ELECTROWEAK RADIATIVE CORRECTIONS IN THE SU(2) x U(1) STANDARD MODEL

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1. INTRODUCTION

The standard SU(2) x U(1) model of the electroweak interaction is in an excellent position after the discovery of the weak vector bosons $W^{\pm}$,Z with masses in the range where they have been expected [1), 2)]. Although this means a strong support of the theory crucial further steps are necessary in order to establish the standard model: the search for the Higgs scalar as the signal for spontaneous symmetry breaking, and precision measurements of the vector boson masses and boson-fermion couplings in the accessible fermionic processes in present and future colliders.

Electroweak processes between fermions can be described with help of essentially 3 parameters each of them related to a typical low energy experiment:
the electric charge $e = \sqrt{4\pi\alpha}$, $\alpha^{-1} = 137.03604$, as obtained e.g. from the Thomson limit of Compton scattering;
the Fermi constant G_F, obtained from the muon lifetime;
the electroweak mixing angle $\sin^2\Theta_W$, obtained from neutrino scattering.

On the other hand the boson masses M_W, M_Z can directly be measured in $p\bar{p}$ collisions. The minimal standard model, where $W^{\pm}$,Z get their masses via a Higgs mechanism with a scalar doublet, predicts relations between M_W, M_Z and the parameters G_F, $\sin^2\Theta_W$, α:

$$1 - \sin^2\Theta_W = M_W^2/M_Z^2, \quad M_W^2 = \pi\alpha/\sqrt{2}\, G_F \sin^2\Theta_W \tag{1.1}$$

Furthermore, if $\sin^2\Theta_W$ or M_W, M_Z are known, all the fermionic neutral current coupling constants are also fixed:

$$\begin{aligned} a_f &= I_3^f/2\sin\Theta_W\cos\Theta_W, \\ v_f &= (I_3^f - 2\, Q_f \sin^2\Theta_W)/2\sin\Theta_W\cos\Theta_W. \end{aligned} \tag{1.2}$$

These coupling constants can best be measured in $e^+e^- \to f\bar{f}$ experiments. Thus we are in a situation where the theoretical relations become more and more encircled by sensitive experiments which subject the standard model to the hard tests it deserves.

Precision experiments require adequate theoretical predictions. The theoretical relations $M_W \leftrightarrow M_Z$, $M_Z \leftrightarrow v_f, a_f$ are in general different for different orders of perturbation theory. The inclusion of higher order effects, the "radiative corrections", becomes a necessity with increasing experimental accuracy. Such precision tests beyond the tree level not only probe the internal consistency of the theory (renormalizability), but are also required to seperate the standard model from possible extensions (supersymmetry, more-Z-models, composite models, ...).

The following sections contain a discussion of the 1-loop renormalization of the standard model and applications of the radiative corrections to fermion processes. Thereby we restrict the discussion to leptonic processes since these allow the cleanest access to the more subtle parts of the theory avoiding theoretical uncertainties as far as possible.

2. RENORMALIZATION

Calculations beyond the tree level require the choice of a renormalization scheme. This defines

- what the free parameters in the Lagrangian are, and
- how these are related to measurable quantities.

The list of work in 1-loop corrections to processes with low momentum transfer q^2 [3) - 13)] and to e^+e^- processes with $q^2 \sim M_W^2$ [14) - 23)] is quite long. The great variety of different renormalization schemes makes it difficult to compare directly the obtained results. It should be pointed out, however, that there is meanwhile a satisfactory agreement between the individual calculations in the questionable points.

Nevertheless there are still several possible sources of confusions when talking about radiative corrections:

- different parameter sets yield different values for physical observables (e.g. the forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$) if the tree level relations are used. Inclusion of radiative corrections with the same set of parameters as in lowest order removes this differences.
- The definition of $\sin^2\Theta_W$ is no longer unique beyond the tree level.

For comprehensive tests it is desirable to have the calculation of all the different processes under consideration within a common renormalization scheme. A scheme with an evident physical interpretation, which is also convenient for practical use, is the on-shell scheme [24), 5), 7), 11)-23)]: The free parameters are the masses M_W, M_Z, M_H, m_f of the gauge and Higgs bosons and fermions together with the electromagnetic fine structure constant α. The renormalization conditions fix the masses as the pole positions of the corresponding 2-point functions.

An example of a non-on-shell scheme is the $\overline{MS}$ scheme [10)] where the renormalization conditions consist simply in subtracting the singular parts of the 1-loop 2- and 3-point functions. The free parameters have to be related to the physical masses in a second step.

Though different renormalization schemes are in principle equivalent due to renormalization group invariance we prefer for the following discussion the on-shell scheme because of its better physical transparency.

Starting point is the classical Lagrangian

$$\mathcal{L}_{c\ell} = \mathcal{L}_G(\vec{W}, B, g_2, g_1) + \mathcal{L}_H(\phi, \mu^2, \lambda) + \mathcal{L}_{FG}(\psi_L, \psi_R, \vec{W}, B) + \mathcal{L}_{FH}(\psi_L, \psi_R, \phi, g_f) \tag{2.1}$$

$\mathcal{L}_G$ is the gauge part with the SU(2) and U(1) fields $\vec{W}_\mu$ and B_μ and the corresponding gauge couplings g_2 and g_1; $\mathcal{L}_H$ is the Higgs part with the scalar doublet ϕ and with μ^2, λ as the potential parameters; $\mathcal{L}_{FG}$ contains the fermion-gauge field interaction with left and right handed fermion fields $\psi_{L,R}$, and $\mathcal{L}_{FH}$ is the Higgs-fermion Yukawa term with coupling constants g_f that induces the fermion masses.

In the fields and parameters of (2.1) the SU(2) x U(1) symmetry of $\mathcal{L}_{c\ell}$ is manifestly apparent. The physical content, however, becomes more transparent after switching to the "physical" fields and parameters

$$W^{\pm}, Z, \gamma;\ e, M_W, M_Z, M_H, m_f \tag{2.2}$$

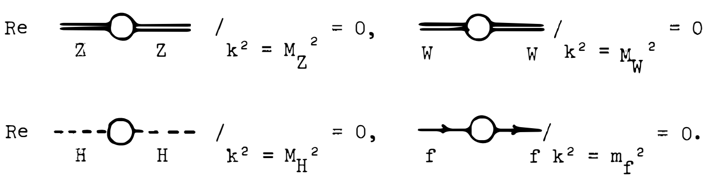
There is no room for $\sin^2\Theta_W$ as an additional independent quantity. The simplest choice in terms of (2.2) that makes the Z-γ mixing term in (2.1) vanish, is

$$\sin^2\Theta_W = 1 - M_W^2/M_Z^2 \tag{2.3}$$

which will be used throughout the forthcoming discussions.

The systematic way for obtaining physical results in higher order is scheduled in table 1 for the method with symmetric field renormalization. Since it is convenient to work in a renormalizable gauge ('t Hooft-Feynman gauge) the gauge fixing term $\mathcal{L}_{fix}$ and corresponding Faddeev-Popov part [25] have to be added to $\mathcal{L}_{c\ell}$. The multiplicative renormalization assigns a field renormalization constant $\sqrt{Z_2}$ to each multiplet of fields and a parameter renormalization constant to each parameter in the symmetric from (2.1). Expanding $Z_i = 1 + \delta Z_i$ yields the renormalized $\mathcal{L}$ which can now be re-expressed in terms of the physical set (2.2) (besides the physical Higgs H also the unphysical Higgs fields $\phi^{\pm}, \chi$ and ghost fields u^{α} are present). From $\delta\mathcal{L}$ follow the counter terms, also re-written in terms of (2.2), which have to be added to the loop integrals. The renormalization constants δZ_i in the counter terms are then fixed by imposing appropriate renormalization conditions. The results are finite Green functions from which the S-matrix elements for the various processes of interest are obtained. The Slavnov-Taylor identities [26] allow to control the consistency of the procedure and to check the final results.

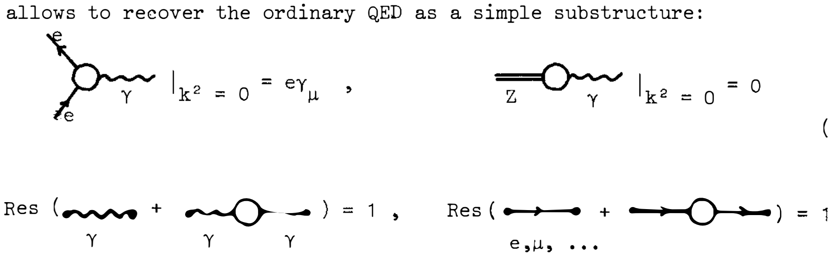
The renormalization conditions give the parameters in (2.2) the physical meaning which we expect them to have. The first class are the on-shell conditions which make the particle content of the theory evident:



(2.4)

(The bubbles mean the 1-loop contributions to the self energies together with the counter terms.)

The second class defines the electric charge in the Thomson limit and allows to recover the ordinary QED as a simple substructure:



(2.5)

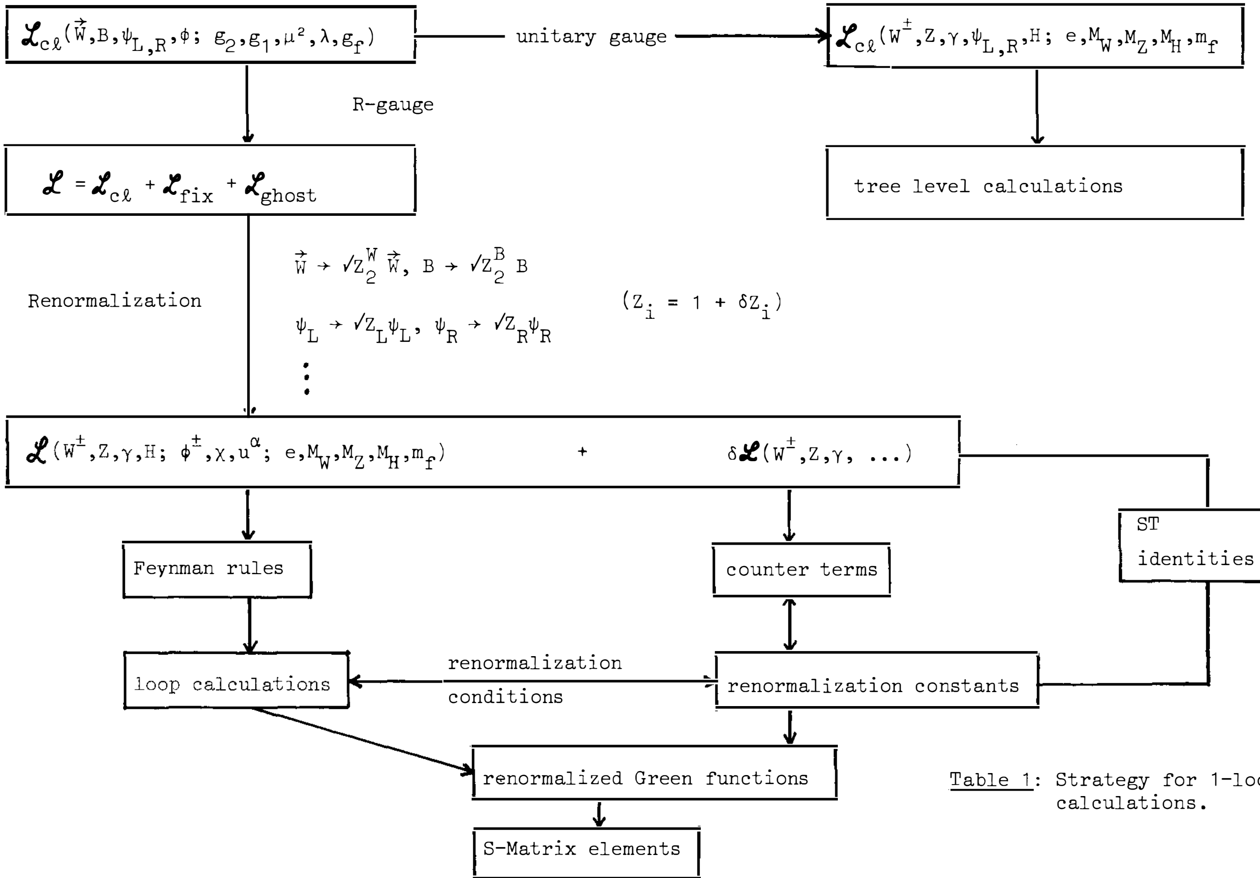


Table 1: Strategy for 1-loop calculations.

(2.4) and (2.5) are sufficient to calculate radiative corrections to fermionic processes with $m_f^2 \ll M_W^2$. A more detailed description can be found in ref. 18). The results can be summarized in terms of the renormalized vector boson propagators

$$\Delta^{\alpha}_{\mu\nu}(k) = (- g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}) \Delta^{\alpha}_{T} + \frac{k_\mu k_\nu}{k^2} \Delta^{\alpha}_{L} , \quad \alpha = \gamma, W, Z, \gamma Z$$

and boson-fermion vertices $\Gamma_\mu^{\alpha ff}(k^2)$.

For external fermion masses $m_f^2 \ll M_W^2$ it is sufficient to deal with the transverse propagators only. These are related to self energies Σ^α in the following way:

$$\Delta^{\alpha}_{T} = i/(k^2 - M_\alpha^2 + \Sigma_\alpha(k^2)), \qquad \alpha = \gamma, Z, W \tag{2.7}$$

$$\Delta^{\gamma Z}_{T} = - \frac{i}{k^2} \Sigma^{\gamma Z}(k^2) \frac{1}{k^2 - M_Z^2} \tag{2.8}$$

The vertex corrections can be expressed in terms of vector and axialvector form factors

$$\begin{aligned} \Gamma_\mu^{\gamma ff} &= - i e Q_f \gamma_\mu + i e \gamma_\mu \left[F_V^{\gamma}(k^2) - F_A^{\gamma}(k^2)\gamma_5\right] \\ \Gamma_\mu^{Zff} &= i e \gamma_\mu (v_f - a_f \gamma_5) + i e \gamma_\mu \left[F_V^{Z}(k^2) - F_A^{Z}(k^2)\gamma_5\right]. \end{aligned} \tag{2.9}$$

A complete list of the self energies and charged and neutral form factors is given in ref. 18). To get an impression 2 examples are displayed: Fig. 2 shows the weak vector form factor of the ee-Z coupling (after splitting off the QED part); other form factors are of similar magnitude. The only sizable weak corrections are the diagonal boson self energies, shown in fig. 1 for Σ_Z (Σ_W is of similar magnitude). The γ-Z mixing is $< 10^{-2}$ up to $k^2 = (200 \text{ GeV})^2$.

In all expressions $\sin^2\Theta_W$ is always to be understood as a book-keeping device in the sense of (2.3).

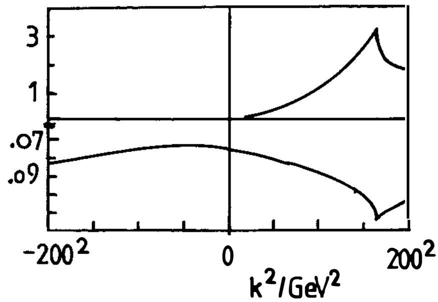


Fig. 1: Z self energy $\Sigma_Z(k^2)$
Lower curve: $\mathrm{Re}\Sigma_Z/(k^2 - M_Z^2)$
Upper curve: $\mathrm{Im}\,\Sigma_Z/M_Z\Gamma_Z$

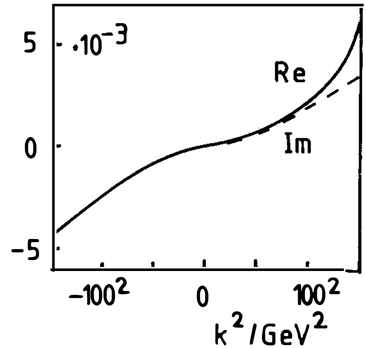


Fig. 2: Weak contribution to the e-Z vector form factor.

3. RADIATIVE CORRECTIONS IN LOW ENERGY PROCESSES

3.1 Muon decay:

The life time τ_μ of the muon decaying via $\mu \to \nu_\mu e \bar{\nu}_e$ is calculated in the Fermi model to be

$$\frac{1}{\tau_\mu} = \frac{G_F{}^2}{192\pi^3}\, m_\mu{}^5 \left(1 - \frac{8m_e{}^2}{m_\mu{}^2}\right)(1 + \delta_{QED}) \tag{3.1}$$

with $G_F = 1.16634(2)\cdot 10^{-5}\,\mathrm{GeV}^{-2}$. $\delta_{QED} = \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)$ is the familiar QED correction in the Fermi model [27].

The standard model in lowest order describes the μ decay by single W exchange (Fig. 3). A comparison of the analytic result for τ_μ with (3.1) leads to the relation

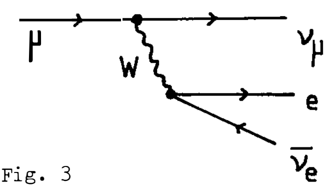


Fig. 3

$$\frac{g_2{}^2}{8M_W{}^2} \equiv \frac{e^2}{8M_W{}^2\,\sin\Theta_W} = \frac{G_F}{\sqrt{2}} \tag{3.2}$$

which allows to calculate M_W if $\sin^2\Theta_W$ is known.

Inclusion of radiative corrections means replacement of the W propagator and the vertices by their renormalized versions (together with ν wave function renormalization) and adding the 2-boson box diagrams (which are finite without renormalization). The dominating weak correction is the W self energy:

$$\frac{1}{q^2 - M_W^2} \rightarrow \frac{1}{q^2 - M_W^2 + \Sigma_W(q^2)} \simeq -\frac{1}{M_W^2} \cdot \frac{1}{1 - \Sigma_W(0)/M_W^2} \tag{3.3}$$

which appears as a multiplicative correction factor to the Born amplitude.

Summing up all 1-loop corrections leads to

$$\frac{1}{\tau_\mu} = \frac{\alpha^2}{384\pi} m_\mu^5 \left(1 - \frac{8m_e^2}{m_\mu^2}\right) \left[\frac{1}{M_W^2 \sin^2\Theta_W(1 - \delta_W)}\right]^2 \cdot (1 + \delta_{QED}) \tag{3.4}$$

with the weak correction

$$\delta_W = \frac{\Sigma_W(0)}{M_W^2} + \frac{\alpha}{4\pi\sin^2\Theta_W}\left(6 + \frac{7-4\sin^2\Theta_W}{2\sin^2\Theta_W} \log\cos^2\Theta_W\right) \cong 0.07. \tag{3.5}$$

Identifying (3.4) with the Fermi result (3.1) yields the corrected version of the relation (3.2):

$$G_F = \frac{\pi\alpha}{\sqrt{2}\, M_W^2 \sin^2\Theta_W(1 - \delta_W)} \quad \text{or} \quad M_W^2 = \frac{(37.281\ \text{GeV})^2}{\sin^2\Theta_W(1 - \delta_W)} \tag{3.6}$$

The simple form (3.6) is a consequence of the factorization of the 1-loop amplitude for $q^2 \cong 0$ into the V-A current-current term times a constant (expressed by the model parameters) which is identified with G_F. Thus G_F includes automatically higher order weak contributions by definition. The use of (3.2), where a tree level quantitiy is put equal to G_F, to derive a value of $\sin^2\Theta_W$ for the NC couplings if M_W (or M_Z) is given has therefore to be treated with caution in general.

δ_W in (3.5) is not really a constant but depends on $\sin^2\Theta_W$, M_W and (via Σ_W) on the other masses in the model. For $M_H = M_Z$, $\sin^2\Theta_W = 0.217$, $m_t = 36$ GeV: $\delta_W = 0.0696 \pm 0.0020$ [28].

The uncertainty is due to the light quark contribution in Σ_W. This hadronic part can be evaluated with help of a dispersion integral over

$\sigma(e^+e^- \rightarrow \text{hadrons})$ [9), 29)]. Recently, an update analysis of the hadronic contribution has been performed by Jegerlehner [30)] who finds a smaller error and a somewhat larger mean value:

$$\delta_W = 0.0711 \pm 0.0007 \tag{3.7}$$

An independent analysis by Cole et al. [31)] yields a hadronic uncertainty of ± 0.0013. An estimate of higher order effects to δ_W by means of the renormalization scheme dependence [30), 32)] adds a further uncertainty even larger than in (3.7). A quadratic summation leads to ± 0.0013, which may be considered as a realistic value for the total uncertainty (without the unknown Higgs and top mass).

M_W in (3.6) can be eliminated in favor of M_Z:

$$M_Z = \frac{(37.281\ \text{GeV})^2}{\sin^2\Theta_W(1 - \sin^2\Theta_W)(1 - \delta_W)} \tag{3.8}$$

If M_Z will be known with high accuracy ($\Delta M_Z/M_Z \sim 5 \cdot 10^{-4}$) from LEP, $\sin^2\Theta_W$ is fixed by inverting (3.8) in terms of α, M_Z, G_F [18), 23)]. Then all observables can be calculated with these parameters which are known to best precision.

The sensitivity to the Higgs mass M_H is not very striking: A variation of M_H from 10 GeV to 500 GeV leads to a shift in $\sin^2\Theta_W$ of 0.0035 (from (3.8) for fixed M_Z). δ_W is, however, more sensitive to a heavy top quark or a next generation with large mass splitting [15), 6), 33)], which reduce the magnitude of δ_W or even reverse its sign (for m_t > 240 GeV).

3.2 Neutrino electron scattering:

A sensitive measurement of $\sin^2\Theta_W$ can be obtained in terms of the ratio of neutrino and antineutrino cross sections

$R = \sigma(\nu_\mu e)/\sigma(\bar{\nu}_\mu e)$, which reads in lowest order:

$$R^o = \frac{1 + \xi + \xi^2}{1 - \xi + \xi^2} \quad \text{with } \xi = 1 - 4\sin^2\Theta_W = \frac{v_e}{a_e} \tag{3.9}$$

The lowest order diagramm consists of single Z exchange (Fig. 4). In higher order those corrections contribute to R which alter the v_e/a_e ratio: the γ-Z mixing propagator, the electromagnetic neutrino vertex (charge radius of ν), and box diagrams with 2 massive boson exchanges. Their inclusion modifies the simple form (3.9): $R^o(\sin^2\Theta_W) \rightarrow R(\sin^2\Theta_W; M_Z, M_H, m_f)$.

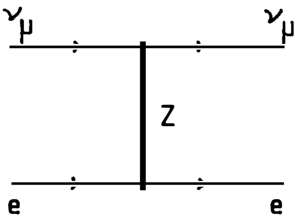


Fig. 4

R becomes now dependent also on the other model parameters. The dependence of R on $\sin^2\Theta_W$ is displayed in Fig. 5. For values $\sin^2\Theta_W \sim 0.23$ the corrections to R are very small [13), 18)]. The influence of the other parameters on the determination of $\sin^2\Theta_W$ (if R is fixed) are:

M_H = 10 - 1000 GeV	0.0024
m_t = 30 - 60 GeV	0.0008
ΔM_Z = ± 5 GeV	± 0.0003
hadronic uncertainty	± 0.0003

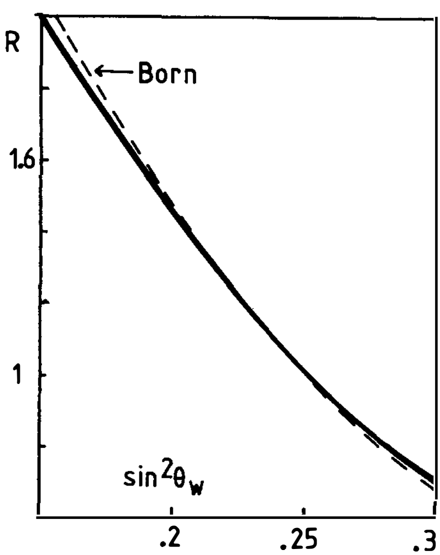
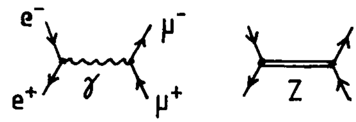


Fig. 5: $R = \sigma(\nu_\mu e)/\sigma(\bar{\nu}_\mu e)$
The thickness of the corrected curve indicates the variation with M_H = 10 - 300 GeV (from ref. 18).

Compared to the expected accuracy of $\Delta\sin^2\Theta_W = 0.005$ in the CHARM experiment the theoretical uncertainties are not very significant. R can therefore be considered as a function of $\sin^2\Theta_W$ only, also in higher order. The present CHARM calue from R is [37] $\sin^2\Theta_W = 0.215 \pm 0.032 \pm 0.012$.

4. RADIATIVE CORRECTIONS IN $e^+e^- \to \mu^+\mu^-$

The two Born diagrams for $e^+e^- \to \mu^+\mu^-$



yield the differential cross section for unpolarized beams:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left(\sigma^\gamma + \sigma^{\gamma Z} + \sigma^Z\right), \quad s = (p_{e^-} + p_{e^+})^2, \tag{4.1}$$

$$\begin{aligned} \sigma^\gamma &= 1 + c^2 \\ \sigma^{\gamma Z} &= \left[2\,v^2\,(1 + c^2) + 2a^2 \cdot 2c\right] \mathrm{Re}\,\chi, \\ \sigma^Z &= \left[(v^2 + a^2)^2\,(1 + c^2) + 4v^2a^2 \cdot 2c\right] |\chi|^2, \end{aligned} \tag{4.2}$$

with

$$\chi = s/(s - M_Z^2 + i\,M_Z\Gamma_Z), \quad c = \cos\Theta, \ \Theta = \sphericalangle\,(e^-,\mu^-), \ v,a \text{ from } (1.2).$$

The 1-loop corrections to (4.1) can be divided into 3 classes:

a) The QED corrections to the γ exchange graph (virtual photons + real photon bremsstrahlung):" reduced QED corrections". Also the QED vacuum polarization is included.

b) The QED corrections (real + virtual photons) also to the Z exchange graph. a+b: "full QED corrections (fig. 6).

c) Weak corrections: non-QED part of the γ vacuum polarization, the Z self energy and γ-Z mixing, non photonic vertex corrections and box diagrams with 2Z and 2W exchange.

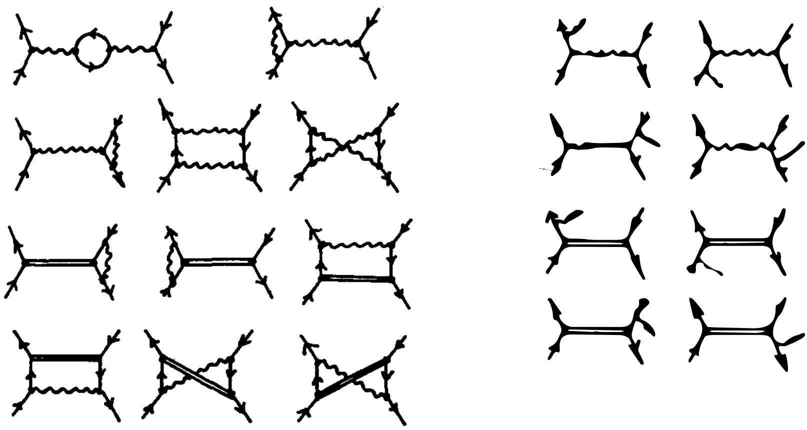


Fig. 6: Full QED corrections to $e^+e^- \to \mu^+\mu^-$

The QED corrections [34)-36)] a, b, include the emission of bremsstrahlung quanta which have to be integrated over their phase space to give an inclusive 2-particle cross section:

$$\frac{d\sigma_B}{d\Omega_\mu} = \int dk_\gamma^o \; d\Omega_\gamma \; \frac{d^5\sigma}{d\Omega_\mu \, d\Omega_\gamma \, dk_\gamma^o}$$

Adding to $d\sigma_B$ the virtual photon corrections the result becomes infrared finite: instead of the IR singularity the details of the γ phase space enter the final result. Conventionally an acollinearity cut to the outgoing $\mu^+\mu^-$ momenta and/or an energy cut to the emitted photon is applied:

$$\measuredangle\,(\mu^-, \mu^+) \leq \delta_{max}, \; k_\gamma^o < \Delta E.$$

This type of corrections therefore depends on the details of the experiments and is conveniently treated by Monte Carlo simulation[35)]. Beyond that, type b) corrections depend on the model parameters v, a, M_Z, Γ_Z. The weak corrections c) are independent of experimental cuts; they include the more subtle parts of the theory beyond the tree level.

An observable of particular interest is the forward-backward asymmetry

$$A_{FB} = \frac{\sigma(\Theta < \pi/2) - \sigma(\Theta > \pi/2)}{\sigma(\Theta < \pi/2) + \sigma(\Theta > \pi/2)}$$

It reads in lowest order (from (4.1)):

$$A_{FB}(|\cos\Theta| < x) = \frac{x}{1+x^2/3}\,\frac{2a^2\mathrm{Re}\chi+4v^2a^2|\chi|^2}{1+2v^2\mathrm{Re}\chi+(v^2+a^2)^2|\chi|^2}\,, \qquad (4.3)$$

x = detector acceptance.

4.1 PETRA/PEP energies:

At energies well below M_Z (4.3) can be approximated by

$$A^o_{FB} \simeq \frac{x}{1+x^2/3}\cdot 2a^2\,\frac{s}{s-M_Z{}^2}\,,\quad a = \frac{-1}{4\sin\Theta_W\cos\Theta_W} \qquad (4.4)$$

The inclusion of the dominant weak correction, the Z boson self energy[17),20)] Σ_Z modifies (4.4) to

$$A^{weak}_{FB} = A^o_{FB}/(1-\delta_Z),\ \delta_Z = -\frac{\mathrm{Re}\Sigma_Z(s)}{s-M_Z{}^2} \approx 0.07\,. \qquad (4.5)$$

The other weak corrections are negligible (table 2). Realistic cuts as used by experimentalists yield an almost cancellation of the QED corrections to Z exchange and the weak correction. Therefore only the model independent reduced QED corrections have to be applied to the data and the experimental result has to be compared with (4.4). This is valied over a wide range of the parameters M_Z and $\sin^2\Theta_W$[18)]. The theoretical prediction for A_{FB} at 34.5 GeV is displayed in Fig. 7. The PETRA data show the tendency to smaller values of $\sin^2\Theta_W$ than derived from M_W,M_Z measurements and ν scattering.

$A_{FB}(\lvert\cos\Theta\rvert < 0.8)$	in %
Born	- 7.62
reduced QED	- 5.80
full QED	- 5.28
Z self energy	- 5.83
γ-Z mixing	- 5.83
vertex corrections	- 5.82
massive boxes	- 5.83

Table 2: A_{FB} at 34.5 GeV

$\delta_{max} = 10^o$, $\Delta E = 0.5\ E_{beam}$

M_Z = 93 GeV, M_W = 82.1 GeV,

M_H = 100 GeV, m_t = 30 GeV

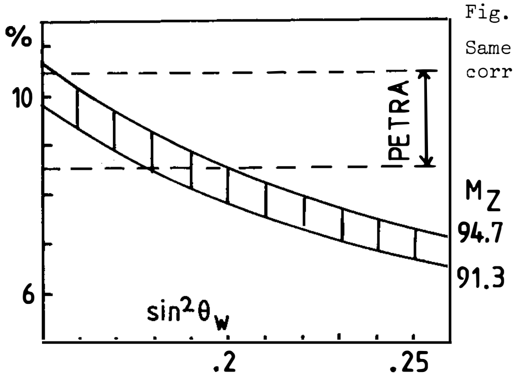


Fig. 7: A_{FB} at 34.5 GeV.
Same cuts as in table 2; the M_Z band corresponds to the 1-σ limit[2].

There is also another way to discuss A_{FB} making use of our knowledge from the μ lifetime: Inserting $\cos\Theta_W = M_W/M_Z$ in (4.4 - 5) together with (3.6) yields:

$$A_{FB}^{weak} = \frac{x}{1+x^2/3} \frac{G_F M_Z^2}{4\pi\alpha\sqrt{2}} \frac{s}{s-M_Z^2} \cdot \frac{1-\delta_W}{1-\delta_Z} \qquad (4.6)$$

In this representation the resulting weak correction is very small[16] ($\Delta A_{FB}^{weak} < 0.001$) since δ_W and δ_Z are of the same magnitude. The tree level formula for A_{FB} in terms of G_F, M_Z includes the main part of the weak corrections. In this case, however, one has to respect the full QED corrections.

4.2 On resonance ($s = M_Z^2$) forward-backward and polarization asymmetry

This simple behaviour of A_{FB} encountered well below the Z resonance is no longer valid for A_{FB} on the Z : due to the on-shell subtraction of the Z self energy there is no large contribution from $\Sigma_Z(M_Z^2)$ which could compensate the correction in (3.6). Thus we find large differences if we eliminate $\sin^2\Theta_W$ in terms of G_F, M_Z either by (3.2) or by (3.8) and use the Born formula for A_{FB} at $s = M_Z^2$:

$$A_{FB}^{o}(x=1) = \frac{3v^2a^2}{(v^2+a^2)^2} = \begin{cases} 0.107 \text{ with } (3.2) \\ 0.039 \text{ with } (3.8) \end{cases}$$

In order to avoid such big differences the use of M_W and M_Z for $\sin^2\Theta_W = 1-M_W^2/M_Z^2$ would be a better method. This is, however, limited by the experimental precision of the W mass.

The cleanest way to discuss A_{FB} is the following: calculate

$$A^o_{FB} + \delta A^{weak}_{FB} = A^{weak}_{FB}\,(\sin^2\Theta_W) \qquad (4.7)$$

as function of $\sin^2\Theta_W$ (besides M_Z, M_H).
δA^{weak}_{FB} is the sum of all weak 1-loop corrections beyond the Z self energy, which is a small quantity (< 0.005). Then extract a value for $\sin^2\Theta_W$ from a measured A^{exp}_{FB} via

$$\delta A^{QED}_{FB} + A^{weak}_{FB}\,(\sin^2\Theta_W) = A^{exp}_{FB} \qquad (4.7')$$

and compare this with $\sin^2\Theta_W$ from other sources like $1-M_W^2/M_Z^2$ or eq. (3.8)

Equivalently: Calculate $\sin^2\Theta_W$ from (3.8) for given M_Z and insert this into (4.7'); we obtain a value which has to be compared with A^{exp}_{FB}. Fig. 8 shows the dependence of A^{weak}_{FB}, eq. (4.7), on $\sin^2\Theta_W$.

Since polarization experiments become feasible at the SLC we also discuss the polarization asymmetry A_L: If the e^- beam is longitudinally polarized (with degree P_L) one can define $A_L = \sigma_U/\sigma_L$ where $\sigma(e^+e^- \to \mu^+\mu^-) = \sigma_U + P_L\sigma_L$. For $s = M_Z^2$ we have in lowest order:

$$A^o_L = \frac{2va}{v^2+a^2}$$

In analogy to (4.7) we calculate

$$A^o_L + \delta A^{weak}_L = A^{weak}_L(\sin^2\Theta_W) \qquad (4.8)$$

with the weak corrections δA^{weak}_L in terms of $\sin^2\Theta_W$. A_L is also displayed in Fig. 8. It can be seen that A_L is more senitive to $\sin^2\Theta_W$ than A_{FB}.

Both types of asymmetries if considered as functions of $\sin^2\Theta_W$ are independent of the light quark uncertainties. Therefore

$$A^{exp}_{FB},\ A^{exp}_L \to \sin^2\Theta_W \leftarrow 1-M_W^2/M_Z^2$$

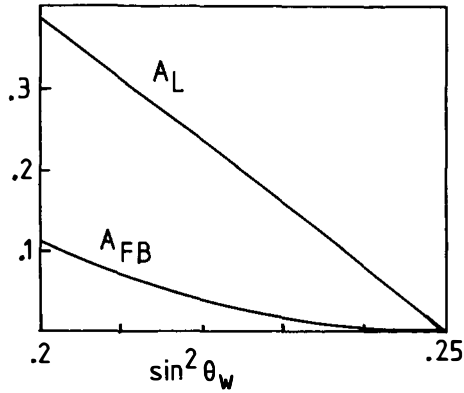


Fig. 8: On-resonance forward-backward and polarization asymmetry, $\sin^2\Theta_W$-dependence.
M_Z = 93 GeV, M_H = 100 GeV (from ref. 22).

would allow a test of the theory with the Higgs mass as the only source of uncertainty: M_H between 10 GeV and 1000 GeV leads to a variation in $\sin^2\Theta_W$ of 0.0014. The method is, however, limited by the accuracy of the mass measurements: If $\Delta M_W/M_W = \Delta M_Z/M_Z = 10^{-3}$ is assumed the experimental uncertainty would be $\Delta\sin^2\Theta_W = 0.002$.

A more precise value for $\sin^2\Theta_W$ can be obtained by (3.8) even with the inherent hadronic uncertainty:

hadronic uncertainty	± 0.0020	± 0.0013	± 0.0007
$\Delta\sin^2\Theta_W$	± 0.0007	± 0.0004	± 0.0002

This accuracy matches the (optimistic?) precision with which $\sin^2\Theta_W$ can be determined from the polarization asymmetry A_L ($\Delta\sin^2\Theta_W \cong \pm 0.0006$). Combinung A_L, eq. (4.8), with (3.8) for fixed M_Z would furthermore exhibit some sensitivity to the Higgs mass: M_H = 10-500 GeV leads to a shift in A_L of - 0.015 Numerical results for various masses can be found in ref. 23 .

A final remark concerns the full QED corrections around the Z°. They depend on the experimental conditions and the Z° parameters in a rather involved way changing the shape of the resonance and of A_{FB}[34)-36)]. The on-resonance value of A_L, however, is relatively stable under QED corrections[36)]. Furthermore, the problem of multiple bremsstrahlung and exponentiation of the leading logs[34),36)] around the Z° deserve more detailed investigations.

5. CONCLUSIONS

Actual measurements of the $W^{\pm}$,Z masses and of $\sin^2\Theta_W$ already indicate the presence of higher order effects in electroweak processes between fermions. More accurate measurements in the near future colliders LEP and SLC will allow to test the standard model beyond the tree level. At the 1-loop level a big amount of work has already been done with a satisfactory agreement between the individual calculations for the standard processes: μ-decays, ν-scattering, and $e^+e^- \to \mu^+\mu^-$.

In the on-shell renormalization scheme the physical meaning of the used parameters (particle masses) and the transparency of the calculations are most evident. In this scheme the mixing angle can be unambigously defined via $\sin^2\Theta_W = 1-M_W^2/M_Z^2$ with the physical boson masses. All experimental information coming from τ_μ, ν-scattering, polarization and forward-backward asymmetries in e^+e^-, ... can be expressed in terms of M_Z, M_W or M_Z, $\sin^2\Theta_W$, if the corresponding radiatively corrected expressions are used. Two independent experiments are needed to fix the input data (e.g. M_Z and τ_μ), the others representing tests of the theory.

The energies up to ∿ $(200\ \text{GeV})^2$ the dominant weak corrections are the diagonal W and Z self energies. Consequently, the weak corrections are

- large in lifetime of the muon (W self energy)
- small in the ratio $\sigma(\nu_\mu e)/\sigma(\bar{\nu}_\mu e)$, where no diagonal self energy corrections are present;

in $e^+e^- \to \mu^+\mu^-$ the magnitude of the weak corrections depends on the parameters used for tree and 1-loop calculations.

Around the Z^0 peak the QED corrections (real + virtual photon corrections) are particularly significant. These do not contain further theoretical information beyond the tree level (except on QED) but they influence the experimental determination of the Z width, cross sections and asymmetries quite remarkably. For reliable discussions of precision experiments on the Z^0 further careful investigations of multi-photon effects are indispensable.

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