

Johnson noise and the thermal Casimir effect

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Abstract. We study the thermal interaction between two nearby thin metallic wires, at finite temperature. It is shown that the Johnson currents in the wires give rise, via inductive coupling, to a repulsive force between them. This thermal interaction exhibits all the puzzling features found recently in the thermal Casimir effect for lossy metallic plates, suggesting that the physical origin of the difficulties encountered in the Casimir problem resides in the inductive coupling between the Johnson currents inside the plates. We show that in our simple model all puzzles are resolved if account is taken of capacitive effects associated with the end points of the wires. Our findings suggest that capacitive finite-size effects may play an important role in the resolution of the analogous problems met in the thermal Casimir effect.

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1. Introduction

In recent years, great advances in experimental techniques have stimulated an intense theoretical and experimental activity on the Casimir effect. By now, the Casimir force has been measured with an accuracy of a few percent (for recent reviews on both theoretical and experimental aspects of the Casimir effect, see [1]). Apart from the fundamental interest raised by this phenomenon, which reveals subtle properties of the quantum vacuum, Casimir forces are now finding new and exciting applications to nanotechnologies [2], to Bose–Einstein condensates [3], to non-contact atomic friction [4] and to superconductors [5].

The increasing number of applications of the Casimir effect, together with the improving level of experimental accuracy reached by recent experiments, calls for a corresponding improved level of accuracy of theoretical predictions. In his seminal paper, Casimir considered the case of two infinite plane-parallel plates made of ideal metal, at zero temperature. In modern experiments, it is necessary to take account of a number of corrections to this ideal situation, arising from geometrical factors, like surface roughness and shape of the plates, finite conductivity of the plates, and a nonzero temperature. While separate consideration of all these corrections presented no particular difficulty, surprisingly it was realized recently that, for the case of metallic plates, the computation of the combined effect of finite metal conductivity and a nonzero temperature poses severe problems. The purpose of the present paper is to clarify the physical origin of these difficulties, and to indicate a possible resolution. To do this, we consider a simple model of two metallic wires at finite temperature. We shall see that the inductive interaction between the Johnson currents in the wires presents all the puzzling features found in the Casimir problem. For our model, a simple resolution of these problems can be obtained by taking account of capacitive effects associated with the end points of the wires. This suggests that also in the Casimir case the analogous difficulties can be resolved in a similar way, by considering capacitive effects resulting from the finite size of the plates.

To understand the problem of the thermal Casimir effect in real metals, it is useful to review briefly the sequence of successive refinements that have been introduced over the years into the theory of the Casimir effect for metallic cavities, in order to describe the effects of temperature and finite conductivity of the plates, paying attention to the *new physical ingredients* that each step adds to the problem. In the zeroth-order approximation the plates are considered as ideal reflectors at zero temperature. Here, the Casimir energy can be interpreted as arising from the zero-point energy of the electromagnetic (e.m.) modes supported by the cavity, a purely quantum phenomenon indeed, that constitutes the genuine essence of the Casimir effect. Including the effect of temperature just adds to the zero-point Casimir energy the contribution of thermally excited e.m. waves. In the next step, one includes the effect of the finite skin depth of e.m. fields in real metals. For experimentally relevant distances between the plates, this can be conveniently done by modeling the metal that constitutes the plates by the well known plasma model, which assumes a permittivity $\epsilon_p(\omega)$

$$\epsilon_p(\omega) = 1 - \Omega_p^2/\omega^2, \quad (1)$$

where Ω_p is the plasma frequency. Note that $\epsilon_p(\omega)$ is real, which means that no dissipation is present (at nonzero frequencies). Therefore, the cavity continues to possess e.m. modes of well defined (real) frequencies, and the physics remains the same as in the ideal case, apart from the fact that now the spectrum of eigenfrequencies is altered, and that the e.m. fields penetrate more or less into the plates. The most important effect of this refinement is seen at short

plate separations, where it produces a significant decrease in the Casimir pressure. Inclusion of temperature corrections is straightforward, of course. Since the skin depth approaches zero for infinite plasma frequencies, one expects to smoothly recover the ideal result, both at zero and nonzero temperature, in that limit. Detailed computations fully confirm this expectation [1]. Then comes the next refinement, the inclusion of dissipation into the picture. Physically, this is an important issue, because all real metals present ohmic losses. As far as the eigenmodes of the cavity are concerned, the presence of a small amount of dissipation just entails a slight broadening of the spectrum (the eigenfrequencies get a small imaginary component), and therefore one expects a small correction to the Casimir energy (i.e. the zero-point contribution). The problem can be studied quantitatively using the famous Lifshitz theory of dispersive forces in real media [6]. All that is needed is a model for the metal that includes dissipation, and the simplest one is of course the classical Drude model $\epsilon_D(\omega)$:

$$\epsilon_D(\omega) = 1 - \frac{\Omega_p^2}{\omega(\omega + i\gamma)}, \quad (2)$$

where γ is the relaxation frequency, that accounts for ohmic losses. If this model is plugged into Lifshitz theory nothing much happens, at $T = 0$, and one finds, as expected, small deviations from the plasma model case. An unexpected result was obtained when the analysis was extended to finite temperature [7], for then large deviations from the plasma model were found, to the extent that the Casimir pressure is almost halved for large separations. A puzzling fact is that the large effect persists also for vanishingly small amounts of dissipation, i.e. for infinitesimal γ 's. The existence of this discontinuity at $\gamma = 0$ attracted a lot of interest from the Casimir community, and stimulated a large number of studies aiming at either disproving it or supporting it.

Important progress was made in [8], where the problem was investigated using an alternative mathematical formulation of the Lifshitz theory, that allows a clean separation of the contribution of thermal photons from that of zero-point fluctuations. In this way, it was realized that the presence of dissipation has little influence on zero-point fluctuations, and that the large deviations from the plasma model discovered in [7] originated from thermally excited evanescent transverse electric (TE) photons that give rise to an additional *repulsive force* between the plates. This thermal contribution is present only for $\gamma \neq 0$, it persists in the limit $\gamma \rightarrow 0$ and at large separations and/or temperatures, it compensates to a large extent for the zero-point Casimir pressure.

At present, the controversy still goes on, with no clear cut answer. For a discussion of alternative points of view, the reader may consult two recent papers by leading experts of the field [9, 10]. We just remark that a serious objection against the result of [7] was raised in [11], where it was shown that the approach of [7] implies a violation of the Nernst heat theorem, if the plates are regarded as perfect lattices. However, this result too has been subjected to criticism (see [10] and references therein). In particular, it has been remarked that thermodynamic inconsistencies might be overcome if space dispersion is considered [12, 13]. That space-dispersion should be taken into account is clear, because the anomalous skin effect becomes important at low temperature. However, it appears that consideration of space-dispersion is not sufficient to provide a complete resolution of thermodynamic problems, because while it ensures that the entropy vanishes in the limit of zero temperature, it does not avoid the problem of negative entropies at intermediate temperatures [12].

Survey of the literature gives the feeling that the real *physical* reason of the puzzling thermal effects brought about by the presence of dissipation in conductors has not yet been

understood. The previous discussion suggests that *when we deal with metals, or more generally with conductors, the simultaneous presence of dissipation and of a finite temperature brings into the problem a qualitatively new phenomenon, that is absent either when dissipation is strictly zero, or when the temperature is zero.* If such a phenomenon exists, the question arises whether it is adequately accounted for in existing treatments of the thermal Casimir effect. Indeed the phenomenon in question has been known for a long time: Johnson noise [14]. As it is well known, Johnson noise is the electronic noise generated by thermal agitation of charge carriers inside a conductor at equilibrium. This really is a new physical ingredient of the problem, which has nothing to do with the physics of cavity modes that exist in a lossless cavity. It is well known that near-field effects associated with Johnson noise are important, for example, for the problem of stability ion-traps [15]. The relevance of Johnson noise for the thermal Casimir effect is discussed in the next section, with the help of a simple system of two conducting wires.

2. Thermal interaction between two wires

As we pointed out in the introduction, the enigmatic features of the thermal Casimir effect arise from the existence of a thermal *repulsive* force, that is present between a pair of metallic plates at finite temperature, when the latter are described by the Drude model. A detailed analysis [8, 16] of the real-frequency spectrum of the thermal interaction between two parallel plates at separation a , as given by the Lifshitz theory, revealed that the repulsive force is due to TE evanescent thermal fluctuations of the e.m. field inside the cavity, with typical (real) frequencies of order $\tilde{\omega} = \gamma(\omega_c / \Omega_p)^2$, where $\omega_c = c/a$ is the characteristic frequency of the cavity. For typical Casimir experiments, and even more so at low temperature, $\tilde{\omega}$ is much less than ω_c . At such low-frequencies, retardation effects are negligible and the relevant e.m. fluctuations basically consist of quasi-static magnetic fields. The physical origin of the resulting interaction between the plates can be clearly understood using Rytov's theory [17] of e.m. fluctuations, which is at the basis of the Lifshitz theory of dispersion forces. According to Rytov's theory, e.m. fluctuations outside material bodies are produced by microscopic fluctuating currents, which are present in the interior of any absorbing medium, according to the fluctuation–dissipation theorem. When the problem is analyzed from this point of view, one realizes that the repulsive force, mediated by quasi-static magnetic fluctuations, arises from the inductive coupling between the Johnson currents, that exist inside the plates. To gain further confidence in the correctness of this simple physical picture, it is a good strategy to devise a simple model which only involves this ingredient of the problem, and see what happens. For this purpose, we consider a system of two identical pieces \mathcal{C}_1 and \mathcal{C}_2 of thin metallic wire at temperature T , displaced by an amount \vec{a} from each other (we consider the orientations of the wires as fixed once and for all, and therefore their mutual position is determined by the displacement \vec{a} connecting an arbitrary point O_1 of \mathcal{C}_1 to an arbitrary point O_2 of \mathcal{C}_2), and we ask what force they exert on each other, as an effect of the respective Johnson currents. Determining this force for two thin wires is indeed much simpler than for two bulk plates. As we are dealing with a quasi-static problem, we can neglect retardation effects and we can regard the force as arising from a direct instantaneous interaction between the currents i_1 and i_2 in the wires. Moreover, we can safely assume that the currents i_1 and i_2 depend only on time, and not on the position along the wires (charge fluctuations decay within a typical time $\tau = \gamma / \Omega_p^2$, which is very short compared with the relevant low frequencies of the problem). Under these simplifying conditions, a simple method to study the interaction between the wires, completely equivalent

to Rytov–Lifshitz theory, consists of replacing the noisy wires by ‘ideal’ noiseless wires, whose end-points are connected to noise e.m.f. generators. Then, our system of two wires is described by the following equations:

$$\mathcal{L} \frac{di_1}{dt} + \mathcal{M}(\vec{a}) \frac{di_2}{dt} + Ri_1 = \mathcal{E}_1(t), \quad \mathcal{L} \frac{di_2}{dt} + \mathcal{M}(\vec{a}) \frac{di_1}{dt} + Ri_2 = \mathcal{E}_2(t). \quad (3)$$

In these equations \mathcal{L} and R , are, respectively, the self-inductances and the resistances of the two wires, $\mathcal{M}(\vec{a})$ is the mutual inductance, and $\mathcal{E}_i(t)$ is the random e.m.f. in the wire \mathcal{C}_i . In principle, \mathcal{L} , \mathcal{M} and R are frequency-dependent quantities, because both the conductivity of the wires and the skin-depth depend on the frequency. However, to keep things simple, we shall neglect this complication and work with constant inductances and resistances, which is correct for sufficiently thin wires and at low enough frequencies. Note that the only quantity that depends on the separation \vec{a} is the mutual inductance $\mathcal{M}(\vec{a})$. As it is well known [18], in general

$$\mathcal{M}^2 \leq \mathcal{L}^2. \quad (4)$$

The power spectrum¹ of the random e.m.f.’s is [19]

$$\langle \mathcal{E}_i(\omega) \mathcal{E}_j^*(\omega') \rangle = 4\pi k_B T R E(\omega/\omega_T) \delta(\omega - \omega') \delta_{ij}, \quad (5)$$

where angle brackets denote statistical averages, $i, j = 1, 2$, $\omega_T = k_B T/\hbar$ and $E(y) = y(e^y - 1)^{-1}$.² The force $\vec{F}_{12}(\vec{a})$ on the wire \mathcal{C}_2 can be written as [20]:

$$\vec{F}_{12}(\vec{a}) = \langle i_1 i_2 \rangle \vec{\nabla}_a \mathcal{M}(\vec{a}). \quad (6)$$

The correlator $\langle i_1 i_2 \rangle$ can be easily computed by taking the time-Fourier transform of equations (3) and using (5). After some computations, we obtain for the force the simple formula:

$$\vec{F}_{12} = -k_B T H \vec{\nabla}_a(m^2), \quad (7)$$

where $m = \mathcal{M}/\mathcal{L}$ and H is the quantity:

$$H = \frac{1}{\pi} \int_0^\infty d\omega \omega E\left(\frac{\omega}{\omega_T}\right) \text{Im} [(\omega_R - i\omega)^2 + \omega^2 m^2]^{-1}, \quad (8)$$

where $\omega_R = R/\mathcal{L}$. As we see, the frequencies ω that contribute to the H are clearly in the range $0 < \omega < \min\{\omega_R, \omega_T\}$, and therefore the low-frequency approximation made in equations (3) is justified if either ω_R or ω_T are low enough.

Let us consider now the features of the force. It is easy to see that H is positive definite, and therefore, since \mathcal{M}^2 decreases as a increases, the force is *repulsive*. Moreover, we note that the force vanishes if we take $R = 0$, i.e. ideal inductances. Both features are analogous to what is found in the thermal correction to the Casimir pressure [8]. Let us go ahead and check the zero-resistance limit. By making the change of variables $x = \omega/\omega_R$ in the integral in equation (8), it is easy to verify that H is only a function of m^2 and ω_R/ω_T . However, the dependence on the latter quantity is only via the Boltzmann factor $E(x \omega_R/\omega_T)$, and since x is of order one or less, we see that for small R 's, $E(x \omega_R/\omega_T)$ becomes one, which represents the classical value. Therefore, in this limit H is a function only of m^2 , $H = f(m^2)$, and we obtain

$$\lim_{R \rightarrow 0} \vec{F}_{12} = -k_B T f(m^2) \vec{\nabla}_a(m^2). \quad (9)$$

¹ The Fourier transform $g(\omega)$ of a function $f(t)$ is normalized here such as $g(\omega) = \int_{-\infty}^\infty dt f(t)$.

² This form of the spectrum is valid for wires that are not exceedingly thin [21].

As we see, the force does not vanish for $R \rightarrow 0$. This is exactly the same kind of discontinuity that occurs in the Casimir case with lossy plates, for vanishing dissipation.

3. Thermodynamic features of the interaction

To verify whether the analogy with the Casimir case is complete, we now consider the thermodynamic features of the interaction between the wires. For this purpose, we need to compute the corresponding free energy \mathcal{F} . Since $\vec{F}_{12} = -\vec{\nabla}_a \mathcal{F}$, we obtain from equations (7) and (8)

$$\mathcal{F} = \frac{k_B T}{\pi} \int_0^\infty \frac{d\omega}{\omega} E\left(\frac{\omega}{\omega_T}\right) \text{Im} \log \left[1 + \left(\frac{\omega m}{\omega_R - i\omega} \right)^2 \right]. \quad (10)$$

Let us consider now the low temperature limit. If the wires have no impurities, at liquid helium temperatures and below, the resistance $R(T)$ approaches zero as T^2 [22]. Therefore near $T = 0$, we always have $\omega_R \ll \omega_T$, and reasoning as before, we can substitute for $E(\omega/\omega_T)$ its classical value, i.e. $H = 1$. But then the integrand in equation (10) becomes independent of T and we find that the free energy is of the form

$$\mathcal{F} \approx g(m^2) k_B T, \quad (11)$$

where $g(z)$ is a positive function (because the imaginary part of the argument of the logarithm in equation (10) is positive definite). Recalling that the entropy S is $S = -\partial \mathcal{F} / \partial T$, we find:

$$\lim_{T \rightarrow 0} S = -k_B g(m^2) \equiv S_0 < 0. \quad (12)$$

Since S_0 depends on the separation between the wires through m^2 , this result represents a violation of the Nernst heat theorem. Again, this is exactly analogous to what is found in the Casimir case. Therefore, our simple model strongly suggests that the troubles with the thermal Casimir effect originate from the Johnson noise in the plates.

The question then is: have we missed anything here? At a closer inspection, we see that we omitted an important ingredient. We did not consider that the wires have a finite length, and therefore have end points, where charges can accumulate and produce capacitive effects. A simple way to account for the effect of the end-points in our elementary approach is to include a capacitance C in the impedance Z of the wires, such that $Z = R - i\omega L + i/(\omega C)$. We might consider also the possibility of a mutual capacitance among the wires, but to keep things simple we shall suppose that the orientation of the wires is such that this effect is minimized (for example, the wires could be at right angles to each other, so that the respective end-points are far from each other). It is clear that the presence of the capacitances makes a big difference at low frequencies, which are the source of the troubles, since they work as ‘high-pass filters’ that block low frequencies. The only modification introduced by the capacitances in the previous formulae is that the quantity $(\omega_R - i\omega)$ occurring inside the square brackets in equations (8) and (10) gets replaced by $\omega_R - i\omega + i\omega_C^2/\omega$, where $\omega_C = 1/\sqrt{L C}$. For a straight wire of length L , ω_C is expected to be of order $2\pi c/(2L)$, i.e. the resonance frequency of a linear antenna.

Consideration of the capacitances indeed resolves all the problems. First, we have verified that the force now vanishes in the zero resistance limit, and thus we recover the ideal metal

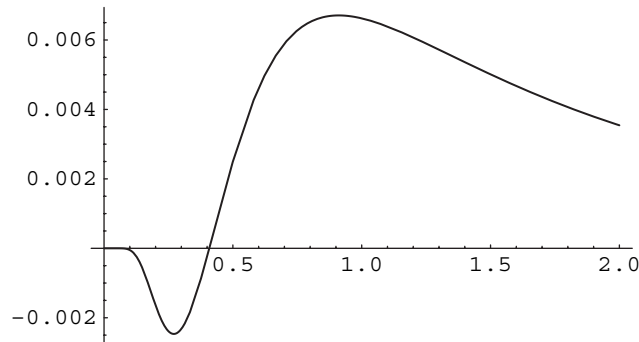


Figure 1. Plot of the free energy (in units of $\hbar\omega_C$) as a function of $t = k_B T / (\hbar\omega_C)$. See text for details.

result. Moreover, we found that even for finite values of the resistance, the value of the force may be affected in an important way, but we shall not discuss this in detail here. What we consider more at length is instead the low temperature limit of the free energy. Since ω_C is independent of T , at sufficiently low temperatures, we always have (for resistors with no impurities) $\omega_R \ll \omega_T \ll \omega_C$. In this limit, it can be verified that

$$\mathcal{F} = -\frac{16\pi^5 m^2}{63} \left(\frac{k_B T}{\hbar \omega_C} \right)^6 \hbar \omega_R. \quad (13)$$

Obviously, the entropy is positive, and vanishes as $T \rightarrow 0$, as required by the Nernst theorem. However, numerical computations show that the entropy is *negative at intermediate* temperatures. This can be seen from the plot of the free energy (in units of $\hbar \omega_C$) as a function of the reduced temperature $t = k_B T / (\hbar \omega_C)$ in figure 1. The curve was computed by taking $m = 0.8$ and $\omega_R(t) = 5 t^2 \omega_C$. As we see, there is a region of temperatures t where the slope of the curve is positive, which corresponds to a negative entropy. This is not necessarily a problem, though, because what needs to be positive is the *total* entropy of the system, which includes the self-entropies of the wires. Each wire, now being an RLC oscillator, has a free energy $\mathcal{F}_{\text{self}}$ equal to:

$$\mathcal{F}_{\text{self}} = k_B T \log[1 - \exp(-\hbar\omega_C/(k_B T))]. \quad (14)$$

As we have checked numerically, inclusion of the wires self-entropies makes the total entropy of the system positive at all temperatures (we could not obtain an analytical proof of this). Therefore, the inclusion of capacitive effects related to the edges of the wire resolves all thermodynamical inconsistencies as well.

4. Conclusions

The results of this paper suggest that the controversial thermal correction to the Casimir interaction between lossy conductors, has its physical origin in the inductive coupling among the Johnson currents existing inside the conductors. This simple physical picture is suggested by Rytov's theory of e.m. fluctuations, which is at the basis of the Lifshitz theory of dispersion

forces. To verify its correctness, we studied the force arising between two thin metallic wires, as a result of Johnson noise. We found that the interaction displays the four basic features found in the thermal Casimir problem, namely:

1. it is repulsive;
2. it persists in the zero resistance limit;
3. it is absent in the case of strictly dissipationless wires;
4. it violates the Nernst heat theorem.

These results appear to us as a clear indication of the important role played by Johnson noise in the thermal Casimir effect for metallic bodies.

In the simple case of two wires studied in this paper, we have shown that all the puzzles raised by this interaction can be resolved by considering capacitive effects arising from the wires finite size, which both ensure smooth convergence to the ideal case in the limit of zero resistance, and resolve as well all thermodynamic inconsistencies at low temperature.

The relevance of capacitive edge effects for resolving the analogous difficulties met in the thermal Casimir effect requires further investigations. Here, we just content ourselves with a few general remarks. Obviously, capacitive effects are expected to be important only if the relevant thermal current-fluctuations have a typical spatial size that is comparable to the plates size L . Recent investigations of the thermal corrections to the Casimir force [8, 16] show that, at room temperature, the troublesome thermal fluctuations are associated with evanescent TE modes, with characteristic frequency of order $\tilde{\omega} = \gamma c^2 / (\Omega_p^2 a^2)$, and characteristic spatial size of the order of the plate separation a . Therefore, we expect that capacitive finite-size effects will be important, for plate separations a not too much smaller than the plates size L . This is the typical case with micro-mechanical devices, that are currently under intense investigation [2].

At low temperature, the situation is more complicated. As the temperature is lowered in the cryogenic range, the increasing degree of spatial correlation between the Johnson currents, implied by the anomalous skin effect, leads to a gradual suppression of fluctuations at small scales. It is conceivable that at very low temperatures, independently of the separation a , the current fluctuations become so correlated as to have a spatial extent comparable with the size of the plates. When this point is reached, edge effects come into play and may become essential for a correct description, as discussed in this paper.

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