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TESTING UNRUH RADIATION WITH ULTRA-INTENSE LASERS*

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ABSTRACT

We point out that using the state-of-the-art (or soon-to-be) intense ultrafast laser technology, violent acceleration that may be suitable for testing general relativistic effects can be realized through the interaction of a high intensity laser with a plasma. In particular, we demonstrate that the Unruh radiation is detectable, in principle, beyond the conventional radiation (most notably the Larmor radiation) background noise, by taking advantage of its specific dependence on the laser power and distinct character in spectral-angular distributions.

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General relativity (GR) is by birth a classical theory. The celebrated discovery by Hawking[1] of the black hole radiation links the GR to quantum mechanics and thermodynamics in one stroke. While the ultimate theoretical understanding of the Hawking radiation, for example through the superstring theory[2], is still in progress, the fundamental importance of the Hawking radiation is hardly questionable. Subsequent to Hawking's discovery, Unruh[3] established that similar radiation can also occur for a "particle detector" under acceleration. Without resorting to detail arguements, one can readily appreciate such a notion intuitively based on the equivalence principle. While the celestial observations of GR effects are clearly important, one wonders if by means of extremely violent acceleration in the laboratory setting these effects can be detected or tested by controlled experiments.

There have been proposals for laboratory detection of the Unruh effect[4]. For example, Yablonovich[5] proposed to detect the Unruh radiation using ionization fronts in solids. Darbinyan et at.[6] proposed to test it through the crystal channeling phenomena. Since the sought-after effects are typically extremely weak , the most severe problem would be the struggle against paramount background signals. Thus the challenge in general is to find a physical setting which can maximally enhance the signal above its competing backgrounds.

It is known that plasma wakefields excited by either a laser pulse[7] or an intense electron beam[8] can in principle provide an acceleration gradient as high as100 GeV/cm, or $10^{23} g_\oplus$. Such acceleration relies on the collective perturbations of the plasma density excited by the driving pulse and restored by the immobile ions, and therefore is an effect arisen over a plasma period. There is in fact another aspect of laser-plasma interaction. Namely, when a laser is ultra-relativistic (i.e., $a_0 \equiv eE_0/mc\omega_0 \gg 1$), the plasma electrons under the direct influence of the laser can be instantly "snowplowed" forward in every laser cycle (which is typically much higher frequency than that of the plasma), resulting in a intermittant acceleration that is much more violent than that provided by the plasma wakefields. For the Petawatt-class lasers currently under development[9], 10 TeV/cm, or $10^{25} g_{\oplus}$, will be possible for these "snowplow" accelerations in the near future.

Although the classical equivalent of the acceleration exerted on a nucleon bound to a nucleus can be as large as $a_{\mathrm{nuclear}} \sim 10^{28} g_{\oplus}$, it is well-known from quantum mechanics that the notion of classical trajectory and acceleration is not justified in subatomic systems. By the same token one should not expect any GR effect to be generated during high energy particle collisions where, if the notion of classical particle motion was wrongly applied, the hard scattering during a very brief moment would suggest an extremely violent acceleration. Furthermore, as will be addressed in more details below, even if the notion of classical acceleration is valid in a physical system, there is also the question of uniformity and duration of such accelerations for the Unruh effect to be applicable. The outstanding character of our system is that the snowplow acceleration is macroscopic and can be well described by classical electrodynamics, and therefore the Unruh effect associated with violent acceleration can be readily applied.

According to Davies[10] and Unruh[3], a uniformly accelerated particle finds itself imbeded in a thermal heat bath with temperature

$$kT = \frac{\hbar \alpha}{2\pi c} \quad , \tag{1}$$

where α is the constant proper acceleration of the particle. In the standard treatment, an internal degree of freedom of the accelerated particle is invoked as a means to detect the Unruh effect. This can be, for example, a monopole moment (interacting with a scalar field)[11,12], or the spin of an electron (interacting with EM fields)[13]. Since the agency that we rely on for the violent acceleration is electromagnetic and acts only on charged particles, we consider an electron, the lightest charged particle, as our particle detector. As was shown by Bell and Leinaas[12], the manifestation of the Unruh effect through the equilibrium degree of spin polarization would require an unphysically long time in the case of a linear acceleration, yet for such an effect in a circular motion the Thomas precession complicates the issue. In our approach, we do not invoke any internal degree of freedom. Rather, we rely on the quivering motion of the electron under the influence of the nontrivial vacuum fluctuations, and look for the emitted photons so induced as our signals.

To be sure, the Unruh radiation is not a "new" radiation. Using the standard field theory (in this case quantum electrodynamics), one should in principle be able to arrive at the same result when properly taking particle radiation reaction into account. Treating the problem in the instantaneous proper frame and invoking the particle response to the thermal vacuum fluctuations, however, help to elucidate the phenomenon through a very intuitive picture in the spirit of the fluctuation-dissipation theorem[14] in thermodynamics.

We assume that in the leading order the accelerated electron is "classical", with well-defined acceleration, velocity and position. Therefore we can introduce a Rindler transformation[15] so that the electron is described in its instantaneous proper frame. Also at this level the linearly accelerated electron will execute a classical Larmor radiation. As a response to the Larmor radiation, the electron reacts to the vacuum fluctuations with a quivering motion in its proper frame. This in turn triggers additional radiation. We assume that this quivering motion is nonrelativistic in the proper frame, and the interaction Hamiltonian can be written as

$$\mathcal{H}_{_I} = -\frac{e}{mc}\vec{p}\cdot\vec{A} = -e\vec{x}\cdot\vec{E} \quad . \tag{2}$$

The probability of the emission of a photon with energy $\omega = \mathcal{E}' - \mathcal{E}$ is

$$\begin{aligned} N(\omega) =& \frac{1}{\hbar^2}\int d\sigma \int d\tau |\langle 1_{\vec{k}}, \mathcal{E}'|\mathcal{H}_{_I}|\mathcal{E}, 0\rangle|^2 \\ =& \frac{e^2}{\hbar^2}\sum_{i,j}^{3}\int d\sigma\int d\tau e^{-i\omega\tau}\langle x_i(\sigma)x_j(\sigma)\rangle\langle E_i(\sigma-\tau/2)E_j(\sigma+\tau/2)\rangle \quad , \end{aligned} \tag{3}$$

where σ and τ are the absolute and relative proper time, respectively. The τ dependence of the position operator has been extracted to the phase due to a unitary transformation. The last bracket is the well-known autocorrelation function for vacuum fluctuations of the electric field,[16] i.e.,

$$\langle E_i(\sigma-\tau/2)E_j(\sigma+\tau/2)\rangle = \delta_{ij}\frac{4\hbar}{\pi c^3}\left(\frac{\alpha}{2c}\right)^4 \sinh^{-4}\left(\frac{\alpha\tau}{2c}\right) \quad , \qquad i,j = 1,2,3. \tag{4}$$

With a change of variable $s = \alpha\tau/2c$, we find

$$\frac{dN}{d\sigma} = \frac{3}{2\pi}\frac{e^2}{\hbar c^3}\left(\frac{\alpha}{c}\right)^3 \langle x_i^2\rangle \int_{-\infty}^{+\infty} ds \exp\Big(-is\frac{2c\omega}{\alpha}\Big)\sinh^{-4}(s-i\epsilon) \tag{5}$$

This integral has poles at $s = n\pi i$, and is periodic every $\Delta s = \pi i$. Thus it can be easily performed by returning the contour along the line Im$s = \pi$, and we get

$$\frac{dN}{d\sigma} = \frac{e^2}{\hbar c^3}\left(\frac{\alpha}{c}\right)^2\langle x_i^2\rangle\Big[2\omega + \left(\frac{c}{\alpha}\right)^2\omega^3\Big]\Big(e^{2\pi c\omega/\alpha}-1\Big)^{-1} \quad . \tag{6}$$

The expectation value of x_i^2 fluctuates due to the random absorption of quanta from the vacuum fluctuation. From the uncertainty principle we have $\langle x_i^2\rangle\langle p_i^2\rangle \gtrsim \hbar^2$.

By absorbing a quanta of frequency ω, the corresponding change of momentum is $\langle p_i^2 \rangle = \langle p^2 \rangle /3 = (2/3) m\hbar\omega$. We shall thus assume that

$$\langle x_i^2 \rangle \sim \frac{3}{2} \frac{\hbar}{m\omega} \quad . \tag{7}$$

Note that this expression is invalid when the quivering motion becomes relativistic, i.e., $\langle p^2 \rangle \gtrsim (mc)^2$. Beyond this limit a fully relativistic treatment is necessary, and higher order processes such as e^+e^- pair production should be included. Taking the typical frequency of the vacuum fluctuation spectrum, $\omega \sim kT/\hbar$, the nonrelativistic approximation corresponds to the constraint that $kT \lesssim mc^2$. Correspondingly, this means the fluctuations of the electron position in our case is larger than the Compton wavelength, i.e., $\langle x^2 \rangle \gtrsim \lambda\!\!\!^{-}{}_c{}^2$, which is consistent with our semi-classical treatment. This range of validity of our approximation is physically unrelated to the well-known issue of *Zitterbewegung* for an inertial electron[17]. But it is interesting to recall that an unaccelerated electron also jiggles under the zero-point fluctuations of the Minkowski vacuum, yet with $\langle x^2 \rangle \lesssim \lambda\!\!\!^{-}{}_c{}^2$ at a frequency $2c/\lambda\!\!\!^{-}{}_c$. This, as we know, will never constitute any radiation.

To find the radiation power, one should insert Eq(7) and further integrate Eq.(6) over $\hbar d\omega$, which diverges in the infrared limit. In reality, however, the duration of acceleration, τ_a, is always finite, which sets a cutoff frequency at $\omega_a \sim 1/\tau_a$. Therefore we introduce a regularization through an infrared cutoff, ω_a, and find

$$\frac{dI_U}{d\sigma} \approx \int_{\omega_a}^{\infty} \hbar d\omega \frac{dN}{d\sigma} \approx \frac{3}{4\pi} \frac{r_e \hbar}{c} \left(\frac{\alpha}{c}\right)^3 \times \begin{cases} (c/\alpha\tau_a)^2 \exp(-2\pi c/\alpha\tau_a), & \tau_a \ll 2\pi c/\alpha, \\ 2\log(\alpha\tau_a/2\pi c) \quad , & \tau_a \gtrsim 2\pi c/\alpha. \end{cases} \tag{8}$$

We see that if the time for acceleration is less than the characteristic time $\tau_c \equiv 2\pi c/\alpha$, then this radiation is exponentially suppressed.

For the sake of simplicity, we treat the laser as a plane EM wave. Let the laser be linearly polarized in x-direction and propagate in the z-direction, with amplitude $E = E_0 \cos k_0\zeta$, where $\zeta = z - v_{ph}t$ is the coordinate of the comoving frame. The normalized vector potential is then $A(\zeta) = [eE_0/mc\omega_0] \sin k_0\zeta \equiv a_0 \sin k_0\zeta$, where a_0 is the conventional dimensionless laser strength parameter. The state-of-the-art, or soon-to-be, laser technology can provide an intensity so high that $a_0 \gtrsim 100$ is attainable.[9] In our conception, the accelerated electrons are provided by a low temperature plasma. This, in principle, induces a collective reaction from the plasma to the laser through the modification of the index of refraction, $n = \sqrt{1 - (\omega_p/\omega_0)^2} \lesssim 1$, where $\omega_p = c\sqrt{4\pi r_e n_p}$ is the plasma frequency.

The Lorentz force equations for a plasma electron driven by a linearly polarized laser , where its magnetic field is related to the electric field by $B_y = nE_x \equiv nE$, can be written as

$$\begin{aligned} \frac{dp_x}{dt} &= -\, en\beta_z E \quad , \\ \frac{dp_z}{dt} &= -\, e(1 - n\beta_x)E \quad . \end{aligned} \tag{9}$$

For a plasma electron initially at rest, its subsequent velocity and energy as a function of ζ in the lab frame can be solved exactly from the above equations. In the regime of our interest it can be shown, with the approximation $\sqrt{[1 + A(\zeta)^2](1 - n^2) + 1} \simeq 1$, that

$$\begin{aligned} \beta_x(\zeta) = \frac{A(\zeta)}{\gamma} \quad ; \qquad \beta_z(\zeta) = \frac{n}{1+n^2}\frac{A(\zeta)^2}{\gamma} \quad ; \\ \gamma(\zeta) = \frac{1 + n^2 + A(\zeta)^2}{1+n^2} \quad . \end{aligned} \tag{10}$$

Note that, as $A(\zeta)$ is periodic, the electron returns to a full stop every half-cycle, where $A(\zeta) = 0$. Taking time derivatives on β_x and β_z, and making Lorentz transformation to the proper frame, it can be shown that the magnitude of the proper acceleration is simply

$$\alpha = ca_0\sqrt{\omega_0^2 + \omega_p^2}\cos k_0\zeta \approx ca_0\omega_0\cos k_0\zeta \quad . \tag{11}$$

Thus in the limit $\omega_p \ll \omega_0$, the plasma effect on the proper acceleration is negligible. It is clear that the maximum acceleration occurs at every half laser cycle, with phases $\eta = k_0\zeta = 0, \pi, 2\pi, \ldots$, which coincide with the phases where the electron comes to rest.

As can be seen from Eq.(10), since $\beta_x \propto A$ while $\beta_z \propto A^2$, the electron is initially accelerated from rest in the transverse direction. But for the case where $a_0 \gg 1$, the motion is rapidly bent towards the direction of laser propagation (in z). This helps the electron to remain in phase with the laser oscillation for a much longer time compared with the nonrelativistic case. There still is, nevertheless, a slight amount of "phase slippage" incured to the electron versus the laser phase. The phase advance of the electron is

$$\frac{d\eta}{dt} = k_0(v_z - v_{ph}) = \frac{\omega_0}{n}\left(\beta_z - \frac{1}{n}\right) \quad . \tag{12}$$

As we discussed earlier, to avoid exponential suppression we look for a minimum acceleration time $\tau_a \gtrsim \tau_c = 2\pi c/\alpha$. The corresponding characteristic laboratory time t_c for acceleration, through Rindler transformation, is then

$$t_c = \frac{c}{\alpha}\sinh(\tau_c\alpha/c) = \frac{c}{\alpha}\sinh(2\pi) \quad . \tag{13}$$

Integrating Eq.(12) from 0 to $\pm t_c/2$, and assuming that the resultant phase slippage $\eta_c \ll 1$ while $a_0\eta_c \gg 1$, we find the phase slippage to be

$$\pm\eta_c = \pm\frac{1}{a_0}(3\sinh 2\pi)^{1/3} \quad . \tag{14}$$

Thus to ensure the uniformity of acceleration during a time t_c, it is necessary that

$\cos\eta_c \approx 1-(\eta_c)^2/2 \approx 1$, or $a_0 \gg (3\sinh 2\pi)^{1/3}/\sqrt{2} \sim 6.6$. As we will discuss below, we assume an ultra-intense laser where $a_0 \sim 100$, thus the nonuniformity of the proper acceleration during this characteristic time is less than 0.5%, and we shall from here on simply assume $\alpha = ca_0\omega_0$ within the time τ_c.

At the classical level, the same linear acceleration induces a Larmor radiation. The total Larmor radiation power is

$$\frac{dI_{_L}}{dt} = \frac{2}{3}\frac{e^2}{m^2c^3}\Big(\frac{dp_\mu}{d\tau}\frac{dp^\mu}{d\tau}\Big) = \frac{2}{3}\frac{r_e m\alpha^2}{c} \quad , \tag{15}$$

where Eqs. (10) and (11), and the identity $d\gamma\beta_z/d\tau = d\gamma/d\tau$, which is a direct consequence of the Lorentz force equations, have been invoked. This means that the contribution to the relativistic Larmor radiation is predominantly from the trasverse acceleration by the laser electric field. As the radiation power is a Lorentz invariant quantity, the relative yield between the Unruh radiation (Eq.(8)) and the Larmor radiation in each half-cycle is

$$\frac{dI_{_U}/dt}{dI_{_L}/dt} \approx \frac{9}{4\pi}\frac{\lambda\!\!\!^{-}_c\alpha}{c^2}\log(\alpha\tau_a/2\pi c) \quad . \tag{16}$$

In our particular setting the phase-slippage increases rapidly due to the hyperbolic dependence of t_c on τ_c and the proper acceleration decreases accordingly. It is therefore adequate to assume that $\tau_a \gtrsim \tau_c = 2\pi c/\alpha$ and $\log(\alpha\tau_a/2\pi c) \sim \mathcal{O}(1)$ in our case. Consider the Petawatt laser currently under development[9], where $\omega_0 \sim 2\times 10^{15}\mathrm{sec}^{-1}$ and $a_0 \sim 100$. This gives $(dI_{_U}/dt)/(dI_{_L}/dt) \sim 3\times 10^{-4}$. To have the Unruh radiation power breaking even with that of Lamor radiation, one would need a laser power ($\propto a_0^2$) more than 7 orders of magnitude larger, or an acceleration as large as $\sim 3\times 10^{31}\mathrm{cm/sec}^2 \sim 3\times 10^{28} g_\oplus$, which is beyond the reach of current laser technology. However, the time structure of these radiations and their different characters in spectral-angular distributions and polarizations help to much relax the demand on acceleration for detectability.

We have shown that the relative phase advance of the electron for emitting typical Unruh photons is a small fraction of the laser half-cycle (cf. Eq.(14)), and have a much sharper temperal profile than that for the Larmor radiation. Because of the snowplow mechanism, the electron rapidly becomes relativistic and is bent forward. As a result the time laps for every period of motion is much longer than the laser period. Roughly, the time separation between successive Unruh signals (for each π-phase slippage) scales as a_0^2:

$$\Delta t \sim \gamma\pi/\omega_0 = (1 + a_0^2/2)\pi/\omega_0 \gg 1/\omega_0 \gg t_c \quad . \tag{17}$$

Therefore it should be possible to set up temporal gates where signals from different periods can be isolated if a thin "film" of plasma is irradiated.

In our treatment the thermal fluctuation is isotropic (cf. Eq.(4))[18] in the electron's proper frame. The radiation induced is therefore also isotropic. Since at each half-cycle by the time when the electron has been accelerated for a time $t_c/2$ from rest, its energy would be $\gamma_c \approx 1 + A^2(\eta_c)/2 \approx 1 + (3\sinh 2\pi)^{2/3}/2 \gg 1$, the Unruh radiation is highly forward boosted in the lab frame. Inserting the proper acceleration $\alpha = ca_0\omega_0$ into Eq.(8), and transforming back to the lab frame with small-angle expansion, the angular distribution becomes

$$\frac{dI_U}{dtd\Omega} \simeq \frac{1}{2\pi^2}\frac{r_e\hbar}{c}\frac{\omega_0^3 a_0^3}{(1+\gamma_c^2\theta^2)^3} \quad . \tag{18}$$

As the Larmor radiation is essentially induced by the transverse acceleration, it is polarized and its angular distribution in the small (θ, ϕ) polar angle expansion is[19]

$$\frac{d^2 I_{_L}}{dt d\Omega} \simeq \frac{2 r_e m c a_0^2 \omega_0^2}{(1+\gamma_c^2\theta^2)^3}\Big[1 - \frac{4\gamma_c^2\theta^2(1-\phi^2)}{(1+\gamma_c^2\theta^2)^2}\Big] \quad . \tag{19}$$

It is clear that the radiation power is minimum at $(\theta, \phi) = (1/\gamma_c, 0)$, where $d^2 I_{_L}/dtd\Omega = 0$. Consider a detector which covers an azimuthal angle $\Delta\phi = 10^{-3}$ around this "blind spot", and an opening polar angle, $\Delta\theta \ll 1/\gamma_c$. Then the partial radiation power for the Unruh signal would dominate over that for the Larmor within this solid angle.

To be sure, there are other types of radiation backgrounds in addition to the Larmor radiation. The snowplowed plasma electrons will interact with the plasma ions and trigger the conventional bremsstrahlung. The cross section of bremsstrahlung for an unscreened hydrogen nucleus per unit photon energy is well-known: $d\chi/d\hbar\omega \sim (16/3)\alpha r_e^2 \ell n(EE'/mc^2\omega)$. Assuming a frequency window of $\Delta\omega/\omega_u \sim 0.1$ and a temporal gate of $\Delta t \sim 2\times 10^{-15}$sec, we find that, for the laser parameters discussed above, the plasma density has to be lower than $n_p \lesssim 10^{18}/\mathrm{cm}^3$ in order that the bremsstrahlung signals be less than that from the Unruh effect, which is not a severe restriction.

We have demonstrated that the Unruh radiation can in principle be detectable against the backgrounds from the conventional radiations using the frontier laser technology and the various experimental techniques. The violent, macroscopic acceleration provided by the snowplow mechanism available from ultra-relativistic lasers can also be a useful tool to test other salient features of general relativity in the laboratory setting. This should open up a brand new window to peek into foundations of physics.

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