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Effective Quark-Quark Interaction in Baryons

B. Sengl

Institute for Theoretical Physics, University of Graz, Universitätsplatz 5, A-8010 Graz, Austria

1 Introduction

In the low-energy regime of QCD, where the fundamental theory is not accurately solvable, one is interested in the effective degrees of freedom that govern the properties of hadrons at this scale. A promising approach to low-energy hadrons consists in constituent-quark models (CQMs). In this context one of the central problems is to find the proper effective interaction between constituent quarks. Traditional CQMs - originally constructed in a non-relativistic framework - adopted one-gluon exchange (OGE) [1] as the hyperfine interaction between constituent quarks (Q). Over the years it has become quite evident that a CQM relying only on OGE Q-Q interactions is not able to describe, e.g., the light and strange baryon spectra. A hyperfine interaction based on OGE leads to the wrong level orderings of positive- and negative-parity excitations specifically in the N and Δ spectra. Furthermore, due to the missing flavor dependence it is not possible to reproduce the N and Λ spectra at the same time. In addition, the OGE interaction produces strong spin-orbit splittings that can hardly be found in the empirical data. Several attempts have been made in order to solve this problem, i.e., one supplemented the color-magnetic interaction by other types of forces, e.g., one introduced an additional meson exchange. These so-called hybrid models, however, did not lead to satisfactory results either [2].

In addition, also other types of CQMs with a different kind of hyperfine interaction have been constructed, such as the ones based on instanton-induced (II) forces [3] or on Goldstone-boson-exchange (GBE) dynamics [4]. Whereas the II CQM is left with the wrong level orderings of the first positive- and negative-parity excitations above the nucleon ground state as well, the GBE CQM is able to reproduce these states in the right places, in accordance with experiment. In this contribution we will mainly be concerned with the extended GBE CQM recently developed by the Graz group.

A considerable number of theoretical and experimental results indicate that QCD at low energies is mainly driven by the mechanism of the spontaneous breaking of chiral symmetry (SBχS). Once we accept this, the original degrees of freedom governing the light-flavor sector of the baryons, namely current quarks and gluons, have to be replaced by effective ones. On the one hand, SBχS leads to constituent quarks with a dynamically generated mass much larger than that of the current quarks. On the other hand, SBχS is at the same time also responsible for the appearance of Goldstone bosons which can be associated with a residual $SU(3)_V$ symmetry. This leads automatically to an effective Lagrangian for the hyperfine interaction in CQMs, which is based on constituent-quark and Goldstone-boson degrees of freedom [5].

The version of the GBE CQM of Ref. [4] relies only on the spin-spin component of the pseudoscalar GBE for the hyperfine interaction of the constituent quarks. This is expected to be the most relevant interaction part with respect to the baryon spectra. Nevertheless, for completeness one must also consider the other possible potential components, i.e., one may also expect multiple Goldstone-boson exchange [6], which brings about even further forces. The extended GBE CQM considers also vector and scalar exchanges and thus contains not only spin-spin but also central, tensor, and spin-orbit forces. In the following we briefly review the extension of the GBE CQM and discuss how the different potential parts of the hyperfine interaction contribute to the total energy of the various light and strange baryon states.

2 Extended GBE CQM

In the extended version of the GBE CQM [7,8] one employs a semi-relativistic Hamiltonian of the form

$$H = \sum_{i=1}^{3} \sqrt{\mathbf{p}_i{}^2 + m_i^2} + \sum_{i<j} \left[V_{conf}(i,j) + V_\chi(i,j)\right]. \tag{1}$$

Here the first term is the relativistic kinetic energy of the constituent quarks and V_{conf} is the linear confinement, which has a strength comparable to the string-tension of QCD. The term V_χ represents the hyperfine interaction (motivated from the SBχS) and contains pseudoscalar (ps), vector (v), and scalar (s) meson exchanges

$$\begin{aligned}
V_\chi(i,j) &= V^{ps}(i,j) + V^{v}(i,j) + V^{s}(i,j)\\
&= \sum_{a=1}^{3} \left[V_\pi(i,j) + V_\rho(i,j) + V_{a_0}(i,j)\right] \lambda_i^a \lambda_j^a\\
&+ \sum_{a=4}^{7} \left[V_K(i,j) + V_{K^*}(i,j) + V_\kappa(i,j)\right] \lambda_i^a \lambda_j^a\\
&+ \left[V_\eta(i,j) + V_{\omega_8}(i,j) + V_{f_0}(i,j)\right] \lambda_i^8 \lambda_j^8 + \tfrac{2}{3}\left[V_{\eta'}(i,j) + V_{\omega_0}(i,j) + V_\sigma(i,j)\right],
\end{aligned} \tag{2}$$

where λ_i denote the Gell-Mann flavor matrices of the individual quarks. The explicit expressions of the individual meson-exchange potentials are for pseudoscalar mesons ($\gamma = \pi, K, \eta, \eta'$)

$$V_\gamma(i,j) = V_\gamma^{SS}(\mathbf{r}_{ij})\, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + V_\gamma^{T}(\mathbf{r}_{ij}) \left[3\,(\hat{\mathbf{r}}_{ij} \cdot \boldsymbol{\sigma}_i)(\hat{\mathbf{r}}_{ij} \cdot \boldsymbol{\sigma}_j) - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right], \tag{3}$$

for vector mesons ($\gamma = \rho, K^*, \omega_8, \omega_0$)

$$\begin{aligned}
V_\gamma(i,j) &= V_\gamma^{C}(\mathbf{r}_{ij}) + V_\gamma^{SS}(\mathbf{r}_{ij})\, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\\
&+ V_\gamma^{T}(\mathbf{r}_{ij}) \left[3\,(\hat{\mathbf{r}}_{ij} \cdot \boldsymbol{\sigma}_i)(\hat{\mathbf{r}}_{ij} \cdot \boldsymbol{\sigma}_j) - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right] + V_\gamma^{LS}(\mathbf{r}_{ij})\, \mathbf{L}_{ij} \cdot \mathbf{S}_{ij}\,,
\end{aligned} \tag{4}$$

and for scalar mesons ($\gamma = a_0, \kappa, f_0, \sigma$)

$$V_\gamma(i,j) = V_\gamma^C(\mathbf{r}_{ij}) + V_\gamma^{LS}(\mathbf{r}_{ij})\,\mathbf{L}_{ij}\cdot\mathbf{S}_{ij}\ , \qquad (5)$$

where $\boldsymbol{\sigma}_i$ are the Pauli spin matrices. The terms V^{SS}, V^T, V^C, and V^{LS} represent the radial potential parts of the spin-spin, tensor, central, and spin-orbit forces, respectively. In case of the vector mesons the real mesons ω and ϕ, being strong mixings of flavor octet and singlet states, are replaced by fictitious pure octet and singlet states ω_8 and ω_0. Similarly, we assume that the scalar mesons f_0 and σ are pure octet and singlet states, respectively, and introduce a light kaonic scalar meson κ to complete the nonet.

The formulae for the radial dependences of the individual potential parts are given explicitly in Refs. [7,8]. They contain a number of parameters which are either fixed or obtained by a fit to the phenomenological baryon spectra. Here we deal with the version of the extended GBE CQM whose parameters have been determined specifically in Ref. [8]; they are quoted in Table 1. The masses of the constituent quarks m_u, m_d, m_s have been fixed a-priori to some standard values known from the literature. For the meson masses μ_γ, which govern the long-range parts of the meson-exchange potentials, one has employed the experimental values. The coupling constants g_γ have been derived from meson-nucleon phenomenology assuming $SU(3)_F$ symmetry; they determine the strengths of all potential parts. In the version of the extended GBE CQM of Ref. [8], however, the spin-orbit forces have been treated as an exception and have been given a different strength g^{LS} determined by a fit to the spectra. Among the free parameters, the cut-offs Λ_γ have been introduced in order to regularize the short-range parts of the meson-exchange potentials. The parameter C represents the strength of the linear confinement and V_0 is needed to fix the lowest eigenvalue to the nucleon mass. Altogether the extended GBE CQM comprises eight free parameters, while all other ingredients can be considered as predetermined.

Table 1. Parameters of the extended version of the CQM based on GBE as given in Ref. [8].

Fixed Parameters		
$m_u = m_d = 340$ MeV	$m_s = 507$ MeV	$\mu_\pi = 139$ MeV
$\mu_K = 493.6$ MeV	$\mu_\eta = 547$ MeV	$\mu_{\eta'} = 958$ MeV
$\mu_\rho = 770$ MeV	$\mu_{K^*} = 892$ MeV	$\mu_{\omega_8} = 869$ MeV
$\mu_{\omega_0} = 947$ MeV	$\mu_\sigma = 680$ MeV	$\mu_{sc} = 980$ MeV
$g_8^2/4\pi = 0.67$	$(g_0/g_8)^2 = 1$	$(g_8^V)^2/4\pi = 0.55$
$(g_8^T)^2/4\pi = 0.16$	$(g_0^V)^2/4\pi = 1.107$	$(g_0^T)^2/4\pi = 0.0058$
$g_s^2/4\pi = 0.67$		
Free Parameters		
$C = 1.935$ fm^{-2}	$V_0 = -334$ MeV	$(g^{LS})^2/4\pi = 0.8$
$\Lambda_\gamma = \Lambda_0 + \mu_\gamma$	$\Lambda_K = 1420$ MeV	$\Lambda_{\eta'} = 1400$ MeV
$\Lambda_0^{ps} = 695$ MeV	$\Lambda_0^v = 375$ MeV	$\Lambda_0^s = 833$ MeV

Among the main achievements of the original (pseudoscalar-exchange) version of the GBE CQM [4] has been the appropriate level ordering of states with positive and negative parities. This typical behaviour is maintained also in the extended model. In our investigations we considered two versions of the extended GBE CQM, namely, the ones with and without spin-orbit forces [8].

In Fig. 1 we demonstrate the effect of the GBE hyperfine interaction in case of the extended version without spin-orbit forces. Starting out from the case with confinement only, the inversion of the lowest positive- and negative-parity states in the N spectrum is gradually achieved when the coupling is increased. At the same time the level crossing of the analogous states in the Λ spectrum is avoided, just as demanded by phenomenology. If spin-orbit forces are included, the same behaviour of the level shifts persists.

In the next section we shall provide evidence how the different potential parts influence the energy levels. Such type of investigations lead to a better understanding of the dynamics stemming from GBE.

3 Influences of Different Force Components

In Ref. [9] we performed a detailed study of the influence of different force components on the light and strange baryon spectra. Here we are going to discuss the most important results, which provide valuable insight into the behaviour of the potential components derived from GBE dynamics.

From phenomenology we observe that the tensor force effects must be minor, as the splittings within LS multiplets (like, e.g., N(1535)-N(1520)) are rather small. Without any tensor and spin-orbit forces the levels within LS multiplets are degenerate. This is evident from Fig. 2a where at the starting point of the plot only spin-spin and central components of the potential act. When the tensor force (of the pseudoscalar exchange) is gradually turned on, there occur rather large splittings in the lowest-lying multiplets of $\frac{1}{2}^-$-$\frac{3}{2}^-$-$\frac{5}{2}^-$ nucleon resonances, namely, of N(1535)-N(1520) and N(1650)-N(1700)-N(1675). Such a behaviour is not seen in the phenomenological spectra. The situation can be remedied by including also the tensor force from vector-meson exchanges, as is done in the extended GBE CQM. They have an effect opposite to the pseudoscalar tensor force [10]. As a result the level splittings within the LS-multiplets get much reduced after the addition of vector-meson exchange (see Fig. 2b), where the action of the vector-meson exchange tensor force is demonstrated as a function of increasing strength. A similar behaviour is found in the splittings of other multiplets, e.g., in the Λ spectrum.

The vector- and scalar-meson exchanges also give rise to spin-orbit forces between the constituent quarks. Their effects on the spectra have been discussed extensively in Refs. [8,9] and they are shown in Fig. 3. The spin-orbit forces allow to improve the description of the practically degenerate $J = \frac{5}{2}^-$ and $J = \frac{5}{2}^+$ nucleon excitations (which are known to a rather good accuracy from experiment). The same is true for the corresponding states in the Λ spectrum. For this purpose, however, one had to use a phenomenological strength g^{LS} in the model of Ref. [8]. Otherwise the spin-orbit forces do not bring much improvement but slightly worsen the description in some cases.

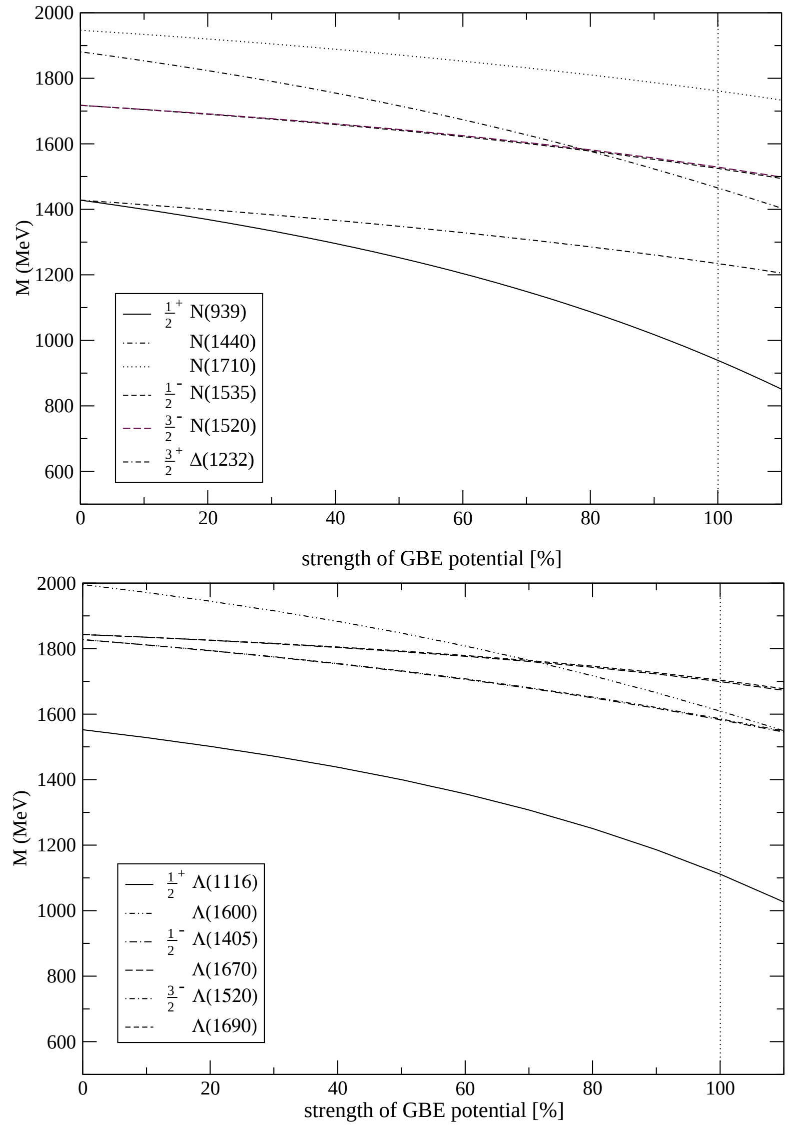

Fig. 1. Level shifts of the nucleon, Δ, and Λ due to the hyperfine interaction in case of the extended GBE CQM (without spin-orbit forces). The full strength of the potential is recovered at 100%.

In summary, in order to reach a good description of the baryon spectra in close agreement with phenomenology (generally small splittings in LS multiplets) one must take into account at least both the pseudoscalar and vector exchanges. For completeness (of the inclusion of multiple GBE) one has also foreseen scalar-meson exchange. However, it plays only a minor role in the level splittings, at least for the moderate magnitude of its coupling deduced from meson-nucleon phenomenology. It has been shown in Ref. [11] that the scalar-meson

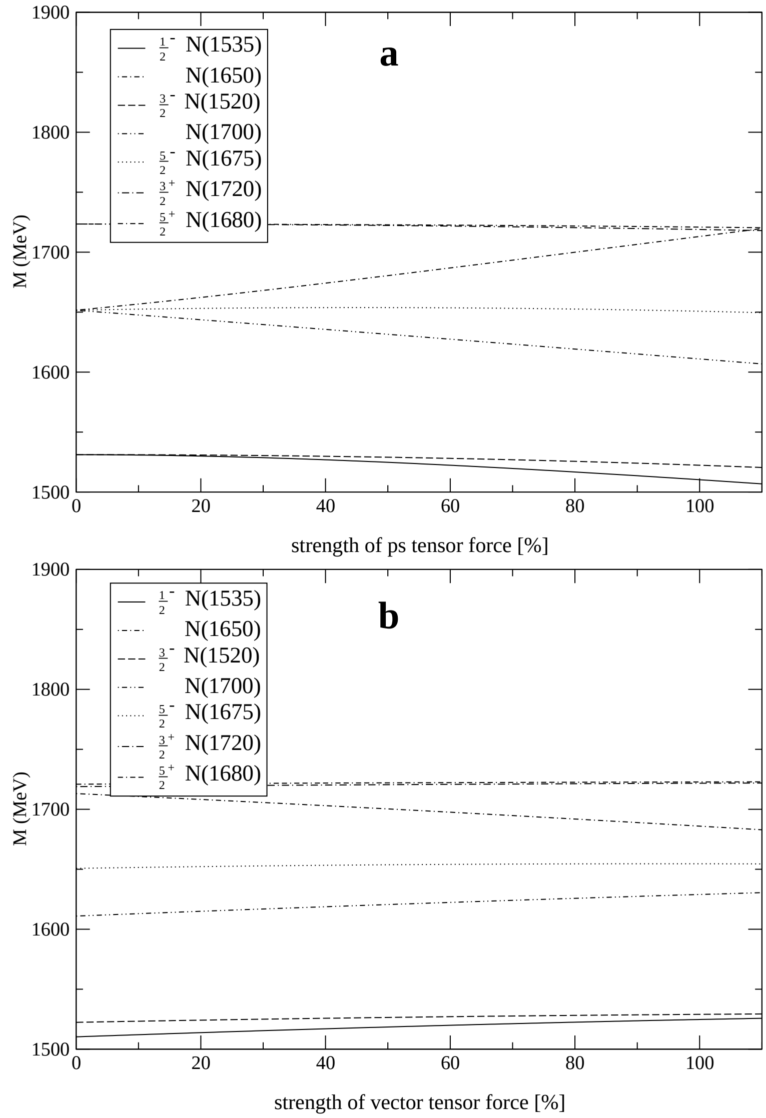

Fig. 2. Level shifts in the nucleon spectrum due to tensor forces: Starting from the case with no tensor force at all, we first turn on only the tensor forces from the pseudoscalar meson exchanges (a), and then in addition the tensor forces from the vector meson exchanges (b).

exchange tends to have a favourable influence on the level splittings of positive- and negative-parity states, however, with a much bigger strength.

4 Conclusion and Outlook

We have discussed the effective quark-quark interactions in baryons within CQMs. In particular, we reported evidences on the behaviour of various potential components along the CQM based on GBE dynamics. We started out from the original

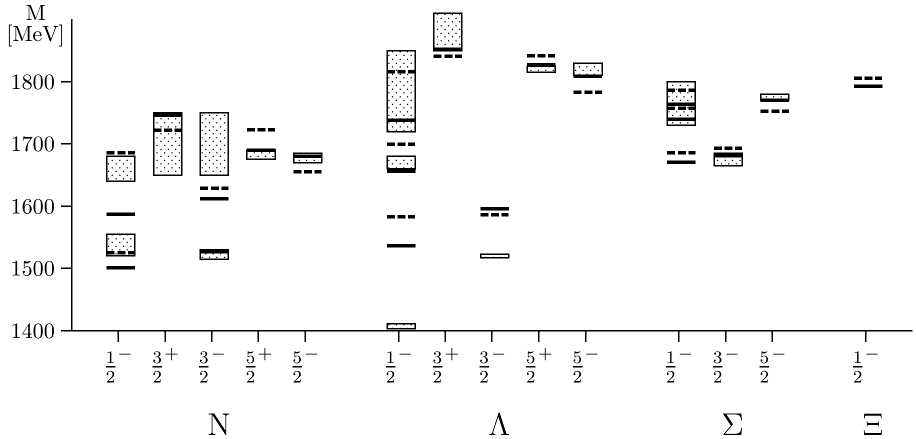

Fig. 3. Influence of spin-orbit forces on selected light and strange baryon states. The solid and dotted bars are the energy levels with and without spin-orbit forces, respectively.

version of the GBE CQM [4], which contains as the hyperfine interaction only the spin-spin componenent of the pseudoscalar-meson exchange. An extension of the GBE CQM to including vector- and scalar-meson exchanges is called for in order to take into account also multiple Goldstone-boson exchanges [7,8]. Thereby the favourable features of the GBE CQM are in general maintained and further improvements in the description of the spectra can be made [9]. Notably, one can reproduce the correct level orderings of the low-lying light and strange baryon spectra with about the same quality as in the original GBE CQM. It will be interesting to apply the extended GBE CQM with its additional force components in the investigation of the electromagnetic structure of the baryons, especially the nucleons, and in other studies of baryon reactions such as mesonic decays of resonances etc. Furthermore, the extended GBE CQM now also brings about the necessary force components for a microscopic derivation of the baryon-baryon interaction, which are missing in the pseudoscalar version [12]; it appears worthwhile to check if the N-N interaction can now be produced directly from the CQM.

Even though the GBE CQM has been quite successful in baryon spectroscopy and in first applications to the elastic electromagnetic and axial form factors of the nucleons [13], one must not forget that the description of the excited states as resonances with finite widths is still not achieved. This is obviously reflected in studies of mesonic N and Δ decays, which have recently been performed with the GBE CQM for the first time in a covariant framework (point form) [14]. One could (consistently) improve on that by extending the GBE CQM beyond $\{QQQ\}$ configurations to including higher Fock states such as $\{QQQ\pi\}$ or $\{QQQ\eta\}$ etc. One may be confident that a more adequate description of the resonances and their (decay) properties will then be achieved.

At this instance the GBE CQM is limited to the sector of light and strange flavors. One could think of extending it also to heavy flavors. Presumably new types of hyperfine interactions will be necessary for this purpose. One will not only need the light-light and heavy-heavy quark-quark forces but notably also the light-heavy flavor interactions. Not much is known about the latter, what will make the attempt of creating a unified CQM of all baryons a rather difficult task.

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