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Secondarily-induced gravitational wave background in modified gravity

Masaaki Morita* and Hirotaka Takahashi

Nagaoka University of Technology, 1603-1 Kamitomioka, Nagaoka, Niigata 940-2188, Japan

E-mail: morita@mst.nagaokaut.ac.jp, hirotaka@oberon.nagaokaut.ac.jp

Abstract. We explore the effect of structure formation on gravitational wave background in dark energy cosmology. Second-order tensor perturbations induced by first-order scalar mode are considered in a modified gravity dark energy model. We present explicitly the procedure for an approximate estimation of the second-order tensor perturbations caused by primary structure formation.

1. Introduction

It is more than ten years since accelerated expansion in the late-time universe was first discovered by observations of type Ia supernovae [1, 2, 3]. In order to account for the cosmic acceleration, theoretical possibilities have been explored with a large number of dark energy models [4]. A 'modified gravity' dark energy model, in which some corrections are added to the Einstein-Hilbert action, is one of theoretically possible models. Although it is of interest to consider the link between gravitational theories and dark energy, modified gravity models have generally severe constraints that come from local gravity tests and cosmological observations [5]. For such constraints, matter density perturbations have been studied extensively [6], but in future measurements of relic gravitational waves will also be useful [7].

In this article, we consider cosmological gravitational wave background generated by primary structure formation. We present how to compute second-order tensor perturbations induced by first-order scalar perturbations in the framework of a modified gravity dark energy model. The appearance of the effect of gravity modification is presumed with the help of the results of the second-order perturbations in a standard, spatially flat FLRW universe model.

2. Basic equations in modified gravity

Let us consider modified gravity models with the action

$$\mathcal{S} = \frac{1}{16\pi G}\int f(R)\sqrt{-g}\ \mathrm{d}^4x\ + \mathcal{S}_{\mathrm{m}}\,, \tag{1}$$

where G is Newton's gravitational constant, R is the four-dimensional scalar curvature, g is the determinant of the space-time metric $g_{\mu\nu}$, and $\mathcal{S}_{\mathrm{m}}$ denotes the action of pressureless matter. The action principle yields the following field equation:

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \mathcal{D}_\mu\mathcal{D}_\nu F(R) + g_{\mu\nu}\Box F(R) = 8\pi G T^{(\mathrm{m})}_{\mu\nu}\,, \tag{2}$$

where $F := \partial f/\partial R$, $\mathcal{D}$ represents the four-dimensional covariant derivative, $\Box := g^{\mu\nu}\mathcal{D}_\mu\mathcal{D}_\nu$, and $T^{(\mathrm{m})}_{\mu\nu}$ is the energy-momentum tensor of pressureless matter. Hereafter we write the function $f(R)$ as $R - \xi(R)$, where $\xi(R)$ plays the role of an effective dark energy component, and then $F(R) = 1 - \xi'(R)$ with a prime $(\,')\,:= \partial/\partial R$.

In a spatially flat, homogeneous and isotropic universe model with the scale factor $a(t)$ and the energy density ρ_m of pressureless matter, the field equation (2) reads

$$3H^2 = 8\pi G\rho_\mathrm{m} + \frac{1}{2}\xi(R_\mathrm{b}) + 3H\left[\xi'(R_\mathrm{b})\right]^{\cdot} - 3\xi'(R_\mathrm{b})\left(\dot{H} + H^2\right), \tag{3}$$

$$2\dot{H} = -8\pi G\rho_\mathrm{m} + 2\xi'(R_\mathrm{b})\dot{H} + \left[\xi'(R_\mathrm{b})\right]^{\cdot\cdot} - H\left[\xi'(R_\mathrm{b})\right]^{\cdot}, \tag{4}$$

where an overdot $(\dot{\ })$ denotes the time derivative $\partial/\partial t$, $H := \dot{a}/a$ is the Hubble parameter, and $R_\mathrm{b} := 6\dot{H} + 12H^2$. Note that these equations are reduced to the Friedmann equations in the standard FLRW universe model if we set $\xi(R) = 2\Lambda$ with a positive cosmological constant Λ.

3. Linear tensor perturbations in a modified gravity model

Let us proceed to linear perturbations in a modified gravity dark energy model. We choose a comoving and synchronous coordinate condition with the line element

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + {}^{(3)}g_{ij}\,\mathrm{d}x^i\mathrm{d}x^j, \tag{5}$$

and the four-velocity $u^\mu = (1, \mathbf{0})$ of pressureless matter. Writing the spatial metric ${}^{(3)}g_{ij}$ as ${}^{(3)}g_{ij} = a(t)^2(\gamma_{ij} + h_{ij})$, where γ_{ij} and h_{ij} are metric components of a background universe and a linearly perturbed universe, respectively, the four-dimensional scalar curvature is written as

$$R = R_\mathrm{b} + R_1 + \mathcal{O}(h^2)\,; \qquad R_1 := \ddot{h} + 4H\dot{h} + \frac{1}{a^2}\left(h^k{}_{\ell|k}{}^{|\ell} - \nabla^2 h\right), \tag{6}$$

where $h := \gamma^{ij}h_{ij}$ is the trace of the metric perturbation h_{ij}, and $\nabla^2(\cdot) := \gamma^{ij}(\cdot)_{|ij}$ (a vertical bar $|$ represents the covariant derivative with respect to γ_{ij}). By applying the Taylor expansion, we also have

$$F(R) = F_\mathrm{b} + F_1 + \mathcal{O}(h^2)\,; \quad F_\mathrm{b} := F(R_\mathrm{b}) = 1 - \xi'(R_\mathrm{b}), \quad F_1 := F'(R_\mathrm{b})R_1 = -\xi''(R_\mathrm{b})R_1\,. \tag{7}$$

Taking the part of order h of the field equation (2) with the help of equation (7), we obtain the evolution equation for h_{ij}:

$$\ddot{h}_{ij} + \left(3H + \frac{\dot{F}_\mathrm{b}}{F_\mathrm{b}}\right)\dot{h}_{ij} + \frac{2}{a^2}\left(\mathcal{R}_{ij} - \frac{1}{4}\mathcal{R}\gamma_{ij}\right) + \frac{1}{F_\mathrm{b}}\left[\ddot{F}_1 + 2H\dot{F}_1 - \left(\dot{H} + 3H^2\right)F_1\right]\gamma_{ij} - \frac{2}{a^2F_\mathrm{b}}F_{1\,|ij} = 0\,, \tag{8}$$

where $\mathcal{R}_{ij} := \frac{1}{2}(h^k{}_{i|jk} + h^k{}_{j|ik} - \nabla^2 h_{ij} - h_{|ij})$, and $\mathcal{R} := \gamma^{ij}\mathcal{R}_{ij}$. The transverse and traceless part of equation (8) gives

$$\ddot{h}^\mathrm{TT}_{ij} + \left(3H + \frac{\dot{F}_\mathrm{b}}{F_\mathrm{b}}\right)\dot{h}^\mathrm{TT}_{ij} - \frac{1}{a^2}\nabla^2 h^\mathrm{TT}_{ij} = 0\,, \tag{9}$$

where h^TT_{ij} is a transverse and traceless part of h_{ij}. Equation (9) tells us that the effect of gravity modification appears only in the friction term in the wave equation for the linear tensor perturbations. Solutions for equation (9) cannot be expressed in terms of analytic functions in general. However, in matter-dominated era, where the effective dark energy component can be neglected, solutions for the linear tensor perturbations in a Fourier space are approximately given as [8]

$$h_{ij}^{\rm TT}(k,a) \propto a^{-3/4}\mathcal{J}_{\pm 3/2}\left(\frac{2ka^{1/2}}{H_0\Omega_{\rm m0}^{1/2}}\right) , \tag{10}$$

and in dark-energy-dominated era, they should become asymptotically

$$h_{ij}^{\rm TT}(k,a) \propto a^{-3/2}\mathcal{J}_{\pm 3/2}\left(\frac{ka^{-1}}{H_0\Omega_{\Lambda 0}^{1/2}}\right) , \tag{11}$$

where $\mathcal{J}_\nu$ is Bessel's function of order ν, k is a wavenumber, and H_0, $\Omega_{\rm m0}$ and $\Omega_{\Lambda 0}$ denote the present values of the Hubble parameter, the matter density parameter and the amount of the effective dark energy component, respectively. These approximate solutions imply that the presence of the effective dark energy component leads to the changes of decaying rate and oscillation periods of the linear tensor perturbations.

4. Second-order tensor perturbations in a modified gravity model

Next let us consider second-order perturbations in a modified gravity dark energy model, and write the spatial metric ${}^{(3)}g_{ij}$ as ${}^{(3)}g_{ij} = a(t)^2(\gamma_{ij} + h_{ij}^{(1)} + h_{ij}^{(2)})$, where $h_{ij}^{(1)}$ and $h_{ij}^{(2)}$ represent the first-order and second-order metric perturbations, respectively. Here we focus on second-order tensor perturbations generated by primary structure formation so that $h_{ij}^{(1)}$ contains only a scalar mode. Following the procedure described in the case of the linear tensor perturbations, we obtain the evolution equation for $h_{ij}^{(2)}$:

$$\ddot{h}_{ij}^{(2)} + \left(3H + \frac{\dot{F}_{\rm b}}{F_{\rm b}}\right)\dot{h}_{ij}^{(2)} - \frac{1}{a^2}\nabla^2 h_{ij}^{(2)} = \mathcal{Q}_{ij} , \tag{12}$$

where $\mathcal{Q}_{ij}$ is a source term that consists of pieces of quadratic terms with respect to $h_{ij}^{(1)}$. In a spatially flat FLRW universe model under Einstein's gravity without a cosmological constant, a scalar-mode solution for linear metric perturbations is written as

$$h_{ij}^{(1)} = \frac{20}{9}\varphi\gamma_{ij} + 2a(t)\varphi_{|ij} , \tag{13}$$

where $\varphi = \varphi(\mathbf{x})$ is an initial gravitational potential. We can adopt the above scalar-mode solution as $h_{ij}^{(1)}$ if primary structure formation proceeds mainly in matter-dominated era, where the effect of dark energy component is not important. Substituting equation (13) into the source term $\mathcal{Q}_{ij}$, we find [9, 10, 11]

$$\mathcal{Q}_{ij} = \frac{3}{7}\left(\nabla^2\mu\gamma_{ij} + \mu_{|ij} - 4\nabla^2\mu_{ij}\right) , \tag{14}$$

where

$$\mu_{ij} := \frac{1}{2}\left(\nabla^2\varphi\varphi_{|ij} - \varphi_{|ik}\varphi^{|k}{}_{|j}\right) , \quad \mu := \gamma^{ij}\mu_{ij} . \tag{15}$$

In the case of Einstein's gravity, equation (12) is solved in a Fourier space using the method of a Green function as [11]

$$h_{ij}^{(2)}(k,t) = a(t)^2\frac{\mathcal{Q}_{ij}(k)}{k^2} + a(t)\frac{28\mathcal{Q}_{ij}(k)}{9k^4} + \left[1 - a^{-3/4}\mathcal{J}_{\pm 3/2}\left(\frac{2ka^{1/2}}{H_0\Omega_{\rm m0}^{1/2}}\right)\right]\frac{280\mathcal{Q}_{ij}(k)}{81k^6} . \tag{16}$$

Note that only the last term in equation (16) is observable as a gravitational wave because of its oscillatory behaviour, and its temporal dependence is the same as that of the solution (10) in the linear case. Hence we presume that the effect of gravity modification will also appear in the same way as the linear case.

5. Summary and outlook

In this article, cosmological gravitational wave background has been considered within a modified gravity dark energy model up to the second order in perturbations. We have shown how to compute the second-order tensor perturbations induced by the first-order scalar perturbations. In the level of linear perturbations, the effect of gravity modification appears only in the friction term in the wave equation, leading to the changes of decaying rate and oscillation periods. In the second-order perturbations, the appearance of the effect of gravity modification is approximately the same as in the linear level, if primary structure formation proceeds mainly in matter-dominated era. In order to make the results more rigorous, the effect of gravity modification should be taken into account in the calculation of the second-order source term $\mathcal{Q}_{ij}$. Also, the effect of scalar-type gravitational waves should be considered, which will arise typically in modified gravity models [7]. These are now under investigation.

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