CDF/ANAL/TOP/CDFR/2392
Version 1.1
January 5, 1994

Calculating the $t\bar{t}$ Production Cross Section

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1 Introduction

In this note we describe the calculation of the top production cross section for the SVX, soft lepton (SLT) and dilepton (DIL) counting experiments. We use the number of candidate events, expected backgrounds, acceptances and uncertainties on these from the latest documentation for these analyses. We discuss several methods for combining the analyses to calculate a combined total cross section which differ in how the acceptance overlaps are treated. Because the number of candidate events tagged by both SVX and SLT is larger than expected (3 common events, 1 expected), these different techniques give somewhat differant results for the total cross section. We include this effect as an additional systematic uncertainty on the measured value.

2 Method

The cross sections and uncertainties are calculated using the following likelihood function:

$$L = e^{-\frac{(\int \mathcal{L}dt - \overline{\int \mathcal{L}dt})^2}{2\sigma_{\mathcal{L}}^2}} \prod_i e^{-\frac{(\epsilon_i - \overline{\epsilon_i})^2}{2\sigma_{\epsilon_i}^2}} e^{-\frac{(b_i - \overline{b_i})^2}{2\sigma_{b_i}^2}} \frac{(\epsilon_i \cdot \sigma_{t\bar{t}} \cdot \int \mathcal{L}dt + b_i)^{n_i}}{n_i!} e^{-(\epsilon_i \cdot \sigma_{t\bar{t}} \cdot \int \mathcal{L}dt + b_i)} \quad (1)$$

Where i=SVX,SLT,DIL, $\overline{\epsilon_i}$ is the total acceptance, $\overline{b_i}$ is the expected background, n_i is the number of observed candidate events, $\sigma_{t\bar{t}}$ is the $t\bar{t}$ production cross section, and $\overline{\int \mathcal{L}dt} = 21.4\ pb^{-1}$ is the integrated luminosity. The $t\bar{t}$ production cross section is found by maximizing $\ln L$. In the maximization, the parameters $\int \mathcal{L}dt$, ϵ_i, and b_i are initialized at their mean values and allowed to vary. The uncertainties on the cross section are the points for which $\Delta(\ln L) = \frac{1}{2}$. Operationally, this maximization (actually minimization of $-\ln L$) is done with the CERNLIB MINUIT package, using MINOS to calculate the uncertainties. Both the individual and the combined cross sections are calculated in this way, though for the individual cross sections there is no product over i and the central value of the cross section, found as the maximum of $\ln L$ is, simply

$$\frac{n - \overline{b}}{\overline{\epsilon_i} \cdot \overline{\int \mathcal{L}dt}} \quad (2)$$

as expected. The formalism is still useful, however, for calculating the uncertainties.

Top quark mass	120 GeV/c^2	140 GeV/c^2	160 GeV/c^2	180 GeV/c^2
ϵ_{SVX}	$1.02 \pm 0.2\%$	$1.5 \pm 0.3\%$	$1.7 \pm 0.3\%$	$1.8 \pm 0.4\%$
ϵ_{SLT}	$0.82 \pm 0.16\%$	$1.0 \pm 0.2\%$	$1.1 \pm 0.3\%$	$1.2 \pm 0.2\%$
$\epsilon_{Dilepton}$	$0.48 \pm .074\%$	$0.65 \pm .064\%$	$0.76 \pm .069\%$	$0.84 \pm .074\%$

Channel	SVX	SLT	Dilepton
Observed Events	6	7	2
Total Bkg.	2.3 ± 0.3	3.1 ± 0.3	0.56 ± 0.14

Table 1: Summary of acceptance for top events, numbers of candidates and backgrounds.

Top quark mass	120 GeV/c^2	140 GeV/c^2	160 GeV/c^2	180 GeV/c^2
$\sigma_{t\bar{t}}^{SVX}$ pb	$17.28^{+14.74}_{-10.19}$	$12.35^{+10.67}_{-7.30}$	$11.53^{+9.82}_{-6.80}$	$10.81^{+9.70}_{-6.41}$
$\sigma_{t\bar{t}}^{SLT}$ pb	$22.22^{+19.11}_{-13.53}$	$17.68^{+15.21}_{-10.75}$	$15.85^{+13.39}_{-9.62}$	$14.70^{+12.37}_{-8.92}$
$\sigma_{t\bar{t}}^{DIL}$ pb	$14.12^{+18.04}_{-10.85}$	$10.41^{+13.08}_{-8.00}$	$8.92^{+11.15}_{-6.85}$	$8.07^{+10.08}_{-6.20}$

Table 2: Calculated $t\bar{t}$ production cross sections for the individual SVX, SLT and DIL results

3 Cross Section Results

3.1 Individual Cross Sections

The calculation of the individual cross sections for the SVX, SLT and DIL results is straightforward. The central value is given by equation 2 above. The uncertainties are calculated as the $\Delta(\ln L) = \frac{1}{2}$ points of the likelihood function of equation 1, where the index i is fixed for SVX, SLT of DIL. The acceptances, number of candidates and background estimates are summarized in Table 1. The results for the individual cross sections are given in Table 2.

3.2 Combined Cross Section Results

The simplest and most straightforward technique for combining the results into a single $t\bar{t}$ cross section is to treat the three counting experiments as completely independent, and maximize the likelihood function 1 using the acceptances and background estimates given in Table 1. Under the independence assumption, the total likelihood is just the product of the three individual experiment likelihoods. This technique is simple and easy to explain but ignores the following effects which we know are present.

1. The SLT acceptance in the documentation does not include the possibility of tagging a real top event with a fake tag and thus underestimates the total efficiency. The tagging efficiency without this effect is about .18 and the fake probability is about 3.3%. Therefore the total tagging efficiency is approximately $0.18 + (1.0 - 0.18) \star 0.033 = 0.21$. The combined cross section in this case is $11.41^{+6.09}_{-4.69}$ pb for $M_{top} = 160 GeV/c^2$, a -4% effect compared to using the standard acceptances.
2. The SLT and SVX backgrounds for fakes, W+bbar, and W+ccbar are calculated by assuming that the entire parent sample of 52 events is not top. Parameterizations of tagging probabilities are applied event-by-event to get the total background. When considering the likelihood of a given top cross section, it is more consistent to calculate the top and non-top components of the parent sample and re-calculate the background based only on the non-top component. In practice, we can only scale the background by the non-top fraction even though the original background was calculated event- by-event. This procedure results in asymptotic backgrounds of 1.5 for the SVX (cf. 2.3 input) and 1.8 for SLT (cf. 3.1 input). The combined cross section in this case is $14.78^{+6.50}_{-5.11}$ pb for $M_{top} = 160 GeV/c^2$, compared with $11.92^{+6.24}_{-4.91}$ pb for the uncorrected case; a +24% effect.
3. Treating all three experiments as independent ignores the overlap in the SVX and SLT. We know that three of the events are double tagged and that this represents an upward fluctuation (See CDF-2310). Ignoring the overlaps will then overestimate the cross section (and including the overlaps will underestimate it). To test the size of the effect, we consider four independent experiments:
a) SVX, no SLT: 3 events observed. For the background, we use the original SVX background, minus the overlap background from CDF-2310. For the acceptance we correct by (1. - SLT tagging efficiency)
b) SLT, no SVX: 4 events observed. Similar to case a) we use the original SLT background minus the overlap background and correct the acceptance by (1. - SVX tagging efficiency).
c) SVX and SLT: 3 events observed. Background from CDF-2310. Acceptance taken to be average of (SVX acceptance)*(SLT tagging efficiency) and (SLT acceptance)*(SVX tagging efficiency)
d) Dileptons, assumed to uncorrelated with above.
The combined cross section in this case is $9.97^{+5.26}_{-4.70}$ pb for $M_{top} = 160 GeV/c^2$, compared with $11.92^{+6.24}_{-4.91}$ pb for the completely uncorrelated case; a -16% effect.

The cross sections from these various techniques are shown in Table 3. The first effect is simple to correct for. As mentioned above, the second effect cannot be corrected for in any more than an approximate manner because we do not know which events are top and which background. Since the background tagging probabilities vary from event to event by fairly large factors, this means we do not know how to correct the background precisely. Instead, we correct by half the estimated +24% and assign a ±12% systematic uncertainty. Finally, it is also not possible to correct exactly for the third effect since it

Top quark mass	120 GeV/c^2	140 GeV/c^2	160 GeV/c^2	180 GeV/c^2
$\sigma_{t\bar{t}}^{SVX}$ pb ; no corrections	$17.28^{+14.74}_{-10.19}$	$12.35^{+10.67}_{-7.30}$	$11.53^{+9.82}_{-6.80}$	$10.81^{+9.70}_{-6.41}$
$\sigma_{t\bar{t}}^{SVX}$ pb ; bkgnd corrections	$20.98^{+15.19}_{-10.45}$	$14.99^{+11.03}_{-7.49}$	$13.99^{+10.13}_{-6.96}$	$13.11^{+10.11}_{-6.61}$
$\sigma_{t\bar{t}}^{SLT}$ pb ; no corrections	$22.22^{+19.11}_{-13.53}$	$17.68^{+15.21}_{-10.75}$	$15.85^{+13.39}_{-9.62}$	$14.70^{+12.37}_{-8.92}$
$\sigma_{t\bar{t}}^{SLT}$ pb ; bkgnd corrections	$29.52^{+20.02}_{-14.02}$	$23.50^{+15.91}_{-11.16}$	$21.05^{+13.93}_{-9.95}$	$19.54^{+12.84}_{-9.23}$
$\sigma_{t\bar{t}}^{SLT}$ pb ; ϵ_{tag} corrections	$19.38^{+19.32}_{-11.96}$	$15.34^{+16.30}_{-9.52}$	$13.81^{+14.77}_{-8.57}$	$12.74^{+13.48}_{-7.90}$
$\sigma_{t\bar{t}}^{DIL}$ pb	$14.12^{+18.04}_{-10.85}$	$10.41^{+13.08}_{-8.00}$	$8.92^{+11.15}_{-6.85}$	$8.07^{+10.08}_{-6.20}$
$\sigma_{t\bar{t}}^{COMBINED}$ pb ; no corrections	$17.89^{+9.36}_{-7.33}$	$13.25^{+6.99}_{-5.46}$	$11.92^{+6.24}_{-4.91}$	$11.03^{+5.87}_{-4.57}$
$\sigma_{t\bar{t}}^{COMBINED}$ pb ; bkgnd corrections	$22.10^{+9.74}_{-7.64}$	$16.48^{+7.32}_{-5.73}$	$14.78^{+6.50}_{-5.11}$	$13.69^{+6.13}_{-4.78}$
$\sigma_{t\bar{t}}^{COMBINED}$ pb ; ϵ_{tag}^{SLT} corrections	$17.11^{+9.11}_{-7.00}$	$12.67^{+6.79}_{-5.20}$	$11.41^{+6.09}_{-4.69}$	$10.54^{+5.69}_{-4.35}$
$\sigma_{t\bar{t}}^{COMBINED}$ pb ; SVX/SLT overlap corrections			$9.97^{+5.26}_{-4.70}$	

Table 3: Comparison of the results from various techniques for calculating the $t\bar{t}$ production cross sections

is inherently a statistical fluctuation. Again, we correct by half of the -16% deviation and assign a ±8% systematic uncertainty. The net correction for these three effects is 0% with a systematic uncertainty of 14%. To be conservative, we inflate this systematic uncertainty to 25%.

Using the 'method 2' background for the SVX of 1.44±0.54 and a psuedo-method 2 background of 2.6 for the SLT (see CDF-2377), results in a shift of the combined cross section by +14% at all masses. A deviation of this size is covered by the 25% systematic uncertainty.