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Thermodynamic Scaling Analysis of Cavitating Fluid Transients in a Cryogenic Environment

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Abstract. Fluid transients resulting from rapid flow acceleration or deceleration can pose a significant risk to the structural strength of fluid handling networks, such as those in cryogenic rocket propulsion systems, LNG transport systems, powerplant flow systems, household water systems and many others. Additionally, due to its oscillating behaviour, fluid transients can lead to the formation and subsequent collapse of vapour cavities in the high-pressure regions, leading to cavitation that may potentially damage equipment if not adequately addressed. The complexity of cavitation-induced fluid transient is heightened in cryogenic fluids due to significant variations in thermophysical properties and the presence of a thermal delay effect, an outstanding phenomenon for cryogenic fluids resulting in cavitation suppression through a reduction in the localized vapour pressure. Cryogenic liquids, operating at extremely low temperatures, require increased maintenance and additional safety precautions. Also, the cryogenic fluids operate near the critical temperature, making them prone to cavitation. In such a situation, experiments using cryogenic fluid become challenging and expensive. Therefore, non-dimensional thermodynamic scaling analysis emerges as a valuable tool, facilitating the substitution of cryogenic fluid with a preferred alternative while maintaining comparable fluid dynamics and thermodynamic characteristics, accounting for the thermal delay effect. This article presents a scaling analysis of the cavitation-induced fluid transient within a fluid network consisting of a fast-closing valve at the downstream end. The study employs cryogenic fluid and hot water at a temperature corresponding to the same thermodynamic parameter, using various scaling models. A comprehensive comparative assessment of the cavitation-induced fluid transient behaviour in cryogenic fluid and hot water is conducted to ascertain the similarity in flow conditions. The similarity approach based on this thermodynamic scaling will be used for a proposed scaled-down experimental setup to study the cryogenic fluid transients at IIT Kharagpur.

1 Introduction

Fluid transients are phenomena that occur due to abrupt changes in the flow parameters within a fluid network. These changes can cause a surge in the pressure, followed by oscillations that propagate at the speed of sound. The pressure depression below the vapour pressure during transient oscillations in pipelines can lead to cavitation, resulting in two-phase fluid transients. Cavitation often occurs at the closed ends of pipelines and areas with changing slopes, where vapour bubbles form and are released into the liquid. The collapse of these bubbles produces localized high-speed liquid jets and shock waves, causing fatigue stress and potential damage. As fluid transients are periodic, the expanding vapour cavity contracts, causing the separated liquid columns to reunite and produce large pressure fluctuations [1].

During cavitation and bubble formation, the latent heat required for the phase change must be extracted from the surrounding liquid. This heat is taken from the neighbouring fluid as sensible heat, causing its temperature to drop (ΔT), which in turn decreases its vapour pressure. This results in the suppression of cavitation caused by bubble formation. This suppression is known as thermal delay, and the phenomenon is commonly referred to as the "thermodynamic effect." The magnitude of this effect can be measured by the temperature drop or ΔT.

Cryogenic fluid transients in pipe flow are critical considerations in various applications, such as delivering cryogenic propellants (e.g., LOX and LH2) in rocket engines. These fluids operate at extremely low temperatures near their critical points, necessitating high maintenance and generation costs along with stringent safety measures. Due to these challenges, experimenting with actual cryogenic fluids in the early design stages of pipe flow systems is often infeasible and costly. Also, cavitation in cryogenic fluids is more complex than in water due to the unique properties of cryogenic fluids, like low liquid-to-vapor density ratio, low latent heat of vaporization, and low thermal conductivity. Therefore, the thermodynamic effect is more prominent in the cryogenic fluids or in water at higher temperatures.

Scaling offers a practical solution for experimentation by replacing the cryogenic fluids with an alternative fluid while maintaining the same thermodynamics and flow behaviour. The thermal and transport properties of cryogenic fluids significantly influence flow characteristics; thus, the thermodynamic effect compensates for the differences when using a substitute fluid.

The purpose of this study is to analyze the thermodynamic scaling of fluid transients caused by sudden valve closure in a cryogenic environment. We use liquid nitrogen and water as reference fluids. By comparing the fluid behaviours of these fluids, we aim to assess the similarity in flow conditions and cavitation behaviour.

2 Methodology

The work of Streeter and Wylie [2], and others culminated in 1-D equations governing fluid transients, expressed in terms of piezometic head H and discharge Q

$$\frac{\partial H}{\partial t} + \frac{a^2}{gA}\frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial H}{\partial x} + \frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{4\tau_t}{\rho g D} = 0 \tag{2}$$

where a is the wave speed of the pressure wave, A is cross-sectional area, g is gravitational constant, ρ is density and τ_t is wall shear stress, The total wall shear stress (τ_t) is a sum of steady (τ_s) and unsteady (τ_u) components:

$$\tau_t = \tau_s + \tau_u \tag{3}$$

Various models are available in the literature to predict this component. Brunone et al. [3] introduced an instantaneous acceleration-based model, which is used in the present study where the unsteady shear stress is proportional to both local and convective acceleration. The equation for unsteady shear stress is:

$$\tau_u = \frac{k\rho D}{4A}\left(\frac{\partial Q}{\partial t} - a \cdot sign(Q)\frac{\partial Q}{\partial x}\right) \tag{4}$$

where k is a damping coefficient and D is diameter of the pipe.

The Method of Characteristics (MOC) is a widely used numerical technique for solving the governing hyperbolic partial differential equations (Eq.1 and Eq.2). MOC is popular because it offers accurate numerical performance, simplicity, and ease of implementation [4].

2.1 Two-phase modelling of fluid transients

Numerical models for approximating two-phase transient flows have been developed by Wylie and Streeter [5], and Bergant and Simpson [6], with the Discrete Vapor Cavity Model (DVCM) and Discrete Gas Cavity Model (DGCM) being widely used.

This study uses the DVCM technique to model two-phase fluid transients. As shown in Fig.1, in DVCM, it is assumed no vapour forms during the steady state, allowing vapour to form only during transient at sections where fluid pressure falls below the vapour pressure. This section is treated as a pressure boundary, set at the vapor pressure for a given temperature. Pure liquid with constant wave speed is assumed between grid points where standard MOC is applicable. The volume of vapour at a computational section is calculated using discharge at the current and previous time steps, with a weighting factor determining stability. The vapour collapses when its volume reaches zero or less, allowing the single-phase flow model to apply using standard MOC. Although DVCM is generally accurate, it can produce unrealistic spikes and oscillations when distributed cavitation occurs [4].

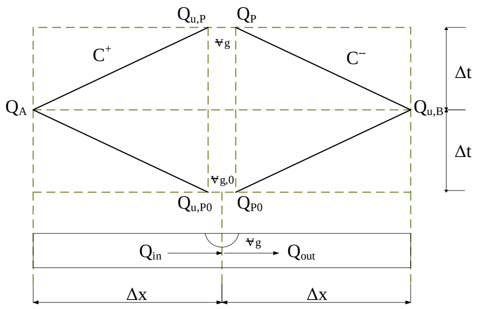

Figure 1: Grid for Two-Phase MOC.

The following equation simulates the change in volume of this gas or vapour cavity:

$$\frac{d\forall_g}{dt} = Q_{out} - Q_{in} \tag{5}$$

Where $\forall_g$ is the volume of the gas cavity being modelled, Q_{out} is a flowrate leaving the node, and Q_{in} is a flowrate entering the node. Integration of Eq. (5) over $t-2\Delta t$ to t result in following equation,

$$\forall_{g,P} = \forall_{g,P0} + 2\Delta t\left(\psi\left(Q_P - Q_{u,P}\right) + (1-\psi)\left(Q_{P0} - Q_{u,P0}\right)\right) \tag{6}$$

Here, $\forall_{g,P}$ is the volume of the vapour cavity or gas pocket at any time t. The subscript P_0 represents points at time $t-2\Delta t$ and P is for time t. Q_P is the flow rate exiting the node and $Q_{u,P}$ is the flow rate entering the node at time t . Similarly, the flow rates with subscript P_0 indicate the conditions at time $t-2\Delta t$.

2.2 Thermodynamic effect

Based on the energy conservation at equilibrium, Stahl & Stepanoff [7] first proposed a B_{factor} to determine the temperature drop (ΔT) as a measure of thermal delay effect as follows:

$$B_{\text{factor}} = \frac{\rho_l C_{pl} \Delta T}{\rho_v h_{fg}} \tag{7}$$

Where C_p is specific heat, h_{fg} is the latent heat of vaporization, and subscripts l and v denote liquid and vapour phase, respectively. The variation in the temperature drop, along with the B_{factor} value, helps assess the significant thermal effects observed in cryogenic fluids.

As an alternative to the B-factor, Brennen [8] proposed a parameter that accounts for the influence of time-dependent thermal boundaries and bubble dynamics using the Rayleigh-Plesset equation. This parameter is defined as

$$\Sigma = \frac{\rho_v^2 h_{fg}^2}{\rho_l^2 \sqrt{\alpha_l} C_{pl} T_l} \tag{8}$$

To evaluate and compare the thermal delay effects in water and cryogenic fluids for turbopump inducers, Ehrlich & Murdock [9] derived a similar dimensionless bubble growth parameter (DB). This parameter incorporates factors like the rotational speed of the inducer Ω and the inducer tip radius r.

$$DB = \frac{r\Omega^{3/2} C_{pl} T_l \rho_l^2 \sqrt{\alpha_l}}{h_{fg}^2 \rho_v^2} \tag{9}$$

To compare the cavitating fluid transients in water and cryogenic fluids, we modified the Ehrlich & Murdock DB parameter for pipe flow by incorporating axial flow velocity instead of the pump inducer speed. The new DB parameter used in this study for pipe flow is defined as:

$$DB_{\text{new}} = \frac{u^{3/2} C_{pl} T_l \rho_l^2 \sqrt{\alpha_L}}{h_{fg}^2 \rho_v^2} \tag{10}$$

3 Results and discussion

The purpose of the current study is to analyse the thermodynamic scaling for cavitating fluid transients occurring due to the sudden closure of the valve in the cryogenic environment. Fig. 2. shows the setup used to perform the numerical simulations. The parameters for the simulations are mentioned in the Table.1. The data used is inspired by the work of Klein et al.[10] using liquid nitrogen as a working fluid.

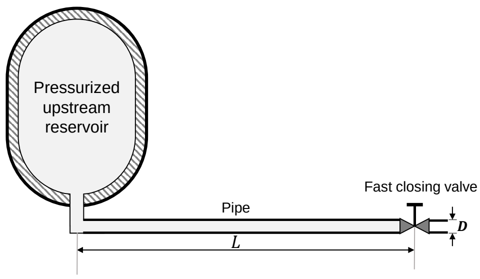

Figure 2: Schematic of the system under study consisting of reservoir, pipeline, and a fast closing valve

Table 1: Simulation parameters for water and liquid nitrogen

Parameters	**Water**	**Liquid Nitrogen**
Pipe length L (m)	9.29	
Pipe diameter D (m)	0.019	
Reservoir pressure (kPa)	1394	
Valve closure time (s)	0.018	
Velocity u (m/s)	**1.375**	**3.25**
Young's Modulus E (GPa)	190	
Pipe roughness ϵ (m)	2×10^{-6}	
Temperature (K)	293-423	87

Fig.3 shows the numerical results obtained using the Method of Characteristics (MOC) with the Discrete Vapor Cavitation Model (DVCM) technique for liquid nitrogen. The y-axis represents the pressure head oscillations at the end of the pipe, just before the downstream valve. The valve closes in 18 milliseconds, as shown in the graph. Due to the sudden closure, the pressure rises to more than twice the initial pressure. As the pressure oscillates, it tries to drop back down but stops at the vapour pressure, causing cavities to form. As time progresses, these cavities collapse due to the increase in pressure, creating a pressure spike higher than the initial peak. This cavitation behaviour continues until the cavities disappear and single-phase oscillations take over.

The aim of this study was to thermodynamically scale the fluid transients of water and nitrogen using a new Ehrlich & Murdock DB parameter (Eq.10). The water temperature was varied from 293 K to 423 K, to find the same DB parameter values ($DB_{H2O|293K} = 4.24$, $DB_{H2O|413K} = DB_{LN2|87K} = 3.9 \times 10^4$), and the results were compared with liquid nitrogen at 87 K. Fig.4 shows the comparison of pressure head oscillations due to sudden valve closure for water at 293 K and 413 K with liquid nitrogen at 87 K. The x-axis is non-dimensionalized with the Joukowsky peak pressure, and the y-axis is non-dimensionalized using the diffusion time as mentioned in Urbanowicz et al.[11]. The figure shows that the pressure head oscillations for water at 293 K do not match the amplitude and phase of the liquid nitrogen case. However, the results for water at 413 K show a comparable degree of agreement with the liquid nitrogen results.

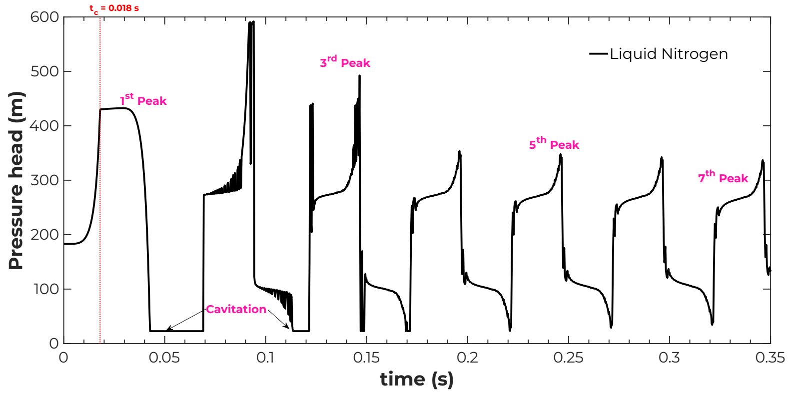

Figure 3: Pressure head oscillations for the liquid nitrogen obtained from numerical simulation using MOC with DVCM

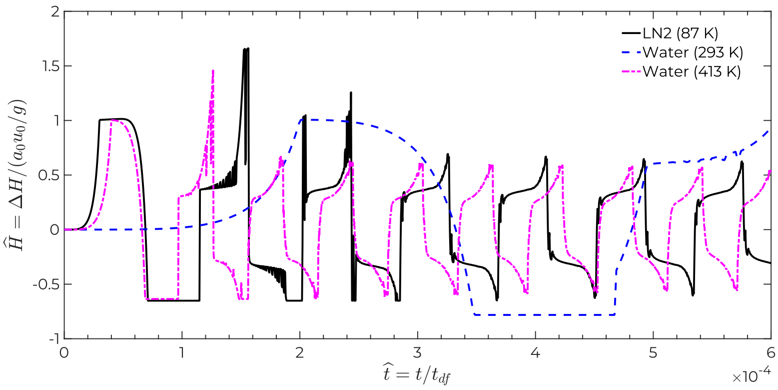

Figure 4: Comparison of dimensionless pressure head oscillations for the water and liquid nitrogen at varying temperatures

This shows the cavitation behaviour of water at the elevated temperature is similar to that of the liquid nitrogen.

Fig. 5 shows the non-dimensional void fraction of vapour at the end of the pipe before the valve for the three cases. The void fraction represents the ratio of the vapour phase to the total volume. From the figure, it can be observed that the cavity persists for a significantly longer period of time in the case of water at 293 K, whereas it shows similar behaviour with liquid nitrogen for water at 413 K, confirming the resemblance in the nature of cavitation.

4 Conclusions

This study effectively demonstrates the application of non-dimensional thermodynamic scaling for analyzing cavitation-induced fluid transients in cryogenic fluids, utilizing hot water as an experimental surrogate. By employing the Method of Characteristics alongside the discrete vapour cavity method, accurate numerical simulations of fluid transients with cavitation were achieved. The modified Ehrlich and Murdock's Dimensionless DB parameter proved instrumental in ensuring thermodynamic scaling, showing a comparable degree of similarity in cavitation behaviour between liquid nitrogen at 87 K and water at an elevated temperature of 413 K. Despite some phase differences and prolonged cavity presence in room temperature water, the comparative analysis confirms the feasibility of using hot water for scaled-down experimental setups. Future refinement of this scaling method will enhance its precision, thereby facilitating the design of a reliable experimental setup at IIT Kharagpur to further study cryogenic fluid transients.

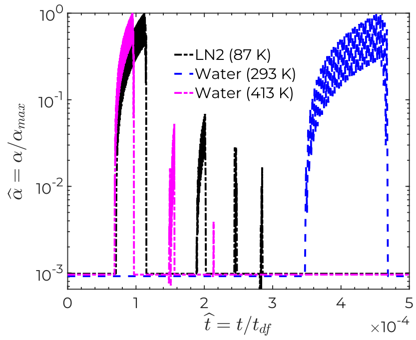

Figure 5: Comparison of dimensionless void fraction of vapour for the water and liquid nitrogen

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