Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY
MNRAS **527,** 11962–11969 (2024) https://doi.org/10.1093/mnras/stad3627
Advance Access publication 2023 December 1

Phenomenological emergent dark energy versus the ΛCDM: ellipticity of cosmic voids

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Accepted 2023 November 21. Received 2023 October 19; in original form 2022 November 27

ABSTRACT

We disclose the influences of the phenomenological emergent dark energy (PEDE) and its generalization (GEDE) on the distribution of tidal ellipticity of the cosmic voids. We also compare our results with that of the standard ΛCDM cosmology. The underlying models consist with recent observations (specially in favour of H_0 tension) and may impact the cosmic voids geometry. We employ the analytic approach based on statistics of the tidal tensor eigenvalues. We confirm a significant sensitivity of the ellipticity distribution function for PEDE and GEDE cosmology. We observe the largest deviation from the standard cosmology for GEDE in the range ($z > 0$, $R_{\rm L} \geq 4\,h^{-1}$ Mpc). Working on these distribution functions we reveal that the mean ellipticity ($\langle\varepsilon\rangle$) profile of GEDE versus redshift lies below than that of PEDE and ΛCDM indicating that GEDE leads to a less elongated society of voids with respect to other models. We also identify a tight correlation between σ_8 and the mean ellipticity evolution. Finally, we explore the mean ellipticity versus $R_{\rm L}$ and estimate that in this case GEDE leads to a different ellipticity curve (up to 11 per cent smaller than that of ΛCDM).

Key words: cosmological parameters – dark energy – large-scale structure of Universe.

1 INTRODUCTION

It is well established that 'ΛCDM' is located at the centre among all models in modern cosmology which can explain, at the same time, the late time acceleration of the cosmic expansion as well as the dark matter puzzle (Riess et al. 1998). Although, this model is compatible with observational data such as cosmic microwave background (CMB; Ade et al. 2016) and bryonic acoustic oscillations (BAO; Beutler et al. (2011; Alam et al. 2017), however, there are some inconsistencies in ΛCDM model with observations. First of all, while the equation of state (EoS) parameter of Λ is fixed, $w_\Lambda = -1$, many observations indicate a tiny variation for EoS parameter of the dark energy (DE). Another challenge is the so-called cosmic tensions including σ_8 (Riess et al. 2019, 2021; Aghanim et al. 2020) and H_0 tensions (Di Valentino et al. 2021). For a detailed list of the cosmic tensions one can see Perivolaropoulos & Skara (2022). Alleviating the aforementioned shortcomings is one of the most important tasks in modern cosmology. Dynamical DE (a model with an evolving EoS parameter) is an important approach in exploring the problems with ΛCDM cosmology. For a wide range of these models, see Bento, Bertolami & Sen (2002), Goswami, Pradhan & Beesham (2019), Bagla, Jassal & Padmanabhan (2003), Ichikawa & Takahashi (2006), Clarkson, Cortes & Bassett (2007), Sahni (2004), Yang et al. (2020), Benevento, Hu & Raveri (2020), Alestas, Kazantzidis & Perivolaropoulos (2021), Solà, Gómez-Valent & de Cruz Pérez (2017), Dutta et al. (2020) and the references therein.

The so-called 'phenomenological' models of DE are those where the EoS parameter are introduced in terms of redshift without any information from microscopic nature of DE. This branch of DE models is widely studied (Astier 2001; Chevallier & Polarski 2001; Linder 2003; Gong & Zhang 2005; Linder & Huterer 2005; Xia et al. 2006; Basilakos & Voglis 2007; Ma & Zhang 2011; Feng et al. 2012). In Li & Shafieloo (2019), a recent example of the phenomenological category is presented with the aim of alleviating the H_0 tension. This model is consistent with plenty of data sets (SNe Ia, BAO, *Lyα* BAO, and CMB) in high and low redshifts. According to the results of this paper, the PEDE seems more consistent with cosmic observations with respect to the ΛCDM ($\Delta\chi^2_{\rm bf} \approx -41.08$, $\Delta{\rm DIC} = -35.38$). This model has neither free parameters nor any serious contributions to the cosmic dynamics during the history of the universe. Although the PEDE reduces the H_0 tension significantly but the sound horizon problem still exist (Li & Shafieloo 2020). Seeking an amendment to the sound horizon problem the authors added a free parameter Δ to PEDE and made a more flexible model. This model is called generalized emergent dark energy (GEDE; Li & Shafieloo 2020) which restore ΛCDM for $\Delta = 0$ and PEDE for $\Delta = 1$. GEDE has been investigated in *Planck 2018, BAO, SNIa*, and *R19* (Yang et al. 2021). Different aspects of PEDE and GEDE were explored in Hernández-Almada et al. (2020) and Sharma (2020). It is worth noting that despite the fact that the nature of PEDE and GEDE models are matter of question but the results in exploring the tensions are completely satisfying.

On the other side, the improvement in the cosmic observations has revealed an important counterpart of the universe, the so-called '*cosmic voids*', surrounded by cosmic sheets and filaments. The

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minimum points in the cosmic density field could be the centre of voids. Since these regions are emptier [for example with a density contrast $\delta_v = -0.9$ (Hoyle & Vogeley 2004) or $\delta_v = -0.8$ (Sheth & van de Weygaert 2004; van de Weygaert & van Kampen 1993a)] than their neighbourhood, they expand faster and form the cosmic voids. According to the void definition up to 95 per cent of the total volume of a galaxy sample could be composed of voids family (Pan et al. 2012). Accurate simulations and observation indicate a large deviation from spherically symmetry in cosmic voids geometry (Shandarin, Sheth & Sahni 2004; Shandarin et al. 2006). It seems that non-sphericity of the voids is a consequence of the large scale cosmic dynamics and local influences (Platen, van de Weygaert & Jones 2008). In a more exact description tidal gravitational effects were introduced responsible for non-spherical shape of the cosmic voids (Shandarin et al. 2006). It was claimed that since the voids are so empty, they have a high chance to be easily deviated from spherical symmetry (Shandarin et al. 2006).

Many features of the voids were discovered through the results of numerical simulations. One interesting case is the void universal density profile (Hamaus, Sutter & Wandelt 2014). In one example the ΛCDM cosmology simulation of voids was performed in Colberg et al. (2005), where a large number of voids ($\approx$80 000) were found. For more examples of the modern simulations of the cosmic voids, we refer to van de Weygaert & van Kampen (1993b), Sheth & van de Weygaert (2004), Gottlöber et al. (2003), Aragon-Calvo & Szalay (2013), Arbabi-Bidgoli & Mueller (2002), Wojtak, Powell & Abel (2016), and Sutter et al. (2014).

Following Ghigna et al. (1996) the void statistics have arisen a lot of attention. In Lee & Park (2006), using the tidal torque theory, an analytic model of cosmic voids was presented and its agreement with the results of *N*-body simulations were discussed. Lee and Park in Park & Lee (2007) showed that ellipticity of voids is sensitive to cosmological parameters and hence the ellipticity could be used to constraint cosmological parameters. They also determined the void shapes by obtaining the positions of halos through an inertia tensor in the voids (Park & Lee 2007). They used the void finder method of Hoyle & Vogeley (2002) which is based on the halo positions to check the validity of their analytic method. The authors of Lee & Park (2009) followed the idea that the DE equation of state could be constrained using the ellipticity of voids. In another work the distribution of ellipticity of voids was used as a precise probe in discriminating DE models (Lavaux & Wandelt 2010). In Bos et al. (2012), the significant sensitivity of cosmic voids ellipticity distribution to dark matter was explained. Beside a direct correlation between the level of clustering(σ_8) and the ellipticity was presented in the last paper. The authors of Bos et al. (2012) proposed that for breaking the degeneracy between $\langle\varepsilon\rangle$ and the σ_8, a simultaneous measurement of both quantities needed. Other applications of the Park and Lee approach can be found in Lavaux & Wandelt (2010) and Biswas, Alizadeh & Wandelt (2010). The effects of the dark matter pressure on the cosmic voids ellipticity distribution was argued in Rezaei (2019). It has been shown that the interaction between dark matter and DE component can increase the mean ellipticity of cosmic voids (Rezaei 2020).

It is worth noting that since DE models are alternatives to ΛCDM, thus, it is of special interest to obtain different aspects of any model with respect to ΛCDM. Therefore we would like to see if PEDE or GEDE leads to a significant different pattern of cosmic voids in comparison to the standard cosmology. Since the most important stages of voids evolution take place at $0.5 \le z \le 2$ (Bos et al. 2012) and the DE models (including PEDE and GEDE) are lately effective, thus, it is more reasonable to see their impacts on the statistics of the cosmic voids. To this end we focus on the ellipticity distribution and the mean ellipticity parameter via the analytic approach by Park & Lee (2007), Lavaux & Wandelt (2010), and Biswas, Alizadeh & Wandelt (2010).

The paper is organized as follows. In the next section we introduce the PEDE and GEDE models and their primary properties. In Section 3 the method of Lee & Park (2009) is applied to PEDE and GEDE. The last section is devoted to the results and discussions.

2 EMERGENT DARK ENERGY

An important challenge in modern cosmology is how to consider the cosmic tensions such as H_0 and σ_8? To avoid these tensions one main idea is to take a correcting agent at early universe. Early DE models are examples of this effort in the literature (Pettorino, Amendola & Wetterich 2013; Karwal & Kamionkowski 2016; Carneiro et al. 2019; Sakstein & Trodden 2020). However, it seems that these efforts fail to solve the tension problem with *R*19 below 3σ (Arendse et al. 2020). In another recent model for facing the cosmic tensions the PEDE (Li & Shafieloo 2019) was presented which does not have any significant impact at early stages of cosmic evolution and dominates the universe at later stages. The fractional density of the PEDE is

$$\Omega_{\rm PEDE}(z) = \Omega_{D0} \times [1 - \tanh(\log_{10}(1+z))], \tag{1}$$

where $\Omega_{\rm D0}$ denotes the present value of the dark energy component and it is worthy to note that this model does not introduce any new degree of freedom. If we take a flat FRW universe filled with two non-interacting component (DE and DM), one can easily find that

$$\Omega_{\rm D}(z) = \Omega_{\rm D0} \exp\left(3\int_0^z \frac{1+w(z')}{1+z'}{\rm d}z'\right), \tag{2}$$

where w is the DE equation of state parameter. From the above equation it is obvious that

$$w(z) = \frac{1}{3}\frac{{\rm d}\ln\Omega_{\rm D}}{{\rm d}z}(1+z) - 1. \tag{3}$$

So using equations (3) and (1) we reach

$$w_{\rm PEDE}(z) = -\frac{1}{3\ln 10} \times [1 + \tanh(\log_{10}(1+z))] - 1. \tag{4}$$

Therefore above parameter($w_{\rm PEDE}$) shows that PEDE will behave as cosmological constant ($w = -1$) at far future. Present value of the EoS equals $w(0) = -\frac{1}{3\ln 10} - 1$ which denotes crossing the phantom line ($w < -1$) in agreement with observational evidences. Consistency of this model with recent observations is investigated in Li & Shafieloo (2019, 2020) and one can see that in some features it is preferred to ΛCDM.

As mentioned in Section 1, the PEDE can solve H_0 tension but there remain doubts about the sound horizon. Seeking an explanation to the sound horizon problem, authors of PEDE try a more flexible version of PEDE by adding a new degree of freedom. The model is named GEDE (Li & Shafieloo 2020). The fractional dark energy density of the GEDE is introduced as

$$\Omega_{\rm GEDE}(z) = \Omega_{\rm D0} \times \frac{1 - \tanh\left(\Delta \times \log_{10}\left(\frac{1+z}{1+z_t}\right)\right)}{1 + \tanh\left(\Delta \times \log_{10}(1+z_t)\right)} \tag{5}$$

where $\Omega_{\rm D0}$ denotes the present value of the fractional density and Δ is a new degree of freedom. For $\Delta = 0$ the model reduces to ΛCDM and the PEDE is recovered for $\Delta = 1$. In equation (5), z_t is the redshift where DE equals DM and hence it is not a free parameter. Using equation (3) one can easily find that the EoS parameter of

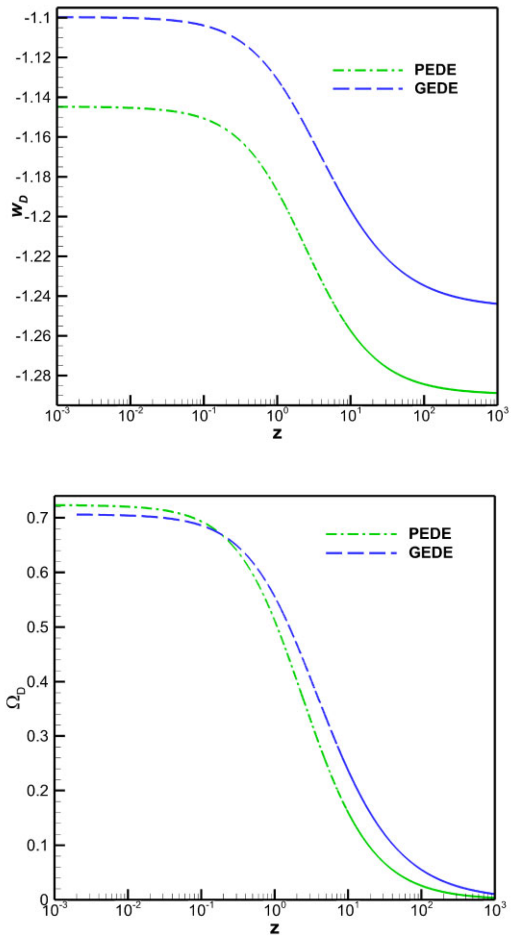

Figure 1. EoS parameter (w) and the fractional density of PEDE and GEDE are depicted versus z.

GEDE is

$$w_{\rm GEDE}(z) = -\frac{\Delta}{3\ln 10}\times\left[1+\tanh\left(\Delta\times\log_{10}\left(\frac{1+z}{1+z_t}\right)\right)\right]-1. \quad (6)$$

In Pan et al. (2020) and Yang et al. (2021), PEDE and GEDE models are constrained by the most recent observational data sets. Along with PEDE (or GEDE), there exists a cold dark matter (CDM) component in the universe. To see the general behaviour of these models the EoS parameter and fractional density for mentioned models are depicted in Fig. 1. This figure implies a more negative pressure with respect to ΛCDM for both models. However the lower panel ($\Omega_{\rm D}$ evolution) of this figure reveals that PEDE (GEDE) contribution to the critical density of the universe is tiny at early epochs and they have a significant contribution in late time. Besides one should pay attention to $w_{\rm eff}=\frac{P_{\rm tot}}{\rho_{\rm tot}}$ which denotes the EoS parameter of the cosmic fluid including all components. This quantity shows when the universe is under domination of gravity or anti gravity. The counterbalance between the cosmic expansion and gravitational effect is a key concept regarding the morphology of cosmic voids. This is the reason why evolution of $w_{\rm eff}$ is depicted in Fig. 2. It is very worthy to note that since these models are highly consistent with the latest observational data sets and specially are successful in facing the H_0 tension thus it is well motivated to consider more subtle issues through these models.

In this paper we would like to discuss another feature of the PEDE-CDM and GEDE-CDM models. Our goal is to investigate the impact

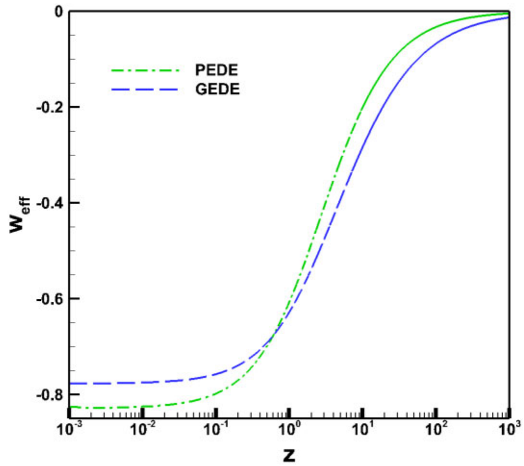

Figure 2. ($w_{\rm eff}$) is depicted versus z for PEDE and GEDE.

of these models on the morphology (ellipticity at present work) of cosmic voids. Thus, we mention best-fitting value of the dynamically free parameters which are important at this occasion in Table 1. Beside we take $\Omega_{\rm r0}=8.6\times10^{-5}$ through this paper.

3 IMPACTS OF PEDE AND GEDE ON THE ELLIPTICITY OF COSMIC VOIDS

3.1 Basic idea

Since the fluctuations of the cosmic density field are seeds of cosmic structures, then the spatial distribution of the resulting structures at early epochs is tightly correlated to the statistics of those fluctuations. At any time, local anisotropy of the fluctuations in the density field could be traced through the eigenvalues and eigenvectors of the local tidal tensor and it is an accepted idea that the spatial distribution of void galaxies follow the pattern of the mentioned eigenvalues and eigenvectors. In order to visualize a cosmic void one can introduce an ellipsoid around any minima of the density field whose principal axes are along the eigenvectors of the tidal tensor and the length of the principal axes correspond to the related eigenvalues. Here our aim is to address the geometry and evolution of such cosmic voids. The distribution of the eigenvalues is available at early times for all points of the density field. In a leading work Park and Lee in Park & Lee (2007) presented an analytic approach which quantifies the statistics of the eigenvalues and eigenvectors of the local tidal tensor and introduces the so-called 'tidal ellipticity' concept. In this work we would like to compare the resulting cosmic voids geometry of the PEDE and GEDE with that of the ΛCDM using the analytic approach presented in Park & Lee (2007).

3.2 Tidal elleipticity

To describe the above picture in a more quantitative manner we take an individual void galaxy whose position could be described by a comoving coordinate **x** at an arbitrary time(τ) as an Eulerian coordinate. Also we introduce **q** as a Lagrangian coordinate, which corresponds to **x** at an early chosen time(τ_0). These two coordinates are related to each other by

$$\mathbf{x}=\mathbf{q}+\mathbf{\Psi}(\mathbf{q},\tau), \quad (7)$$

where $\mathbf{\Psi}$ is the displacement field, responsible for dynamics of the system. The mentioned local tidal tensor is related to the

Table 1. Observational constraints at 68 per cent and 95 per cent CL on the free parameter of the ΛCDM, PEDE, and GEDE.

Parameters	ΛCDM (Pan et al. 2020)	PEDE (Pan et al. 2020)	GEDE (Yang et al. 2021)
$\Omega_{\rm m}$	$0.295^{+0.0050+0.010}_{0.0051-0.0095}$	$0.276^{+0.0049+0.0010}_{-0.0049-0.0094}$	$0.293^{+0.0065+0.013}_{-0.0067-0.013}$
Δ	0	1	$0.55^{+0.20+0.42}_{-0.21-0.43}$
H_0	$68.77^{+0.4+0.79}_{-0.40-0.80}$	$71.98^{+0.46+89}_{-0.46-0.91}$	$69.86^{+0.75+1.4}_{-0.74-1.4}$
σ_8	$0.815^{+0.0086+0.017}_{-0.0086-0.017}$	$0.847^{+0.0087+0.017}_{-0.0087-0.017}$	$0.834^{+0.012+0.025}_{-0.012-0.025}$

displacement field as

$$T_{ij} = \frac{\partial \Psi_i(\mathbf{q})}{\partial q_j}. \quad (8)$$

The most interesting feature of this tensor is its eigenvalues and eigenvectors. In Shandarin et al. (2006), the authors claimed that gravitational tidal effects could be responsible for non-spherical shape of the voids. A spherically symmetric tidal effect (equivalent to a spherical geometry) is a consequence of the same set of equal eigenvalues (λ_i, $i = 1, 2, 3$) and any non-equal set could describe a non-spherically symmetric geometry. This idea was first developed quantitatively by Park and Lee in Park & Lee (2007) to study cosmic voids. Since the primary matter density field inherit its fluctuations from a primary highly symmetric field(probably the inflaton), thus we deal with a Gaussian density field. Since at early epochs of cosmic evolution the fluctuations are small, their growth could be explored as a perturbation and the primary Gaussian distribution keeps its Gaussian form. As we already mentioned, we wish to obtain the distribution of eigenvalues of the tidal tensor of the initial fluctuations. To this end one can use the probability distribution of the ordered eigenvalues of the tidal tensor as (Doroshkevich 1970)

$$p(\lambda_1, \lambda_2, \lambda_3; \sigma_{R_{\rm L}}) = \frac{3375}{8\sqrt{5}\pi\sigma^6_{R_{\rm L}}} \exp\left(-\frac{3K_1^2}{\sigma^2_{R_{\rm L}}} + \frac{15K_2}{2\sigma^2_{R_{\rm L}}}\right) K_3, \quad (9)$$

where $K_1 = \lambda_1 + \lambda_2 + \lambda_3$, $K_2 = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1$, $K_3 = (\lambda_1 - \lambda_2)(\lambda_2 - \lambda_3)(\lambda_1 - \lambda_3)$, and σ_R is the linear rms fluctuation of the matter density field smoothed on a filtering scale $R_{\rm L}$. Although the distribution function of the eigenvalues is non-Gaussian but it corresponds to a Gaussian form of density perturbations. In small regime of fluctuations one can find

$$\delta_v = \frac{\rho_v - \bar{\rho}}{\bar{\rho}} = \lambda_1 + \lambda_2 + \lambda_3, \quad (10)$$

where δ_v is the density contrast threshold of the void formation. Visualizing the tidal tensor eigenvalues and relating this concept to an ellipsoid geometry, one can write (Park & Lee 2007)

$$\nu = \sqrt{\frac{1-\lambda_1}{1-\lambda_3}},$$

$$\mu = \sqrt{\frac{1-\lambda_2}{1-\lambda_3}},$$

$$\varepsilon = 1 - \nu = 1 - \sqrt{\frac{1-\lambda_1}{1-\lambda_3}}, \quad (11)$$

where two first cases of the above relations are the two axial ratios of the ellipsoid and ν is called sphericity. The last parameter ε denotes the ellipticity of the void which shows deviation of the ellipsoid from spherical symmetry. Based on this ordered eigenvalues,($\lambda_1 > \lambda_2 > \lambda_3$), of the tidal tensor one can determine the shape and dynamics of the tidal voids(the voids which we assume according to the eigenvalues of the tidal tensor). Depending on the values of λ_i one can find several types of geometry of voids. True voids correspond to (all $\lambda_i > 0$), while filament voids are those with (only $\lambda_3 < 0$) and pancake ones are (only $\lambda_1 > 0$) (Lavaux & Wandelt 2010). Besides considering equation (10) the independent variables of the distribution function $P(\lambda_1, \lambda_2, \lambda_3)$ reduce to two variables. Thus conditional version of the probability density distribution should be taken such that to account as (Rezaei 2020; Lee & Park 2009; Bos et al. 2012)

$$p(\mu, \nu|\delta_v, \sigma_{R_{\rm L}}) = \frac{3375\sqrt{2}}{\sqrt{10\pi}\sigma^5_{R_{\rm L}}}$$
$$\exp\left[\frac{-5\delta_v^2 + 15\delta_v(\lambda_1+\lambda_2)}{2\sigma^2_{R_{\rm L}}}\right] \exp\left[\frac{-15(\lambda_1^2 + \lambda_1\lambda_2 + \lambda_2^2)}{2\sigma^2_{R_{\rm L}}}\right]$$
$$\times(2\lambda_1 + \lambda_2 - \delta_v)(\lambda_1 - \lambda_2)(\lambda_1 + 2\lambda_2 - \delta_v)\frac{4(\delta_v - 3)^2\mu\nu}{(\mu^2+\nu^2+1)^3}. \quad (12)$$

Definition of $\sigma_{R_{\rm L}}$ read

$$\sigma^2_{R_{\rm L}} = \int_0^\infty \frac{k^2 {\rm d}k}{2\pi^2} P(k) W^2_{kR_{\rm L}}, \quad (13)$$

where $W_{kR_{\rm L}}$ is the window function and $R_{\rm L}$ is the Lagrangian filtering scale which is directly related to size of the voids. The window function is

$$W_{kR_{\rm L}} = 3\left[\frac{\sin(kR_{\rm L})}{(kR_{\rm L})^3} - \frac{\cos(kR_{\rm L})}{(kR_{\rm L})^2}\right]. \quad (14)$$

In (13), $P(k)$ is the matter power spectrum (Park & Lee 2007; Bardeen et al. 1986). The matter power spectrum can be defined as

$$P(k) = AkT(k)^2, \quad (15)$$

where $T(k)$ is the transfer functionRezaei (2019).

In order to use equation (15), we use the observational constraints from Li & Shafieloo (2019, 2020). Beside we obtain A regarding the constraint $\sigma(R_{\rm L} = 8\,h^{-1}) = \sigma_8$ for each model of interest.

Up to now we have introduced the mathematical description of the statistics of eigenvalues at early epochs, however our main interest is to consider the issue at low redshift epochs where the structure formation process enters non-linear era. Due to this fact the distribution of the tidal eigenvalues of the density field the equation (9) is not valid anymore. Thus we should clarify that if we are allowed to use the formalism developed so far for late universe and also see what the effect of time evolution on the distribution function is. Fortunately, it could be seen that although the density field is highly non-linear at late epochs but this non-linear effects are less manifested in the displacement fields(which relates $\mathbf{x}(\tau)$ to $\mathbf{q}(\tau)$) (Biswas, Alizadeh & Wandelt 2010). It is also worth to note that the non-linearity behaviour is more effective in Eulerian manner (which describes the physical quantities localy) while switching to Lagrangian description (which takes the cosmic fluid as a set of mass elements), one can ignore the non-linear aspects

(Feldbrugge & van de Weygaert 2023). Therefore, even at low-redshift regime before shell-crossing, the evolution of the cosmic voids from initial conditions (around the minima in density field) in the Lagrangian coordinate could be largely explored through the Zel'Dovich approximation in where the displacement field could be decomposed to time and spatial parts ($\mathbf{\Psi}(\mathbf{q}, \tau) = D(\tau)/D(\tau_0)\mathbf{\Psi}(\mathbf{q}, \tau_0)$), where $D(\tau)$ is the linear growth factor. In this approximation one can also find that $\lambda_i(\tau) = D(\tau)/D(\tau_0)\lambda_i(\tau_0)$. Applying these equations to distribution function of eigenvalues, equation (9), to obtain the distribution function at an arbitrary time τ, one finds that the resulting distribution function is exactly the same as equation (9) where $\sigma_{R_{\rm L}}$ being replaced by $D(\tau)/D(\tau)0)\sigma_{R_{\rm L}(\tau_0)})$ and any eigenvalue is replaced by its value at time τ. Biswas et.al. in Biswas, Alizadeh & Wandelt (2010) showed that the above approximation works well. They found that studying the eigenvalue evolution based on the ellipsoidal collapse method and the Zel'Dovich approximation have a negligible difference. Thus, although voids are non-linear objects at recent epochs, using the Lagrangian description in a Zel'Dovich approximation is largely valid. For a detailed and clear description about Eulerian and Lagrangian approaches as well as linearity versus non-linear effects in cosmic web formation one can see Feldbrugge & van de Weygaert (2023).

Hence, to our aim in this paper we replace $\sigma_{R_{\rm L}}$ by $\sigma(z, R_{\rm L})$ in equation (12) where the definition of $\sigma(z, R_{\rm L})$ reads

$$\sigma(z, R_{\rm L})^2 = D(z)^2\sigma^2_{R_{\rm L}}. \tag{16}$$

Taking the above relation, the equation (12) and equations (11) one can easily find that

$$p(\nu, z, R_{\rm L}) = p(1-\varepsilon, z, R_{\rm L}) = \int_{\nu}^{1} p(\mu, \nu|\delta_v, \sigma(z, R_{\rm L}))\mathrm{d}\mu. \tag{17}$$

Beside, the linear growth factor($D(z)$) in the case of the standard cosmology(Λ CDM) is

$$D(a) = \frac{5}{2}\frac{H(a)}{H_0}\Omega_{\rm m0}\int_0^a \frac{\mathrm{d}a'}{(a'E(a'))^3}, \tag{18}$$

while in a dynamical DE cosmology($w_{\rm DE}$ changes versus time), we are not allowed to use (18). In the presence of an evolving DE the growth factor read

$$D(z) \propto \exp\left[\int_z^{z_i} \frac{\Omega_{\rm m}(z')^{\gamma}}{1+z'}\mathrm{d}z'\right], \tag{19}$$

where $\gamma = 0.545 + 0.05(1 + w_{\rm DE}(z = 1))$ (Linder & Huterer 2005).

Now we are ready to consider the main goal of this paper which is the impact of PEDE and GEDE on the cosmic void ellipticity. Normalized $p(\varepsilon, z, R_{\rm L})$ at hand, the mean value of the ellipticity is

$$\langle\varepsilon\rangle(z, R_{\rm L}) = \int_0^1 \varepsilon\, p(\varepsilon)\mathrm{d}\varepsilon \tag{20}$$

In obtaining the results we set the free parameters to the values presented in Table 1. The threshold density contrast for void formation is set at $\delta_v = -0.9$ (Hoyle & Vogeley 2004). At first we consider the impacts of PEDE and GEDE on the probability distribution function(PDF). The results are summarized in Fig. 3. From this figure one can find that the distribution functions tend to less values of ε with increasing z which seems to be an odd result because it was expected to find more elongated voids at higher values of redshift. However the results of Fig. 3 implies that the clustering process is in progress and leads to more elongated voids at lower redshifts. Actually at present time the domination of dark energy tends to formation of more spherical voids but the PDF shows that still the clustering process is stronger and the total trends of the ellipticity is growing and the presence of the dark energy component just slows down the process. A close look to Fig. 3 reveals almost in all cases PEDE keeps close to ΛCDM while for $z = 0.5$, 1 and $R_{\rm L} \geq 4(8)h^{-1}$, GEDE has a significant different ellipticity distribution function with respect to other models of interest. The overall trend of these figures is completely in agreement with what we expect and those previously presented in Park & Lee (2007), Lee & Park (2009), Bos et al. (2012), and Lavaux & Wandelt (2010).

In Fig. 4, we presented evolution of $\varepsilon_{\rm max}$ against redshift for all models. $\varepsilon_{\rm max}$ is the ellipticity at which most cosmic voids have at any redshift. It can bee seen from Fig. 4 that at present epoch the resulting voids in ΛCDM look more spherical than PEDE and GEDE while in earlier epochs, GEDE is the model with most spherical (smallest $\varepsilon_{\rm max}$) voids. From Fig. 4, we find that the fastest rate of evolution in $\varepsilon_{\rm max}$ belongs to $R_{\rm L} = 4\,h^{-1}$. GEDE in all parts of this figure lead to smaller values of $\varepsilon_{\rm max}$ except in late universe which is an obvious consequence of Fig. 2.

Mean ellipticity of voids $\langle\varepsilon\rangle$ is another key quantity which is significantly sensitive to DE component. Since, the main stages of voids evolution happen at $0.5 \leq z \leq 2$ Bos et al. (2012), thus we plot the related diagrams around this range. To this aim equation (3.2) is used and the quantity is calculated versus z and $R_{\rm L}$. The results are depicted in Fig. 6. At first it is worth to note that the general behaviour of $\langle\varepsilon\rangle$ follows the pattern of Lee & Park (2009). Beside $\langle\varepsilon\rangle$ reduces as the redshift increases which is predicted for evolving dark energy models in a *N*-body simulations of structure formation (Bos et al. 2012; Rezaei 2020). In the left panel of Fig. 6 one can find that the voids are less elongated at higher redshifts in agreement with what we found from distribution function figures. This panel also suggest that although the mean ellipticity of the models at present era are very close to each other but the GEDE has significant smaller values(up to 13 per cent) of ellipticity with respect to standard cosmology at higher redshifts. Thus it seems that the ellipticity evolution could be used to discriminate GEDE-CDm from PEDE-CDM and ΛCDM. It is also worth to mention the degeneracy between $\langle\varepsilon\rangle$ and the σ_8 according to results of Bos et al. (2012). Here, we also find the same degeneracy between $\langle\varepsilon\rangle$ and σ_8. To emphasis the point, we depicted $\langle\varepsilon\rangle$ versus $\sigma 8$ and also σ_8 against redshift in Fig. 5. Left panel of Fig. 5, reveals a one to one correlation between these variables. Besides the σ_8 evolution in right panel of Fig. 6 shows a very close trend to $\langle\varepsilon\rangle$ evolution (Fig. 6) for all models of interest. This result is another confirmation of what already reported in Bos et al. (2012). According to these results, $\langle\varepsilon\rangle$ doesn't look to be more informative than σ_8. However, the analytic approach of Park and LeePark & Lee (2007) leaves a chance to obtain $\langle\varepsilon\rangle$ versus $R_{\rm L}$. The result is depicted in the right panel of Fig. 6. Fig. 6) implies that Λ CDM and PEDE are highly degenerated in all smoothing scales and apparently these models are not distinguishable very accurately by this method. However the GEDE has a different manner of evolution and $\langle\varepsilon\rangle$ in this case(versus $R_{\rm L}$) seems to be a suitable quantity for discriminating between the models. However this point(smoothing scale dependency of $\langle\varepsilon\rangle$) deserves more accurate investigations via simulations and a clear discussion on the void finding techniques which is beyond the scope of present work.

3.3 Relation to observations and simulations

One important part of our work is to check the consistency of our model with observations. Since there are different approaches in facing the void concept and its geometry (Lavaux & Wandelt 2010), we should clarify how our results could be compared to the *N*-body simulations and galaxy surveys. In identifying a void one can

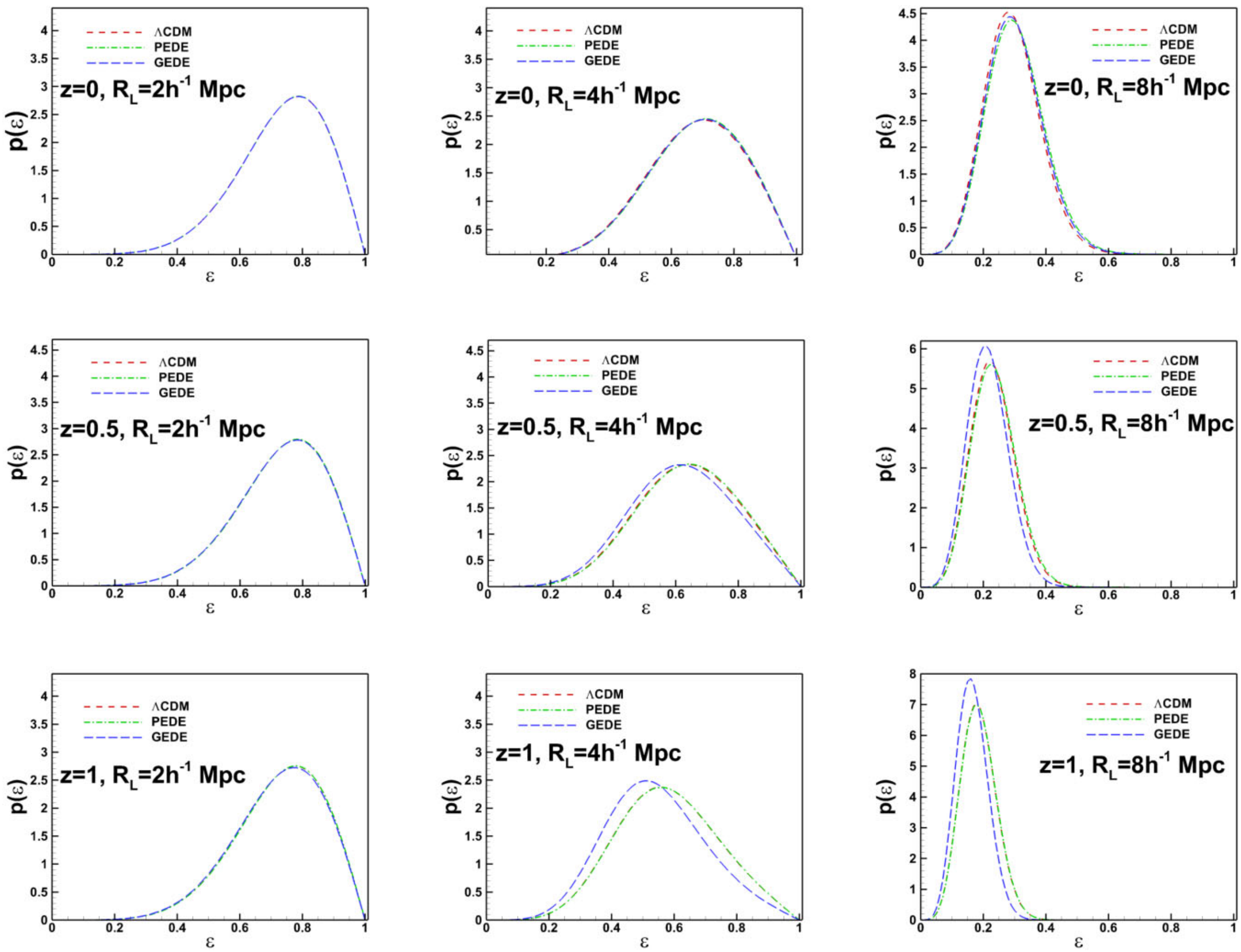

Figure 3. The normalized distribution function, $p(\varepsilon)$, is plotted against ε for ΛCDM, PEDE, and GEDE.

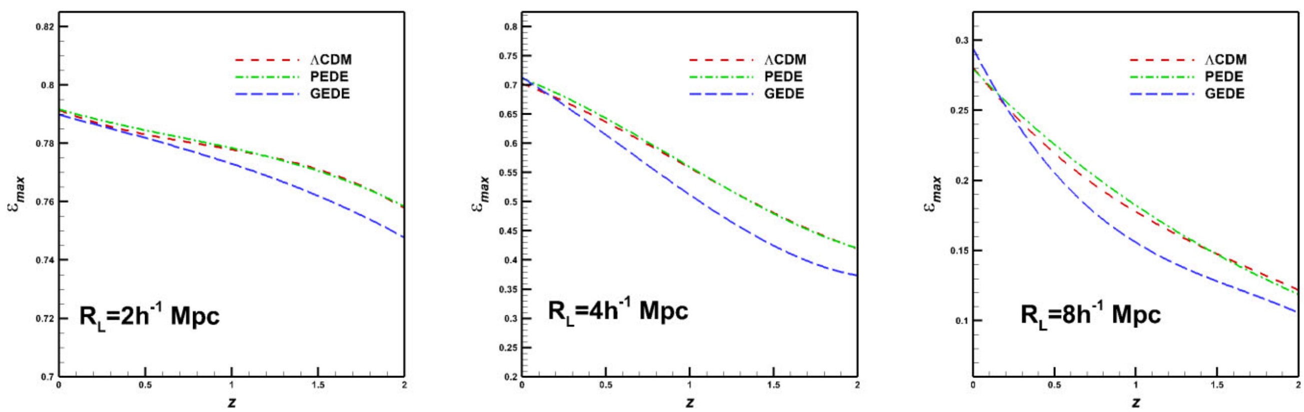

Figure 4. Redshift dependency of $\varepsilon_{\max}$ is depicted for all models.

follow three main approaches. In each class people define different void finder methods. In one approach empty space between galaxies were introduced as a void (Kauffmann & Fairall 1991; Hoyle & Vogeley 2004; Patiri et al. 2006). In another class the geometrical structure which was traced by galaxies in the dark matter distribution was accounted for a void (Plionis & Basilakos 2002; Colberg et al. 2005; Shandarin et al. 2006; Platen, van de Weygaert & Jones 2007). The other approach identifies structures dynamically by considering gravitationally unstable points in the dark matter distribution (Hahn et al. 2007; Forero-Romero et al. 2009). Depending on the void finder method one can identify the voids from the results of simulations and galaxy surveys. In any case of void finding methods we obtain a society of voids with different sizes and shapes. After identifying the voids, following special techniques one can obtain the mean ellipticity of the voids society. This work basically could be performed at different redshift(z) epochs. Next the results could

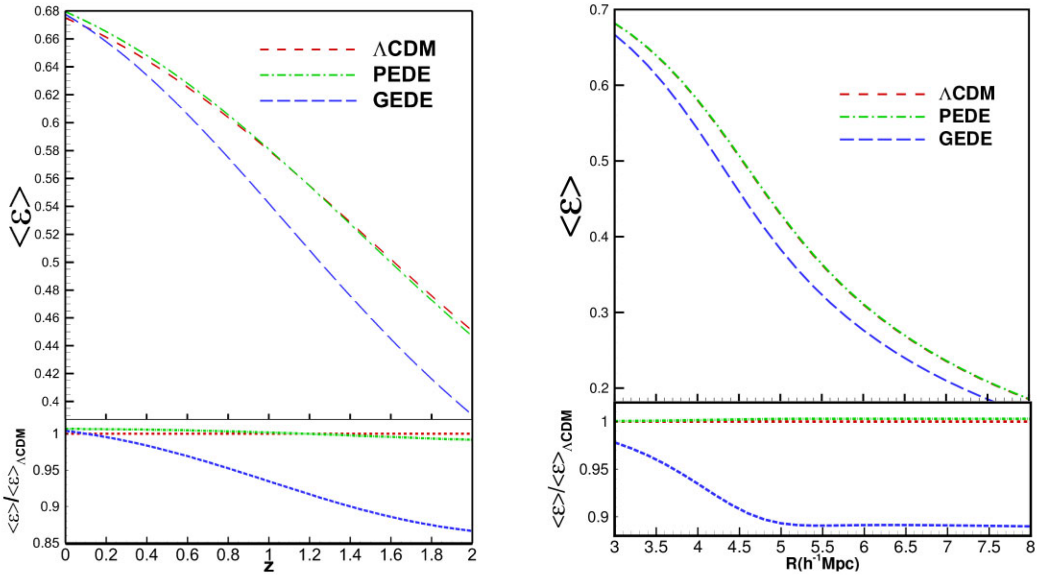

Figure 5. Mean ellipticity parameter is plotted against z, $R_{\rm L}$. In the left panel, $R_{\rm L} = 4\,h^{-1}$, and in the right panel, $z = 1$.

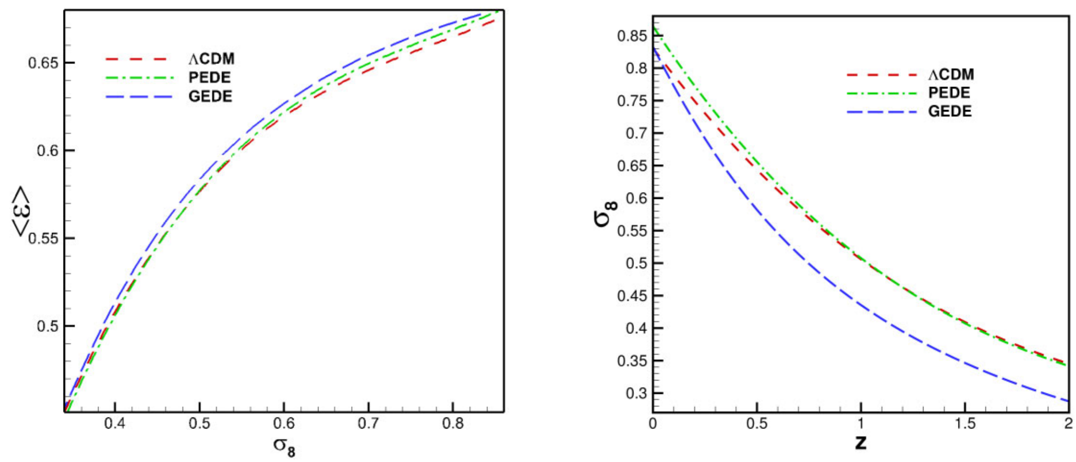

Figure 6. Mean ellipticity parameter is plotted against σ_8. The right panel shows evolution of σ_8 versus redshift.

be compared to what is calculated through pure theoretical approach such as present work. Examples of such a prescription are already discussed in Park & Lee (2007), Lavaux & Wandelt (2010), Biswas, Alizadeh & Wandelt (2010), and Bos et al. (2012). Comparison with the results of simulations and observations is an interesting and important part of this work which will be addressed in future.

4 SUMMARY AND CONCLUSIONS

In this paper, we have investigated the phenomenological emergent dark energy (PEDE). We also generalized our study to the case with an additional degree of freedom, compared to PEGE, which called GEDE. We have disclosed the influences of PEGE and GEDE on the geometry and statistics of the cosmic voids. Following Park & Lee (2007), we have employed an analytic approach and explored the evolution of the ’tidal ellipticity’. Our basic assumptions include Zel’Dovich approximation and Doroshkevich form of eigenvalue distribution of the local tidal tensor. We found out that for smaller smoothing scale ($R_{\rm L} = 2\,h^{-1}$ Mpc) all models (PEDE, GEDE, and ΛCDM) have an almost identical ellipticity distribution for redshift choices ($z = 0, 0.5, 1$) which seems to be a result of clustering process. In this region voids look more elongated (larger ε) due to larger tidal effects of gravity. On larger smoothing scale($R_{\rm L} = 4, 8\,h^{-1}$ Mpc), the distinction between PEDE, GEDE, and ΛCDM is clearly evident. GEDE in the older epochs and for $R_{\rm L} > 2$ tends to a less elongated society of voids with respect to PEDE and ΛCDM. We have also considered the mean ellipticty ($\langle\varepsilon\rangle$) of the models. The mean ellipticity of PEDE looks very close to that of ΛCDM through history of the universe while mean ellipticity in GEDE cosmology shows a lower valued profile (up to 13 per cent at $z = 2$) with respect to other models. All of these results are in agreement with what we expect from PEDE and GEDE according to Fig. 2. Furthermore, we have found a tight correlation between $\langle\varepsilon\rangle$ and σ_8 which is consistent with the arguments given in Bos et al. (2012). Finally, we calculated the mean ellipticity evolution versus Lagrangian smoothing scale ($R_{\rm L}$) and revealed that GEDE has a distinct manner of evolution compared to Λ CDM and PEDE models. When $R_{\rm L} \geq 4\,h^{-1}$ Mpc, this difference reaches to 11 per cent smaller compared to PEDE and ΛCDM.

We have also calculated the distribution function of ellipticity and the mean ellipticity factor of GEDE which differ from those of

PEDE and ΛCDM. The latter leads to a less elongated society of cosmic voids in large scale structure of the universe. Our analysis confirmed the sensitivity of the ellipticity parameter to the underlying cosmology (including a CDM and a DE component). However, we could not identify a direct and one to one correspondence between the EoS parameter and the tidal ellipticity. The last result is also identified in other models of DE which are currently under investigation and will be reported soon.

Finally, it is worth noting that a complete investigation of the presented models needs more comprehensive studies including simulations and comparing the results with galaxy surveys such as SDSS. Given a wide range of observational data available, in the future we shall focus on this issue and the results will be appeared elsewhere.

ACKNOWLEDGEMENTS

I deeply thank the referee for valuable comments and feedback. I would like to appreciate A. Sheykhi and A. Poostforush for helping me with the text. I am grateful to Z. Rezaei for constructive comments and useful discussions.

DATA AVAILABILITY

The mathematica code will be shared on reasonable request to the author.

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