

## Review

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# Dynamical Asymmetries, the Bayes' Theorem, Entanglement, and Intentionality in the Brain Functional Activity

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# Dynamical Asymmetries, the Bayes' Theorem, Entanglement, and Intentionality in the Brain Functional Activity

David Bernal-Casas <sup>1,†</sup> and Giuseppe Vitiello <sup>2,\*,†</sup> 
<sup>1</sup> Department of Genetics, Microbiology and Statistics, Faculty of Biology, University of Barcelona, 08028 Barcelona, Spain

<sup>2</sup> Department of Physics "E.R. Caianiello", University of Salerno, 84084 Fisciano, Salerno, Italy

\* Correspondence: givitiello@unisa.it; Tel.: +39-089968220

† The authors contributed equally to this work.

**Abstract:** We discuss the asymmetries of dynamical origin that are relevant to functional brain activity. The brain is permanently open to its environment, and its dissipative dynamics is characterized indeed by the asymmetries under time translation transformations and time-reversal transformations, which manifest themselves in the irreversible "arrow of time". Another asymmetry of dynamical origin arises from the breakdown of the rotational symmetry of molecular electric dipoles, triggered by incoming stimuli, which manifests in long-range dipole-dipole correlations favoring neuronal correlations. In the dissipative model, neurons, glial cells, and other biological components are classical structures. The dipole vibrational fields are quantum variables. We review the quantum field theory model of the brain proposed by Ricciardi and Umezawa and its subsequent extension to dissipative dynamics. We then show that Bayes' theorem in probability theory is intrinsic to the structure of the brain states and discuss its strict relation with entanglement phenomena and free energy minimization. The brain estimates the action with a higher Bayes probability to be taken to produce the aimed effect. Bayes' rule provides the formal basis of the intentionality in brain activity, which we also discuss in relation to mind and consciousness.

**Keywords:** brain dynamical asymmetries; arrow of time; spontaneous breakdown of symmetry; coherent states; Bayes' theorem; entanglement; free energy; heart-brain dialog; mind; consciousness



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## 1. Introduction

In this paper, we focus on the brain asymmetries of dynamical origin, rather than on discussing the asymmetries of anatomical and cellular nature, which are also crucial to the brain's functional activity.

We discuss the dynamical asymmetries under time translation transformations and time-reversal transformations, which manifest themselves in the irreversible "arrow of time". As we will see, these arise from the dissipative character of the brain dynamics.

We also discuss the asymmetry of dynamical origin resulting from the breakdown of the rotational symmetry of the molecular electric dipoles. Such a breakdown of symmetry is triggered by the incoming stimuli, and generates the dynamical formation of long-range dipole-dipole correlations, favoring neuronal correlations, their synchronous oscillations, and net formation. Dealing with elementary constituents such as molecules, their properties and their electric dipoles implies the use of the formalism of quantum field theory (QFT), as originally proposed by Ricciardi and Umezawa in their study of the brain as a many-body system [1–5], and also adopted in its subsequent extension to dissipative dynamics [6–8].

A system possesses symmetry properties under certain transformations when these leave it unchanged, i.e., they do not induce observable changes in it. For simplicity, consider a set of identical objects, e.g., letters  $AAAA \dots$ . The  $A$ 's can be exchanged between them without modifying the 'system' (symmetry under permutation),  $AAA \dots \rightarrow AAA \dots$ . Similarly, the  $A$ 's in the  $ABA$  system can be exchanged without observable changes (bilateral

symmetry). The permutation of the  $A$ 's does not produce changes in the system since (the information of) the position of each of the  $A$ 's is irrelevant. In general, symmetry denotes indeed a lack of information.

Suppose now to “break the symmetry” by introducing a suffix to the  $A$ 's,  $A_1A_2A_3 \dots$  or  $A_1BA_2$ . Differences then appear when the  $A$ 's positions are exchanged,  $A_1A_2A_3 \dots$  is different from  $A_2A_1A_3 \dots$ , etc., and  $A_1BA_2 \neq A_2BA_1$ . The different “ordering” of the  $A$ 's corresponding to their indices now carries non-empty information.

Thus, we see that the (indices) diversity allows ordering, and vice versa, order implies diversity, i.e., the possibility of distinguishing elements in the set. On the other hand, it is well known that patterns of differently ordered elements carry different information contents (an example in the Italian words: amor, Roma, ramo, Omar, etc.). In conclusion, the breaking of symmetry generates ordering; order is a lack of symmetry.

Asymmetries, or lack of symmetry, arising through the process of spontaneous breakdown of symmetry (SBS) in QFT are widely studied in condensed matter physics and high energy physics, with experimentally confirmed results. Observable dynamical ordering of the elementary constituents of the system is generated by the spontaneous breakdown of continuous symmetry.

Ordering is a relational property of the elements, or, in the physics jargon, it is a collective property of the elements, it is a system property, not of the individual elements. The relationship between the spontaneous breakdown of continuous symmetries and order reveals itself in the form of the dynamics of coherence, based on the being-in-phase of the elementary components of the physical systems. The dynamics of coherence appears to be a fundamental ruling law of nature.

In their work, Ricciardi and Umezawa do not contrast QFT techniques with those based on biochemistry commonly used in neuroscience. They rather explore the basic dynamics that support neuronal activity and can explain the extraordinary functional efficiency of the brain, otherwise impossible to explain, then as today, only as a result of the intrinsic randomness that regulates biochemical interactions. The detailed study of nerve cells and their connections in the brain structure is certainly *necessary*, but it is *not sufficient* to explain the high efficiency of brain activity. Schrödinger had already underlined in his lectures [9] in *What is life?*, in 1944, that in biological systems, the “regularities only on average” emerging from the “statistical mechanisms” (p. 78) are not enough to explain the “enigmatic biological stability” (p. 47), so that the “expectation of the classical physicist [...] far from being trivial, it is wrong” (p. 19). These observations by Schrödinger on biological systems in general are even more relevant in the case of the brain. In the same years, Karl Lashley became aware of these problems from his laboratory observations and raised the alarm in the form of a “dilemma” [10]:

“[...] Here is the dilemma. Nerve impulses are transmitted [...] from cell to cell through definite intercellular connections. Yet, all behavior seems to be determined by masses of excitation [...] within general fields of activity, without regard to particular nerve cells [...] What sort of nervous organization might be capable of responding to a pattern of excitation without limited specialized path of conduction? The problem is almost universal in the activity of the nervous system”.

Umezawa asked [5] the question of whether there was a long-range correlation associated with the brain. If there is such a correlation, “each constituent of the system should be trapped by this correlation and its individual behavior should not be freely exhibited and should instead be controlled by the correlation. In that case, we do not observe individual cells, but quasi-cells (in analogy to the term quasi-particles)”. The experimental observation [1] of the “existence of similar and almost simultaneous responses in several regions of the brain (a kind of *long-range correlation*) [...] does not find any explanation in terms of activity of the single nerve cells: new *non-classical* mechanisms have to be looked for [...] It then arises naturally a possibility: [...] one could attempt to give a *more general* description of brain dynamics [...] it is strongly suggestive of a *quantum model*” (the authors’ emphasis).

On the other hand, the profound difference between artificial neural networks and the natural brain has been well described by John von Neumann in 1958 [11]: “The mathematical or logical language truly used by the central nervous system is characterized by a less logical and arithmetical depth than what we are normally used to. [...] We need exquisite numerical precision over many logical steps to achieve what brains accomplish in very few short steps”.

The plan of the paper is the following. We review in Section 2 the quantum field theory model of the brain by Ricciardi and Umezawa, and in Sections 3 and 4 its subsequent extension to the dissipative dynamics. We discuss the spontaneous breakdown of the continuous dipole rotational symmetry, of continuous time translational symmetry, and the breakdown of the time-reversal symmetry, the role played by the entropy, and the processes of memory recording, recalling, and memory forgetting. Some aspects of the quantum dissipative model of the brain are discussed with particular reference to the dynamic nature of memory, the activity of the mind, and the heart–brain dialog. Aspects related to dreams, deterministic chaos, fractal self-similarity, and the social dimension of brain activity are also very briefly mentioned. We then show that Bayes’ theorem or rule [12,13] in probability theory is intrinsic to the structure of the brain states, and it provides the formal basis of the intentional activity of the brain. In Section 5, we discuss the strict relation of Bayes’ rule with entanglement and free energy minimization in the brain memory states. In Section 6, we comment on the intentionality, mind, and consciousness. Remarks on the brain–heart dialog are presented in Section 7. There, the relevance of Bayes’ rule, entanglement phenomena, and free energy minimization in general physical systems is also mentioned, together with concluding remarks and perspectives for further works. Some of the formal details are in Appendix A.

## 2. The Many-Body Model of the Brain

Ricciardi and Umezawa in their 1967 work, and also in 1978 and 1979 in their works with Stuart and Takahashi [3,4], observe that artificial neural network models are certainly of great use in computation and control theory, “however in the case of natural brains, it might be pure optimism to hope to determine the numerical values for the coupling coefficients and the thresholds of all neurons by means of anatomical or physiological methods” [1], considering that there are myriads of complex components, approximately  $10^{11}$  neurons, each one connected to other  $10^4$  neurons,  $10^{15}$  synapses, without considering the glial cells, all in a bath of water molecules, which in number constitute over 90% of the present molecules. Ricciardi and Umezawa therefore ask themselves [1]:

“[...] First of all, at which level should the brain be studied and described? In other words, is it essential to know the behavior in time of any single neuron in order to understand the behavior of natural brains? Probably the answer is negative. The behavior of any single neuron should not be significant for the functioning of the whole brain, otherwise higher and higher degree of malfunctioning should be observed unless to assume the existence of “special” neurons, characterized by an exceptionally long half-life: or to postulate a huge redundancy in the circuitry of the brain. However, up to our knowledge, there has been no evidence which shows the existence of such “special” neurons, and to invoke the redundancy is not the best way to answer the question”.

Walter Freeman notes [14] that laboratory observations suggest that perception cannot be attributed solely to the properties of the single neuron, as happens in the “microscopic approach that currently dominates neuroscience research. We have found that perception depends on the simultaneous, cooperative activity of millions of neurons spread throughout expanses of the cortex. Such global activity can be identified, measured and explained only if one adopts a macroscopic view alongside the microscopic one”.

Clearly, these observations are on the same line of what Lashley said in his “dilemma”. We will see below that the “masses of excitations” observed by him and also highlighted with the modern experimental techniques of fMRI and PET, find their description in the QFT formalism in terms of “collective modes” of the elementary components of the brain system.

The dynamical mechanism that Ricciardi and Umezawa (RU) place at the basis of their quantum (many-body) model is the one of SBS in QFT [15–18]. To illustrate this, consider that the equations of motion, which describe the temporal evolution (of the fields) of our system, generally have symmetry properties with respect to certain transformations characterizing the system itself. When the state of minimum energy (the ground state, or vacuum) does not have the same symmetry properties of the equations of motion, then spontaneous breakdown of symmetry (SBS) occurs. This can happen as a result of a stimulus, even a very weak one, to which the system is subjected. The ‘breakdown’ is called spontaneous because the system situates itself in the asymmetric ground state on the basis of its internal dynamics, in the given boundary conditions, for example, temperature.

The Goldstone theorem in QFT [19,20] states that when spontaneous breakdown of a continuous symmetry occurs, the dynamics of the system generates long-range correlations between its elementary components (long-range compared to the linear dimensions of the elementary components), thus producing ordered patterns. Ordering is possible as long as the correlations do not interfere negatively, i.e., they are coherent.

In quantum theories, one can describe these long-range correlations in terms of quanta, the Nambu–Goldstone (NG) quanta. They are bosons and satisfy the Bose–Einstein quantum statistics. The ordering is then described in terms of coherent condensation of the NG quanta in the ground state, which is therefore a coherent state [21], i.e., there is phase coherence between the microscopic variables of the system. Their dynamics is that of a collective mode. Examples of the NG quanta in condensed matter physics are magnons in magnets and phonons in crystals. A measure of the order is provided by the “order parameter”, which is a classical field, i.e., not subject to quantum fluctuations. In QFT ordering from coherent condensation of NG quanta is observed in a wide range of temperatures (cf. comments in the Appendix A).

This is a crucial point. The phenomenon of SBS and the consequent formation of coherent structures in the ground state allows the dynamic transition from the quantum (microscopic) level to the classical (macroscopic) one. We then speak of macroscopic quantum systems, in the sense that quantities characterizing classical behavior (for example, the order parameter) can only be computed by resorting to the quantum dynamics of the microscopic level.

In the RU model, “memory recording” is dynamically generated by the coherent condensation process of long-range correlations (the NG quanta) in the asymmetric ground state [1,3–5,22]. These are dynamical correlations, not “hard connections” (wiring); they provide the physical realization of the plasticity of the brain, those “general fields of activity, without regard to particular nerve cells”, the configurations “without a limited and specialized conduction pathway”, mentioned by Lashley in his “dilemma”. And, as Freeman reiterates [23], “bioelectric waves in the brain can be stopped by cold treatments, electric shocks or drugs, without memory loss after recovery, and furthermore, memory is not lost in many ablation experiments or when a brain is cut in many directions so that certainly some pre-existing networks are destroyed”. He adds that “memories” do not reside in circuits and neuronal networks (they are not wired). In this sense, with reference to the long-range correlations observed between neurons, Karl Pribram suggested [24] to adopt the concepts of hologram and coherence developed in laser physics.

Neurons, glial cells, and other biological components are considered classical, not quantum structures in the RU model (and in the dissipative quantum model illustrated below), contrary to what happens in other models in the literature on quantum brain and consciousness [8,25]. The problem of what the quantum variables are in the RU model is left by the authors for a later moment. The model shows how crucial is the role of the SBS leading to the asymmetries realized by NG condensation domains.

The RU model, however, has a very limited memory capacity, contrary to what is observed in the natural brain. Once a memory is recorded as a process triggered by an external stimulus, the subsequent stimulus produces an “overprinting” of the new memory over the previous one, and therefore its cancellation. In the dissipative quantum model

of the brain, this difficulty is overcome, and the quantum variables are identified with molecular dipole vibrational fields.

### 3. Dissipation and the Asymmetry of the Arrow of Time

In 1983, inspired by the works of Fröhlich [26], Del Giudice et al. proposed [27–29] to use the QFT formalism of the spontaneous breakdown of the rotational symmetry of the electric dipoles of water molecules in the study of biological systems. In the human body, water is present in the amount of 70% by weight and 90% by number of molecules. The chemical and electrochemical activity of a large number of macromolecules are in fact not possible except in the presence of water. We just mention that in 1986, one of the results obtained in this approach was the derivation of the inner diameter of the microtubules, computed to be of about 14.6 nm [29], in very good agreement with the observed one of about 15 nm. As far as we know, there is no other theoretical derivation in the literature, in spite of the great relevance of the role of microtubules in biological systems. Subsequently, in 1992, Jibu and Yasue [30,31] proposed that water should be the elementary constituent in the construction of the RU model. Independently, the dissipative model of the brain was proposed [6], where quantum variables are identified in the electric dipole fluctuations of the water molecule.

In the dissipative model, the memories are recorded as in the RU model and described by the condensation of the NG quanta associated with the dipole wave correlations (the dipole wave quanta, DWQ) [6] (see also [32–37]). Observations made with EEG, ECoG, fNMR, and other techniques reveal that domains of myriad neurons enter into coherent oscillations modulated in amplitude (AM) and phase (PM). They form within a few milliseconds (ms), persist over a range of 80–120 ms, have carrier frequencies in the range of 12–80 Hz (the beta–gamma range), and resynchronize with frequencies in the theta–alpha band (3–12 Hz), extend from a few mm up to 19 cm in linear size in humans and most of the hemisphere in rabbits and cats.

The analysis [23,33] of the observed time scales and spatial extension leads to the exclusion of the possibility that they are generated by the propagation of chemical vectors, which would be too slow. Furthermore, cortical domains are observed to abruptly jump from a receiving state to an active transmitting state. Correlations at large distances are not created by the exchange of neurotransmitters, whose propagation is therefore *not the cause, but rather the effect* of the formation of correlated domains; long-range correlations also support [38] ephaptic neuronal connection [39,40] and extra-synaptic “wireless” signaling recently reported in [41,42], where neuronal exchanges due to “neuropeptides released from dense-core vesicles and acting on longer temporal and spatial scales” have been observed to occur in the worm *C. elegans*. On the other hand, the magnetic field and electric current density supported by the extracellular dendritic trees are too weak to be responsible for the rich neuronal pattern. Neuronal radio waves (combined electric and magnetic field propagation) are also excluded due to the imbalance between the electrical permittivity and magnetic permeability (80:1) of the neural tissue. Electromagnetic waves are also excluded due to the low frequency (<100 Hz) and long wavelengths (on the order of km) at the EEG frequencies.

For a “closed” system, energy conservation is ensured by symmetry under continuous time translation (Noether’s theorem), which means that the origin of the time axis is arbitrary: it can be arbitrarily shifted (translated). This does not happen for biological systems and for the brain, whose origin, or birth, is well identified in time. For these systems, therefore, there is breakdown of the symmetry under continuous temporal translations, and the symmetry also breaks under time inversion: the asymmetry between past and future in the flow of time thus characterizes brain functional activity, the emerging *arrow of time* cannot be inverted, the temporal evolution of the brain is intrinsically irreversible.

In fact, obtaining information from the outside world introduces by itself the division of the temporal axis into “before” and “after” having obtained the information [6]:



“Now you know it”, that is, it introduces “the present” (the *Now*), which testifies of the irremediable, definitive asymmetry between past and future.

The canonical formalism (the only one available!) is developed for closed systems. It therefore requires that the open system (the brain) is considered together with its environment so that the whole behaves like a closed system, with possible flows of energy between the two. It is therefore necessary to double the number of degrees of freedom: each one of the degrees of freedom describing the system, say  $A_{\mathbf{k}}$ , is associated with a corresponding degree of freedom of the environment,  $\tilde{A}_{\mathbf{k}}$ . The subscript  $\mathbf{k}$  denotes the momentum and/or other kinematic or charge variables. A similar strategy is used in QFT at finite temperature (thermo field dynamics (TFD)) in the study of unstable systems and quantum dissipation [15–17,43].

Since the flows exiting (out) from the system are entering (in) the environment, and vice versa, the set  $(A_{\mathbf{k}}, \tilde{A}_{\mathbf{k}})$  is a closed set, and the environment describes the image “inverted in the time” of the system, as in a ‘mirror of time’, its “Double” [6,7].

We note that the doubling of the degrees of freedom consists in the doubling of the state space  $\mathcal{H} \rightarrow \mathcal{H} \times \mathcal{H}$ , and of the operator algebra  $A \rightarrow A \times A$ . We have then the  $q$ -deformed Hopf algebra with non-commutative co-product  $\Delta A = A \times q + q^{-1} \times A$ , where  $q = e^{\theta}$  for bosons (or  $q = e^{i\theta}$  for fermions).

The asymmetry between  $(A \times q)$  and  $(q^{-1} \times A)$ , implicit in the non-commutativity of  $\Delta A$  under exchange of  $(A \times q) \leftrightarrow (q^{-1} \times A)$ , allows the possibility to distinguish between the system and its environment [44], and plays thus a crucial role in the self-identification process characterizing the brain activity in relation with its environment.

Since the Bogoliubov transformations (Equations (A6) and (A7)), on which the construction of the dissipative model is based, can be obtained from the co-product  $\Delta A$ , with  $q = e^{\theta}$ , we see that its non-commutativity is at the origin of the brain dynamical asymmetries under time translation and time-reversal transformations (the arrow of time).

#### 4. Memory States and Memory Capacity

In order to make the flow of the reading easier, most of the mathematical details are reported in Appendix A. The brain memory state at the initial time  $t_0$  and finite volume  $V$ , is given by the asymmetric ground state [6]:

$$|0(\theta)\rangle_0 = \prod_{\mathbf{k}} \frac{1}{\cosh \theta_{\mathbf{k}}} \exp \left( \tanh \theta_{\mathbf{k}} A_{\mathbf{k}}^{\dagger} \tilde{A}_{\mathbf{k}}^{\dagger} \right) |0\rangle, \quad (1)$$

where  $A_{\mathbf{k}}^{\dagger}$  and  $\tilde{A}_{\mathbf{k}}^{\dagger}$  are the hermitian conjugate of  $A_{\mathbf{k}} = (1/\sqrt{2})(a_{\mathbf{k}} + \tilde{a}_{\mathbf{k}})$  and  $\tilde{A}_{\mathbf{k}} = (1/\sqrt{2})(a_{\mathbf{k}} - \tilde{a}_{\mathbf{k}})$ , with  $a_{\mathbf{k}}^{\dagger}$ ,  $\tilde{a}_{\mathbf{k}}^{\dagger}$  and  $a_{\mathbf{k}}$ ,  $\tilde{a}_{\mathbf{k}}$ , the creation and annihilation DWQ operators, respectively. We recall that the DWQ are generated by the SBS of the dipole rotational symmetry induced by the incoming stimulus. The subscript  $\mathbf{k}$  denotes the momentum,  $\theta_{\mathbf{k}}$ , will be constrained by the minimization of the free energy (see below). The Hamiltonian  $\hat{H}_0$  (Equation (A5)) is symmetric under transformations induced by  $\mathcal{G}$  in Equation (A3). The state  $|0\rangle$  is however not symmetric under  $\mathcal{G}$ , as shown by Equation (1). The asymmetry of such a state is expressed by the condensation in it of the pairs of  $(A_{\mathbf{k}}, \tilde{A}_{\mathbf{k}})$ .

We also note that  $\mathcal{G}$  is a generator of the  $SU(1,1)$  group (cf. Equations (A9) and (A10)).

The vacuum state  $|0\rangle \equiv |0, \tilde{0}\rangle \equiv |0\rangle \times |0\rangle$  is annihilated by  $a_{\mathbf{k}}$  and  $\tilde{a}_{\mathbf{k}}$ ,  $a_{\mathbf{k}}|0\rangle = 0 = \tilde{a}_{\mathbf{k}}|0\rangle$ . Thus,  $A_{\mathbf{k}}|0\rangle = 0 = \tilde{A}_{\mathbf{k}}|0\rangle$  for any  $\mathbf{k}$ . The canonical commutation relations (CCR) for the  $A$  and  $\tilde{A}$  operators are the same as the ones for the  $a$  and  $\tilde{a}$ , for any  $\mathbf{k}$  (cf. Equation (A1)).

The Hamiltonian controlling time evolution is assumed to be [6]

$$\hat{H} = \hat{H}_0 + H_I = \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} (A_{\mathbf{k}}^{\dagger} A_{\mathbf{k}} - \tilde{A}_{\mathbf{k}}^{\dagger} \tilde{A}_{\mathbf{k}}) + i \sum_{\mathbf{k}} \hbar \Gamma_{\mathbf{k}} (A_{\mathbf{k}}^{\dagger} \tilde{A}_{\mathbf{k}}^{\dagger} - A_{\mathbf{k}} \tilde{A}_{\mathbf{k}}), \quad (2)$$

with  $\hat{H}_0|0(\theta)\rangle_0 = 0$  and  $[\hat{H}_0, H_I] = 0$ . However, the vacuum  $|0(\theta)\rangle_0$  is asymmetric (not invariant) under  $H_I$ , which formally expresses the asymmetries under continuous time translations and time-reversal. It is

$$\begin{aligned} |0(\theta(t))\rangle &= \exp\left(-it\frac{\hat{H}}{\hbar}\right)|0(\theta)\rangle_0 \\ &= \prod_{\mathbf{k}} \frac{1}{\cosh(\Gamma_k t - \theta_k)} \exp\left(\tanh(\Gamma_k t - \theta_k) A_{\mathbf{k}}^\dagger \tilde{A}_{\mathbf{k}}^\dagger\right) |0\rangle, \end{aligned} \quad (3)$$

with  $\langle 0(\theta(t)) | 0(\theta(t)) \rangle_0 = 1$  and  $\theta_k(t) \equiv \Gamma_k t - \theta_k$ , and the initial condition  $\theta_k(0) \equiv -\theta_k$ . The state  $|0(\theta(t))\rangle$  is a generalized  $SU(1, 1)$  ( $q$ -deformed) squeezed coherent state [15,17,44,45], and an isomorphism exists between (deformed) squeezed coherent states and fractal self-similarity [15,46].

The number of condensed  $A_{\mathbf{k}}$  bosons in the state  $|0(\theta(t))\rangle$  is

$$\mathcal{N}_{A_{\mathbf{k}}}(\theta(t)) = \langle 0(\theta(t)) | A_{\mathbf{k}}^\dagger A_{\mathbf{k}} | 0(\theta(t)) \rangle = \sinh^2(\Gamma_k t - \theta_k) \quad (4)$$

and we also have  $\mathcal{N}_{\tilde{A}_{\mathbf{k}}}(\theta(t)) = \sinh^2(\Gamma_k t - \theta_k)$ , as expected since  $|0(\theta(t))\rangle$  contains pairs of  $A_{\mathbf{k}}$  and  $\tilde{A}_{\mathbf{k}}$ , for any  $\mathbf{k}$ , at any  $t$ , i.e.,  $\mathcal{N}_{A_{\mathbf{k}}}(\theta(t)) = \mathcal{N}_{\tilde{A}_{\mathbf{k}}}(\theta(t))$ , so that  $(\mathcal{N}_{A_{\mathbf{k}}} - \mathcal{N}_{\tilde{A}_{\mathbf{k}}}) = 0$ , for any  $\mathbf{k}$ , is a constant of motion. We denote by  $\mathcal{N} \equiv \{\mathcal{N}_{A_{\mathbf{k}}}(\theta(0)) = \mathcal{N}_{\tilde{A}_{\mathbf{k}}}(\theta(0)), \forall \mathbf{k}, \text{ at } t = 0\}$  the “memory code” corresponding to the initially recorded memory, at  $t = 0$ .  $\mathcal{N}_{A_{\mathbf{k}}}(\theta(t))$  is conditioned to satisfy the Bose–Einstein distribution function at any  $t$ , for any  $\theta(t) \equiv \{\theta_k(t), \forall \mathbf{k}\}$ , i.e., in each one of the infinitely many unitarily inequivalent representations  $\{|0(\theta(t))\rangle\}$ , thus ensuring that multiple different memories can be retained, one for each inequivalent  $\theta$ -representation (see also below).

From Equation (4), we see that the coherent condensation of  $A$  and  $\tilde{A}$  vanishes at time  $t = \tau$ , with  $\tau$  the largest among the values  $\tau_k = \theta_k/\Gamma_k$ , for all  $\mathbf{k}$ . At times  $t' \geq \tau$ ,  $|0(\theta(t'))\rangle$  is reduced to the vacuum  $|0\rangle$ . The memory recorded in  $|0(\theta)_0$  has been “forgotten”. The initial or a different stimulus may then restore it, or else construct from  $|0\rangle$  a different  $|0(\theta')\rangle_0$ . Notice that  $\Gamma_k$  may depend in general on several parameters possibly dependent on specific conditions and properties of the system (the brain). We thus see that different time scales  $\tau$  can be allowed, and a hierarchy of memories with different lifetimes (long and short lifetime memories) can be described by the model [47].

Equation (4) also shows that the memory code  $\mathcal{N}$  is associated to the set  $\theta \equiv \{\theta_k\}$ . Since states  $|0(\theta(0))\rangle$  with different  $\theta$ -sets,  $\theta, \theta', \theta'' \dots$ , belong to different, unitarily inequivalent representations of the CCR among the infinitely many ones existing in QFT [15–17,48], we see that at  $t = 0$ , there is a *huge memory recording capacity*, each different memory being recorded in a different  $\theta$ -representation. This is an advantage with respect to the RU model offered by considering dissipation (not achievable in quantum mechanics (QM) where all the representations of the CCR are unitarily equivalent due to the von Neumann theorem).

The orthogonality among states with different  $\theta$ , and/or different time  $t$  can be indeed shown by using of the continuous limit relation  $\sum_{\mathbf{k}} \mapsto \frac{V}{(2\pi)^3} \int d^3\kappa$ . Indeed we have [6]

$$\langle 0 | 0(\theta(t)) \rangle \rightarrow 0, \text{ for } V \rightarrow \infty, \quad \forall t, \quad (5)$$

$$\langle 0(\theta'(t')) | 0(\theta(t)) \rangle \rightarrow 0, \text{ for } V \rightarrow \infty, \quad \forall t, t', \theta, \theta', \quad t \neq t', \theta \neq \theta'. \quad (6)$$

These equations formally imply that time evolution of  $|0(\theta(t))\rangle$  in the infinite volume limit is through unitarily inequivalent representations of the CCR, and cannot be inverted, which is denoted as the *irreversibility of the arrow of time*, and that the origin of temporal axis cannot be arbitrarily changed (translated), confirming the asymmetry of  $|0(\theta(t))\rangle$  under continuous time translation and time-reversal transformations.

Since the Hilbert spaces for different times  $t \neq t'$  are in the infinite volume limit unitarily inequivalent spaces, there is no unitary transformation connecting them. This constitutes a “protection” against interferences between different memory codes (against



*confusion* of memories). The unitary inequivalence, however, may be smoothed since in realistic situations the infinite volume limit may not be reached due to defects or impurities and/or to the system boundary effects [6,15,28,29,49]. In that case, “association” of memories may be obtained, or else “paths” through memories in the memory space (the space of the memory states). As a matter of fact, the evolution of  $|0(\theta(t))\rangle$  describes trajectories in the space of the representations of the CCR. In this connection, without entering into details for brevity, we only mention that they are classical trajectories in a regime of deterministic chaos [50]. Small variations in boundary conditions determine divergent trajectories, which allows the brain to respond immediately to rapidly varying or unexpected stimuli, a behavior that cannot be described solely on the basis of the randomness of biochemical activity. Chaotic aspects in brain dynamics have been observed in neuroscience [14], and it has been stressed its relevant role in the “readiness” of the brain to shift the attention from one input to another one, from scenario to scenario. Freeman observes [14]: “[...] The brain transforms sensory messages into conscious perceptions almost instantly. Chaotic, collective activity involving millions of neurons seems essential for such rapid recognition. [...] Our studies have led us as well to the discovery in the brain of chaos–complex behavior that seems random but actually has some hidden order. The chaos is evident in the tendency of vast collections of neurons to shift abruptly and simultaneously from one complex activity pattern to another in response to the smallest of inputs. [...] In fact, we propose it is the very property that makes perception possible. We also speculate that chaos underlies the ability of the brain to respond flexibly to the outside world and to generate novel activity patterns, including those that are experienced as fresh ideas”. The activity of thinking is indeed closely linked to wandering [51].

In closing this section, we comment on the stimulus triggering the SBS. Such a stimulus can also be very weak. However, it must be “in phase”, resonant with the system, and “significant” for it so as to be able to stimulate a response from it. The set of states  $|0(\theta)\rangle_0$  constitutes the so-called attractor landscape. The SBS incoming stimulus may lead the system to sit in one of the already existing attractors, or, in the negative case, it may generate a new attractor. The “meaningfulness” of the stimulus is determined by the possibly modified relations between the attractors in the landscape. The creation of a new attractor is never a simple “addition” to the landscape, like the one of a new item to a list; it always implies the reshuffling of the relations among the attractors of the landscape, which amounts to a further comprehension of the past and present perceptual experiences. The meaningfulness of the incoming stimulus arises from such a “contextualization”.

In the RU model and in the dissipative quantum model, the recall of memory consists in the excitation of the modes  $A_{\mathbf{k}}$  in the memory code  $\mathcal{N}$ . Since this is determined by an infinite number of  $A_{\mathbf{k}}$  for each  $\mathbf{k}$ , the recall stimulus may be “similar”, not necessarily the same as the one that generated the memorization. It is sufficient that the difference between the excited modes  $A_{\mathbf{k}}$  and those of the code  $\mathcal{N}$  consists of a set of zero measure [51].

## 5. Entanglement, the Bayes Theorem, and Free Energy

The  $A_{\mathbf{k}}$  and  $\tilde{A}_{\mathbf{k}}$  within the coherent condensate of the state  $|0(\theta(t))\rangle$  are entangled modes [51,52]. The degree of entanglement is provided by the nonvanishing value of covariance,  $\text{cov}(N_{A_{\mathbf{k}}}, N_{\tilde{A}_{\mathbf{k}}}) \equiv \langle N_{A_{\mathbf{k}}} N_{\tilde{A}_{\mathbf{k}}} \rangle - \langle N_{A_{\mathbf{k}}} \rangle \langle N_{\tilde{A}_{\mathbf{k}}} \rangle$  [13,53,54], where expectation values  $\langle * \rangle$  are computed in the state  $|0(\theta(t))\rangle$ :

$$\text{cov}(N_{A_{\mathbf{k}}}, N_{\tilde{A}_{\mathbf{k}}}) \equiv \langle N_{A_{\mathbf{k}}} N_{\tilde{A}_{\mathbf{k}}} \rangle - \langle N_{A_{\mathbf{k}}} \rangle \langle N_{\tilde{A}_{\mathbf{k}}} \rangle = \frac{1}{4} \sinh^2 2\theta_{\mathbf{k}}(t) \neq 0, \quad \text{for } t < \tau. \quad (7)$$

It is zero for uncorrelated modes, and since then  $\langle N_{A_{\mathbf{k}}} N_{\tilde{A}_{\mathbf{k}}} \rangle = \langle N_{A_{\mathbf{k}}} \rangle \langle N_{\tilde{A}_{\mathbf{k}}} \rangle$ . The entanglement is also revealed by the impossibility to factorize in single-mode states Equation (3), as it appears by writing it as

$$|0(\theta(t))\rangle = \left( \prod_{\mathbf{k}} \frac{1}{\cosh \theta_{\mathbf{k}}(t)} \right) \times \left[ |0\rangle \otimes |0\rangle + \sum_{\mathbf{k}} \tanh \theta_{\mathbf{k}}(t) (|A_{\mathbf{k}}\rangle \otimes |\tilde{A}_{\mathbf{k}}\rangle) + \dots \right], \quad (8)$$

for any  $t$  (a similar expansion holds at  $t = 0$  for  $|0(\theta)\rangle_0$ ).

An analysis in terms of Bell's disequalities can also be conducted, since the expansion Equation (8) also contains Bell type states  $|\psi_{\pm}\rangle \propto (|A_{\mu,\mathbf{k}}\rangle |\tilde{A}_{-\mu,-\mathbf{k}}\rangle \pm |A_{-\mu,-\mathbf{k}}\rangle |\tilde{A}_{\mu,\mathbf{k}}\rangle)$ , for any  $\mathbf{k}$ , with  $\mu$  denoting quantum numbers that in full generality may also be present. The higher-power terms signal, however, that the entanglement is due to the  $(A, \tilde{A})$ -pair coherent condensate (the summation in Equation (8) contains infinitely many terms in the continuum  $\mathbf{k}$  limit). Its robustness against disentanglement is assured by the fact that the transformation  $|0\rangle \rightarrow |0(\theta)\rangle$  is nonunitary in the infinite volume limit in QFT [15]. As already recalled in the previous Section, in QM all the representations of the CCR are unitarily equivalent (the von Neumann theorem), and therefore the transition between entangled and disentangled phases is induced by a unitary operator, in contrast to what happens in QFT, as described above. In a similar way, the existence of unitarily inequivalent representations of CCR in QFT also guarantees the quite stable ordered coherent structures for long lifetimes even at high temperatures. For example, iron magnetization is lost at 770 °C. See the comment at the end of Appendix A for further examples and remarks.

A measure of the entanglement is also given by the entropy (introduced below, Equations (12) and (A14)). The 'Bogoliubov angle'  $\theta_k(t)$  determining the covariance and the entropy has to satisfy the minimization of the free energy leading to the Bose–Einstein distribution (Equation (A16)). This sheds also light on the role played by temperature.

Entanglement is also signaled by the fact that the only contribute to the expectation value in  $|0(\theta)\rangle$  of the number  $\mathcal{N}_{A_{\mathbf{k}}}$  comes from the tilde modes (which act as the 'address', or the 'truth evaluation function' for the non-tilde modes):

$$\mathcal{N}_{A_{\mathbf{k}}} = \langle 0(\theta) | A_{\mathbf{k}}^{\dagger} A_{\mathbf{k}} | 0(\theta) \rangle = \langle 0(\theta) | \tilde{A}_{\mathbf{k}}(\theta) \tilde{A}_{\mathbf{k}}^{\dagger}(\theta) | 0(\theta) \rangle \sinh^2 \theta_k = \sinh^2 \theta_k. \quad (9)$$

and vice versa, with  $\theta_k = \theta_k(t)$  for any  $t$ .  $\tilde{A}_{\mathbf{k}}(\theta)$  (and  $A_{\mathbf{k}}(\theta)$ ) are given in Equation (A7) (and (A6)). Moreover, the entanglement is also manifest in the relations

$$\frac{1}{\cosh \theta_k} A_{\mathbf{k}}^{\dagger} | 0(\theta) \rangle = \frac{1}{\sinh \theta_k} \tilde{A}_{\mathbf{k}} | 0(\theta) \rangle, \quad (10)$$

$$\frac{1}{\cosh \theta_k} \tilde{A}_{\mathbf{k}}^{\dagger} | 0(\theta) \rangle = \frac{1}{\sinh \theta_k} A_{\mathbf{k}} | 0(\theta) \rangle, \quad (11)$$

as implied by  $\tilde{A}_{\mathbf{k}}(\theta) | 0(\theta) \rangle = 0 = A_{\mathbf{k}}(\theta) | 0(\theta) \rangle$  and Equation (A6) and (A7), showing that the creation of a mode  $A_{\mathbf{k}}$  is equivalent to the destruction of a mode  $\tilde{A}_{\mathbf{k}}$ , up to convenient factors, and vice versa. Notice the asymmetry between the  $A_{\mathbf{k}}$  and  $\tilde{A}_{\mathbf{k}}$  coefficients in these equations, reflecting the already mentioned (in Section 3) asymmetry in the non-commutative co-product  $\Delta A$  of the  $q$ -deformed Hopf algebra at the basis of the Bogoliubov transformations Equations (A6) and (A7).

In summary, phase correlations between the modes  $A_{\mathbf{k}}$  and  $\tilde{A}_{\mathbf{k}}$  in the coherent condensate of the ground state  $|0(\theta(t))\rangle$  are at the origin of the entanglement [38,52]. As it happens in condensed matter physics, the "element of reality" is provided by the observable properties of the condensate, without need of introducing "spooky forces at a distance".

It is shown in Appendix A that for the state  $|0(\theta)\rangle$  it can be introduced the entropy  $\mathcal{S}_A$  (Equations (A14)) [15–17,43]

$$\mathcal{S}_A \equiv \langle 0(\theta) | \mathcal{S}_A | 0(\theta) \rangle = - \sum_{n=0}^{+\infty} W_n \ln W_n. \quad (12)$$

where

$$W_n = \prod_{\mathbf{k}} \frac{\sinh^{2n_{\mathbf{k}}} \theta_k}{\cosh^{2(n_{\mathbf{k}}+1)} \theta_k}, \quad 0 < W_n < 1, \quad \sum_{n=0}^{+\infty} W_n = 1, \quad (13)$$

with  $n_{\mathbf{k}} \equiv n_{A_{\mathbf{k}}}$  and the set  $\{n_{\mathbf{k}}\}$  is denoted by  $n = \tilde{n}$  entering in the expansion of the state  $|0(\theta)\rangle$  as summation of  $n$ - and  $\tilde{n}$ -components states (cf. Equation (A13)):

$$|0(\theta)\rangle = \sum_{n=0}^{+\infty} \sqrt{W_n} (|n\rangle \otimes |\tilde{n}\rangle). \quad (14)$$

In Appendix A, we also show how a specific “target” state  $|0(\theta)\rangle$  (a specific  $\theta$ ) is reached in our formalism (Equations (A18)–(A23)).

For the  $\tilde{A}_{\mathbf{k}}$  modes, a similar expression for the entropy  $S_{\tilde{A}}$  may be obtained.

The last two equations show that the probability of finding  $n_{\mathbf{k}}$  quanta  $A_{\mathbf{k}}$  in the  $|n_{\mathbf{k}}\rangle$  component state of  $|0(\theta)\rangle$  given the presence of  $\tilde{n}_{\mathbf{k}}$  quanta  $\tilde{A}_{\mathbf{k}}$  in the  $|\tilde{n}_{\mathbf{k}}\rangle$  state is:

$$P(n_{\mathbf{k}}|\theta_k) = W_{n,k} = \frac{\tanh^{2n_{\mathbf{k}}} \theta_k}{\cosh^2 \theta_k} = e^{-S_{n_{\mathbf{k}}}}, \quad \forall \mathbf{k}. \quad (15)$$

with obvious notation (cf. Equation (13)). Since  $\theta_k$  is conditioned to satisfy the Bose–Einstein distribution Equation (A16), we may denote  $P(n_{\mathbf{k}}|\theta_k)$  as:  $P(n_{\mathbf{k}}|\theta_k) \equiv P(n_{\mathbf{k}}|\tilde{n}_{\mathbf{k}}) \equiv P(n_{\mathbf{k}}|\beta, \omega_k)$ .

In  $|0(\theta)\rangle$ ,  $A_{\mathbf{k}}$  and  $\tilde{A}_{\mathbf{k}}$  come in pairs  $\forall \mathbf{k}$ , then also for  $\tilde{A}_{\mathbf{k}}$ , we have:

$$P(\theta_k|n_{\mathbf{k}}) = \frac{\tanh^{2n_{\mathbf{k}}} \theta_k}{\cosh^2 \theta_k} = e^{-S_{n_{\mathbf{k}}}}, \quad \forall \mathbf{k}. \quad (16)$$

By construction, the probability  $P(\theta_k)$  of finding the specific  $\theta_k$  equals the one of finding  $n_{\mathbf{k}}$  particles in  $|0(\beta)\rangle$  with the Bose–Einstein distribution, Equation (A16), and we have:  $P(n_{\mathbf{k}}) = P(\theta_k) = P(\tilde{n}_{\mathbf{k}})$ ,  $\forall \mathbf{k}$ . Thus, Equation (15) can be written as:

$$P(n_{\mathbf{k}}|\theta_k) = \frac{P(\theta_k|n_{\mathbf{k}})P(n_{\mathbf{k}})}{P(\theta_k)}, \quad (17)$$

i.e., the Bayes relation.

In conclusion, we see that the same value of the Bogoliubov phase  $\theta_k$  constitutes the formal link between entanglement, Bayes’ theorem, and free energy minimization (cf. Equations (7), (15) and (A16)). In connection with the role of free energy, I would like to observe that since 1995, in the first formulation of the dissipative quantum model of the brain [6], free energy has been recognized to be relevant to the brain’s functional activity in a crucial way, as it has been confirmed in all the subsequent studies of the dissipative model, see e.g., [34,49], which is of course not surprising since the free energy principle is one of the basic laws of nature. Subsequently, its role has been recognized in other QM and classical brain modeling [55], particularly in Friston’s works, see e.g., [56,57].

## 6. Intentionality, Mind, and Consciousness

The Bayes rule and its relation to the entanglement and free energy minimization discussed in the previous section accounts formally for *intentionality* in the brain functional activity. As stressed by Walter Freeman and Karl Pribram, neuronal nonlinear dynamics is always aimed at realizing “intentional actions” in the environment [7,24,32,33,58–60].

The Bayes rule [12] is linked to the probability of causal aspects given the observed effects, which is of course highly relevant for science in all its branches. In the case of the brain’s continuous interaction with the world, such a permanent “dialog” with its “Double” develops through the action–perception cycle, and the choice of the most successful action in reaching the desired task is performed by estimating the action with higher Bayes probability to be taken in order to produce the aimed effect. This is the meaning of the intrinsic relation of Bayes’ rule with the memory state described in the previous section.

The actions are attributable to the activity of the brain forward in time, according to the arrow of time, while the Double is responsible for the research and reconstruction in the backward (mirror) time, where it lives, of the perception acquired in the past for the

purposes of planning the action judged as necessary for survival, for being-in-the-world. At time  $t_0$ , the action is “determined” by what the brain wants to happen at time  $t > t_0$ . This is the *mind* activity [38].

The Double, going back in time, in the reconstruction of a past perceptual experience, “provides the imagination from which to build the hypothesis to be tested by the action” [38]. The neural activity ahead in time guides the “intentional” action that follows the perceptual experience and is planned on the basis of the hypothesis provided by the Double. “It is the Double that imagines the world outside, free from the chains of thermodynamic reality” (the arrow of time) [38]. Brains test the hypothesis using the action–perception loop. The experience that the action, in repeated trials, is infallibly followed by changes in the environment creates the perception of time and causality simultaneously [38]. Neuronal activity, therefore, evolves “along parallel timelines”, one corresponding to the reconstruction of the past in memory (the *mind*, the *mental activity*), the other directed towards the future with the intention of obtaining a specific objective and of planning the actions to take.

In the state  $|0(\theta(t))\rangle$ , the entanglement of the pairs  $(A, \tilde{A})$  indicates that the relationship, the “dialog” between the self (the brain) and its Double cannot be interrupted (disentangled). The entanglement between  $A$  and  $\tilde{A}$  therefore translates into the entanglement between brain activity and mental activity, without the possibility that they can be separated. There are not two entities, there is not a double level of existence, matter and mind, but a single indivisible entity. The dissipative model does not describe mental states separate from the brain. The *consciousness* resides in the dialog of the self with its Double [6]. In this dialog, the relationships between perceptual experiences within the landscape of attractors are established, and in this way, they acquire a *meaning* through the action–perception cycle that realizes Merleau-Ponty’s [61,62] intentional arc. The credibility of a meaning is verified on the basis of the adequacy of the resulting action, which is the content of the concept of pragmatic information [63,64]. The landscape of attractors, built in previous experiences, is continually remodeled with each new perceptual input. *Memory is not memory of information, it is memory of meanings*.

The conclusion is that the *closure* of the brain is lethal. Pathological behaviors may arise, for example with depressive states deriving from ‘detachment’ from the world (from the loss of interest in the world) [7,47], including forms of narcissism in which *self-reflection* leads to the *reduction* of the world to oneself, to the “identification with the Double”, or to a multiplicity of discordant but coexisting worldviews with associated multiplicity of Doubles and consequent multiple personalities (dissociative identity disorder (DID)).

The brain open to the world is “awake”, connected to the world by a number of links, by the dialog with its Double, which is recognized as other than the self (for a study on the alpha-wave modulations in the awake resting-state see [65]). However, when the ties with the world are reduced in number, or weakened, e.g., during sleep, or as a result of drugs, in certain altered states of consciousness, then the ability is also weakened to distinguish the Double as other than self, and the perception of the ordered evolution of time may be lost, and with it the dissipative nature of brain dynamics, from which the orthogonality between representations (memory states) derives. Hence, in such states it may occur an unexpected overlapping and mixing of images and times, which do in fact happen in dreams [59].

The openness of the brain to the world and its being intrinsically linked to it also means openness to “other brains”, involvement in the formation of *cultural networks*, in the sharing of a “social brain” [66]. The term “culture” refers in general to the heritage of meanings, their concrete representation in works, the history of their formation, the structure of the shared language, the “cultural fabric” that identifies communities and peoples, designs their future, and permeates the practice of their social and economic organization. It is therefore interesting to study the dynamics and the conditions of stability and the phase transitions that lead to processes with greater or lesser criticalities, with the establishment of new, different correlations between the components (the brains) of the community. A higher level of “phase correlations between brains” then manifests itself in the social dimension [52,67]. Supranational and military organizations work on

operational projects to control the social dimension of the brain and induce convenient phase transitions, which support 'superior' interests and reasons to promote, for example, the uniform establishment of behaviors and needs in large regions, with the removal, or construction, of barriers and cultural traditions (urbanization, globalization) [68].

## 7. Further Discussion and Concluding Remarks

Considering the dialog of the brain with the world, it is interesting to consider also the dialog with the inner-body world, e.g., the dialog brain–heart.

In the “neuro-visceral integration model” [69–71], it is observed that the heart receives and releases complex hormonal signals, cytokines and microvesicles into the bloodstream with which to modulate the dialog with the brain and the entire organism. The brain does not appear to be hierarchically “superior” to the heart. From embryology, it is known that it begins to beat before the brain is formed. The “intrinsic cardiac nervous system” influences the activity of the fronto-cortical areas and the motor cortex, with effects on the level of attention, perceptive sensitivity and intuitive-emotional processes, awareness, and feelings, in an active heart–brain communication (“heart–brain symphony”). The heart is connected to the intrathoracic and extrathoracic nerve ganglia, the spinal cord, and through the brainstem, to the hypothalamus, thalamus, amygdala, and cerebral cortex. Plasma water constitutes approximately 55% of blood volume, and SBS, with the resulting coherent dipole correlations, is induced by cardiac systolic contraction and rotational motion that induce vorticity in the blood flow [71].

The observed immediacy of the response of the heart–brain system to perceptual experiences (despite the delays due to biochemical activity) may be described in terms of “in-phase” coherence between neuronal correlations and those of blood flow. Neurological and cardiovascular diseases could be two sides of the same coin.

The heart, not just the brain, appears dynamically involved in a continuous dialogic relationship with the Double.

Data collected in vivo and in vitro show that sound vibrations can influence the growth, structure, and behavior of cells [72–74]. This suggests that psychic activity and its *language* can determine biochemical changes (and vice versa), and therefore have a biological impact on specific diseases and therapies [72–74], confirming the connection between neuronal activity and somatic, phonetic, perceptive, and emotional activity [75]. Acoustic and electromagnetic waves, such as those of the heart–brain axis, can determine the dynamics and structure of cells and their components. The meaning of this complex dialogic process lies in the harmony that is generated in *being-in-the-world* (adaptive response), in which the *aesthetic experience* [76,77] ultimately consists, or in the conflict with the Double (illness). In this case, it is vital to retrace the passage from *feeling* to *comprehending*, up to the recovery of the ‘missing data’, to *knowing*, and, in the cycle, together with the Double, from *knowing*, to *comprehending*, to *feeling* [78], towards knowledge.

To conclude, we have discussed dynamical asymmetries arising from the openness of the brain to its environment, namely from its dissipative dynamics. We have reviewed the model by Ricciardi and Umezawa, where the formalism of QFT has been used to describe the microscopic molecular brain dynamics as in condensed matter physics. Its extension to dissipative dynamics has been also reviewed, showing how dissipation is responsible for the breakdown of continuous time translation transformations and time-reversal transformations, which manifest in the irreversibility of the “arrow of time”. Non-equilibrium brain dynamics, also related to the holographic picture and to modifications of consciousness states induced, e.g., by anesthetics, are objects of analysis in current literature [79–81].

The SBS of continuous spherical rotational symmetry of molecular electric dipoles has been discussed, with the consequent dynamical generation of long-range correlations and the associated dipole wave quanta  $A_{\mathbf{k}}$ . The QFT canonical formalism requires the doubling of the degrees of freedom and then the memory state turns out to be a coherent condensed  $SU(1,1)$  state of pairs  $(A_{\mathbf{k}}, \tilde{A}_{\mathbf{k}})$ , where  $\tilde{A}_{\mathbf{k}}$  are the “mirror in time” modes



describing the environment. This is denoted as the system “Double”, i.e., the mirror in time image of the system. The properties of memory states have been discussed, such as the entanglement between the pairs  $A_k$  and  $\tilde{A}_k$  and the free energy minimization leading to the Bose–Einstein distribution function. One thus realizes that the formalism is the temperature-dependent QFT formalism of Thermo Field Dynamics (TFD) (which is strictly related to the  $C^*$ –algebra formalism [82]).

Since there is an isomorphism between fractal self-similarity and the coherent nature of the dynamics, the coherence properties of the state  $|0(\theta(t))\rangle$  reveals its fractal self-similarity structure [46], which agrees with experimental observations [83–86]. In addition, the dissipative dynamics also implies chaotic deterministic trajectories in the space of the memory states [50], which again is in agreement with neuroscience laboratory observations [14,83].

In this paper, the new result we have obtained is that the canonical formalism of QFT for the closed system  $(A, \tilde{A})$  has *built in* the Bayes rule structure and that its formal and physical basis is in the entanglement between  $A$  and  $\tilde{A}$  under the dynamical constraints of the minimization of the free energy. In turn, this tells us that the intentionality in the brain’s functional activity is formally expressed by and follows Bayes’s rule.

It is then interesting to observe that in Bayes’ rule, the conditional probability distribution for the system  $A$  given system  $\tilde{A}$  (and the one for the system  $\tilde{A}$  given system  $A$ ) implicitly assumes that a *correlation* exists between  $A$  and  $\tilde{A}$  able to determines reciprocal “constraints” so that, within some reasonable approximation, the couple  $(A, \tilde{A})$  might be considered to behave as a “closed” system.

Incidentally, we recall that one of the many applications of Bayes’ rule is indeed in cognitive science. Just in the last few years, Bayesian models have addressed symbolic reasoning [87], causal learning and inference [88–90], human inductive learning [91], visual scene perception [92], motor control [93], semantic memory [94], language processing and acquisition [95,96], social cognition [97], human decision-making [55], among other research questions. Remarkably, our result, within the frame of the dissipative quantum model of the brain, confirms the relevance of Bayes’ rule in cognitive science.

In our discussion, we have remarked that laboratory observations show that many aspects of the functional activity of the brain arise from the systemic, cooperative activity of molecular and cellular components. Several recent theoretical models are motivated by such laboratory observations, for example in [98], the activity of the perception and consciousness are proposed to arise from the global electromagnetic field emerging from the cooperative, rather than from the single neuronal firing. By resorting to the QFT formalism developed for biological systems [27–29,99,100], the coherent domains in the brain are studied in [101] in relation to the cortical dynamics, also in the perspective of a description of consciousness.

We have seen that the dissipative quantum model of the brain provides a mathematical description of the act of consciousness in terms of the dialog of the self with the Double and of its self-identification as being distinct from its Double. The model is further supported by the novel result presented in this paper on the Bayesian formal character of intentionality, found to be intrinsic to the coherent structure of the asymmetric memory states.

We also remark that  $SU(1, 1)$  generalized coherent states occur in many systems, in condensed matter physics, elementary particle physics, quantum optics, etc. In all the cases, they are manifestly dependent on temperature, so QFT appears to have an intrinsic thermal nature [15]. Our conclusions in this paper on the formal relation of TFD states with entanglement, its temperature dependence (see, e.g., Equation (A16)), Bayes’ rule, free energy, and quantum boson statistics have, therefore, a very general validity, not limited to brain studies, that extends in fact to many physical systems.

It is in our plans to extend our present QFT analysis to some of the sectors where already Bayes’ theorem has given promising results, such as in “decision-making”, “governance activity”, quantum optics, quantum computing, and also to some pathology in neurology, for example, in the study of schizophrenia viewed from an open/closed system perspective. Applying to such sectors the conclusions here reached might lead to interesting

novel results. With reference to the intense discussions on artificial intelligence (AI) [102], its usefulness, its wide horizon of applications, and its influence in every sector of human activities, we conclude by saying that, as already mentioned in [51], if ever it happens to construct an AI device capable of consciousness, it must have all the best properties characterizing the human, the ability to learn, unpredictable behavior, infidelity, inevitably linked to the world, inalienable freedom. It must be named *Spartacus*.

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## Abbreviations

The following abbreviations are used in this manuscript:

DOAJ	Directory of Open Access Journals
MDPI	Multidisciplinary Digital Publishing Institute
CAR	Canonical Anticommutation Rules
CCR	Canonical Commutation Rules
DWQ	Dipole Wave Quanta
QFT	Quantum Field Theory
SBS	Spontaneous Breakdown of Symmetry
TFD	Thermo Field Dynamics
NG	Nambu–Goldstone
QM	Quantum Mechanics
RU	Ricciardi and Umezawa

## Appendix A

For the reader's convenience, we report here some of the mathematical details [15–17,43]. The canonical commutation relations (CCR) for the  $a_{\mathbf{k}}$ ,  $\tilde{a}_{\mathbf{k}}$ ,  $A_{\mathbf{k}}$  and  $\tilde{A}_{\mathbf{k}}$  boson operators are

$$[a_{\mathbf{k}}, a_{\mathbf{q}}^{\dagger}] = \delta_{\mathbf{k}, \mathbf{q}} = [\tilde{a}_{\mathbf{k}}, \tilde{a}_{\mathbf{q}}^{\dagger}]; \quad [a_{\mathbf{k}}, \tilde{a}_{\mathbf{q}}^{\dagger}] = 0 = [a_{\mathbf{k}}, \tilde{a}_{\mathbf{q}}]. \quad (\text{A1})$$

$$[A_{\mathbf{k}}, A_{\mathbf{q}}^{\dagger}] = \delta_{\mathbf{k}, \mathbf{q}} = [\tilde{A}_{\mathbf{k}}, \tilde{A}_{\mathbf{q}}^{\dagger}]; \quad [A_{\mathbf{k}}, \tilde{A}_{\mathbf{q}}^{\dagger}] = 0 = [A_{\mathbf{k}}, \tilde{A}_{\mathbf{q}}]. \quad (\text{A2})$$

The asymmetric state in Equation (1) is obtained by using the generator  $\mathcal{G}$  [15–17,43]

$$\mathcal{G} \equiv \mathcal{G}(\theta) = -i \sum_{\mathbf{k}} \theta_{\mathbf{k}} (A_{\mathbf{k}}^{\dagger} \tilde{A}_{\mathbf{k}}^{\dagger} - A_{\mathbf{k}} \tilde{A}_{\mathbf{k}}), \quad (\text{A3})$$

$$|0(\theta)\rangle_0 = e^{i\mathcal{G}} |0\rangle. \quad (\text{A4})$$

$|0(\theta)\rangle_0$  is a state with equal number of  $A_{\mathbf{k}}$  and  $\tilde{A}_{\mathbf{k}}$  condensed in it. It is thus the vacuum state (the zero energy) for the Hamiltonian  $\hat{H}_0$ ,  $\hat{H}_0 |0(\theta)\rangle_0 = 0$ , with

$$\hat{H}_0 \equiv H_0 - \tilde{H}_0 = \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} (A_{\mathbf{k}}^{\dagger} A_{\mathbf{k}} - \tilde{A}_{\mathbf{k}}^{\dagger} \tilde{A}_{\mathbf{k}}). \quad (\text{A5})$$

Note that  $\hat{H}_0$  is symmetric under  $\mathcal{G}$ ,  $[\hat{H}_0, \mathcal{G}] = 0$ . However, the vacuum state  $|0\rangle$  is asymmetric, as shown by Equation (A4). At finite volume  $V$ ,  $A_{\mathbf{k}}$  and  $\tilde{A}_{\mathbf{k}}$  are transformed as

$$A_{\mathbf{k}}(\theta) = \exp(i\mathcal{G}) A_{\mathbf{k}} \exp(-i\mathcal{G}) = A_{\mathbf{k}} \cosh \theta_{\mathbf{k}} - \tilde{A}_{\mathbf{k}}^{\dagger} \sinh \theta_{\mathbf{k}}, \quad (\text{A6})$$

$$\tilde{A}_{\mathbf{k}}(\theta) = \exp(i\mathcal{G}) \tilde{A}_{\mathbf{k}} \exp(-i\mathcal{G}) = \tilde{A}_{\mathbf{k}} \cosh \theta_{\mathbf{k}} - A_{\mathbf{k}}^{\dagger} \sinh \theta_{\mathbf{k}}, \quad (\text{A7})$$

and similarly for hermitian conjugates. They are Bogoliubov transformations with a Bogoliubov angle (phase)  $\theta_k$ . They are canonical transformations since the CCR for  $A_{\mathbf{k}}$ ,  $A_{\mathbf{k}}^\dagger$  and  $\tilde{A}_{\mathbf{k}}$ ,  $\tilde{A}_{\mathbf{k}}^\dagger$  are preserved for  $A_{\mathbf{k}}(\theta)$ ,  $A_{\mathbf{k}}^\dagger(\theta)$ , and the tilde ones, and should be thought of as inner automorphism of the algebra  $su(1,1)_{\mathbf{k}}$ . One is thus dealing with  $\oplus_{\mathbf{k}} su(1,1)_{\mathbf{k}}$ . It can be shown [44] that Bogoliubov transformations are obtained by using the co-product  $\Delta A$  of the  $q$ -deformed non-commutative Hopf algebra (cf. Section 3). Equation (A4) and  $A_{\mathbf{k}}|0\rangle = 0 = \tilde{A}_{\mathbf{k}}|0\rangle$  show that

$$A_{\mathbf{k}}(\theta)|0(\theta)\rangle_0 = \exp(i\mathcal{G})A_{\mathbf{k}}\exp(-i\mathcal{G})|0(\theta)\rangle_0 = 0, \quad (\text{A8})$$

and similarly,  $\tilde{A}_{\mathbf{k}}(\theta)|0(\theta)\rangle_0 = 0$ , for any  $\mathbf{k}$ , i.e.,  $|0(\theta)\rangle_0$  is the vacuum for  $A_{\mathbf{k}}(\theta)$  and  $\tilde{A}_{\mathbf{k}}(\theta)$ .

Equations (A6) and (A7) show that  $|0(\theta)\rangle_0$  is not annihilated by  $A_{\mathbf{k}}$  and  $\tilde{A}_{\mathbf{k}}$ .

Notice that  $(A_{\mathbf{k}}^\dagger A_{\mathbf{k}} - \tilde{A}_{\mathbf{k}}^\dagger \tilde{A}_{\mathbf{k}})$  is proportional to the Casimir operator of the  $SU(1,1)$  group commuting with all its generators [15,16]. For completeness, we also report the generators and the algebra of the  $SU(1,1)$  group, omitting for notational simplicity the subscript  $\mathbf{k}$ . We have

$$J_+ = A^\dagger \tilde{A}^\dagger, \quad J_- = J_+^\dagger = A \tilde{A}, \quad J_3 = \frac{1}{2}(A^\dagger A + \tilde{A}^\dagger \tilde{A} + 1), \quad (\text{A9})$$

$$[J_+, J_-] = -2J_3, \quad [J_3, J_\pm] = \pm J_\pm. \quad (\text{A10})$$

It is  $\mathcal{C}^2 \equiv \frac{1}{4} + J_3^2 - \frac{1}{2}(J_+ J_- + J_- J_+) = \frac{1}{4}(A^\dagger A - \tilde{A}^\dagger \tilde{A})^2$ , with  $\mathcal{C}$  the Casimir operator. In the representation labeled by the value  $j \in \mathbb{Z}_{\frac{1}{2}}$  of  $\mathcal{C}$ , one can express the eigenstates of  $\hat{H}_0$  in terms of the basis of simultaneous eigenstates of  $\mathcal{C}$  and of  $(J_3 - \frac{1}{2})$ ,  $\{|j, m\rangle; m \geq |j|\}$ :

$$\mathcal{C}|j, m\rangle = j|j, m\rangle, \quad j = \frac{1}{2}(\mathcal{N}_A - \mathcal{N}_{\tilde{A}}); \quad (\text{A11})$$

$$(J_3 - \frac{1}{2})|j, m\rangle = m|j, m\rangle, \quad m = \frac{1}{2}(\mathcal{N}_A + \mathcal{N}_{\tilde{A}}). \quad (\text{A12})$$

The state  $|0(\theta)\rangle_0$  corresponds to the choice  $j = 0$  (for all  $\mathbf{k}$ ) and there are  $m \equiv \{m_{\mathbf{k}}\}$  coexisting, independent eigenstates of  $\mathcal{C}$ , each one corresponding to  $m_{\mathbf{k}} = \mathcal{N}_{A_{\mathbf{k}}} = \mathcal{N}_{\tilde{A}_{\mathbf{k}}}$ .

The Hamiltonian  $\hat{H}$ , Equation (2), is introduced and the time evolution of  $|0(\theta)\rangle_0$  leads to  $|0(\theta(t))\rangle$ , Equation (3), which for simplicity we also denote as  $|0(\theta)\rangle$ , using  $\theta \equiv \theta(t)$ .

For any  $t$  (and  $\mathbf{k}$ ),  $|0(\theta)\rangle$  can also be written as [15–17]

$$|0(\theta)\rangle = \exp\left(-\frac{1}{2}S_A\right)|\mathcal{I}\rangle = \exp\left(-\frac{1}{2}S_{\tilde{A}}\right)|\mathcal{I}\rangle, \quad (\text{A13})$$

with

$$S_A \equiv -\sum_{\mathbf{k}} \left\{ A_{\mathbf{k}}^\dagger A_{\mathbf{k}} \ln \sinh^2 \theta_k - A_{\mathbf{k}} A_{\mathbf{k}}^\dagger \ln \cosh^2 \theta_k \right\}, \quad (\text{A14})$$

and  $|\mathcal{I}\rangle \equiv \exp(\sum_{\mathbf{k}} A_{\mathbf{k}}^\dagger \tilde{A}_{\mathbf{k}}^\dagger)|0\rangle$ , which is a condensate of (infinitely many) pairs of  $A_{\mathbf{k}} \tilde{A}_{\mathbf{k}}$  modes, for any  $\mathbf{k}$ .  $S_{\tilde{A}}$  is obtained by replacing in Equation (A14)  $A_{\mathbf{k}}$  and  $A_{\mathbf{k}}^\dagger$  with  $\tilde{A}_{\mathbf{k}}$  and  $\tilde{A}_{\mathbf{k}}^\dagger$ , respectively. Note that  $S_A - S_{\tilde{A}} = -\sum_{\mathbf{k}} \left\{ (A_{\mathbf{k}}^\dagger A_{\mathbf{k}} - \tilde{A}_{\mathbf{k}}^\dagger \tilde{A}_{\mathbf{k}}) \ln \tanh^2 \theta_k \right\}$  and  $S_A - S_{\tilde{A}} \equiv \langle 0(\theta) | S_A - S_{\tilde{A}} | 0(\theta) \rangle = 0$  [15,16,43]. Equation (A13) gives Equations (12)–(14).

At any  $t$ , during its time evolution, the system has to satisfy the basic requirement of the minimization of free energy. Thus, introduce the free energy  $\mathcal{F}_A$  [16,17,43]:

$$\mathcal{F}_A \equiv \langle 0(\theta) | \left( H_A - \frac{1}{\beta} S_A \right) | 0(\theta) \rangle, \quad (\text{A15})$$

where  $H_A = \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} A_{\mathbf{k}}^\dagger A_{\mathbf{k}}$ , and  $S_A$  is given by Equation (A14) (and similarly for the  $\tilde{A}$  modes). The minimization of the free energy at any  $t$   $\partial \mathcal{F}_A / \partial \theta_{\mathbf{k}} = 0, \forall \mathbf{k}$ , gives then  $\ln \tanh^2 \theta_{\mathbf{k}} = -\beta \omega_{\mathbf{k}}$  and the Bose–Einstein distribution function follows:

$$\sinh^2 \theta_{\mathbf{k}} = \frac{1}{e^{\beta(t)\omega_{\mathbf{k}}} - 1}, \quad (\text{A16})$$

which means that  $\theta_{\mathbf{k}} = \theta_{\mathbf{k}}(\beta)$  for any  $t$ . The density matrix is  $\rho_{\mathbf{k}} = f_{\mathbf{k}}^{A_{\mathbf{k}}^\dagger A_{\mathbf{k}}}$ , with  $f_{\mathbf{k}} \equiv e^{-\beta \omega_{\mathbf{k}}}$ . Time dependence of  $\beta = \beta(t)$  leads to

$$dE_A = \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} \dot{\mathcal{N}}_{A_{\mathbf{k}}}(t) dt = \frac{1}{\beta} dS_A = dQ, \quad (\text{A17})$$

for  $\partial \beta / \partial t \approx 0$  and where  $\dot{\mathcal{N}}_{A_{\mathbf{k}}}$  denotes time-derivative. Since  $S_A - S_{\tilde{A}} = 0$ , i.e.,  $dS_A = dS_{\tilde{A}} = \beta dQ$ , we see that the tilde system acts as the heat bath (the reservoir) for the non-tilde system, and vice versa.

Suppose that the entropy operator in Equation (A14), obtained for the memory state  $|0(\theta)\rangle$ , is computed in the state  $|0(\theta')\rangle$ . We have

$$S_{n'_{\mathbf{k}}} = -\left\{ n'_{\mathbf{k}} \ln \sinh^2 \theta_{\mathbf{k}} + (1 + n'_{\mathbf{k}}) \ln \cosh^2 \theta_{\mathbf{k}} \right\}, \quad (\text{A18})$$

where  $n'_{\mathbf{k}} = \sinh^2 \theta'_{\mathbf{k}}$  is the number operator  $A_{\mathbf{k}}^\dagger A_{\mathbf{k}}$  computed in the  $|0(\theta')\rangle$  state, with Bose–Einstein distribution function  $n'_{\mathbf{k}} = 1/(e^{\beta \omega'_{\mathbf{k}}} - 1)$ , and  $\omega'_{\mathbf{k}} = \omega_{\mathbf{k}}(\theta'_{\mathbf{k}}) = -(1/\beta) \ln \tanh^2 \theta'_{\mathbf{k}}$ . We want to reach the target  $n'_{\mathbf{k}} = n_{\mathbf{k}}$  starting from the system in the  $S_{n'_{\mathbf{k}}}$  configuration, where we have

$$S'_A \equiv \langle 0(\theta') | S'_A | 0(\theta') \rangle = - \sum_{n'=0}^{+\infty} W_{n'} \ln W_{n'}. \quad (\text{A19})$$

with

$$W_{n'} = \prod_{\mathbf{k}} \frac{\sinh^{2n'_{\mathbf{k}}} \theta_{\mathbf{k}}}{\cosh^{2(n'_{\mathbf{k}}+1)} \theta_{\mathbf{k}}}, \quad 0 < W_{n'} < 1, \quad \sum_{n'=0}^{+\infty} W_{n'} = 1, \quad (\text{A20})$$

The target is reached by tuning the  $\theta$ -set,  $\theta = \{\theta_{\mathbf{k}}, \forall \mathbf{k}\}$  (i.e., by searching in the memory space, among the inequivalent representations  $\{|0(\theta)\rangle\}$  in the landscape of the attractors), which is equivalent to minimizing with respect to  $\theta_{\mathbf{k}}$  the free energy with  $H'_A$  and  $S'_A$  (cf. Equation (A15)) computed in the  $|0(\theta')\rangle$  state. Considering that  $\omega'_{\mathbf{k}} = \omega_{\mathbf{k}}(\theta'_{\mathbf{k}})$ , this equivalent to compute the derivative of  $S_{n'_{\mathbf{k}}}$  with respect to  $\theta_{\mathbf{k}}$ , for all  $\mathbf{k}$ ,  $\partial S_{n'_{\mathbf{k}}} / \partial \theta_{\mathbf{k}} = 0$ , from which we obtain  $n'_{\mathbf{k}} = n_{\mathbf{k}}$ . Thus, for  $n'_{\mathbf{k}} = n_{\mathbf{k}} = 0$  in Equation (A20) we have  $W_{n'}(k) = 1/\cosh^2 \theta_{\mathbf{k}}$  and for  $n'_{\mathbf{k}} = n_{\mathbf{k}} \neq 0$  it is  $W_{n'}(k) = W_n(k) = \sinh^{2n_{\mathbf{k}}} \theta_{\mathbf{k}} / \cosh^{2(n_{\mathbf{k}}+1)} \theta_{\mathbf{k}}$ , i.e., the “target” state  $|0(\theta)\rangle$  is obtained.

We can proceed similarly for the tilde modes. Note that the Bayes rule is still valid in the  $\theta'$  configuration:

$$P(n_{\mathbf{k}} | \theta'_{\mathbf{k}}) = \frac{P(\theta'_{\mathbf{k}} | n_{\mathbf{k}}) P(n_{\mathbf{k}})}{P(\theta'_{\mathbf{k}})}. \quad (\text{A21})$$

In general, the difference between the two configurations is

$$\sinh^2 \theta'_{\mathbf{k}} - \sinh^2 \theta_{\mathbf{k}} = \sinh(2\theta_{\mathbf{k}}) \delta \theta_{\mathbf{k}}, \quad (\text{A22})$$

which holds for small  $\delta \theta_{\mathbf{k}}$ , for any  $\mathbf{k}$ , with  $\delta \theta_{\mathbf{k}} \equiv \theta'_{\mathbf{k}} - \theta_{\mathbf{k}}$  assumed to be positive. For time-dependent  $\theta = \theta(t)$ , the variation in time of the difference  $n'_{\mathbf{k}} - n_{\mathbf{k}}, \forall \mathbf{k}$  in Equation (A22) may be written in terms of energy and entropy variations, according to Equation (A17), as [15,50]

$$dE'_A - dE_A = \Delta \left( \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} \dot{\mathcal{N}}_{A_{\mathbf{k}}}(t) dt \right) = \frac{1}{\beta} (dS'_A - dS_A). \quad (\text{A23})$$

Finally, note that coherent condensation is observed over a wide range of temperatures: in iron, magnetization is lost at 770 °C; the diamond crystal melts at 3545 °C; sodium chloride (table salt) melts at 804 °C; the superconductivity of niobium compounds vanishes at −153 °C, and in copper and bismuth compounds at −252 °C. Of course, these systems can be fully described, with detailed experimental confirmations, only in QFT, not in QM. They are long-lasting, quite stable systems (“the diamond is forever!” according to a popular commercial) and provide experimental confirmations that they are not subject to the well-known decoherence phenomenon, which occurs in QM, not in QFT. Indeed this has been shown in ref. [103] by explicit quantitative computations. The criticism by Max Tegmark on the decoherence of brain states had as its object the QM brain modeling (cf. his paper [104]), not the dissipative model, on which our discussions have been here based, which is a QFT model and is thus free from decoherence phenomenon.

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