ACCELERATED UNIVERSE DOMINATED BY HOLOGRAPHIC DARK ENERGY, SUPERGRAVITY INFLATIONARY POTENTIAL*

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Holographic dark energy cosmology free of the Q-ball formation, with accelerated expansion and an observable cosmological constant considered as the sum of the vacuum (Λ_{vac}) and an induced term ($\Lambda_{ind} = -3m^2/4$) with m being the ultra-light masses (ULM) ($\approx$ Hubble parameter) implemented in the theory from supergravities arguments and non-minimal coupling, is constructed and discussed in this letter.

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Two of the greatest mysteries in modern physics today are what drives the accelerated expansion of the universe and why the cosmological constant (lambda) is positive and extremely small (the cosmological constant problem) [1–3]. The origin of the first problem is theoretically explained by adding exotic dark energy matter with an equation of state $p = \omega_\Lambda \rho$, $\omega_\Lambda < -1/3$ (ω_Λ is a parameter, not necessarily constant). This negative-pressure dark energy density gives rise to cosmic acceleration and its origin remains a mystery from the point of view of Einstein general relativity, standard particle theory or even quantum gravity. This phenomenological explanation unfortunately didn't succeed to give a realistic explanation of the second problem that is, why the observed lambda is 120 orders of magnitude less than the theoretical estimates [4]. Several theories have been proposed including scalar fields (quintessence with single field or with N coupled field), complex scalar fields, $1/R$ gravity theory, a phantom field, k-essence, etc. [5, 6]. Of particular interest for us is the holographic dark energy model with event horizon as IR cutoff scale L and $\rho_\Lambda = 3c^2/8\pi G L^2$ proposed in the literature based on holographic principle (G is Newton's gravitational constant and c is a constant) [7–12]. It was argued that this model can produce an accelerating cosmic expansion and that holographic principle can provide a natural physical and realistic solution to the cosmological constant problem. In this letter, we study a new holographic dark energy model with a non-minimally coupled complex scalar field with a modified supergravity dark energy density. In recent years, a lot of works have been devoted to the investigation of the cosmological models with the non-minimal coupling between gravity and inflaton scalar field and to their connection with inflationary cosmology and phase transitions in the early Universe. It has also been noted that spontaneous symmetry breaking and phase transitions can be induced by curvature via the non-minimal coupling with the external gravitational field [13–15]. Recently, we have investigated a particular cosmological model with complex scalar self-interacting inflation field non-minimally coupled to gravity, based on supergravities argument [16–18]. It was shown that in the case of non-minimal coupling between the scalar curvature and the density of the scalar field such as $L=-(\xi/2)\sqrt{-g}R\phi\phi^*$ (R is the scalar curvature or the Ricci scalar) and for a particular scalar complex potential field $V(\phi\phi^*)=(3m^2/4)(\omega\phi^2\phi^{*2}-1)$, ($\omega$ is a tiny parameter that can be time-dependent and to be differentiate from ω_Λ), inspired from supergravity inflation theories, ultra-light masses m are implemented naturally in the Einstein field equations (EFE), leading to an effective cosmological constant Λ in accord with observations. The metric tensor of the spacetime is treated as a background and the Ricci scalar in the non-minimal coupling term, regarded as an external parameter, was found to be $R=4\Lambda-3m^2=4\Lambda_{eff}$ where $\Lambda_{eff}=\Lambda-(3m^2/4)$ is the effective cosmological constant and Λ is the vacuum cosmological constant. That is to say, there is another induced contribution to the vacuum cosmological constant with $\Lambda_{\text{induced}}=-3m^2/4$. These ultra-light masses are in fact too low while they may have desirable feature for the description of the accelerated universe [19–21]. We consider the non-minimally coupled theory described by the action

$$S=S_G+S_{\text{int}}+S_{\phi\phi^*}=$$
$$=\int\sqrt{-g}d^4x\left(\left(\frac{1}{2\kappa}-\frac{\xi\phi\phi^*}{2}\right)R+\frac{1}{\kappa}\Lambda-\frac{1}{2}g^{\mu\nu}\left(\partial_\mu\phi^*\partial_\nu\phi+\partial_\mu\phi\partial_\nu\phi^*\right)-\tilde{V}\left(\phi\phi^*\right)\right). \tag{1}$$

g is the absolute value of the determinant of the metric tensor, $\kappa=8\pi G$, $S_G=(2\kappa)^{-1}\int d^4x\sqrt{-g}\left(R+2\Lambda\right)$ is the Einstein-Hilbert gravitational part of the action, $S_{\text{int}}=-(\xi/2)\int d^4x\sqrt{-g}R\phi\phi^*$ is the non-minimal interaction term between the gravitational and the complex scalar fields, and finally $S_{\phi\phi^*}$ describes the material part of the action associated with the complex scalar field.

The variations of the above action with respect to the scalar field and the metric tensor yield the Klein-Gordon field Eq. (1) and the Einstein equations:

$$\left(\frac{1}{\kappa}-\xi\phi^*\phi\right)G_{\mu\nu}+\left(\partial_\mu\phi^*\partial_\nu\phi+\partial_\mu\phi\partial_\nu\phi^*\right)-g_{\mu\nu}\partial_\lambda\phi^*\partial^\lambda\phi+ \\ +g_{\mu\nu}\left(\Lambda-\frac{3}{4}m^2+\frac{3}{4}m^2\omega\phi^2\phi^{*2}\right)+\frac{1}{3}\left(g_{\mu\nu}\Box\phi^*\phi-D_\nu\partial_\mu\phi^*\phi\right)=0, \tag{2}$$

where $\Box$ denotes the covariant derivatives and D_ν is the covariant derivative. The last three terms in the bracket are due to the variations of the term $-(1/2)\sqrt{-g}\xi R\phi\phi^*$ in the action. These terms modify the gravitational constant and the cosmological constant as $\kappa_{eff}^{-1}=\kappa^{-1}-\xi\phi^*\phi$ and $\Lambda_{\text{mod}}=\Lambda-$ $-(3/4)m^2\left(1-\omega\phi^2\phi^{*2}\right)\equiv\Lambda+V\left(\phi\phi^*\right)$. The ultra-light masses were shown to contribute to the dark energy problem. We propose now the effective holographic dark energy density (EHDED)

$$\rho_\Lambda=\frac{3H^2}{\kappa_{eff}}=\frac{3H^2}{\kappa}\left(1-\xi\kappa\sqrt{\frac{1}{\omega}\left(\frac{4V(\phi\phi^*)}{3m^2}+1\right)}\right), \tag{3}$$

$$=\rho_c+\rho_{m+}, \tag{4}$$

following the idea where the IR cutoff is chosen at Hubble scale in order to be consistent with recent astronomical observations and $c=1$ [6]. In case $\phi=0$, $\rho_\Lambda=\rho_c=3H^2/8\pi G$ the critical density of the four-dimensional FRW standard cosmology [22]. The extra density ρ_{m+} is positive unless the non-minimal coupling constant is negative. The main feature of the EHDED is that at the critical value of the scalar field $\phi_c^2\phi_c^{*2}=\omega^{-1}$ where the potential vanishes, $\rho_{m+}=-3\xi\phi_c\phi_c^*H^2$ [17]. In other words, $\rho_\Lambda=3H^2\kappa^{-1}\left(1-\kappa\xi\omega^{-1/2}\right)$. This corresponds to a slow change of the scalar field in nonsingular cosmological model. If in addition, $\omega=(\kappa\xi)^2$ then $\rho_\Lambda=0$. In what follows, we take a four dimensional FRW spatially flat universe favored by recent observations with metric $ds^2=-dt^2+a^2(t)dx_idx^i$, where $a(t)$ is the scale factor. Instead of complex scalar field, we would like to use alternative field variables. In order to construct equations which relate the SIP to the recent astronomical observations suggesting the accelerating expansion of the cosmos, we consider simple real quintessence field by letting $\phi(t)=\varphi(t)\exp(ik\alpha)$ and assuming that there is no contributions from the angular-motion at a first level, that is, we neglect centrifugal terms. In other words, we consider a local phase transformation for the scalar field such that ϕ and φ represent the same physical object (the universe tends to the de Sitter-like regime through chaotic inflation for the real field). The field cosmological equations corresponding to the real inflationary supegravity decaying potential $V(\varphi)=(3m^2/4)(\omega\varphi^4-1)$ with the ansatz $\omega=\omega_0 t^{-2}$ (ω_0 is the value of ω at $t = 1$) chosen, $\kappa_{eff}^{-1}=\kappa^{-1}-\xi\varphi^2$ and $\rho_\Lambda=3H^2\kappa^{-1}\left(1-\kappa\xi\varphi^2\right)$ are as follows [6, 23–27]:

$$H^2=\frac{\kappa}{3\left(1-\xi\kappa\varphi^2\right)}\left(\rho+\rho_\Lambda+\dot{\varphi}^2+6\xi H\varphi\dot{\varphi}+\frac{3}{4}m^2\left(\omega\varphi^4-1\right)\right), \tag{5}$$

$$\ddot{\varphi}+3H\dot{\varphi}+6\xi\left(2H^2+\dot{H}\right)\varphi+3m^2\omega\varphi^3=0, \tag{6}$$

where $H=\dot{a}/a$ is the Hubble parameter. Remember that the ULM with scalar kinetic energy of order of $1/2\,\dot{\varphi}^2$ were shown to contribute to the dark energy and cosmological constant problem, the reason we assume their kinetic energy contribution in Eq. (5) [16, 28]. In order to obtain solutions to these field equations, we adopt power-law solutions for the scale factor $a(t)=a_o t^r$ and by making the ansatz $\varphi(t)=\varphi_0 t^p$ corresponding to the attractive solution in the phase space where r and p are real parameters to be determined. From Eqs. (6) and (5) we obtain simply:

$$p(p-1)+3rp+6\xi r(2r-1)+3m^2\omega\varphi_0^2 t^{2p+2}=0, \tag{7}$$

$$p^2+6\xi pr+\frac{3}{4}\omega m^2\varphi_0^2 t^{2p+2}-\frac{3}{4}m^2\varphi_0^{-2}t^{2-2p}=0. \tag{8}$$

The model requires, for a consistent solution of Eqs. (7) and (8) to exist, that ultra-light particles (decaying cold dark matter) are unstable and decays as $m=m_0 t^{-1}$, m_0 is a constant chosen such that at $t = 1$, $m = m_0$ (explaining the effective cosmological smallness constant behaving in this case as $\Lambda_{eff}\propto t^{-2}$) [29, 30]. In fact, one expects that cold dark matter particles with time decreasing masses may have an important measurable effect in the dynamical motion of the halo of spiral galaxies. At clusters scale this could have important consequences on dark matter halos (axions and mass varying neutrinos are good candidate for the cold dark matter of our universe) [31–33]. Moreover, recent astronomy observations suggest that our universe has a critical energy density which consists of $1/3$ the matter density and $2/3$ the dark energy density with negative pressure [34–38]. As a consequence, we can consider the simplest case of dark energy dominated universe with $m \ll 1.$ In order to have a consistent acceptable solution, we choose $p = 1$ [23, 39, 40]. In other words we have the following ansatz: $\varphi(t)=\varphi_0 t$, $m=m_0\,\varphi_0/\varphi$, $\omega=\omega_0\,\varphi_0^2/\varphi^2$ and a supergravity scalar field potential that behaves as an inverse power-law $V(\varphi)=3/4\,m_0^2\varphi_0^2\left(\varphi_0^2\omega_0-1/\varphi^2\right)\approx$ $\approx V_0\left(\varphi_0/\varphi\right)^2$, $V_0=-(3/4)m_0^2$. It is easy to check that, in this case, $3r+6\xi r(2r-1)+3\alpha\approx 0$ and $(1+6\xi r)+(3/4)\alpha\approx 0$ respectively where $\alpha\equiv\omega_0\varphi_0^2 m_0^2$ for simplicity (φ_0 is the present value of the scalar field at $t=1$). As a result, one can easily prove that the coefficient r depends only on the coupling coefficient ξ. Two cases appear: for $\xi<<1$, $r\approx 5/4$, while for $\xi>>1$, $r\approx 5/2$ and both correspond to an acceleration universe. The second case seems more accelerated. In reality, as follows from cosmological applications, we expect ξ to be small [41]. It is worth mentioning that Hsu has already point out that there is no accelerated expansion in the holographic dark energy model with $\rho_m=3(1-c^2)M_p^2H^2$, here c is a integral constant, while, Li argued that the holographic dark energy model with $\rho_m=3c^2M_p^2R_h^2$ (R_h is the future event horizon) succeeds in accelerating the universe [42–44]. The model in this paper is similar to the former model but accelerating expansion still occurs if a non-minimally coupled complex scalar field behaving like $\varphi(t)\propto t$, a modified supergravity dark energy density and a decaying ultra-light masses are present in the theory. It can be seen that despite the slow roll in inflation theory, $\varphi\to\infty$ for large t and the universe expansion accelerated forever while the effective supergravity potential $V(\varphi)\to(3/4)\alpha<<1$ (minimum value) at late times. Note that the conservation of the energy-momentum of the field in a dark energy dominated universe reduces to $\dot{\rho}_\Lambda+3H(\rho_\Lambda+p_\Lambda)=0$ and the dark energy restriction at actual time $-1<(\omega_\Lambda)_0<-2/3$ is easily checked to be verified [29]. An additional interesting feature comes from the effective Ricci scalar tensor $R_{eff}=\kappa\left(4\Lambda-3m^2+3m^2\omega\varphi^4\right)\left(1-\xi\kappa\varphi^2\right)^{-1}$ of our non-minimal coupling theory. It behaves as [17, 45]:

$$R_{eff}=\kappa\frac{4\Lambda-3m_0^2t^{-2}+3\alpha}{1-\xi\kappa\varphi_0^2t^2}\to 0 \tag{9}$$

as time grows up. That is $R_{eff}\to 0$ at late times while it is finite at $t=1$. This later could contribute on some astrophysical problems (for example the CMB spectrum) [46–50]. In particular for $\xi<<1$, the cosmological scenario discussed does not suffered from the Q-ball formation, some kind of non-topological soliton whose stability is guaranteed by some conserved charge Q [32, 51–59].

In reality, the formation of Q-ball is very generic for complex scalar field. Their formation is some kind of trouble for spintessence which is a canonical complex scalar field. The hessence model can avoid this difficulty but no holographic dark energy is present in its basic scenario [57, 58, 60–67]. In addition, we believe that stable Q-balls produce the present baryon asymmetry of the universe by their decay and as a result they will produce dangerous relics at the same time when they decay to produce the baryons. Their decay temperature becomes in general much lower than the freeze-out temperature of the dangerous lightest supersymmetric particle, which causes serious and unrealistic constraints [66]. To avoid this problem, we need to have a proper potential possessing a $O(N)$ internal symmetry and obeying a certain critical condition of instability for the potential against Q-ball formation [61]. One of these potential is of inverse power-law as the one appeared in this letter, *e.g.* $V(\varphi) \approx V_0 \left(\varphi_0/\varphi\right)^2$ [39, 40]. Note that in Q-ball theory, a term $Q^2/a^6\varphi^3$ is associated to Eq. (7) and a term $-Q^2/2a^6\varphi^2$ is associated to Eq. (8). In our scenario, both will behave as $Q^2/a_0^6\varphi_0^4 t^{6r+4p-2}$ where $Q \equiv a^3\varphi^2\dot{\theta} \propto t^{3r+2}\dot{\theta}$, θ is a second variable introduced to describe the hessence scenario [57, 59]. It is not difficult to notice that they will not modify crucially the cosmological holographic scenario. For homogenous φ and θ, in order to have a total conserved charge within the physical volume, θ must decreases with time as $\theta \propto t^{-3r-1}$, otherwise, the charge increases with time and changes its fate, that is it will be difficult to be evaporated or to be dissociated. There is a critical value of charge say Q_c for which Q is constant depending on the time-evolution of θ and on r. If in the other side $\theta \propto t^m$ with $m < -3r-1$, then Q-ball lose their charge and disappear in a burst of particles resulting on some inhomogeneity that could be of important cosmological consequences. Q_c corresponds to $m = -3r-1$. The effective charge due to non-minimal coupling will behaves as $Q_{eff}^2 = Q^2/\left(1-\xi\kappa\varphi^2\right) \to Q^2$ at late times in particular for $\xi << 1$. These results encourage a thorough study of the dark energy problem of the universe and further investigations of the model described are in progress.

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REFERENCES

1. J. L. Tonry *et al.* (2003), *Astrophys. J.* **594** 1.
2. R. A. Knop *et al.* (2003), *Astrophys. J.* **598** 102.
3. T. Manabhan and T. R. Choudhury (2003), *Mon. Not. Roy. Astron. Soc*. **344** 828.

4. P. J. Peebles and B. Ratra (2003), *Rev. Mod. Phys*. **75** 599.
5. C. G. Shao, R. G. Cai, B. Wang and R. K. Su (2006), *Phys. Lett*. **B 633** 164.
6. M. Ito (2005), *Europhys. Lett.* **71** 712.
7. P. Horava and D. Mimic (2000), *Phys. Rev. Lett*. **85** 1610.
8. J. Polchinski, L. Susskind and N. Toumbas (1999), *Phys. Rev*. **D 60** 084006.
9. G. Guberina, R. Horvat and H. Nikolic (2005), *Phys. Rev*. **D 72** 125011.
10. Q. G. Huang and Y. G. Gong (2004), *JCAP* **0408** 006.
11. Q. G. Huang and M. Li (2004), *JCAP* **0408** 013.
12. G. M. Shore (1980), *Ann. Phys*. **128** 376.
13. K. Ishikawa (1983), *Phys. Rev*. **D 28** 2445.
14. I. L. Buchbinder and S. D. Odintsov (1985), *Class. Quant. Grav*. **2** 721.
15. I. L. Buchbinder, S. D. Odintsov and I. L. Shapiro (1992), *Effective Action in Quantum Gravity* (*Bristol: IOP Publication*).
16. R. A. El-Nabulsi (2005), *Phys. Letts.* **B 619** 26.
17. R. A. El-Nabulsi (2006), *Chin. Phys. Lett.* Vol.**23**, No.**5**, 1124.
18. R. A. El-Nabulsi (2005), *EJTP* **10** 15.
19. H. F. M. Goenner (1984), *Found. Phys*. **14** 865.
20. S. M. Caroll, (1998), *Phys. Rev. Lett*. **81** 3067.
21. R. Horvat (1999), *Mod. Phys. Lett*. **A 14** 2245.
22. S. Weinberg (1972), *Gravitation and Cosmology*, John Wiley & Sons.
23. J. P. Uzan (1999), *Phys. Rev*. **D 59** 123510.
24. T. Chiba (1999), *Phys. Rev*. **D 60** 083508.
25. O. Bertolami and P. J. Martins, *Phys. Rev*. **D** 61, 064007 (2000).
26. X. Zhai and T. Zhao (2005), *Nuovo Cim*. **120 B** 1007.
27. R. Rosenfeld and J. A. Frieman (2005), *JCAP* **0509** 003.
28. G. Dvali and M. Zaldarriaga (2002), *Phys. Rev. Lett*. **88** 091303.
29. A. Arbab (2003), *Class. Quantum Grav*., **20** 93.
30. M. Birkel and S. Sarkar (1997), *Astropart. Phys*. **6** 197.
31. M. Beltran, J. Garcia-Bellido, J. Lesgourgues and Riazuelo (2004), *Phys. Rev.* **D 70** 103530.
32. J. A. Gu and W. Y. P. Hwang (2001), *Phys. Lett*. **B 517** 1.
33. R. Fardon, A. E. Nelson and N. Weiner (2004), *JCAP* **0410** 005.
34. A. G. Riess *et al.* (1998), *Astron. J*. **116** 1009.
35. S. Perlmutter *et al.* (1999), *Astrophys. J*. **517** 565.
36. R. Jimenez, L. Verde, T. Treu and D. Stern (2003), *Astrophys. J*. **593** 622.
37. A. Linde (1990), *Particle Physics and Inflationary Cosmology*, (Harwood, Chur, Switzerland).
38. J. P. Abreu, P. Crawford and J. P. Mimoso (1994), *Class. Quant. Grav*. **11** 1919.
39. I. Zlatev, L. Wang and P. Steinhardt (1999), *Phys. Rev. Lett*. **82** 896.
40. B. Ratra and P. Peebles (1988), *Phys. Rev.* **D 37** (12) 3406.
41. R. Aldrovandi, G. E. A. Matsas, S. F. Novaes and D. Spehler (1994), *Phys. Rev*. **D 50** 2645.
42. S. D. H. Hsu (2004), *Phys. Lett*. **B 544** 13.
43. M. Li (2004), *Phys. Lett*. **B 606** 1.
44. K. Ke and M. Li (2005), *Phys. Lett*. **B 606** 173.
45. A. D. Rendall (2005), *Class. Quant. Grav*. **22** 1655.
46. H. K. Jassal, J. S. Bagla and T. Padmanabhan (2005), *Mon. Not. Roy. Astron. Soc*. **356** L11.
47. G. Efstathiou (2003), *Mon. Not. Roy. Astron. Soc*. **343** L95.
48. J. P. Luminet, J. Weeks, A. Riazuelo, R. Lehoucq and J. Uzan (2003), *Nature* **425** 593.
49. Y. Gong, B. Wang and Y. Z. Zhang (2005), *Phys. Rev*. **D 72** 043510.
50. Y. S. Myung (2005), *Phys. Lett*. **B 610** 18.
51. T. D. Lee and Y. Pang (1992), *Phys. Rep*. **221** 251.

52. S. Coleman (1985), *Nucl. Phys*. **B 262** 263.
53. A. Kusenko (1997), *Phys. Lett.* **B 404** 285.
54. A. Kusenko and M. Shaposhnikov (1998). *Phys. Lett*. **B 418** 46.
55. G. Dvali, A. Kusenko and M. Shaposhnikov (1997), *Phys. Lett*. **B 417** 99.
56. T. Ghergetta, C. Kolda and S. P. Martin (1996), *Nucl. Phys.* **B 468** 37.
57. H. Wei and R. G. Cai (2005), *Phys. Rev*. **D 72** 123507.
58. L. A. Boyle, R. R. Caldwell and M. Kamionkowski (2002), *Phys. Lett*. **B 545** 17.
59. H. Wei, R. G. Cai and D.-F. Zeng (2005), *Class. Quant. Grav.* **22** 3189.
60. T. Multamaki and I. Vilja (2002), *Phys. Lett*. **B 542** 137.
61. X. Z. Li, J. G. Hao and D. J. Liu (2002), *Class. Quant. Grav.* **19** 6049.
62. F. Wang and J. M. Yang (2005), *Nucl. Phys*. **B 709** 409.
63. F. Wang and J. M.Yang (2006), *Eur. Phys. J.* **C 45** 815.
64. W. Zhao and Y. Zhang (2006), *Phys. Rev*. **D 73** 123509.
65. D. Lohiya and M. Sethi (1999), *Class. Quant. Grav*. **16** 1545.
66. T. Matsuda (2002), *Phys. Rev*. **D 65** 103501.
67. T. Multamaki and I. Vilja (2000), *Nucl. Phys*. **B 574** 130.