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GRAVITATIONAL RADIATION FROM SUPERNOVA EXPLOSIONS*

Ronald J. Adler

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

Departamento de Fisica
Universidade Federal de Pernambuco
50.000 Recife, Brasil

Bostjan Zeks

Departamento de Fisica
Universidade Federal de Pernambuco
50.000 Recife, Brasil

and

The Josef Stefan Institute, Ljubljana
- Yugoslavia -

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ABSTRACT

As a simple model of a supernova explosion we study a point mass exploding into two equal point masses moving back to back at high velocity, or a superposition of such elementary explosions. We employ two calculational methods. In one method we assume an infinite acceleration for zero time and calculate the gravitational field and the distribution of emitted energy. A cutoff frequency in the power spectrum is necessary to obtain a finite total energy output. The spectrum is constant from zero to this cutoff frequency. For nonrelativistic ejecta velocities the angular distribution is typical of quadrupole radiation, while for ultra relativistic ejecta it is isotropic. To justify the infinite acceleration approximation we also calculate with the quadrupole approximation, using a smooth finite acceleration, and verify the previous qualitative results in the nonrelativistic limit. A brief table of fields, energies, and fluxes is given for some reasonable supernova parameters.

I. INTRODUCTION

Gravitational radiation is of intrinsic interest since it is one of the few qualitatively novel predictions of general relativity theory, having no classical analogue; moreover the specific nature of such radiation could, in principle, distinguish between conventional general relativity and such variations as the scalar-tensor theory of Brans and Dicke.[1-3] On the other hand, if we assume the correctness of general relativity theory, gravitational radiation presents an observational window for astronomy which can provide new information on highly energetic astrophysical phenomena such as supernova explosions or quasars. The general subject of gravitational radiation astronomy has been discussed by a number of authors, e.g., Thorne and Press,[4] Ruffini and Wheeler,[5] Misner, Thorne, and Wheeler,[3] and Weber.[6]

In this paper we will study in detail the gravitational radiation from a very simple model of a supernova: a point mass exploding into two equal point masses moving apart at high velocity. Such a model recommends itself for its simplicity, and should provide a reasonable description of some types of supernova explosions as evidenced for example by the asymmetric ejecta in the crab nebula; a superposition of such explosions should provide an even better description. Our model clearly ignores details such as the implosion preceding the ejection of material, and all effects of rotation. Rotational effects might be expected to give interesting structure to the frequency spectrum, e.g., peaks at characteristic frequencies. We also ignore radiation from the remnants of the supernova, for example from stellar fragments in short period orbits, or rapidly rotating eccentric young neutron stars; these have been discussed, for example, by Ruffini and Wheeler.[5]

We calculate the gravitational radiation in two complementary ways. First, using a method of Weinberg,[1] we assume the acceleration is infinite and of zero duration. For this highly idealized situation we find the gravitational radiation field and the distribution of energy in frequency and angle; the field has a theta function time dependence, and the energy distribution is independent of frequency. A frequency cutoff is thus necessary to achieve a finite total energy. The angular distribution of energy for nonrelativistic ejecta is the typical $\sin^4\theta$ of quadrupole radiation. This calculation however is valid for arbitrary velocities of ejecta and would be applicable to a supernova explosion in which a large mass of high energy protons and electrons is emitted. For ultra relativistic ejecta the gravitational radiation is isotropic.

The calculation with infinite acceleration contains the implicit assumption of an infinite stress energy tensor, which is contrary to the basic idea of linearized general relativity theory. We thus verify our results for the nonrelativistic limit by studying finitely accelerated ejecta in the quadrupole approximation.[6] For this purpose we choose an arbitrary convenient smooth function to describe the ejecta motion during explosion. The results are of course finite with no cutoff required. We find that the net change in the radiation field is the same as with the previous calculation, the power spectrum is very similar, and the physical significance of the cutoff frequency is made explicit.[1]

Finally we discuss and present a short numerical table of fields, energies, and fluxes for some reasonable supernova parameters. These values are in substantial agreement with previous estimates.[4]

Our purpose in this work is two-fold: (1) we display in some detail the qualitative nature of radiation expected from actual supernovas—properties such as time dependence and power spectrum; (2) we verify the applicability of the infinite acceleration approximation, which is very convenient but whose basic physical assumption may be considered suspect, as noted above.[1]

Many of our conclusions have been obtained previously.[3,4,5] The novel features, we believe, are: (1) Our results are valid for arbitrary ejecta velocity, e.g., the radiation is isotropic for ultra relativistic ejecta. (2) The infinite acceleration method employed is demonstrated to be applicable, at least in the nonrelativistic limit. (3) The time structure of the energy emission is displayed, and for the motion we have studied is seen to contain several pulses.

II. INFINITE ACCELERATION

We first review briefly the solution of the standard equations of linearized general relativity theory as discussed in detail by numerous authors.[1,3,6] The perturbation of the metric, $h_{\mu\nu}$, is given in terms of the energy-momentum tensor $T_{\mu\nu}$ by

$$h_{\mu\nu}(\vec{x},t) = 4G \int \frac{S_{\mu\nu}(\vec{x}\,',t_r)}{|\vec{x}\,'-\vec{x}\,|}\, d^3x' \cdot \quad S_{\mu\nu}(\vec{x},t) = T_{\mu\nu}(\vec{x},t) - \frac{1}{2}\eta_{\mu\nu}\, T_\alpha{}^\alpha(\vec{x},t) \qquad (2.1)$$

Here G is the gravitational constant, $t_t = t - |\vec{x}\,' - \vec{x}\,|$ is the retarded time, $\eta_{\mu\nu}$ is the Lorentz metric with signature (+, -, -, -), and we have set the velocity of light c=1. Asymptotically far from the source the solution becomes

$$h_{\mu\nu}(\vec{x},t) = \frac{4G}{r} \int S_{\mu\nu}(\vec{k},\omega)\, e^{-i\omega(t-r)}\, d\omega + \text{c.c.} \qquad (2.2)$$

where $2r = |\vec{x}\,|$, $k = \omega\vec{x}$, $S_{\mu\nu}(\vec{k},\omega)$ is the Fourier transform of $S_{\mu\nu}(\vec{x},t)$, defined explicitly by

$$S_{\mu\nu}(\vec{x},t) = \int_0^\infty d\omega \int \frac{d^3k}{(2\pi)^3}\, S_{\mu\nu}(\vec{k},\omega)\, e^{-i\omega t} + \text{c.c.} \qquad (2.3)$$

In a small volume of the asymptotic region the field $h_{\mu\nu}$ approximates a plane wave. If the propagation direction is taken as the z axis it can be shown that all the components of $h_{\mu\nu}$ except $h_{11} = -h_{22}$ and $h_{12} = h_{21}$ can be made zero by a coordinate transformation, which leaves h_{11} and h_{12} unchanged.[2] This may also be interpreted as a gauge transformation, i.e., we use the traceless transverse gauge.[1,3] Accordingly we need only calculate S_{11} and S_{12} to find the radiation field.

As in Ref. 1 we now consider a system of freely moving point particles with four-momenta p_i^μ, energies E_i, and three-velocities $\vec{v}_i$, which change abruptly at t = 0 to corresponding primed quantities; the energy-momentum tensor for this system of infinitely accelerated particles is

$$T^{\mu\nu}(\vec{x},t) = \sum_i \left[\frac{P_i^\mu P_i^\nu}{E_i} \delta^3(\vec{x}-\vec{v}_i t)\,\theta(-t) + \frac{P_i'^\mu P_i'^\nu}{E_i'} \delta^3(\vec{x}-\vec{v}_i' t)\,\theta(t) \right] \tag{2.4}$$

The Fourier transform $S_{\mu\nu}$ is then

$$S^{\mu\nu}(\vec{k},\omega) = \sum_i \left[\frac{P_i^\mu P_i^\nu - \frac{1}{2}\eta^{\mu\nu} m_i^2}{2\pi i E_i(\omega-\vec{v}_i \cdot \vec{k}-i\epsilon)} - \frac{P_i^\mu P_i^\nu - \frac{1}{2}\eta^{\mu\nu} m_i^2}{2\pi i E_i'(\omega-\vec{v}_i' \cdot \vec{k}+i\epsilon)} \right] \tag{2.5}$$

which leads to the radiation field

$$h^{\mu\nu}(\vec{x},t) = \frac{4G}{r} \sum_i \left[\frac{P_i^\mu P_i^2 - \frac{1}{2}\eta^{\mu\nu} m_i^2}{E_i(1-v_i\cos\theta_i)}\,\theta(r-t) + \frac{P_i'^\mu P_i'^\nu - \frac{1}{2}\eta^{\mu\nu} m_i'^2}{E_i'(1-v_i'\cos\theta_i')}\,\theta(t-r) \right] \tag{2.6}$$

where $v_i = |\vec{v}_i|$ and θ_i is the angle between $\vec{v}_i$ and $\vec{k}$, the wave direction. Note that the field changes abruptly, and that the metric before and after the radiation pulse is constant. A transformation can of course make the perturbation zero before the pulse so only the <u>change</u> in $h_{\mu\nu}$ is physically significant.

Using the standard expression for the energy density of the radiation field[1,3] we can then obtain the distribution of emitted energy in frequency and angle

$$\begin{aligned} \frac{dE}{d\Omega\, d\omega} &= 2G\omega^2 \left[T^*_{\alpha\beta}(\vec{k},\omega) T^{\alpha\beta}(\vec{k},\omega) - \tfrac{1}{2} |T_\lambda{}^\lambda(\vec{k},\omega)|^2 \right] \\ &= 2G\omega^2 \left[S^*_{\alpha\beta}(\vec{k},\omega) S^{\alpha\beta}(\vec{k},\omega) - \tfrac{1}{2} |S_\lambda{}^\lambda(\vec{k},\omega)|^2 \right] \end{aligned} \tag{2.7}$$

Since $S_{\mu\nu}$ is proportional to ω^{-1} as evident in (2.5) we see that the power spectrum (2.7) is independent of ω and the total energy E is infinite unless a cutoff frequency is introduced.

We now introduce our model as discussed in the introduction. That part of the mass of the pre-supernova star which will be lost in the explosion we denote by 2M. The remainder of the stellar mass we assume remains or is emitted in such a way as to produce no radiation, i.e., spherically symmetrically. The two ejected masses are assumed to be of equal mass $m < M$, and to have momenta $\vec{p}$ and $-\vec{p}$. We view the explosion then in terms of a system of freely moving but infinitely accelerated particles with four-momenta

$$\begin{aligned} P_1^{\mu} &= (M,0,0,0)\ , \qquad P_1'^{\mu} = (M,0,p\sin\theta,\ p\cos\theta) \\ P_2^{\mu} &= (M,0,0,0)\ , \qquad P_2'^{\mu} = (M,0,-p\sin\theta,\ -p\cos\theta) \end{aligned} \tag{2.8}$$

where $M^2 = m^2 + p^2$. For convenience we have oriented the axes so the motion is in the y,z plane and the z-axis is the radiation direction. It is evident that $S_{12} = 0$ and hence $h_{12} = 0$, so the radiation is totally polarized in the + mode. We find the field from (2.6)

$$h_{11}(\vec{x},t) = \frac{4GM}{r}\left[\theta(r-t) + \frac{m^2}{M^2 - p^2\cos^2\theta}\,\theta(t-r)\right] \tag{2.9}$$

As previously noted only the net change in $h_{\mu\nu}$ has physical significance.

$$\Delta h_{11} = -\frac{4GM\,p^2\sin^2\theta}{M^2 - p^2\cos^2\theta} \tag{2.10}$$

The time derivative $\dot{h}_{11}$ is a measure of emitted energy. It is an infinitely narrow pulse

$$\dot{h}_{11}(\vec{x},t) = -\frac{4GM\,p^2\sin^2\theta}{M^2 - p^2\cos^2\theta}\,\delta(r-t) \tag{2.11}$$

Finally the energy distribution is very easily obtained from (2.7) since there is only one polarization

$$\frac{dE}{d\Omega\, d\omega} = 4G\omega^2 |S_{11}(\vec{k},\omega)|^2 = \frac{GM^2 p^4 \sin^4\theta}{\pi^2 (m^2+p^2\sin^2\theta)^2} \tag{2.12}$$

As noted before the total energy diverges unless a cutoff frequency ω_c is introduced.

In the nonrelativistic limit we write the kinetic energy of one ejecta mass as $T = p^2/2m$ and have

$$\frac{dE}{d\Omega\, d\omega} = \frac{4GT^2 \sin^4\theta}{\pi^2}$$

$$\frac{dE}{d\Omega} = \frac{4GT^2 \sin^4\theta\, \omega_c}{\pi^2} \tag{2.13}$$

$$E = \frac{32\, GT^2 \omega_c}{15\pi}$$

The $\sin^4\theta$ angular dependence is characteristic of quadrupole radiation as may be expected. It is easy to verify these results by order of magnitude dimensional arguments as we will discuss shortly.

In the ultrarelativistic limit the radiation is isotropic and very simple.

$$\frac{dE}{d\Omega d\omega} = \frac{GM^2}{\pi^2}, \qquad E = \frac{4GM^2\omega_c}{\pi}. \tag{2.14}$$

As has been discussed by a number of authors[3] the power output of a low velocity source with internal power flow P_i is of order

$$P \sim P_i^2/P_0, \qquad P_0 = G^{-1} = 3.6 \times 10^{59}\ \text{erg/sec} \tag{2.15}$$

In the low velocity limit of our model the internal kinetic energy is T and the characteristic frequency is ω_c so the internal power flow is of order $T\omega_c$ which must have a characteristic duration of order $1/\omega_c$. Thus

$$P \sim GT^2\omega_c^2 \quad , \qquad E \sim GT^2\omega_c \tag{2.16}$$

which is consistent with (2.13). For a high velocity source the relation (2.15) still holds if P_i is suitably interpreted as the total energy of the source times ω_c.[7,8] Then

$$P \sim GM^2\omega_c^2 \quad , \qquad E \sim GM^2\omega_c \quad , \tag{2.17}$$

which is consistent with (2.14).

Let us summarize our main results:

(1) The field changes abruptly with a theta function dependence on retarded time.

(2) The power spectrum is constant up to the cutoff frequency ω_c.

(3) In the nonrelativistic limit the angular dependence is that of a quadrupole source.

In the following section we will study these points further by comparison with a calculation using the quadrupole approximation.

III. QUADRUPOLE APPROXIMATION

The calculation of the preceding section contains the convenient but dangerous assumption that linearized general relativity theory is applicable to the infinite acceleration approximation with a cutoff frequency ω_c. However such an infinite acceleration implicitly corresponds to an infinite stress energy tensor, contrary to the basic ideas of the linearized theory. To verify the applicability of the infinite acceleration approximation method we now study the same simple physical model with the quadrupole approximation.[6] A smooth and finite acceleration is used in this calculation and no divergences occur.

To obtain the quadrupole approximate solution to (2.1) we suppose that radiation from all points in the source is in phase. This will be so if the relevant wavelengths are much greater than the source size and if the source motion is nonrelativistic. Thus we ignore time retardation across the source and write the retarded time as $t_r = t-r$. The field is then a simple integral over $s_{\mu\nu}$

$$h_{\mu\nu}(\vec{x},t) = \frac{4G}{r}\int s_{\mu\nu}(\vec{x}',t-r)\, d^3x' \tag{3.1}$$

As noted previously we only need the space components S_{11} and S_{12}. For this we use the conservation of $T^{\mu\nu}$ to obtain the well-known result[6]

$$\int T_{ij}(\vec{x},t) d^3x = \frac{1}{2}\frac{d^2}{dt^2}\int e x^i x^j d^3x \tag{3.2}$$

where $e = T^{oo}$ is the mass density of the low velocity source. Then for propagation in the z direction

$$\begin{aligned} h_{11}(\vec{x},t) &= \frac{G}{r}\left[\frac{d^2}{dt^2}\int e(x^2-y^2)\, d^3x\right]_{t_r} \\ h_{12}(\vec{x},t) &= \frac{2G}{r}\left[\frac{d^2}{dt^2}\int e xy\, d^3x\right]_{t_r} \end{aligned} \tag{3.3}$$

If the axes are oriented as before, with motion in the y,z plane, we see that $h_{12} = 0$ and that h_{11} is proportional to $\sin^2\theta$. We may write (3.3) for the 2-point particles as

$$h_{11}(\vec{x},t) = -\frac{2Gm\sin^2\theta}{r}\left[\frac{d^2}{dt^2}\,\xi^2(t)\right]_{t_r} \tag{3.4}$$

where ξ describes the linear motion of one ejecta mass.

We now choose a convenient function ξ to represent accelerated motion from $t = 0$ to $t = t_c$ and uniform motion at velocity v thereafter. Since the third time derivative of ξ will enter the expression for the energy of the radiation field we choose a function with a finite third derivative

$$\begin{aligned} \xi(t) &= 0 \quad , \quad t < 0 \\ \xi(t) &= \frac{v}{2}(t - \sin\omega_0 t/\omega) \quad , \quad 0 < t < t_c \\ \xi(t) &= tv - t_c v/2 \quad , \quad t_c < t \\ \omega_0 &\equiv \pi/t_c \end{aligned} \tag{3.5}$$

This has convenient derivations, integrals, and a simple Fourier transform. We obtain the field, with $t_r = t-r$,

$$\begin{aligned} h_{11}(\vec{x},t) &= 0 \quad , \quad t_r < 0 \\ h_{11}(\vec{x},t) &= -\frac{4Gmv^2\sin^2\theta}{r}\Big[(2\cos^2\omega_0 t - 2\cos\omega_0 t + \\ &\qquad + \omega_0 t\sin\omega_0 t)\tfrac{1}{4}\Big]_{t_r} \quad , \quad 0 < t_r < t_c \end{aligned} \tag{3.6}$$

$$h_{11}(\vec{x},t) = -\frac{4Gmv^2\sin^2\theta}{r} \quad , \quad t_c < t_r$$

The field rises smoothly from zero, reaches a peak at about $\omega_0 t_r = 2.75$, and becomes constant after t_c. The net change is the same as for the infinite acceleration calculation in the nonrelativistic limit, (2.10), with $m \equiv M$ and

$p^2 << M^2$. (See Fig. 1.)

The average energy density of a radiation field with + polarization is given by[1,3]

$$< t_{00} > = \frac{|\dot{h}_{11}(\vec{x},t)|^2}{16\pi G} \tag{3.7}$$

from which we obtain the energy output

$$\begin{aligned} \frac{dE}{d\Omega\, dt} = r^2 < t_{00} > = \frac{4GT^2 \sin^4\theta\omega_0^2}{16\pi} \Big[& 16 \sin^2 \omega_0 t \cos^2 \omega_0 t \\ & + 9 \sin^2 \omega_0 t + \omega_0^2 t^2 \cos^2 \omega_0 t - 24 \sin^2 \omega_0 t \cos \omega_0 t \\ & - 8\, \omega_0 t \cos^2 \omega_0 t \sin \omega_0 t + 6\, \omega_0 t \sin \omega_0 t \cos \omega_0 t \Big]_{t_r} \end{aligned} \tag{3.8}$$

where $T = mv^2/2$ is the kinetic energy of one ejected particle. This function is shown in Fig. 2. Due to the peak occurring in h_{11} (Fig. 1) the energy output has a peak followed by a zero, and then a narrow spike. The integrated energy is

$$\frac{dE}{d\Omega} = \frac{4\, GT^2 \sin^4\theta}{\pi^2} \left[\frac{\pi^2}{16\, t_c} \left(\frac{\pi^3}{6} + \frac{11\pi}{4} \right) \right] = \frac{4\, GT^2 \sin^4\theta}{\pi^2} \left(\frac{8.52}{t_c} \right) . \tag{3.9}$$

This is identical with the infinite acceleration result (2.13) if we identify the cut-off frequency as $\omega_c = 8.52/t_c$. This is close to $2\pi/t_c$ as we might expect on dimensional grounds, and provides a verification of the obvious physical interpretation of the cutoff ω_c.[1]

Next we obtain the power spectrum. For this we need the Fourier transform of h_{11}, which we denote by $g(\omega)$.

$$\begin{aligned} h_{11}(\vec{x},t) &= \int_0^\infty g(\omega) e^{-i\omega t} \frac{d\omega}{2\pi} + \text{c.c.} \\ g(\omega) &= \int_{-\infty}^\infty h_{11}(\vec{x},t)\, e^{i\omega t} dt \end{aligned} \tag{3.10}$$

Note that g is simply $4GS_{11}(\vec{k}, \omega)/r$. In terms of g we can write the power spectrum as

$$\frac{dE}{d\Omega\, d\omega} = \frac{r^2 \omega^2 |g(\omega)|^2}{16\, \pi^2\, G} \tag{3.11}$$

We find g to be

$$g(\omega) = \frac{-8GT \sin^2\theta}{r\omega}\left[ie^{i\omega t}A(\omega) - iB(\omega) + e^{i\omega t}C(\omega)\right] = \frac{-8GT \sin^2\theta}{r\omega} F(\omega)$$

$$A(\omega) = \frac{6\omega_0^2 - \omega^2}{2(4\omega_0^2 - \omega^2)} - \frac{\omega^4}{2(\omega_0^2 - \omega^2)^2}$$

$$B(\omega) = \frac{\omega^2 - 2\omega_0^2}{2(4\omega_0^2 - \omega^2)} + \frac{\omega^4}{2(\omega_0^2 - \omega^2)^2} \tag{3.12}$$

$$C(\omega) = \frac{\pi\omega}{4(\omega_0^2 - \omega^2)}$$

Thus

$$\frac{dE}{d\Omega\, d\omega} = \frac{4GT^2 \sin^4\theta}{\pi^2} |F(\omega)|^2 \tag{3.13}$$

The function $|F|^2$ is plotted in Fig. 3. We see that this power spectrum is qualitatively very similar to that of the preceding section.

In summary of the quadrupole calculation, we have obtained fields and energy distributions which justify the infinite acceleration results of Sec. 2. Moreover, the time structure and frequency structure are displayed explicitly.

IV. NUMERICAL ESTIMATES

Our calculation contains as parameters the mass m, velocity v, and frequency ω_c. These must be estimated from observation and from explicit supernova models. For example, v is of order 10^3 km/sec and m is perhaps $\sim 10^{33}$ gm for the Crab nebula.[9] For a range of reasonable values of these parameters we have calculated the field change (2.10), the total energy (2.13), and the average energy flux at the earth given by

$$\langle F \rangle = \frac{dE}{r^2 t_c d\Omega} = \frac{2GT^2\omega_c^2}{r^2\pi^3} \tag{4.1}$$

where we use $\theta = \pi/2$ and $t_c = 2\pi/\omega_c$. These are given in the table for r = 5,000 light years and $\omega_c = 10^4$ $\sec^{-1}$. This frequency corresponds to the time required for a shock wave moving at $\sim .3c$ to cross a stellar core of size ~ 6 km. As is evident in Fig. 3, the peak flux will probably be several times the average $\langle F \rangle$.

It is amusing to rephrase the energy output in a dimensionally convenient form as follows. For the relevant parameters we define

$$m = \alpha M, \quad v = \beta c, \quad t_c = \frac{2\pi}{\omega_c} = \lambda\left(\frac{GM}{c^2}\right)/\beta_s c \tag{4.2}$$

where we have written c explicitly for later convenience. Here α is the fraction of the total stellar mass M ejected in the explosion, β is the dimensionless ejecta velocity, λ is the size of the exploding stellar core in units of half the Schwarzschild radius of the star, and β_s is the characteristic dimensionless velocity for the explosion to proceed across the stellar core. Then the total energy output may be written as a fraction of the stellar rest energy as

$$E = \frac{16}{15}\left(\frac{\alpha^2\beta^4\beta_s}{\lambda}\right) Mc^2 \tag{4.3}$$

As an example we set $\alpha = .5$, $\beta = 10^{-2}$, $\lambda = 4$, and $\beta_s = .3$, a typical speed of sound in a neutron star. Then (4.3) gives for a star of mass $M = 2 \times 10^{34}$ gm an energy output of $.4 \times 10^{46}$ ergs, approximately the same conditions and result as the third example in the table.

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REFERENCES

1. S. Weinberg, Gravitation and Cosmology (John Wiley and Sons, Inc., New York, 1972), Chap. 10.
2. R. J. Adler, M. Bazin, M. M. Schiffer, Introduction to General Relativity (McGraw-Hill Book Co., New York, 1975), 2nd ed., Chap. 9.
3. C. W. Misner, K. S. Thorne, J. A. Wheeler, Gravitation (W. H. Freeman & Co., San Francisco, 1973), Part 8.
4. W. H. Press and K. S. Thorne, Ann. Rev. Astron. Astrophys. 10, 335 (1972).
5. R. Ruffini and J. A. Wheeler, "Relativistic Cosmology and Space Platforms," Proc. Conf. on Space Physics, ESRO, Paris, 1971.
6. J. Weber, General Relativity and Gravitational Waves (Interscience, New York, 1961), Chap. 8.
7. R. J. Adler, to be published in Phys. Rev. D (about May 1975).
8. P. J. Westevelt, ZhETF Pis. Red. 4, 333 (1966) [JETP Lett. 4, 225 (1966)].
9. L. Caroff and J. Scargle, private communication.

TABLE

m	v/c	$\lvert\Delta h_{11}\rvert$	E (erg)	$\langle F \rangle$ ($erg/cm^2 sec$)
10^{33}	10^{-3}	$.6 \times 10^{-22}$	$.4 \times 10^{40}$	1.4×10^{-1}
10^{33}	10^{-2}	$.6 \times 10^{-20}$	$.4 \times 10^{44}$	1.4×10^{3}
10^{34}	10^{-2}	$.6 \times 10^{-19}$	$.4 \times 10^{46}$	1.4×10^{5}
10^{34}	10^{-1}	$.6 \times 10^{-17}$	$.4 \times 10^{50}$	1.4×10^{9}
10^{35}	10^{-1}	$.6 \times 10^{-16}$	$.4 \times 10^{52}$	1.4×10^{11}

FIGURE CAPTIONS

1. The time structure of the gravitational radiation field h_{11} in the quadrupole approximation. The line ---- is the θ function dependence in the infinite acceleration approximation.
2. The time structure of the energy emission in the quadrupole approximation.
3. The power spectrum of the radiation field in the quadrupole approximation compared with the infinite acceleration approximation. Normalized to $4GT^2 \sin^4 \theta/\pi^2$ at $\omega = 0$. The ---- line is the infinite acceleration result.

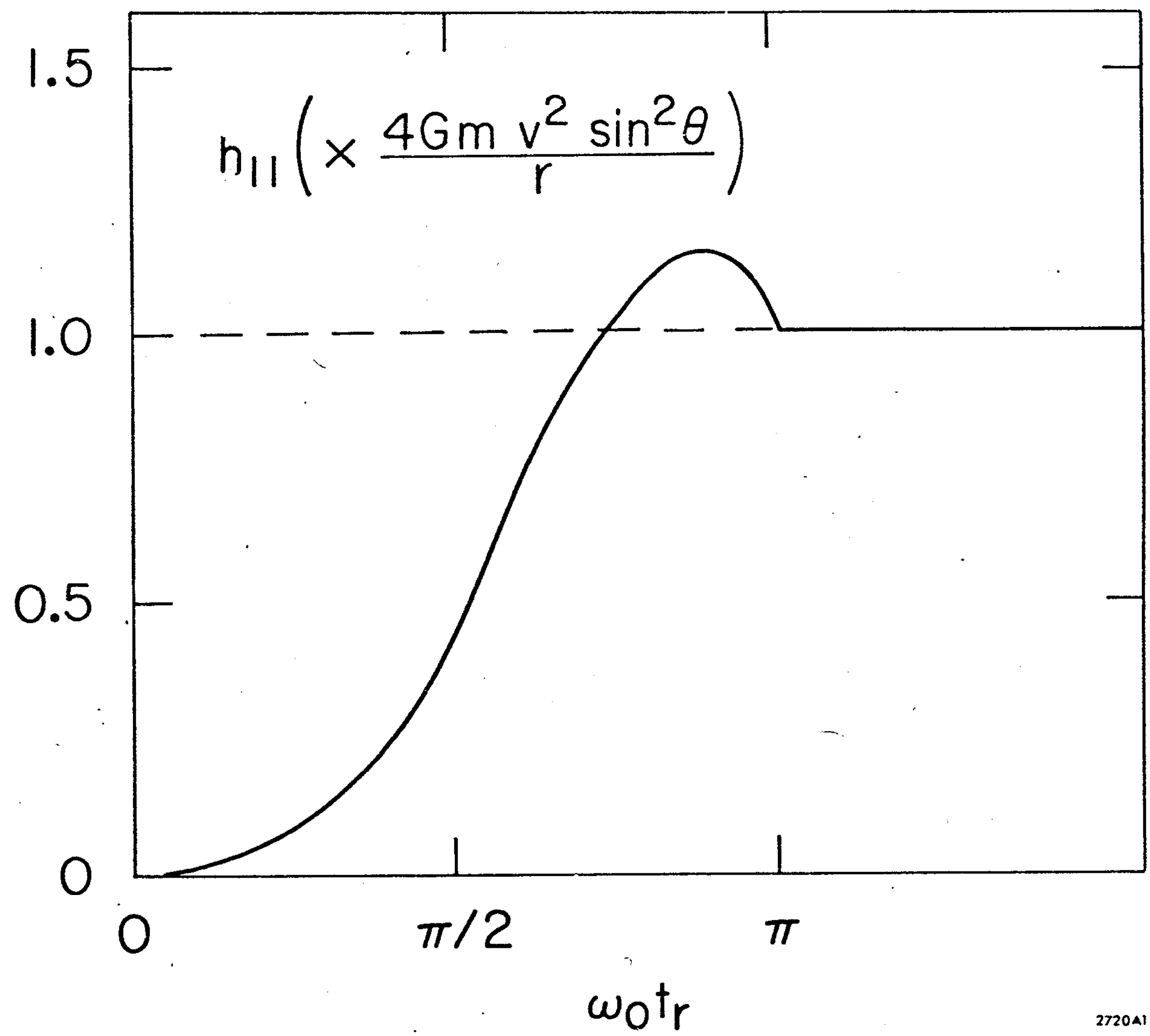
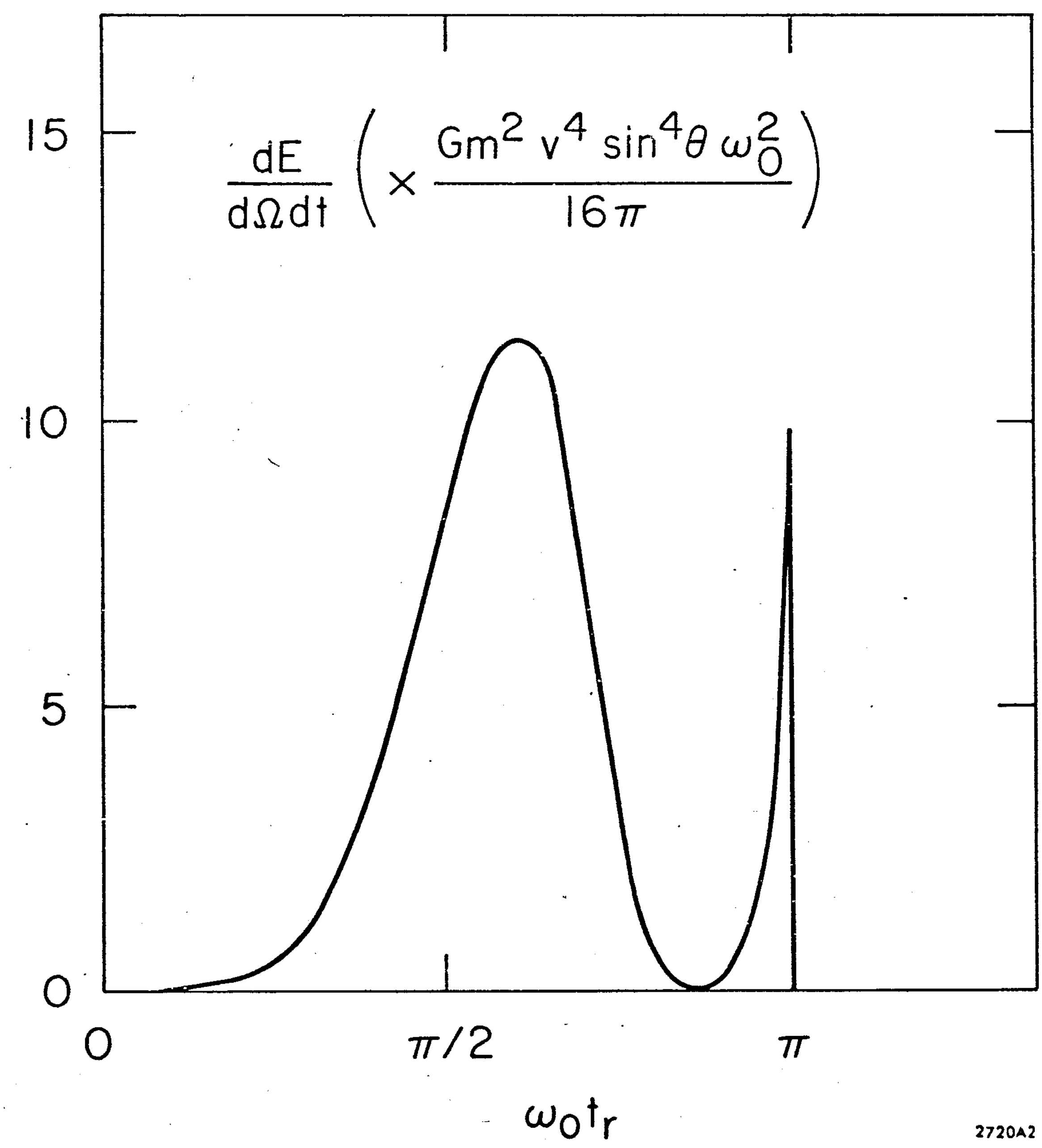

1.5
1.0
0.5
0
$h_{11}\left(\times \frac{4Gm\, v^2 \sin^2\theta}{r}\right)$
0
$\pi/2$
π
$\omega_0 t_r$
2720A1

Fig. 1



15
10
5
0
$\frac{dE}{d\Omega dt}\left(\times\frac{Gm^2 v^4 \sin^4\theta\,\omega_0^2}{16\pi}\right)$
0
$\pi/2$
π
$\omega_0 t_r$
2720A2

Fig. 2

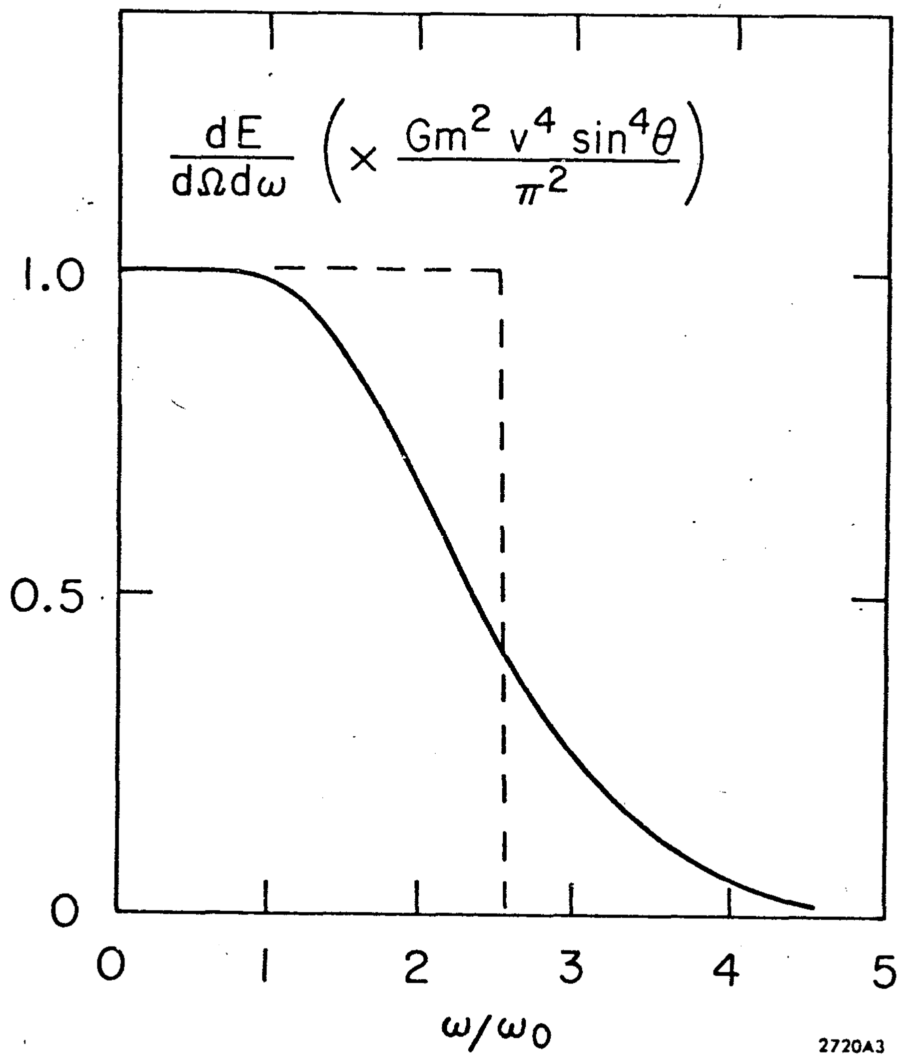

$\frac{dE}{d\Omega d\omega} \left(\times \frac{Gm^2 v^4 \sin^4\theta}{\pi^2} \right)$
1.0
0.5
0
0
1
2
3
4
5
ω/ω_0
2720A3

Fig. 3