Black Holes in Effective String Theory

Takashi Torii[1](a) and Nobuyoshi Ohta[2](b)

(a) *Department of General Education, Osaka Institute of Technology, Asahi-ku, Osaka 535-8585*
(b) *Department of Physics, Kinki University, Higashi-Osaka, Osaka 577-8502*

Abstract

We study black hole solutions in the Einstein gravity with Gauss-Bonnet term, the dilaton and the Maxwell gauge field in various dimensions. The spacetime is asymptotically flat and the solutions have static and spherical symmetry. We focus on the effect of the higher order correction term of the dilaton field and found that it gives little effect on the solutions except for the five-dimensional case. The charged black hole solution does not have the extreme limit by the coupling to the dilaton field.

1 Introduction

One of the long-standing problems in theoretical physics is how to reconcile gravity with quantum theory. Superstring theory is the leading candidate for quantum gravity. In order to study the geometrical properties and strong gravitational phenomena, it is still difficult to apply full superstring theory itself. In this situation, it is appropriate to investigate these problems by using the effective low-energy field theories including string quantum corrections.

Many works have been done on black hole solutions in dilatonic gravity, and various properties have been studied since the work in Refs. [1]. On the other hand, it is known that there are higher-order quantum corrections from string theories. It is thus important to ask how these corrections may modify the results. Several works have studied the effects of higher order terms, but most of the work considers theories without dilaton, which is one of the most important ingredients in the string effective theories. Hence it is most significant to study black hole solutions and their properties in the theory with the higher order corrections and dilaton. The simplest higher order correction is the Gauss-Bonnet (GB) term coupled to dilaton in heterotic string theories.

In our previous paper [2], we have studied asymptotically flat black hole solutions with the GB correction term and dilaton without a cosmological constant in various dimensions from 4 to 10 with $(D-2)$-dimensional hypersurface of curvature signature $k=+1$. We have then presented our results on black hole solutions with the cosmological constant with $(D-2)$-dimensional hypersurface with $k = 0, \pm 1$ [3–5]. In the string perspective, it is also interesting to examine asymptotically non-flat black hole solutions with possible application to AdS/CFT and dS/CFT correspondences in mind. However, in our previous work, we do not consider the higher order term of the dilaton field as in many other works. Hence in this work we investigate the effects of such term on the properties of the black holes. We also include the U(1) gauge field in the system to examine whether coupling of the dilaton field affect the gauge field.

2 Dilatonic Einstein-Gauss-Bonnet-Maxwell theory

We consider the following low-energy effective action for a heterotic string in one scheme:

$$S = \frac{1}{2\kappa_D^2}\int d^Dx\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{-\gamma\phi}F^2 + \alpha_2 e^{-\gamma\phi}\left\{R_{\rm GB}^2 + \frac{3}{16}\mu(\partial\phi)^4\right\}\right], \tag{1}$$

where κ_D^2 is a D-dimensional gravitational constant, ϕ is a dilaton field, $\gamma = 1/2$, F is a gauge field, $\alpha_2 = \alpha'/8$ is a numerical coefficient given in terms of the Regge slope parameter, and $R_{\rm GB}^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} -$

[1]Email address: torii@ge.oit.ac.jp
[2]Email address: ohtan@phys.kindai.ac.jp

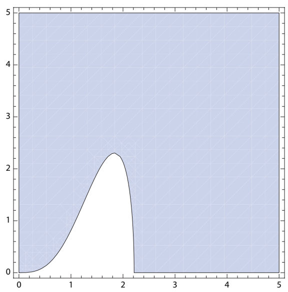

Figure 1: The region where the discriminant of the quadratic equation for ϕ'_H is positive in $D = 4$. The horizontal axis is $e^{\gamma\phi_H/2} r_H$ and the vertical axis is $e^{\gamma\phi_H} e_1$.

$4R_{\mu\nu}R^{\mu\nu} + R^2$ is the GB correction. In the original derivation of the effective action, it was first derived in the Einstein frame from the S-matrix calculation in string theory and then transformed into the string frame. Also it is convenient to interpret our results in the Einstein frame. Hence we have transformed this into the Einstein frame, reduced to D dimensions, and used the field redefinition ambiguity $\delta g_{\mu\nu} = \alpha'[a_1 R_{\mu\nu} + a_2 \nabla_\mu\phi\nabla_\nu\phi + g_{\mu\nu}\{a_3 R + a_4(\nabla\phi)^2 + a_5\nabla^2\phi\}]$ and $\delta\phi = \alpha'[b_1 R + b_2(\partial\phi)^2 + b_3\nabla^2\phi]$, up to higher order terms. We have also set $H = 0$. The constant μ is introduced to see the difference of the solutions with and without the higher order term of the dilaton field. In the above process, higher order terms are dropped. This is allowed because the effective low-energy action can be determined up to field redefinition when it is read off from the scattering amplitudes computed in string theories. All this means that there is no absolutely preferred form of the action if they are related this way, and one cannot claim which system is better.

Let us consider the static spacetime and adopt the metric and field strength

$$ds_D^2 = -B(r)e^{-2\delta(r)}dt^2 + B^{-1}dr^2 + r^2 h_{ij}dx^i dx^j, \quad F_{0r} = f(r)', \tag{2}$$

where $h_{ij}dx^i dx^j$ represents the line element of a $(D-2)$-dimensional hypersurface with constant curvature $(D-2)(D-3)k$ and volume Σ_k for $k = \pm 1, 0$. A prime denoted the derivative with respect to r.

In this paper we study spherically symmetric solutions ($k = 1$). The field equation for the Maxwell field is easily integrated to give

$$f' = \frac{e_1}{r^{D-2}} e^{\gamma\phi - \delta}, \tag{3}$$

where e_1 is a constant corresponding to the charge. Hence our task is reduced to setting boundary conditions for the fields B, δ and dilaton ϕ and integrate the above set of equations.

Let us first examine the boundary conditions of the black hole spacetime. We assume the following boundary conditions for the metric functions:

1. The existence of a regular black hole horizon r_H: $B_H = 0, \quad B'_H > 0, \quad |\delta_H| < \infty, \quad |\phi_H| < \infty$.

2. The nonexistence of singularities outside the event horizon: $B(r) > 0, \quad |\delta| < \infty, \quad |\phi| < \infty$.

Here and in what follows, the values of various quantities at the horizon are denoted with subscript H.

3. Asymptotic flatness at spatial infinity (as $r \to \infty$):

$$B \sim 1 - \frac{2M}{r^{D-3}}, \quad \delta(r) \sim \frac{\delta_1}{r}, \quad \phi \sim \frac{\phi_1}{r}, \tag{4}$$

with finite constants M, δ_1, ϕ_1, where M corresponds to the mass parameter of the black hole.

By the equation of the dilaton field with the regularity condition at the horizon, we obtain the quadratic equation determining ϕ'_H. The $(\partial\phi)^4$-term in Eq. (1) does not contribute to the boundary

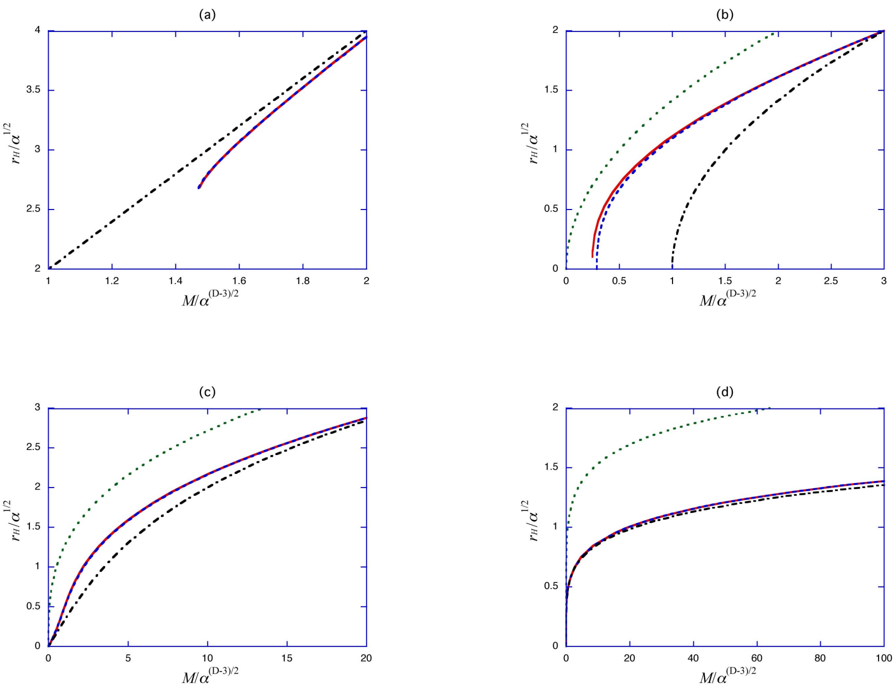

Figure 2: The M-r_H diagram of the neutral solution ($e_1 = 0$) in Einstin-GB-dilaton system with the higher order term of the dilaton field (red solid line), in Einstin-GB-dilaton system without the higher order term of the dilaton field (blue dashed line), in Einstin-GB system (black dot-dashed line), and in GR (green doted line) (a) $D = 4$, (b) $D = 5$, (c) $D = 6$, and (d) $D = 10$.

condition of the dilaton field at the horizon. For the $D = 4$ case, there is a parameter region where the quadratic equation has no real solution. The region where the discriminant is positive is depicted in Fig. 1. For $e = 0$, the boundary is $r_H e^{\gamma\phi_H/2} > 24^{1/4} \approx 2.192$. For $e^{\gamma\phi_H} e_1 > 2.3032$, the discriminant is positive for all r_H.

It is useful to consider several scaling symmetries of our field equations (or our model). Firstly, α_2 can be scaled out by the following scaling as $r \to r/{\alpha_2}^{1/2}$, $M \to M/{\alpha_2}^{(D-3)/2}$, $e_1 \to e_1/{\alpha_2}^{(D-2)/2}$. The field equations also have a shift symmetry:

$$\phi \to \phi - \phi_*, \quad r \to e^{\gamma\phi_*/2} r, \quad e_1 \to e^{(D-2)\gamma\phi_*/2} e_1. \tag{5}$$

where ϕ_* is an arbitrary constant. The third one is the shift symmetry under

$$\delta \to \delta - \delta_*, \quad t \to e^{-\delta_*} t, \tag{6}$$

with an arbitrary constant δ_*, which may be used to shift the asymptotic value of δ to zero.

3 Black hole solutions

It will be instructive to compare our results with the non-dilatonic case. When the dilaton field is absent (i.e., Einstein-GB-Maxwell system), we substitute $\phi \equiv 0$ and $\gamma = 0$. For the $D \geq 5$ case, the field equations can be integrated to yield [7]

$$B = 1 - \frac{2m(r)}{r^{D-3}}, \quad \delta = 0, \tag{7}$$

where

$$m(r) = \frac{r^{D-1}}{4\alpha_2 (D-3)_4} \left[-1 \pm \sqrt{1 + \frac{8(D-3)_4 \bar{M}}{r^{D-1}} - \frac{(D-4)e_1^2}{8(D-2) r^{2(D-2)}}} \right], \tag{8}$$

and $\bar{M}$ is an integration constant corresponding to the asymptotic value $m(\infty)$ for the plus sign in Eq. (8). In the $\alpha_2 \to 0$ limit, the solutions with the plus sign approach the Reissner-Nortström solutions. This means that they can be considered to be the solution with GB correction to GR. On the other hand, the solutions with the minus sign do not have such a limit. For these reasons, we call the solutions with plus (minus) sign the (non-)GR branch.

In the dilatonic case, since the basic equations do not have non-trivial analytical solutions, we have to resort to the numerical method. Firstly we consider the neutral solution ($e_1 = 0$). The relation between the mass and the horizon radius of the black holes in various dimensions are shown in Fig. 2. We find that the relations are qualitatively same as those of solutions without the higher order term of the dilaton field ($\mu = 0$) except for the 5-dimensional case[2].

In the $D = 5$ case, there is the lower limit for the mass and the horizon radius of the black holes as in the 4-dimensional case. In $D = 4$ the lower limit is determined by the condition at the horizon shown in Fig. 1, while the second derivative of the dilaton field diverges at some radius outside of the event horizon in the lower limit in $D = 5$.

For the charged solutions, we depict the relation between the mass and horizon radius in $D = 5$ in Fig. 3. The electric charge is $e_1 = 5$. The charged Boulware-Deser solution [7] have the extreme limit where the black hole horizon and the inner horizon coincide. The extreme solution has the lowest mass and is the smallest black hole solution. The dilatonic solution also has the lowest limit solution. It is not, however, the extreme solution and the horizon is not degenerate. At this solution, the field variables diverges as in the neutral black hole solution. As a result, there is no extreme solution even for the charged black hole in the dilatonic case.

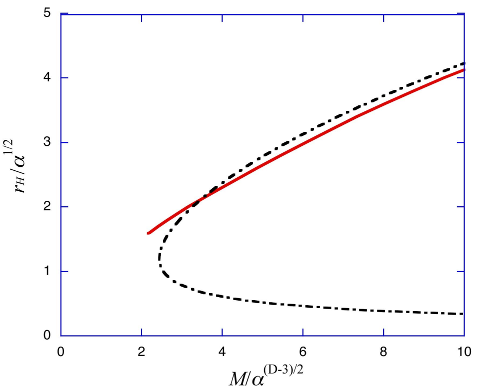

Figure 3: The M-r_H diagram of the charged solution ($e_1/\alpha_2{}^{(D-2)/2} = 5$) in Einstin-GB-dilaton system with the higher order term of the dilaton field (red solid line) and in Einstin-GB system (black dot-dashed line) in $D = 5$.

References

[1] G. W. Gibbons and K. Maeda, Nucl. Phys. **B298** (1988) 741; D. Garfinkle, G. T. Horowitz, and A. Strominger, Phys. Rev. D **43** (1991) 3140.

[2] Z. K. Guo, N. Ohta and T. Torii, Prog. Theor. Phys. **120** (2008) 581 [arXiv:0806.2481 [gr-qc]].

[3] Z. K. Guo, N. Ohta and T. Torii, Prog. Theor. Phys. **121** (2009) 253 [arXiv:0811.3068 [gr-qc]].

[4] N. Ohta and T. Torii, Prog. Theor. Phys. **121** (2009) 959 [arXiv:0902.4072 [hep-th]].

[5] N. Ohta and T. Torii, Prog. Theor. Phys. **122** (2009) 1477 [arXiv:0908.3918 [hep-th]].

[6] N. Ohta and T. Torii, Prog. Theor. Phys. **124** (2010) 207 [arXiv:1004.2779 [hep-th]].

[7] D. G. Boulware and S. Deser, Phys. Rev. Lett. **55** (1985) 2656; Phys. Lett. B **175** (1986) 409.