

The principle of finiteness – a guideline for physical laws

Abraham Sternlieb

Ariel University Center for Samaria, ISRAEL

E-mail: asternlieb@ariel.ac.il

Abstract. I propose a new principle in physics—the principle of finiteness (FP). It stems from the definition of physics as a science that deals with measurable dimensional physical quantities. Since measurement results including their errors, are always finite, FP postulates that the mathematical formulation of legitimate laws in physics should prevent exactly zero or infinite solutions. I propose finiteness as a postulate, as opposed to a statement whose validity has to be corroborated by, or derived theoretically or experimentally from other facts, theories or principles.

Some consequences of FP are discussed, first in general, and then more specifically in the fields of special relativity, quantum mechanics, and quantum gravity. The corrected Lorentz transformations include an additional translation term depending on the minimum length epsilon. The relativistic gamma is replaced by a corrected gamma, that is finite for $v=c$. To comply with FP, physical laws should include the relevant extremum finite values in their mathematical formulation.

An important prediction of FP is that there is a maximum attainable relativistic mass/energy which is the same for all subatomic particles, meaning that there is a maximum theoretical value for cosmic rays energy. The Generalized Uncertainty Principle required by Quantum Gravity is actually a necessary consequence of FP at Planck's scale. Therefore, FP may possibly contribute to the axiomatic foundation of Quantum Gravity.

1. Introduction: the principle of “finiteness”

The notion of finiteness of physical entities has appeared in the scientific literature for quite some time, as opposed to either the notion of absolute zero or to the notion of infinity. The Born and Infeld model [1], trying to deal with the infinite self-energy of a point charge, was founded on a “principle of finiteness” that says (in their words) that a satisfactory theory should avoid physical quantities becoming infinite (this was before the arrival of renormalization). Albert Einstein [2], referring to the meaning of R_s , the Schwarzschild radius, and the black-hole singularity, wrote that physical models, cannot exhibit such a singularity, including the need to modify his own general relativity in the future, when new relevant evidence will be available.

A. Vilenkin [3] has proposed a cosmological model that is intended to avoid the Big-Bang singularity.

T. D. Lee [4] mentions the necessity of time discretization because of the finite number of measurements that can be performed in a finite time.



Gambini and Pullin [5] discuss loss of coherence and unitarity in the measurement process and consequently obtain an expression for a minimum possible clock accuracy:

$$\delta t \propto t_{pl}^{2/3} \bullet t^{1/3} \quad (t_{pl} = \text{Planck time}).$$

Refs. [6] and [7] mention that infinities generally arise because of the point-like definition of elementary particles, and that string theory introduces finiteness (finite-sized strings) as a possible remedy. Finally, to conclude this literature sampling, in Ref. [8] mention is made of the (until now unknown) contributions of Ettore Majorana to the ideas of elementary (finite) length and time scales. The central theme evolving from the above-mentioned references, is that there is no physical reality besides measurable entities. In other words, there is no place in physical models for zero-size (point-like) entities or for infinite (singular) entities.

It is clear that the measurement process and its outcome are of outmost importance in defining the “legitimate” content of physics, as we shall see in the following. We begin our discussion by mentioning summarily the axiomatic basis of special relativity.

Albert Einstein developed his special relativity theory on the basis of two principles (postulates): a) the constancy of the speed of light, and b) the principle of special relativity, that requires invariance of the mathematical expressions of “legitimate” physical laws (under appropriate transformations) when passing from one inertial frame of reference to another which is moving at a constant velocity with respect to the first. The two postulates were chosen because of being based on experimental evidence and on its logical consequences, as available at that time.

By adhering to his postulates and studying carefully the measurement process, Einstein derived the Lorentz Transformations (which had been invented before him, ad-hoc, by Lorentz), which explained naturally the length contraction and time dilation. As a consequence, the immovable ether frame of reference became superfluous.

New principles (postulates) in physics do not appear frequently. They should be based on well-established experimental facts and/or on reasonable logical thinking. Also, they should be as general as possible, in order to be relevant to most of physics (Special and General Relativity, Quantum Mechanics, etc.)

Accordingly, it seems desirable to derive such a postulate from the very definition of physics. This would be a fundamental postulate, to which the laws of physics should obey.

I choose to (ad-hoc) define physics as follows: physics is the science that deals (not exclusively) with well-defined physical quantities, including theories about their functional relationships. It also deals with measurement methodologies relevant to the measurable quantities. Concerning the complex relationship between theory and measurement (experiment), one should consult books on the philosophy of science [9, 10], which is outside the scope of this paper.

In measurement methodology there is always a need to include a measurement error (“accuracy”, or “error bar”) when stating an experimental result R:

$$R = x \pm \Delta x \quad (1)$$

where x is the expected value (single or average) of a measurement result of any physical measurable dimensional quantity/entity, and Δx (>0) is the measurement error (accuracy).

It is not within the scope of this paper to discuss the theory of measurement error. At this point I would like to mention that errors can be improved (reduced) by increasing the number

of measurements, or, by improving the measurement methods. However, even if measurement errors (accuracy) of measurable quantities do improve in the course of time, they have never become exactly zero, because of finiteness of resolution of measurement devices and because of finiteness of possible number of measurements.

It would be safe to assume that errors will never become exactly zero (for example, as saying that “*c*”— the speed of light in vacuum—is never going to change). This is basically because non-zero accuracy is a consequence of the finiteness of human capabilities, and this is a rather permanent and fundamental limitation.

The summary of the discussion above is that accuracy (error) of any physical measurable dimensional quantity is always finite (non-zero):

$$\Delta x \geq \varepsilon > 0 \quad (2)$$

ε is the lower limit value of error for the physical entity called x . In this section (section 1), ε is positive.

Usually there is a finite ratio between a measurement result and its error Δx . It follows that measurement results of any physical measurable dimensional quantity cannot be exactly zero, i.e. they are finite. Even if $x = 0$ (which might occur), for the net results R , $|R| = \Delta x \geq \varepsilon > 0$. Therefore we assert that the result R can never be $R = 0 \pm 0$. Therefore:

$$|R| \geq \varepsilon > 0 \quad (3)$$

According to the definition of physics, as stated above, we conclude that values of exactly zero should not be allowed as acceptable solutions of mathematical expressions describing legitimate physical laws, because exactly zero experimental results (0 ± 0) do not occur. A corollary would be that physical laws should include ε in their formulation.

Next, we discuss very shortly the infinity problem. For reasons similar to those stated above, measurement results are always finite, this time in the sense of being non-infinite:

$$|R| \leq E < \infty \quad (4)$$

E is the upper limit value for the physical entity called x . To summarize our discussion till now, measurement results are always finite, namely non-zero and non-infinite:

$$0 < \varepsilon \leq |R| \leq E < \infty \quad (5)$$

Physical laws are formulated by means of mathematical expressions. As a consequence of the previous discussion, I propose that a new basic principle (postulate) of physics should be formulated, as follows:

“A legitimate law of physics is one whose mathematical expression does not allow exactly zero or infinite solutions as possible values for the measurable dimensional physical quantities present in the expression.”

I call this the “Principle of Finiteness”. By logical terminology, this is established as a necessary condition for the “legitimacy” of physical laws.

In the following section, we explore some implications of the finiteness principle when applied to several well-known laws of physics. Deliberately, I do this in a mostly deductive way, independently of any other physical theory or principle, besides, perhaps, the “simplicity” principle, as we shall see later.

2. Some consequences of the finiteness principle

2.1. General consequences

There are some immediate general consequences that can be derived from the finiteness principle:

a) Every measurable dimensional physical entity (quantity) has a minimum finite (non-zero) value (ε) and a maximum finite (non-infinite) value (E).

The minimum finite (non-zero)—or—the maximum finite (non-infinite) values of any measurable dimensional physical quantity are by definition impenetrable (irreducible) limits, namely no lower, or respectively no higher, values, are allowed.

b) Mathematical expressions of physical laws, which do not comply with the finiteness principle, should be changed accordingly in the simplest possible way (the “simplicity” principle), desirably without violating existing well-established principles.

2.2. Consequences of the finiteness principle in special relativity

I follow Einstein’s derivation of the Lorentz Transformation ([11], 1920). We use two coordinate systems, k and k' , whose x -axes permanently coincide. We consider only events on the x -axis. An event is represented in system k by distance x (from k -origin) and time t , and in system k' by distance x' (from k' -origin) and time t' .

The origin of k' is moving relative to k with velocity v , along the x -axis. We assume that at $t = t' = 0$, the origins of k and k' coincide, and a light-signal is transmitted from the origin along the positive x -axis. The choice of $t=0$ is, of course, arbitrary. We can set $t=t_0$, where t_0 is non-zero, and translate to zero by a variable change.

We follow faithfully Einstein’s derivation, with the difference that I include in my derivation a minimum finite length error ε in his initial equations, as required by the Finiteness Principle (ε is the “Gedanken” minimum length error ([11], 1905)).

In this section (2.2), we assume that ε is a real invariant number (same in k and k').

Einstein’s original equations for the light signal are:

$$\left. \begin{aligned} x - ct &= 0 \\ x' - ct' &= 0 \end{aligned} \right\}$$

for the positive x -axis.

My corrected equations become:

$$\left. \begin{aligned} x - ct &= \varepsilon \\ x' - ct' &= \varepsilon \end{aligned} \right\} \quad (6)$$

Similar relations are obtained for the negative x -axis.

After some calculations we get the corrected Lorentz Transformations:

$$\left\{ \begin{aligned} x' &= \gamma x - v\gamma t + \varepsilon(1 - \gamma) \\ t' &= -\frac{v\gamma}{c^2}x + \gamma t + \frac{\varepsilon v\gamma}{c^2} \end{aligned} \right\} \quad (7)$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \quad (8)$$

It is important to notice that the length measurement error ε is an integral part of the Lorentz Transformations.

The corrected relativistic length contraction factor Γ_{CL} becomes:

$$\Gamma_{CL} \equiv \frac{L_C}{L_0} = \frac{1}{\gamma} \left[1 - \frac{\varepsilon(1 - \gamma)}{L_0} \right], \quad (9)$$

where $\frac{1}{\gamma}$ is the usual contraction factor ($\varepsilon = 0$), L_0 is length measured at rest, and L_C is contracted length (should be $\geq \varepsilon$ according to the Finiteness Principle, where ε is the minimum length).

Two remarks are in order:

- a) The maximal possible contraction of any distance L_0 (from (9)) is (ε/L_0) , as it should be, as required by the finiteness principle.
- b) If $L_0 = \varepsilon$, $\Gamma_{CL} = 1$ (as it should be).

Next, the corrected relativistic time-dilation factor is:

$$\Gamma_{CT} \equiv \frac{\tau_D}{\tau_0} = \gamma \left[1 - \frac{v\varepsilon}{\tau_0 c^2} \right] \quad (10)$$

τ_0 = time duration measured at rest

τ_D = dilated time duration

γ = usual time dilation ($\varepsilon = 0$)

Two remarks are in order:

- a) one can test (10) by trying to detect changes in lifetimes of ultra-high velocity ($v \sim c$) and/or very short-lived subatomic particles. In this case we may use:

$$\frac{\tau_D}{\tau_0} \cong \gamma \left[1 - \frac{\varepsilon}{\tau_0 c} \right] \quad (11)$$

to detect departure from γ which is larger than γ 's measurement error.

- b) We notice that Γ_{CT} goes to infinity when $v \rightarrow c$. In this case and in general, the finiteness principle requires that γ should be replaced by a corrected function γ_c that satisfies the following requirements:
 - i. γ_c should be finite for $v=c$ (consequently we should get finite extremum values for all physical quantities).
 - ii. γ_c should be identically equal to 1 (one) for all velocities when rest-frame (proper) measured values of physical quantities are equal to their extremum values.
 - iii. Otherwise, γ_c should be identical to γ (of course, this should include all known physics and obviously much beyond).

It is easy to show that a simple function satisfying these requirements is:

$$\gamma_C \equiv \gamma \left[\frac{1 - e^{-\frac{\alpha}{\gamma}}}{1 - e^{-\alpha}} \right] \quad (12)$$

where α is a non-negative dimensionless number. We do not know yet how to derive eq.(12) **ab initio**, and we do not know if this equation is simplest or unique.

We require the replacement of γ by γ_C in Lorentz transformations and in the Special Relativity Lagrangian. Lorentz invariance and the Lorentz transformation group property are not observed for cases (i) and (ii) above, but are observed for case (iii) which includes all known physics.

In general, α is a function of the rest-frame values and extremum values of physical quantities. Usually α is extremely large, besides case (ii) above, where α is zero.

We will estimate later in the paper the value of α for subatomic particles

In this case γ_C is practically identical to γ , unless v is extremely close to c (ultra-high velocities). In this case equation (10) becomes:

$$\frac{\tau_D}{\tau_0} = \gamma \left[\frac{1 - e^{-\frac{\alpha}{\gamma}}}{1 - e^{-\alpha}} \right] \left[1 - \frac{v\varepsilon}{\tau_0 c^2} \right] \quad (13)$$

When $\gamma \gg \alpha$, we obtain (ultra-high velocities regime):

$$\frac{\tau_D^{\max}}{\tau_0} = \left[\frac{\alpha}{1 - e^{-\alpha}} \right] \left[1 - \frac{\varepsilon}{\tau_0 c} \right] \quad (14)$$

Since α is very large:

$$\frac{\tau_D^{\max}}{\tau_0} \approx \alpha \left[1 - \frac{\varepsilon}{\tau_0 c} \right] \quad (15)$$

τ_D^{\max} = maximum allowable time dilation (!)

Assuming $\tau_0 \gg \frac{\varepsilon}{c}$, we get:

$$\tau_D^{\max} \approx \alpha \tau_0$$

This and eq. (15) may be tested under appropriate conditions.

We now use (12) to correct Einstein's mass/energy equation such as to obey the Finiteness Principle.

Einstein's equation:

$$m = m_0 \gamma \quad (16)$$

m_0 = rest mass

The corrected equation becomes:

$$m = m_0 \gamma \left[\frac{1 - e^{-\frac{\alpha}{\gamma}}}{1 - e^{-\alpha}} \right] \quad (17)$$

Then, for $\gamma \gg \alpha$, we obtain:

$$m_{\max} \approx m_0 \alpha \quad (18)$$

m_{\max} =maximum attainable relativistic mass (energy) (!)

In Section 2.3 we estimate α for sub-atomic particles.

As we shall see, m_{\max} is a fixed value for all sub-atomic particles, irrespective of m_0 . In this case, the maximum possible relativistic mass/energy for all sub-atomic particles is a constant:

$$E_{\max} = m_{\max}c^2 \quad (19)$$

We conclude that length contraction and time dilation depend on ε , γ , and also on rest values of length and time interval being measured. Finally it is easy to show that the relativistic velocity addition law and $E = mc^2$ do not change, because the new translational terms in (7) are time-independent.

2.3. Consequences in quantum mechanics

I choose to apply the Finiteness Principle to the uncertainty principle, because of its central significance in Quantum Mechanics.:

$$\Delta x \cdot \Delta p \sim h \quad (20)$$

$$\bullet\bullet \Delta x \sim \frac{h}{\Delta p} \quad (21)$$

h is Planck's constant

In this discussion, Δx and Δp are assumed positive.

According to the Finiteness Principle, we have to correct the above equation such that Δx cannot become less than ε , the minimum length error (same ε as in Section 2.2) , for any finite value of Δp .

Note: in this section, (2.3), $\varepsilon > 0$.

Equation (21) represents Δx as a decreasing function of Δp . According to the Finiteness Principle, Δx should become a corrected function of Δp , which has a minimum finite value $\varepsilon > 0$, for a finite value of Δp .

The simplest (increasing) function to ensure this and to be added to (21) is: $a\Delta p$, where a is a positive number, to be estimated in the following.

Therefore the necessary corrected equation is (see paragraph 2.1.b above) :

$$\Delta x \sim \frac{h}{\Delta p} + a\Delta p \quad (22)$$

To find a , we set the derivative of (22) to zero, which gives the minimum:

$$-\frac{h}{(\Delta p)_{\min}^2} + a = 0 \quad (23)$$

So:

$$(\Delta p)_{\min} = \sqrt{\frac{h}{a}} \quad (24)$$

$(\Delta p)_{\min}$ is the Δp value for which Δx is a minimum ($\Delta x = \varepsilon > 0$).

We insert (24) in (22), and remember that: $(\Delta x)_{\min} = \varepsilon > 0$. We get:

$$\varepsilon = \frac{h}{\sqrt{\frac{h}{a}}} + a\sqrt{\frac{h}{a}} \quad (25)$$

Then we get:

$$a = \frac{\varepsilon^2}{4h} \quad (26)$$

Therefore the simplest corrected form of the uncertainty principle that obeys the Finiteness Principle is:

$$\Delta x \sim \frac{h}{\Delta p} + \frac{\varepsilon^2}{4h} \Delta p \quad (27)$$

We summarize the minimum coordinates of (27):

$$\left. \begin{aligned} (\Delta x)_{\min} &= \varepsilon \\ (\Delta p)_{\min} &= \frac{2h}{\varepsilon} \end{aligned} \right\} \quad (28)$$

Now we estimate α (see (12)) for particles obeying the corrected uncertainty principle. We assume that $(\Delta p)_{\min}$ is also the maximum value of Δp if we want equation (27) to be a single-valued monotonous function.

We obtain:

$$\frac{2h}{\varepsilon} = \max(\gamma_C m_0 v) \quad (29)$$

(since γ_C replaces γ for $v \sim c$, see equation (12)); therefore:

$$\max(\gamma_C m_0 v) = m_0 \max(\gamma_C v) = m_0 \max(\gamma_C c) = m_0 c \max(\gamma_C) = m_0 c \alpha$$

and we have:

$$\begin{aligned} \frac{2h}{\varepsilon} &= \alpha m_0 c \\ \bullet\bullet \alpha &= \left(\frac{2h}{\varepsilon c} \right) \frac{1}{m_0} \end{aligned} \quad (30)$$

Then equation (18) becomes:

$$m_{\max} = m_0 \alpha = \frac{2h}{\varepsilon c} \quad (31)$$

and equation (19) becomes:

$$E_{\max} = \alpha m_0 c^2 = \frac{2h}{\varepsilon c} c^2 = \frac{2hc}{\varepsilon} \quad (32)$$

Therefore we conclude that all sub-atomic particles have the same maximum possible relativistic mass/energy, irrespective of their rest mass/energy. They obtain this value at $v \approx c$ (ultra-high velocities).

3. Discussion

Before continuing, we remark parenthetically that contradictions may arise between the consequences of the finiteness principle and the consequences of other well-established principles. In this case, the discrepancies might occur because different principles may deal with different regimes of the parameters (e.g., different velocity ranges).

Returning to the discussion on the results obtained in this paper, we first note that equation (27), with $\varepsilon = l_p$ (Planck's length) is required in quantum gravity as a replacement for the usual uncertainty principle, and it is referred to as the "generalized uncertainty principle" [12]. In this context a generalized quantum-mechanical Hamiltonian is constructed, on the basis of the generalized uncertainty principle, which becomes important at the Planck scale. In Ref. [13] a generalized uncertainty principle is derived from micro-black hole Gedanken experiment, using only the uncertainty principle and R_s (Schwarzschild radius).

Historically, a generalized uncertainty principle was first derived in [14] and then in [15] with what corresponds to our $\varepsilon = l_p$, and $l_p^2 \approx \alpha'$ (string tension).

From the results obtained in our paper, it is clear that the Finiteness Principle explains in a fundamental way the additional term in the generalized uncertainty principle ($\varepsilon = l_p$ is a special case, representing the inevitable gravitational interference uncertainty). It is indeed very encouraging to find that quantum gravity is consistent with the Finiteness Principle.

It is also important to mention that the T-Duality in string theory requires a minimum observable length $R_{\min} \sim \alpha'^2$ and brings about a generalized uncertainty principle [16, 17]. This means that T-Duality (and maybe other aspects of string theory) is intimately connected with the Finiteness Principle.

Next, we note that neutrino measurements data are very important in testing the consequences of the Finiteness Principle, because of their $m_0 \neq 0$, and $v \sim c$ [see eqs. (17), (18), (19), (31), (32) with $\varepsilon = l_p$ (Planck length) and $m_{\max} = m_p$ (Planck mass)]. It is especially significant to have experiments giving each at least two of the three following parameters: energy, rest-mass, speed.

However, the usefulness of the existing neutrino data [18, 19, 20, 21, 22, 23] is limited, because of lack of ultra-high energy measurements, lack of absolute mass measurements, and finally, lack of sufficient accuracy of speed measurement at $v \sim c$.

Finally, because of the importance of the fundamental constants to the Planck scale limits, astronomical measurements are being made and evaluated extensively, in order to determine the accuracy and constancy of various parameters [24].

4. Conclusions

I propose a new postulate in physics, the Principle of Finiteness. I believe it is a very basic postulate, to which any "legitimate" physical law should obey. I explore the implications of the finiteness principle for some simple relations of special relativity and quantum mechanics.

The corrected relativistic length contraction and time dilation factors depend on: ε , γ , τ_0 , L_0 , and m_0 (through α).

An important prediction is that the maximum relativistic mass/energy attainable is the same

for all sub-atomic particles (independent of m_0).

It also seems that the generalized uncertainty principle required by quantum gravity (and T-duality) as a replacement for the usual Heisenberg uncertainty principle, is actually a necessary consequence of the Finiteness Principle. It follows that quantum gravity is consistent with the Finiteness Principle. The Finiteness Principle apparently explains in a fundamental way the need of quantum gravity to replace the usual uncertainty principle with a corrected (“generalized”) version, at the minimum length scale.

Therefore, the Finiteness Principle may contribute to the axiomatic foundation of quantum gravity, in addition to many other consequences.

Acknowledgments

It is my pleasure to acknowledge useful remarks by: Prof. J. Bekenstein, Prof. L. Dorman, Prof. N. Fisch, Prof. S. Cohen, Prof. A. Yahalom, Prof. L. Horwitz, Dr. A. Yarom, and Prof. A. Polyakov.

I would like to thank J. A. Baumgaertel and L. Peterson for carefully preparing the manuscript for publication.

References

- [1] Born M 1934 *Proc. R. Soc. A.* **143** 410
Born M and Infeld L 1934 *Proc. R. Soc. A.* **144** 425
- [2] Einstein A 1939 *Annals of Mathematics* **40** (4) 922-936
- [3] Vilenkin A 1982 *Phys. Lett. B* **117** (1-2) 25-28
- [4] Lee T D 1983 *Phys. Lett. B* **122** 217
Lee T D 1987 *J. Stat. Phys.* **46** 843
- [5] Gambini R and Pullin J. 2007 *Found. Phys.* **37** (7) 1074-1092
- [6] Nicolai H 2009 *Physics* **2** 70
- [7] Bern Z, Carrasco J J, Dixon LJ, Johansson H and Roiban R 2009 *Phys. Rev. Lett.* **103** 081301
- [8] Esposito S and Salesi G 2010 *Ann. Phys.* Berlin 1-11
- [9] Popper KR 1968 *The Logic of Scientific Discovery* Hutchinson, London
- [10] Margenau H 1950 *The Nature of Physical Reality* McGraw-Hill, New-York
- [11] Einstein A 1920 *Relativity: The Special and General Theory* Henry Holt, New York
Einstein A 1905 On the Electrodynamics of Moving Bodies *Annalen der Physik*, **17** footnote No.3.
Footnote text: “We shall not here discuss the inexactitude which lurks in the concept of simultaneity of two events at approximately the same place, which can only be removed by an abstraction.” If the Principle of Finiteness is accepted then this abstraction is prohibited and the inexactitude remains.
- [12] Das S and Vagenas EC 2009 *Can. J. Phys.* **87** 233-240
- [13] Scardigli F 1999 *Phys. Lett. B* **452** (1-2) 39-44
- [14] Veneziano G 1986 *Euro Phys. Lett.* **2** (3) 199
- [15] Gross DJ and Mende P F 1987 *Phys. Lett. B*, **197** 129
Gross DJ and Mende P F 1988 *Nucl. Phys. B* **303** 407
- [16] Witten E, April 1996 *Phys. Today* 24
- [17] Ne’eman Y *Ann. Phys.* Leipzig, **8** (1) 3-17
- [18] Spolyar D, Buckley M, Freese K, Hooper D and Murayama H 2009, High Energy Neutrinos as a Test of Leptophilic Dark Matter, *arxiv: 0905.4764 v1 Astro-ph.co*
- [19] *Rev. of Particle Physics* <http://pdg.lbl.gov>
- [20] Witten E 2002 *Nature* **415** 969
- [21] Bahcall JN 1989 *Neutrino Astrophysics* Cambridge Univ. Press, ISBN 052137975x
- [22] Dorman LI 2004 *Cosmic Rays in the Earth’s Atmosphere and Underground* (Kluwer Academic Publishers (Astroph. And Space Sci. Library)) 52-74.

- [23] Pagliaroli G, Rossi-Torres F and Vissani F 2010 *Astroparticle Phys.* **33** 287-291
- [24] Garcia-Berro E , Isern J and Kubyshin YA 2007 *Astron. Astroph. Rev.* **14** 113-170