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Favored neutrino mass mechanisms of the $0\nu\beta\beta$-decay unified by an interpolating formula

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Abstract. The neutrino mass mechanisms of the neutrinoless double beta decay ($0\nu\beta\beta$-decay) generated within the left-right symmetric models are discussed by taking advantage of an interpolating formula for nuclear matrix element depending on the neutrino mass. Its application is justified by a weak dependence of mean square neutrino momentum, which is a properly scaled ratio of nuclear matrix elements for the light and heavy neutrino mass mechanisms, on decaying isotope. Due to this fact there is a single unified lepton number violating parameter including dependendence on all neutrino masses in the decay rate and the light and heavy neutrino mass mechanisms can be hardly distinguished by observing the $0\nu\beta\beta$-decay on different isotopes. Further, a see-saw type neutrino mixing, which relates neutrino mixing in light and heavy neutrino sectors, is proposed. By considering the current limit on the $0\nu\beta\beta$-decay half-life of ^{136}Xe the regions of dominance of exchange of light and heavy neutrinos are identified.

1. Introduction

The main aim of experiments on the search for the neutrinoless double beta decay process ($0\nu\beta\beta$-decay) [1],

$$(A, Z) \rightarrow (A, Z+2) + 2e^-, \tag{1}$$

is to prove that neutrinos are Majorana particles and to measure the effective Majorana neutrino mass

$$m_{\beta\beta} = \left| \sum_{j=1}^{3} U_{ej}^2 m_j \right|, \tag{2}$$

where U_{ej} is the element of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix and m_j is the mass of neutrino. It is assumed that the conventional light neutrino exchange mechanism generated by left-handed V-A weak currents is the dominant mechanism of the $0\nu\beta\beta$-decay.

The value of $m_{\beta\beta}$ can be evaluated with help of neutrino oscillation parameters by making an assumption about the mass of lightest neutrino and by choosing a type of spectrum (normal or inverted) and values of CP violating Majorana phases. In future experiments, a sensitivity a few tens of meV to $m_{\beta\beta}$ is planned to be reached. This is the region of the inverted hierarchy of neutrino masses. In the case of the normal mass hierarchy $m_{\beta\beta}$ is too small, a few meV, to be probed in the $0\nu\beta\beta$-decay experiments of the next generation [1].

However, there is a possibility that the $0\nu\beta\beta$-decay is governed by another mechanism having origin in theories beyond the Standard model of particle physics. Currently, the attention is paid to the left-right symmetric theories [2, 3], which allow to understand the smallness of neutrino masses in a natural way. In this scenario, there are additional contributions to the $0\nu\beta\beta$-decay rate due right-handed currents and heavy neutrinos.

In this contribution we show that the observation of the $0\nu\beta\beta$ half-life with different isotopes will not allow to distinguish light and heavy neutrino mass mechanisms of this process as the effective neutrino momentum, which is determined through the ratio of corresponding nuclear matrix elements, depends only weakly on a given isotope. This fact allows to introduce a general lepton number violating parameter (LNV) of these mechanisms by taking the advantage of the interpolating formula for the nuclear matrix element depending on neutrino mass. Further, the dominance of light and heavy neutrino mass mechanisms is studied by considering the see-saw type neutrino mixing in which mixing of heavy neutrinos is related to the mixing of light neutrinos due to unitary condition.

2. The $0\nu\beta\beta$-decay expressed with interpolating formula

The Majorana neutrino mass mechanisms of the $0\nu\beta\beta$-decay within left-right symmetric models (LRSM) [2, 3] are considered. The left-handed ν_{eL} and right-handed ν_{eR} weak eigenstate electron neutrinos are expressed as superpositions of the light and heavy mass eigenstate Majorana neutrinos ν_j and N_j as follows:

$$\nu_{eL} = \sum_{j=1}^{3}\left(U_{ej}\nu_j + S_{ej}N_j^C\right), \qquad \nu_{eR} = \sum_{j=1}^{3}\left(T^*_{ej}\nu_j^C + V^*_{ej}N_j\right). \tag{3}$$

The 3×3 block matrices in flavor space U, S, T, V form a 6×6 unitary neutrino mixing matrix [4]

$$\mathcal{U} \;=\; \begin{pmatrix} U & S \\ T & V \end{pmatrix}. \tag{4}$$

It is a generalization of the PMNS matrix, which diagonalizes the general 6×6 Dirac-Majorana neutrino mass matrix in the Langrangian. As a result one ends up with 3 light m_i (i=1,2 and 3) and 3 heavy M_i (i=1,2 and 3) neutrino masses.

For sake of simplicity the possible $W_L - W_R$-mixing is not taken into account and the subdominant left-right interference term for the $0\nu\beta\beta$-decay half-life is neglected. Then, we have

$$[T^{0\nu}_{1/2}]^{-1} \;=\; G^{0\nu}g_{\rm A}^4 m_p^2\left(\left|\sum_{i=1}^{3}\left(U^2_{ej}\; m_i\; M'^{0\nu}(m_i) \;+\; S^2_{ej}\; M_i\; M'^{0\nu}(M_i)\right)\right|^2 \right.$$
$$\left. + \lambda^2\left|\sum_{i=1}^{3}\left(T^2_{ej}\; m_i\; M'^{0\nu}(m_i) \;+\; V^2_{ej}\; M_i\; M'^{0\nu}(M_i)\right)\right|^2\right).$$

Here, $\lambda = (M_{W_L}/M_{W_R})^2$, where M_{W_L} and M_{W_R} ($M_{W_L} < M_{W_R}$) are masses of W_L and W_R gauge bosons, respectively. $m_{\rm p}$ is the proton mass and $g_{\rm A}$ is the unquenched value of axial-vector coupling constant ($g_{\rm A} = 1.269$). The phase-space factor $G^{0\nu}$ is tabulated for various $0\nu\beta\beta$-decaying nuclei in Ref. [5]. The nuclear matrix element $M'^{0\nu}$ is given by [6]

$$M'^{0\nu}(m) = \frac{1}{m_{\rm p}m_{\rm e}}\,\frac{R}{2\pi^2 g_A^2}\sum_n\int d^3x\,d^3y\,d^3p e^{i{\rm p}\cdot({\bf x}-{\bf y})}\frac{\left\langle 0_F^+\right| J^{\mu\dagger}({\bf x})\left|n\right\rangle\left\langle n\right| J_\mu^\dagger({\bf y})\left|0_I^+\right\rangle}{\sqrt{p^2+m^2}(\sqrt{p^2+m^2}+E_n-\frac{E_I-E_F}{2})}\,. \quad (5)$$

Here, m_e and R are the mass of electron and the nuclear radius, respectively. Initial and final nuclear ground states with energies E_I and E_F are denoted by $\left|0_I^+\right\rangle$ and $\left|0_F^+\right\rangle$, respectively. The summation runs over intermediate nuclear states $|n\rangle$ with energies E_n. For sake of simplicity we assume that coupling constants of left- and right-handed weak one-body nuclear charged currents are the same, $J^\dagger_{LR\mu} = J^\dagger_{L\mu} \equiv J^\dagger_\mu$. $J^\dagger_\mu$ depends on the effective value of axial-vector coupling constant $g_{\rm A}^{\rm eff}$ of the nucleon in a nucleus.

Nuclear matrix elements associated with light and heavy neutrino mass mechanisms, $M'^{0\nu}_\nu$ and $M'^{0\nu}_N$, are limiting cases of $M'^{0\nu}(m)$ in Eq. (5):

$$M'^{0\nu}(m_i \to 0) = \frac{1}{m_{\rm p}m_{\rm e}}M'^{0\nu}_\nu, \qquad M'^{0\nu}(M_{\rm i}\to\infty) = \frac{1}{M_{\rm i}^2}M'^{0\nu}_{\rm N}. \quad (6)$$

To a good accuracy $M'^{0\nu}(m)$ can be approximated by the formula

$$M'^{0\nu}(m) \simeq M'^{0\nu}_{\rm N}\frac{1}{\langle p^2\rangle + m^2} \quad (7)$$

interpolating two limiting cases in (6). The parameter of this "interpolating formula",

$$\langle p^2\rangle = m_{\rm p}m_{\rm e}\frac{M'^{0\nu}_{\rm N}}{M'^{0\nu}_\nu} \quad (8)$$

is interpreted as the mean square neutrino momentum in a nucleus. We note that the "interpolating formula" in Eq. (7) reproduces the "exact" QRPA result with rather good accuracy except for the transition region where its deviation amounts 20% - 25% [6].

The current values of the matrix elements $M'^{0\nu}_\nu$ and $M'^{0\nu}_{\rm N}$ calculated within the interacting shell model (ISM) [7], interacting boson model (IBM)[8], quasiparticle random phase approximation (QRPA) [9, 10]), projected Hartree-Fock Bogoliubov approach (PHFB)[11], and covariant density functional theory (CDFT) [12] are presented, e.g., in Tables 6 and 7 of Ref. [1]. The value of corresponding parameter $\sqrt{\langle p^2\rangle}$ is shown for various isotopes in Table 1. The unquenched value of axial-vector coupling constant and the Argonne two-nucleon short-range correlations are assumed.

For the averaged value $\sqrt{\langle p^2\rangle_a}$ and variance σ associated with considered nuclear structure approaches we find

$$\begin{aligned}\sqrt{\langle p^2\rangle_a}(\sigma) &= 155(17)\ \text{MeV}, \quad (\text{ISM})\\ &= 120(17)\ \text{MeV}, \quad (\text{IBM})\\ &= 175(11)\ \text{MeV}, \quad (\text{QRPA})\\ &= 128(4)\ \text{MeV}, \quad (\text{PHFB})\\ &= 132(5)\ \text{MeV} \quad (\text{CDFT}).\end{aligned} \quad (9)$$

Table 1. The values of the parameter $\sqrt{\langle p^2 \rangle}$ of the interpolating formula specified in Eq. (10) for a given isotope calculated within different nuclear structure approaches: interacting shell model (ISM) [7], interacting boson model (IBM)[8], quasiparticle random phase approximation (QRPA) [9, 10]), projected Hartree-Fock Bogoliubov approach (PHFB)[11], and covariant density functional theory (CDFT) [12]. The Argonne short-range correlations are taken into account. The non-quenched value of weak axial-vector coupling g_A is assumed.

Method	$\sqrt{\langle p^2 \rangle}$ [MeV]										
	^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{116}Cd	^{124}Sn	^{128}Te	^{130}Te	^{136}Xe	^{150}Nd
ISM	178	134	138				153		159	170	
IBM	113	103	103	129	136	130	109	109	109	107	155
QRPA	189	163	164	180	174	157	186	178	180	183	
PHFB				130	127			131	132		121
CDFT	122	129	131	129	131	133	138		138	137	138

We notice a large spread of values of the parameter $\sqrt{\langle p^2 \rangle_a}$ due to different nuclear structure approaches. The largest and smallest value (175 MeV and 120 MeV) is obtained within the QRPA with isospin restoration [9, 10]) and the IBM [8], respectively. However, in the case of all nuclaer structure approaches the variance σ is very small of the order of 3-10 %. It allows to conclude that $\langle p^2 \rangle$ is practically the same for all isotopes of experimental interest and can be replaced with averaged value $\langle p^2 \rangle_a$. This conclusion is also supported by the statistical treatment of $M'^{0\nu}_{\nu}$ and $M'^{0\nu}_{\mathrm{N}}$ performed in Ref. [13].

By taking the advantage of the "interpolating formula" (7) with the parameter $\langle p^2 \rangle_a$ to a good accuracy the $0\nu\beta\beta$-decay half-life in Eq. (5) can be rewritten as

$$[T^{0\nu}_{1/2}]^{-1} = \eta^2_{\nu N}\, g^4_{\mathrm{A}} \left| M'^{0\nu}_{\nu} \right|^2 G^{0\nu}. \tag{10}$$

with

$$\eta^2_{\nu N} = \left| \sum_{j=1}^{3} \left(U^2_{ej} \frac{m_j}{m_e} + S^2_{ej} \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M^2_j} \frac{M_j}{m_e} \right) \right|^2 \tag{11}$$
$$+ \lambda^2 \left| \sum_{j=1}^{3} \left(T^2_{ej} \frac{m_j}{m_e} + V^2_{ej} \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M^2_j} \frac{M_j}{m_e} \right) \right|^2 .$$

Here, the parameter $\eta_{\nu N}$ is a general lepton number violating parameter for light and heavy neutrino mass mechanisms in the context of the left-right symmetric models, which does not depend on the isotope under consideration.

From Eqs. (10) and (11) it follows that the dominance of light or heavy neutrino mechanisms of the $0\nu\beta\beta$-decay can not be established by an observation of this process on different isotopes. This task requires an additional information or assumption concerning neutrino masses and mixing.

3. Study of the dominance of light and heavy neutrino mass mechanisms

For sake of simplicity the flavor universal mixing between the active and sterile neutrino sectors of the seesaw mixing matrix $\mathcal{U}$ is proposed to study the dominance of light and heavy neutrino mass mechanisms in the $0\nu\beta\beta$-decay rate. We have

$$\mathcal{U} = \begin{pmatrix} U_0 & \zeta\, \mathbf{1} \\ -\zeta\, \mathbf{1} & V_0 \end{pmatrix}. \tag{12}$$

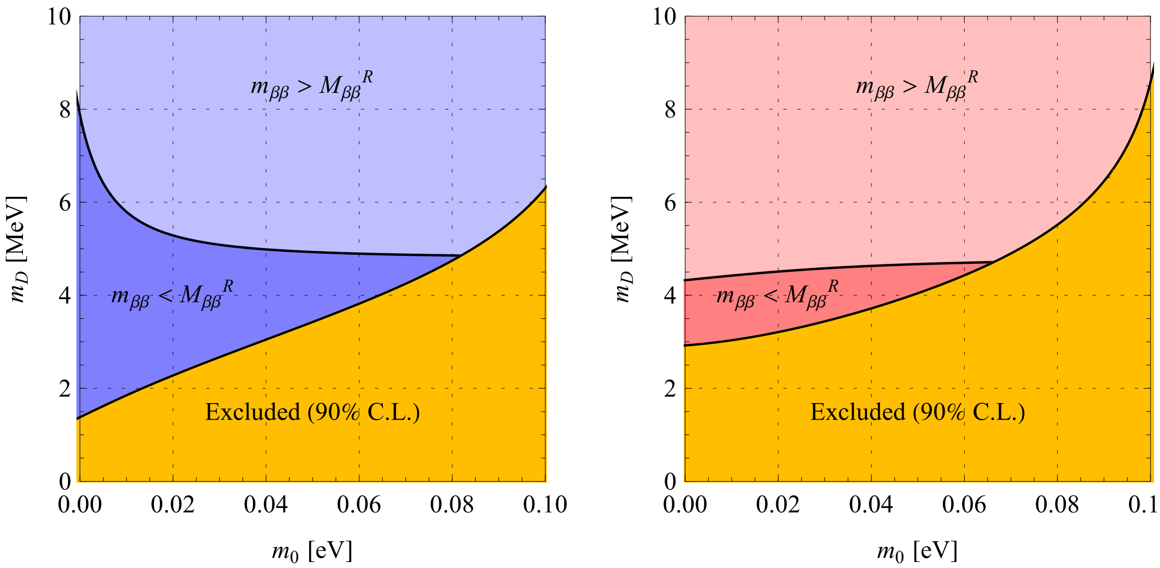

Figure 1. The region of the dominance of the $M^R_{\beta\beta}$ contribution over $m_{\beta\beta}$ contribution to the LNV parameter $\eta_{\nu N}$ for the see-saw type of neutrino mixing matrix given in Eq. (12) and by assuming $m_i \simeq m_D^2/M_i$ The cases of the normal and inverted hierarchy of neutrino masses are presented in the left and right pannels, respectively. The current constraint on the $\eta_{\nu N}$ parameter is deduced from the lower limit on the $0\nu\beta\beta$-decay half-life of ^{136}Xe [15] by using nuclear matrix element calculated within the QRPA [9]. For square ratio of masses of left and right vector bosons $\lambda = 7.7 \cdot 10^{-4}$ is considered.

Here, $\zeta = \frac{m_D}{m_{LNV}}$, where m_D is the typical scale of the charged leptons masses and m_{LNV} is the LNV scale of the order of the Majorana masses M_i of the heavy neutrinos. U_0 can be identified with the PMNS U matrix ($U = U_0$). From the unitarity conditions we find

$$V_0 = U_0^\dagger, \qquad U_0 U_0^\dagger = (1-\zeta^2)\mathbf{1}, \qquad V_0 V_0^\dagger = (1-\zeta^2)\mathbf{1}. \tag{13}$$

It is assumed that a small violation of the unitarity of U_0 and V_0 matrices is beyond the current accuracy of phenomenological determination of elements of the PMNS matrix.

Further, we assume that $m^2_{LNV} \gg \langle p^2 \rangle_a$ and the sees-saw relations for neutrino masses $m_i \simeq m_D^2/m_{LNV}$ and $M_i \simeq m_{LNV}$. We find

$$\eta^2_{\nu N} \;=\; \frac{1}{m_e^2}\left(m^2_{\beta\beta} \;+\; (M^{\rm R}_{\beta\beta})^2\right), \qquad M^{\rm R}_{\beta\beta} = \lambda\,\frac{\langle p^2\rangle_a}{m_D^2}\left|\sum_{j=1}^{3}(U_0^\dagger)^2_{ej}\, m_j\right|. \tag{14}$$

The matrix $U^\dagger$ expressed with three mixing angles θ_{12}, θ_{13} and θ_{23} and three CP violating phases α_1, α_2 and δ takes the form

$$V_0 = \begin{pmatrix} c_{12}\,c_{13}\,e^{-i\alpha_1} & \left(-s_{12}\,c_{23} - c_{12}\,s_{13}\,s_{23}\,e^{-i\delta}\right)e^{-i\alpha_1} & \left(s_{12}\,s_{23} - c_{12}\,s_{13}\,c_{23}\,e^{-i\delta}\right)e^{-i\alpha_1} \\ s_{12}\,c_{13}\,e^{-i\alpha_2} & \left(c_{12}\,c_{23} - s_{12}\,s_{13}\,s_{23}\,e^{-i\delta}\right)e^{-i\alpha_2} & \left(-c_{12}\,s_{23} - s_{12}\,s_{13}\,c_{23}\,e^{-i\delta}\right)e^{-i\alpha_2} \\ s_{13}\,e^{i\delta} & c_{13}\,s_{23} & c_{13}\,c_{23} \end{pmatrix}. \tag{15}$$

Here, $c_{ij} \equiv \cos(\theta_{ij})$ and $s_{ij} \equiv \sin(\theta_{ij})$. We note that each element of the first row is multiplied by the same phase $e^{-i\alpha_1}$, i.e., the Majorana phase α_1 (and α_2 as well) does not affect the heavy neutrino LNV parameter $M^R_{\beta\beta}$ within considered scenario. On the contrary, the Dirac phase δ affects $M^R_{\beta\beta}$ and has no impact on $m_{\beta\beta}$.

In Fig. 1 the region of dominance of $M^R_{\beta\beta}$ mechanism over $m_{\beta\beta}$ mechanism in the $0\nu\beta\beta$-decay rate is displayed in the plane of parameters m_D and m_0 (the lightest neutrino mass). The cases of the normal (left panel) and inverted hierarchy (right panel) of neutrino masses are discussed. We see that in the case of normal (inverted) hierarchy $m_D \leq 1.5$ MeV ($m_D \leq 2.9$ MeV) is already excluded by the present experimental lower bound on the $0\nu\beta\beta$-decay half-life of ^{136}Xe.

4. Conclusions

In summary, the light and heavy neutrino mass mechanisms of the $0\nu\beta\beta$-decay generated within the left-right symmetric models were analyzed. It was found that properly scaled ratio of nuclear matrix elements for the light and heavy neutrino mass mechanisms, which can be identified with square mean neutrino momentum $\langle p^2 \rangle$ of the interpolating formula, exhibits practically no dependence on isotope for all favoured nuclear structure methods. This fact allows to introduce new lepton number violating parameter $\eta_{\nu N}$, which is a coherent sum of square lepton number violating parameters $m_{\beta\beta}$ and $M^R_{\beta\beta}$ associated with light and heavy neutrino exchange mechanisms, respectively. It is concluded that the observation of the $0\nu\beta\beta$-decay on two and more nuclear isotopes will allow to decide about the dominance of light or heavy neutrino-mass mechanism in the decay rate.

Further, a viable see-saw type 6×6 neutrino mixing matrix (see Eq.(12)), where due to unitarity condition the 3×3 mixing matrix of heavy neutrinos is just hermitian conjugate of the 3×3 mixing matrix (PMNS matrix) of light neutrinos, was propsed. Within this neutrino mixing scenario the dominance of light and heavy neutrino mass mechanisms in the plane of parameters of Dirac mass m_D and lightest neutrino mass m_0 was established by using the experimental contraint on the $0\nu\beta\beta$-decay of ^{136}Xe and assuming $\lambda = 7.7 \cdot 10^{-4}$.

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