Frascati Physics Series Vol. 69 (2019)
PHOTON 2019 - International Conference on the Structure and the Interactions of the Photon
Jun 3-7, 2019

THE $\gamma^*\gamma^* \to \eta_c(1S, 2S)$ TRANSITION FORM FACTOR FROM QUARKONIUM WAVE FUNCTIONS

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Abstract

We discuss $\gamma^*\gamma^* \to \eta_c(1S)\,,\, \eta_c(2S)$ transition form factor for both virtual photons. The general formula is given. We use different models for the $c\bar{c}$ wave function obtained from the solution of the Schrödinger equation for different $c\bar{c}$ potentials: harmonic oscillator, Cornell, logarithmic, power-law, Coulomb and Buchmüller–Tye. We showed some examples of wave functions in the Light Front representation as well as in the rest frame of $c\bar{c}$. We compare our results to the BaBar experimental data for $\eta_c(1S)$, with one real and one virtual photon, and to the values collected by the Particle Data Group for the form factor $F(0,0)$, decay width $\Gamma_{\gamma\gamma}$ and decay constant f_{η_c}. We also consider the non-relativistic limit for $F(0,0)$ with the wave function evaluated at the origin $R(0)$.

1 Introduction

In the last few years, the pseudoscalar charmonium states $\eta_c(1S)$ and its radial excitation $\eta_c(2S)$ have been paid a lot of attention from both theoretical [1, 2] and experimental [3, 4, 5] sides. So far the collaboration CLEO, BABAR, Belle, L3 Collaboration have extracted the transition form factor for light mesons $(\pi^0\,,\eta\,,\eta^{'})$ from events, where only one of the leptons in the final state could be measured. Similar researches were done for $\eta_c(1S)$ by the BABAR collaboration. The study of transition form factor for both-off shell photons is motivated by the possibility of accurate measurement of the double-tag mode, considering the high luminosity for Belle2.

The matrix element for the $\gamma^*\gamma^* \to \eta_c$ fusion can be written with the help of the form factor $F(Q_1^2, Q_2^2)$:

$$\mathcal{M}_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_c) = 4\pi\alpha_{\rm em}\,(-i)\varepsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta\, F(Q_1^2, Q_2^2)\,, \tag{1}$$

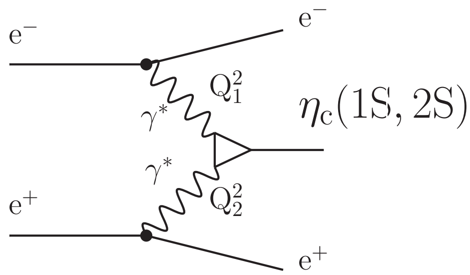

Figure 1: *Feynman diagram for the process $e^+e^- \to e^+e^-\eta_c(1S, 2S)$.*

where $Q_i^2 = -q_i^2 > 0\,, i = 1, 2$ are space like virtualities of the photon. To describe the $\gamma^*\gamma^*$ transition we used Light-Front Wave Function $\psi(z, k_\perp)$ and $F(Q_1^2, Q_2^2)$ takes the form [6]:

$$F(Q_1^2, Q_2^2) = e_c^2\sqrt{N_c}4m_c \cdot \int \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3}\psi(z, \mathbf{k})\Big\{\frac{1-z}{(\mathbf{k}-(1-z)\mathbf{q}_2)^2 + z(1-z)\mathbf{q}_1^2 + m_c^2} + \frac{z}{(\mathbf{k}+z\mathbf{q}_2)^2 + z(1-z)\mathbf{q}_1^2 + m_c^2}\Big\}. \quad (2)$$

Here $(z, \mathbf{k})$ are light-front variables, z and $1-z$ is are fraction of longitudinal momentum of η_c and $\mathbf{k}$ is the relative momentum of the quark and antiquark in the center-of-mass of the $c\bar{c}$ system. In Fig. 2 we present the dependence of the transition form factor onthe photon virtualities Q_1^2 and Q_2^2.

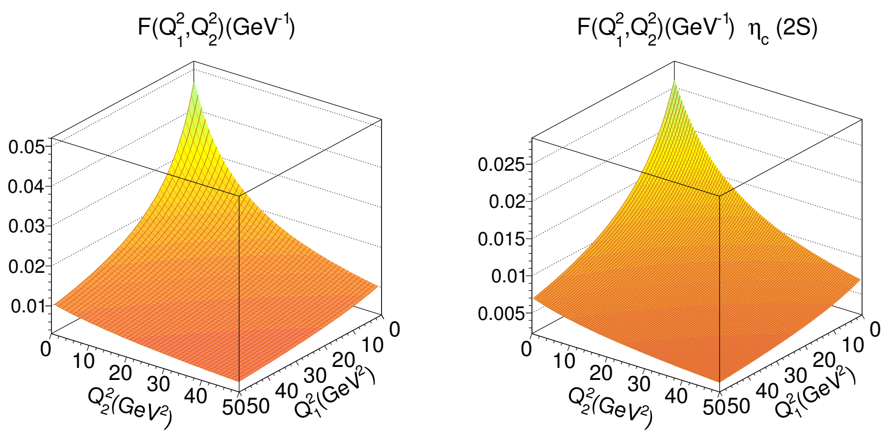

Figure 2: *Transition form factor for $\eta_c(1S)$ and $\eta_c(2S)$ for Buchmüller–Tye potential.*

2 Radial momentum-space wave function and Terentev prescription

The radial wave function in the rest frame of the quark-antiquark system is obtained from the Schrödinger equation:

$$\frac{\partial^2 u(r)}{\partial r^2} = (V_{\text{eff}}(r) - \epsilon)u(r)\,, \quad (3)$$

where $u(r) = \sqrt{4\pi}\, r\psi(r)$, and $V_{\rm eff}(r)$ is the effective potential, as described in Ref [7].

Then we transform $u(r)$ into the momentum space wave function

$$\int_0^\infty |u(r)|^2 dr = 1 \quad \Rightarrow \quad \int_0^\infty |u(p)|^2 dp = 1\,. \tag{4}$$

One can notice in Fig. 3 that each wave function $u(p)$ has slightly different behaviour, dependent on the applied effective potential and related to the model c quark mass. For further calculation we used the

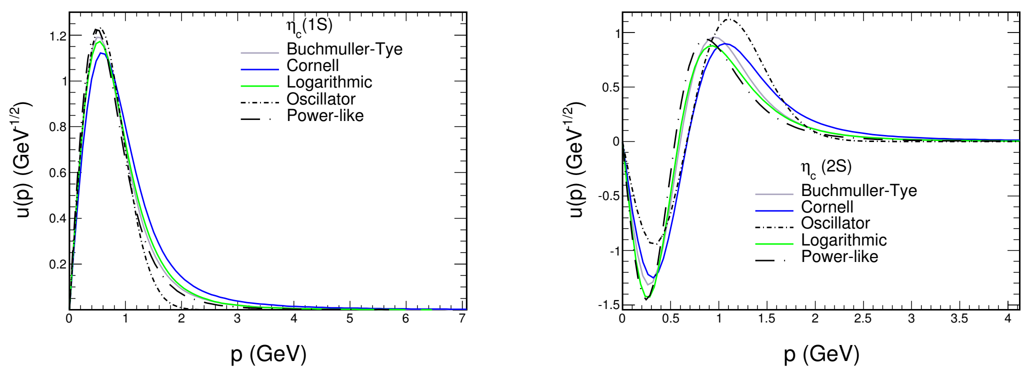

Figure 3: *Radial momentum-space wave function for different potentials. On the left-hand side the $\eta_c(1S)$ is presented, on the right-hand side the $\eta_c(2S)$.*

Terentev prescription, in order to obtain the Light-Front wave function:

$$\psi(z, k_\perp) = \frac{\pi}{\sqrt{2M_{c\bar{c}}}} \frac{u(p)}{p}\,, \tag{5}$$

using

$$p_\perp = k_\perp, \quad p_z = \left(z - \frac{1}{2}\right) M_{c\bar{c}}\,, \quad M_{c\bar{c}}^2 = \frac{k_\perp + m_c^2}{z(1-z)}\,. \tag{6}$$

Eq. (5) includes also the Jacobian factor of changing the variables of the integration. An example of the light cone wave function is shown in Fig. 4, for the Buchmüller–Tye potential model. One can observe that the wave function is strongly peaked around z equal to 1/2.

3 $F(0,0)$ transition for both on-shell photons

In order to write the formula for both on-shell photons, we can simplify Eq. (2):

$$F(0,0) = e_c^2 \sqrt{N_c}\, 4m_c \cdot \int \frac{dz d^2k_\perp}{z(1-z)16\pi^3} \frac{\psi(z,k_\perp)}{k_\perp^2 + m_c^2}\,, \tag{7}$$

and then the relation between the two-photon decay width and $F(0,0)$ can be expressed by:

$$\Gamma(\eta_c \to \gamma\gamma) = \frac{\pi}{4} \alpha_{\rm em}^2 M_{\eta_c}^3 \, |F(0,0)|^2\,. \tag{8}$$

The so-called decay constant f_{η_c} can be extracted numerically by integrating over variable z in the equation:

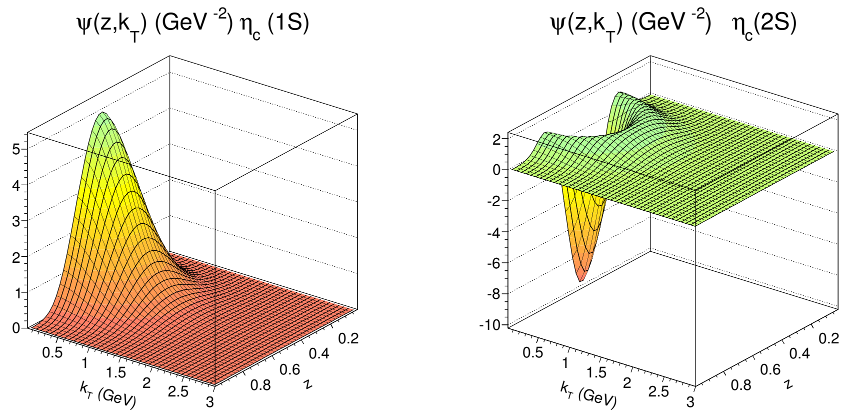

Figure 4: Radial light-front wave function for the Buchmüller–Tye potential.

$$f_{\eta_c}\varphi(z,\mu_0^2) = \frac{1}{z(1-z)}\frac{\sqrt{N_c}\,4m_c}{16\pi^3}\int d^2k_\perp\,\theta(\mu_0^2-k_\perp^2)\,\psi(z,k_\perp)\,, \tag{9}$$

with the following normalization of the distribution amplitude: $\int_0^1 dz\,\varphi(z,\mu_0^2) = 1$.

$F(0,0)$ can be rewritten in terms of the radial momentum-space wave function $u(p)$:

$$F(0,0) = e_c^2\sqrt{2N_c}\,\frac{2m_c}{\pi}\int_0^\infty \frac{dp\,p\,u(p)}{\sqrt{M_{c\bar{c}}^3}(p^2+m_c^2)}\,\frac{1}{2\beta}\log\left(\frac{1+\beta}{1-\beta}\right)\,, \tag{10}$$

In the non-relativistic (NR) limit, where $p^2/m_c^2 \ll 1, \beta \ll 1$, and $2m_c = M_{c\bar{c}}$ or $2m_c = M_{\eta_c}$, we obtain

$$F(0,0) = e_c^2\sqrt{N_c}\sqrt{2}\frac{4}{\pi\sqrt{M_{\eta_c}^5}}\int_0^\infty dp\,p\,u(p) = e_c^2\sqrt{N_c}\frac{4\,R(0)}{\sqrt{\pi M_{\eta_c}^5}}\,, \tag{11}$$

where $\beta = p/\sqrt{p^2+m_c^2}$ and R(0) is the radial wave function at the origin. The values of the transition form factor with both photons on-shell, decay constant as well as decay width $\Gamma_{\gamma\gamma}$ are collected in Table 1 for $\eta_c(1S)$ and in Table 2 for $\eta_c(2S)$.

Table 1: *Transition form factor $|F(0,0)|$ for $\eta_c(1S)$ at $Q_1^2 = Q_2^2$ =0.*

potential type	m_c [GeV]	$\lvert F(0,0)\rvert$ [GeV^{-1}]	$\Gamma_{\gamma\gamma}$ [keV]	f_{η_c}[GeV]
harmonic oscillator	1.4	0.051	2.89	0.2757
logarithmic	1.5	0.052	2.95	0.3373
power-like	1.334	0.059	3.87	0.3074
Cornell	1.84	0.039	1.69	0.3726
Buchmüller–Tye	1.48	0.052	2.95	0.3276
experiment	-	0.067 ± 0.003 [8)]	5.1 ± 0.4 [8)]	0.335 ± 0.075 [9)]

We calculated the normalized transition form factor: $F(Q^2,0)/F(0,0)$ with the aim of comparing our results with the experimental data obtained by the BABAR collaboration [10)], see Fig. 5. The

Table 2: *Transition form factor* $|F(0,0)|$ *for* $\eta_c(2S)$ *at* $Q_1^2 = Q_2^2 = 0$.

potential type	m_c [GeV]	$\|F(0,0)\|$ [GeV^{-1}]	$\Gamma_{\gamma\gamma}$ [keV]	f_{η_c} [GeV]
harmonic oscillator	1.4	0.03492	2.454	0.2530
logarithmic	1.5	0.02403	1.162	0.1970
power-like	1.334	0.02775	1.549	0.1851
Cornell	1.84	0.02159	0.938	0.2490
Buchmüller-Tye	1.48	0.02687	1.453	0.2149
experiment [8]	-	0.03266 ± 0.01209	2.147 ± 1.589	-

right panel of Fig. 5 presents the prediction for the normalized transition form factor for $\eta_c(2S)$. Rather different results are obtained with each potential model. We noticed that the best description of the data is given by the model with $m_c = 1.334$ GeV. Moreover, we observed a strong dependence on the quark mass.

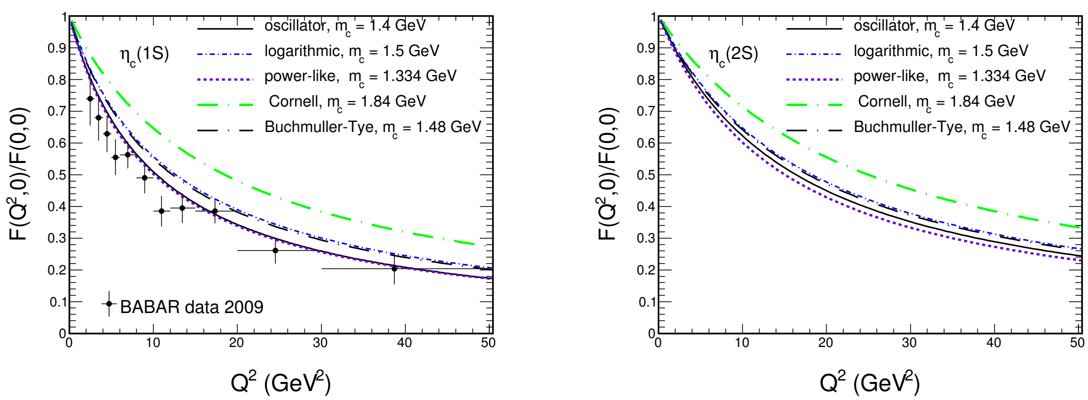

Figure 5: *Normalized transition form factor:* $F(Q^2, 0)/F(0,0)$ *as a function of photon virtuality* Q^2. *The BABAR data are shown for comparison* [10]

4 Conclusion

The transition form factors for different wave functions, obtained as a solution of the Schrödinger equation for the $c\bar{c}$ system for different phenomenological $c\bar{c}$ potentials from the literature, were calculated in Ref [6], where more details and results can be found. We have studied the transition form factors for $\gamma^*\gamma^* \to \eta_c(1S, 2S)$ for two space-like virtual photons, which can be accessed experimentally in future measurements of the cross section for the $e^+e^- \to e^+e^-\eta_c$ process in the double-tag mode. The transition form factor for only one off-shell photon as a function of its virtuality has been studied and compared to the BaBar data for the $\eta_c(1S)$ case. The dependence of the transition form factor on the virtuality has been studied as well.

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