt{7}/2$, the Hamiltonian exhibits a first-order phase transition between the $U(5)$ and $SU(3)$ limits separated by the critical point $\xi_c = \frac{8}{17}$. For $0 \leq \xi < \xi_c$ the equilibrium shape is spherical ($\beta_0 = 0$) and for $\xi_c < \xi \leq 1$ it is axially deformed ($\beta_0 > 0$) with prolate symmetry for $\chi = -\sqrt{7}/2$ and oblate symmetry for $\chi = +\sqrt{7}/2$.

In the general case, for $0 < |\chi| \leq \sqrt{7}/2$ one has a first-order phase transition between the spherical and deformed phase. For $\xi < \xi_c$ the equilibrium shape is spherical ($\beta_0 = 0$), and for $\xi > \xi_c$ the equilibrium shape is deformed ($\beta_0 > 0$) with prolate symmetry for $\chi < 0$ ($\gamma_0 = 0°$) and oblate symmetry for $\chi > 0$ ($\gamma_0 = 180°$). Here, the z-axis is chosen along the symmetry axis.

For the case of a pure quadrupole-quadrupole interaction ($\xi = 1$) the equilibrium value of the deformation parameter is given by [6]

$$\beta_0 = \frac{1}{2}\left(\sqrt{\frac{2}{7}}|\chi| + \sqrt{\frac{2}{7}\chi^2 + 4}\right)\,, \tag{4}$$

which increases from the $SO(6)$ value $\beta_0 = 1$ for $\chi = 0$ to the $SU(3)$ value $\beta = \sqrt{2}$ for $|\chi| = \sqrt{7}/2$.

Fig. 1 shows the equilibrium value of the deformation parameter as a function of the control parameter, ξ, and the structure of the quadrupole operator, χ.

Excited bands arise as intrinsic excitations of the ground state band

$$\begin{aligned} |\beta\rangle &= \frac{1}{\sqrt{N}}\,b_\beta^\dagger b_c|g\rangle\,, & b_\beta^\dagger &= \frac{1}{\sqrt{1+\beta^2}}\left(-\beta s^\dagger + d_0^\dagger\right)\,, \\ |\gamma\rangle &= \frac{1}{\sqrt{N}}\,b_\gamma^\dagger b_c|g\rangle\,, & b_\gamma^\dagger &= \frac{1}{\sqrt{2}}\left(d_2^\dagger + d_{-2}^\dagger\right)\,. \end{aligned} \tag{5}$$

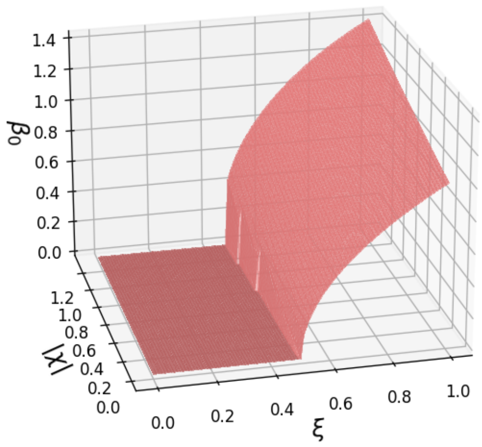

Figure 1. Equilibrium value of the deformation parameter β.

The intrinsic energies of the β- and γ-band can be obtained in the large N limit [7]. The ratio

$$E_{\gamma\beta} = \frac{E_\gamma}{E_\beta} = \frac{9\sqrt{\frac{2}{7}}|\chi|\beta_0}{4 + \sqrt{\frac{2}{7}}|\chi|\beta_0(\beta_0^2 + 3)}, \tag{6}$$

does not depend explicitly on the control parameter ξ, but implicitly it does through the determination of the equilibrium value of the deformation parameter β_0 for each combination of ξ and χ.

In the consistent Q-formalism, the same quadrupole operator is used for the Hamiltonian and the $E2$ transition operator [8]. The ratio of intrinsic transition rates depends on the intrinsic matrix elements for the transitions between the ground state band and the β- and γ-band [9]

$$R_{\beta\gamma} = \frac{|\langle\beta|T_0(E2)|g\rangle|^2}{|\langle\gamma|T_2(E2) + T_{-2}(E2)|g\rangle|^2} = \frac{1}{2(1+\beta_0^2)}\left(\frac{1 + \sqrt{\frac{2}{7}}|\chi|\beta_0 - \beta_0^2}{1 - \sqrt{\frac{2}{7}}|\chi|\beta_0}\right)^2. \tag{7}$$

Fig. 2 shows the relative intrinsic energies and transition probabilities for the deformed region $\xi_c < \xi < 1$ and $0 \leq |\chi| \leq \sqrt{7}/2$. For a given value of χ, the ratio of intrinsic energies $E_{\gamma\beta}$ increases with ξ. In the $SU(3)$ limit ($\chi = \mp\sqrt{7}/2$ and $\xi = 1$) the intrinsic energies of the β- and γ-band are the same, whereas in the rest of the parameter space in the $\xi\chi$-plane the ratio $E_{\gamma\beta} < 1$.

For the large majority of the parameter space the behavior of the intrinsic transition rates is the opposite: for a given value of χ, the ratio of intrinsic energies $R_{\beta\gamma}$ decreases with ξ until it vanishes for the case of a pure quadrupole-quadrupole interaction ($\xi = 1$). Only for values $1 < |\chi| < \sqrt{7}/2$ the ratio first increases before decreasing to 0. For the $SU(3)$ value $|\chi| = \sqrt{7}/2$ the ratio of intrinsic transition rates continues to increase with ξ to reach the value of $R_{\beta\gamma} = 3/2$ in the exact $SU(3)$ limit. The general trend is that for the larger part of

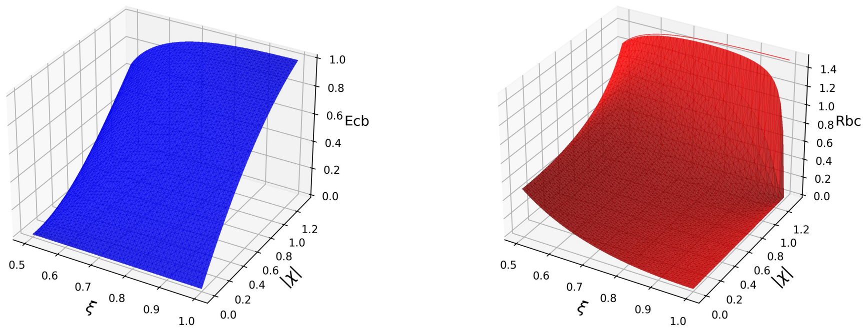

Figure 2. Ratio of intrinsic energies E_γ/E_β (left) and intrinsic transition probabilities R_β^2/R_γ^2 (right).

the parameter space $g \to \gamma$ transitions are stronger than $g \to \beta$ transitions. Only when χ is getting close to the $SU(3)$ value of $\mp\sqrt{7}/2$ the two transitions become comparable, and the $g \to \beta$ transitions may even become stronger than the $g \to \gamma$ transitions.

At this stage it is important to emphasize that the results for the intrinsic energies and the quadrupole transitions are based on the large N limit of the IBM in which the coupling between rotations and vibrations is neglected.

3 Case studies

As an illustration we apply the above analysis in the intrinsic IBM to three deformed nuclei, ^{156}Gd, ^{162}Dy and ^{168}Er. These nuclei have a $R_{42} = E_{\rm exc}(4_1^+)/E_{\rm exc}(2_1^+)$ ratio of 3.24, 3.29 and 3.31, respectively, all very close to the rotational value of 10/3. The intrinsic energies are determined by subtracting the rotational contribution to the experimental energies: $E_\beta = E_{0_\beta^+}$ and $E_\gamma = E_{2_\gamma^+} - 6/2\mathcal{I}_\gamma$ where $\mathcal{I}_\gamma$ is the moment of inertia of the γ-band. The intrinsic transition rates can be determined from the ratio of $B(E2)$ values

$$R_{\beta\gamma} = \frac{B(E2; 0_g^+ \to 2_\beta^+)}{B(E2; 0_g^+ \to 2_\gamma^+)} \; . \tag{8}$$

The results are summarized in Table 1.

The nucleus ^{168}Er has been the subject of an extensive study in IBM-1 in terms of a mixture of the $SU(3)$ and the $SO(6)$ quadrupole-quadrupole interactions [14] which in the present analysis effectively translates into a smaller (absolute) value of χ. The value of the

Table 1. Ratio of intrinsic energies and transitions. The theoretical values are calculated according to Eqs. (6) and (7). The experimental values are taken from [10–13].

				Intrinsic IBM		Experiment	
Nucleus	R_{42}	ξ	χ	$E_{\gamma\beta}$	$R_{\beta\gamma}$	$E_{\gamma\beta}$	$R_{\beta\gamma}$
^{156}Gd	3.24	0.98	−1.29	0.998	0.131	1.019	0.135 ± 0.009
^{162}Dy	3.29	0.84	−0.65	0.584	0.063	0.581	$0.063^{+0.024}_{-0.063}$
^{168}Er	3.31	0.93	−0.64	0.611	0.012	0.613	0.012 ± 0.002

control parameter ξ is close to that for a pure quadrupole-quadrupole interaction, $\xi = 0.93$ and the structure of the quadrupole operator is given by $\chi = -0.64$. Fig. 3 shows that there is only a very small region in the parameter space of the $\xi\chi$-plane where both the ratio of intrinsic energies (green) and transitions (blue) can be reproduced. The properties of ^{168}Er have also been analyzed in the framework of partial dynamical symmetries of the IBM [15].

The nucleus ^{156}Gd is usually presented as the standard example of a nucleus with $SU(3)$ symmetry with almost degenerate β- and γ-bands [5]. We find a good description of the ratio of intrinsic energies and transition rates for $\xi = 0.98$ and $\chi = -1.29$. Just as for ^{168}Er there is only a very small range in the $\xi\chi$-plane that can reproduce the experimental values. It is important to note that the robust prediction made in Ref. [16] of very small values of $R_{\beta\gamma}$ only holds for a pure quadrupole-quadrupole interaction, $\xi = 1$ and $\chi \neq \mp\sqrt{7}/2$. In this case the intrinsic matrix element for $\beta \to g$ transitions vanishes in the large N limit. Inspection of Fig. 2 shows that a small breaking of the $SU(3)$ limit, $\xi = 0.98$ and $\chi = -1.29$, makes it possible to reproduce the experimental value $R_{\beta\gamma} = 0.13$.

The final example is that of the nucleus ^{162}Dy. This analysis is based partially on as of yet unpublished data on lifetime measurements. In [12] it was concluded that the first excited $K^P = 0^+$ band at 1400.2 keV is a genuine β-vibration. The ratio of intrinsic transion rates was extracted from the quadrupole transitions of the 2^+_γ state at 888 keV and the 2^+_β state at 1453 keV to the ground state band [13]

$$\begin{aligned} B(E2; 2^+_\gamma \to 0^+_g) &= 4.68 \pm 0.21 \text{ WU} \\ B(E2; 2^+_\beta \to 4^+_g) &= 0.76^{+0.29}_{-0.76} \text{ WU} \end{aligned} \tag{9}$$

In this case, the area of allowed values in the $\xi\chi$-plane is a bit larger because of the experimental uncertainty in the $2^+_\beta \to 4^+_g$ transition, especially in the ξ direction.

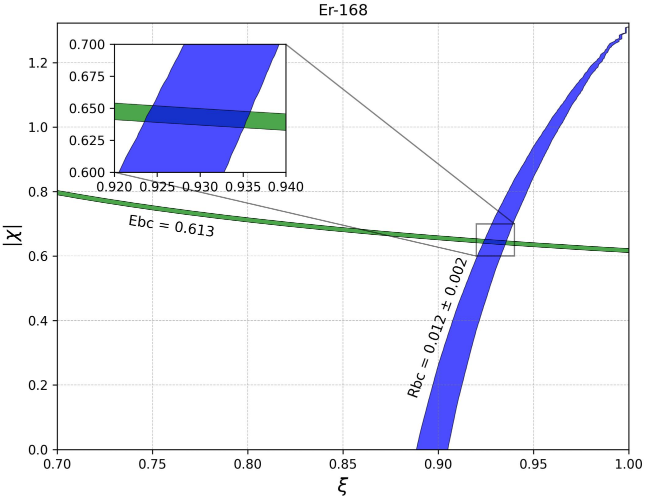

Figure 3. Ratio of intrinsic energies and quadrupole transitions in ^{168}Er.

4 Summary and conclusions

In this contribution, we presented the first results of an analysis of intrinsic energies and quadrupole matrix elements in the consistent-Q formalism of the IBM. We considered a schematic Hamiltonian that contains all of the richness of the dynamical symmetries and quantum phase transitions between spherical and deformed nuclei.

An analysis in the consistent-Q formalism shows that for almost the entire parameter range of the control parameter ξ and the structure of the quadrupole operator χ the $g \to \gamma$ transitions are stronger than the $g \to \beta$ transitions, with the only exception for χ close to the $SU(3)$ value $|\chi| = \sqrt{7}/2$. This indicates that the usual treatment of quantum phase transitions between vibrational and rotational nuclei in terms of a mixture of $U(5)$ and $SU(3)$ interactions cannot reproduce the observed dominance of $g \to \gamma$ transitions over $g \to \beta$ transitions in deformed nuclei.

As an application we applied the intrinsic IBM to three deformed nuclei: ^{168}Er, ^{156}Gd and ^{162}Dy. In all cases it was found that there is relatively small region in the parameter space of the $\xi\chi$ plane in which the experimental intrinsic energies and quadrupole transitions can be reproduced. A more extensive study of rare-earth nuclei in the intrinsic IBM will be discussed in a future paper [17].

Acknowledgments

This work was supported in part by PAPIIT-DGAPA (Mexico) grant IG101423.

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