# TOPOLOGY OF HIGGS FIELDS

J. Arafune*, P.G.O. Freund+ and C.J. Goebel§

*National Laboratory for High Energy Physics
Oho-Machi, Tsukuba-Gun, Ibaraki 300-32, Japan

+Enrico Fermi Institute and Department of Physics
University of Chicago, Chicago, Ill 60637, U.S.A.

§Department of Physics
University of Wisconsin, Madison, Wis 53706, U.S.A.

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Abstract It is shown that the conserved magnetic charge discovered by 't Hooft in nonabelian gauge theories with spontaneous breaking is not associated with the invariance of the action under a symmetry group. Rather, it is a topological characteristic of an isotriplet of Higgs fields in a three-dimensional space: the Brouwer degree of the mapping from a large sphere in the configuration space to the unit sphere in the field space provided by the normalized Higgs fields $\hat{\phi}^a = \phi^a(\phi^b\phi^b)^{-1/2}$. The use of topological methods in determining magnetic charge configuration is outlined. A peculiar interplay between Dirac strings and zeros of the Higgs fields under the gauge transformations is pointed out. The monopole-antimonopole system is studied. ( to be published in Journal of Mathematical Physics )

## DISCUSSIONS

(1) Question by B.Schroer: Are the conservation laws a property of a particular solution, a property of a class of solutions or an intrinsic property of the dynamics?
Answer: They are due to the dimension of the space and the special number of the Higgs fields. They are true for any reasonable solutions.

(2) Question by B.Schroer: Do you expect this classical solution to be useful in a future quantum field theory?
Answer: Yes, I think so. But it is not clear to me whether the conservation law has a positive meaning in the quantized theory.

(3) Question by N.Nakanishi: I am afraid that "$\dot{Q}_{mag} = 0$" does not necessarily follow from $\partial_\mu j^\mu_{mag} = 0$, because in this derivation one has to assume the vanishing of $j^\mu_{mag}$ at very distant points. This assumption may not be satisfied in this case, because the normalized field $\hat{\phi}^a = \phi^a / |\vec{\phi}|$, ( $|\vec{\hat{\phi}}| = 1$ ), is used in the definitions of $j^\mu_{mag}$ . Furthermore, I think that $Q_{mag}$ may change discontinuously if all the components of $\vec{\phi}$ vanish at a particular *time* $x^0 = a^0$.
Answer: Fortunately we are dealing with fields with spontaneous breaking. In order that the energy of a given system is to be finite $|\vec{\phi}|$ must approach a non-vanishing constant at distant points. Therefore vanishing of $\vec{\phi}$ for all the space at a time $a^0$ is not allowed; for it gives infinite energy at that time. For simplicity we examined only a case where there are finite number of zeros of Higgs fields. Since $j^0_{mag} = 0$ for non-vanishing $|\vec{\phi}|$, the boundary condition is satisfied to obtain $\dot{Q}_{mag} = 0$ in such a case.