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PAPER

From Newton to universal Planck natural units – disentangling the constants of nature

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Abstract

This study exploits a historical gap in the evolution of metric systems that resulted from the incomplete implementation of the 'rationalization' concept published by Heaviside in 1893 and by ignoring the suggestion of Maxwell in 1873 to use the simplest form of Newton's gravitational law expression with no proportionality constant. Bridging this gap required deriving an experimental Rationalized Metric System (RMS) and a corresponding Universal Planck Natural Unit System (UPNUS) in [LT] units. The described solution combines Heaviside's rationalization of Newton's law and makes the unit of mass dimensions [L^3T^{-2}], as suggested by Maxwell. Consequently, the modified Coulomb's law, changes the unit of electric charge to the same dimensions as those of mass. The elimination of the kilogram and ampere has a disentangling effect on the dependencies among the constants of nature and opens new horizons. New systems have the potential to become powerful exploratory tools in fundamental research and education because of the simplification of the relationships among physical quantities. Some highlights from the analyzed examples are worth mentioning. The well-known expression for the Stoney mass (m_S) when converted to RMS units is reduced to the electron charge quantity, whereas traditional metric systems entangle the charge, speed of light, and gravitational constant, forming an entity in the dimension of mass, as first presented by Stoney in 1874. Furthermore, a well-substantiated conjecture is proposed, wherein the Stoney energy $E_S = m_S c^2$ is likely the long-sought, finite electric field energy of the electron, and the gravitational constant appears to be the limiting factor. In UPNUS, the most disentangled fundamental expression, apart from the Stoney mass, is the elementary charge $\breve{\boldsymbol{e}}$ as the function of the fine structure constant $\boldsymbol{\alpha}$ and the Planck mass $\breve{\boldsymbol{m}}_{\boldsymbol{P}}$, namely $\breve{\boldsymbol{e}} = \breve{\boldsymbol{m}}_{\boldsymbol{P}}\sqrt{\boldsymbol{\alpha}} \approx 1.073476469$, with $\breve{\boldsymbol{e}}$ and $\breve{\boldsymbol{m}}_{\boldsymbol{P}}$ of the same [L^3T^{-2}] dimensions in Planck units, where $\breve{\boldsymbol{m}}_{\boldsymbol{P}} = \boldsymbol{4\pi}$.

1. Introduction

The importance of the units of measure cannot be overstated. Correctly defined systems of units no matter what they are, describe the same physical reality; however, some are more useful than others depending on needs. This is similar to sets of lenses ranging from microscopes, through wide angle camera lenses to astronomical telescopes. However, systems of units unlike lenses may have different number of base units of measure which can be referred to as dimensions. The evolution of systems of units has lasted for at least four centuries, closely aligned with the progress of physical sciences. The question often arises of how many dimensions are needed and what choice is the best. Although no system of measure changes physical reality some can enhance or obscure the view of natural processes. The results of this research show that the choice of units of measure can lead to new discoveries.

The International System of Units (SI) was established in 1960 with seven base units of measure and the corresponding definitions of fundamental constants as in [1]. These include the unperturbed ground state

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hyperfine transition frequency of the cesium 133 atom $\Delta\nu Cs$, speed of light in vacuum c, Planck constant h, elementary charge e, Boltzmann's constant k, Avogadro's constant N_A, and luminous efficacy K_{cd}.

The unit of mass in the SI is the kilogram, derived from the earlier unit of weight called the 'gramme.' The kilogram prototype was then defined as 'the weight of a cubic decimeter of distilled water at its temperature of maximum density and weighed in vacuo'(Hallock [2] p.62). This prototype was replicated in the famous platinum-iridium alloy cylinder, which served as the legal standard for the kilogram, along with the meter standard, which were established in 1799 (Hallock [2] p.63). This historical event led to the development of the SI we have today.

The electric charge, originally referred to as 'the quantity of electricity' (Everett [3] p.161), was assigned a unit of measure in the 19th century in the Centimeter-Gramme-Second (CGS) system actively supported by Maxwell and Thomson [4] in 1873. Before that, in 1686, Newton defined the 'quantity of matter' as 'arising from its density and bulk conjunctly,' which he also called 'mass' in *The Mathematic Principles of Natural Philosophy* [5] (also referred to as '*The Principia* '). According to Jammer, the concept of mass was still debated issue at the time [6] pp.64–74. In the first (1873) edition of J. C. Maxwell's *Treatise on Electricity and Magnetism* [7], it was suggested that the unit of mass should be defined based solely on time and length units:

> In the dynamical theory of astronomy, the unit of mass is deduced from the units of time and length, combined with the fact of universal gravitation. The astronomical unit of mass is that mass, which attracts another body, placed at the unit of distance so as to produce in that body the unit of acceleration. In framing a universal system of units we may either deduce the unit of mass in this way from those of length and time already defined, and this we can do to a rough approximation in the present state of science; […] If, as in the astronomical system, the unit of mass is defined with respect to its attractive power, the dimensions of [M] are [L^3T^{-2}]. For the acceleration due to the attraction of a mass **m** at a distance **r** is by the Newtonian Law $\mathbf{m}/\mathbf{r^2}$ ([7] sect 5 p.4).

It is worth noting Maxwell's preference for the simplest possible representation of physical law evident from the absence of any gravitational constant in this citation. This is consistent with the fact that Newton's laws of gravitation being represented verbally, as Haug [8] points out (e.g., in Proposition LXXVI, Theorem XXXVI, and Corollary 3 in *The Principia* [5]). Considering this, it is possible to infer the simplest Newton's law expression, consistent with Maxwell's comments, as shown in equation (1). Newton's law can be transformed into the equation of mass as a function of the angular velocity ω and radius of orbit r, and the dimensions of the mass and force emerge naturally.

$$F_N = \frac{M_N m_N}{r^2} = m_N a \Rightarrow \frac{M_N}{r^2} = \omega^2 r \Rightarrow M_N = \omega^2 r^3, \tag{1}$$

where F_N, M_N, and m_N are the Newtonian gravitational force and masses, respectively.

In this case, the mass of an object may be referred to as the 'Newtonian mass' M_N, which has the dimensions of [L^3T^{-2}]. For example, if we assume that the Moon moves on a circular orbit around Earth using average parameters derived from NASA tables [9] such that $r \approx 3.844\,000 \times 10^8\ m$ and $\omega \approx 2.661\,695 \times 10^{-6} s^{-1}$, we can calculate the Earth's mass as $M_N \approx 4.024\,083 \times 10^{14}\ m^3 s^{-2}$. This value is approximately 0.955% different from the accurately measured Earth's geocentric gravitational constant GM, which is $3.986\,004\,418 \times 10^{14} m^3 s^{-2} \pm 8 \times 10^5 m^3\ s^{-2}$ (McCarthy [10]).

In electrostatics, the CGS Electrostatic Unit (ESU) is a derived unit of the electric charge, constructed using the Coulomb force equation in the form proposed by Maxwell [7] pp.42–43 without any additional constants.

$$F_{ECGS} = \frac{q^2}{r^2}. \tag{2}$$

Given that the CGS-derived unit of force dimensions are [dyne] = [MLT^{-2}] (Everett [3] p.161), we can use dimensional analysis to derive the dimensions of the electric charge q to find that [q] = [$M^{1/2}L^{3/2}T^{-1}$]. However, in SI, the CGS tradition has not been followed, and the dimensions of the electric charge and Coulomb's law are expressed differently.

$$F_{ESI} = \frac{1}{4\pi\varepsilon_0}\frac{q_{SI}^2}{r^2}. \tag{3}$$

The equation form of (3) is referred to as 'rationalized,' and includes the $1/4\pi$ factor, which was influenced by Giorgi [11] and originally insisted upon by Heaviside [12]. This rationalized form of the law causes 4π to appear in physical equations in the context of spherical electric fields but not in uniform fields. As pointed out by Heaviside [12] p.116, the number 4π becomes 'obtrusively prominent' in unexpected physical contexts, leading to the impression of it being an 'essential part of all electric and magnetic theories.'

Heaviside [12] p.117 criticized the form of equation (2) without the $1/4\pi$ factor, as preferred by Maxwell, for being 'the origin of 4π absurdity' in the CGS framework. Heaviside concluded quoting Maxwell, that equation (2), which is the point source central force field law without a 'useless and absurd factor'([12] p.118), with the addition of $1/4\pi$ being the best approach. The addition of $1/4\pi$ indicates that the spherical surface increases with distance r, causing a reduction in the central force field intensity by attenuation over the growing area; for example, as for the flux of heat [12] p.119. Thus we conclude that inverse square laws without a surface area context are, simply, a mystery. Why 'rationalization' has not been introduced in the SI for the law of universal gravitation, while it was for the electric charge, as seen in equation (3), is puzzling. This would only require scaling the gravitational constant value by multiplying it by 4π and adding the factor $1/4\pi$ in front of SI Newton's law expression. Following Maxwell and Heaviside, it is possible to 'rationalize' the metric system to achieve consistency and simplicity by eliminating the base units of mass and charge. This approach extends to the standard Planck unit system, referred to as the Universal Planck Natural Unit System (UPNUS). It is demonstrated that removing two base units of the kilogram and ampere does not lead to defective systems of units, but, on the contrary, offers a better perspective on the relations among the constants of nature and physical quantities, and a promising platform for unification theories.

The rationality of the absence of a base unit of mass, as advocated by Maxwell, has been repeatedly discussed in the literature. For example, in 1914, Buckingham [13] p.373 working from a different angle, rediscovered the previously established fact that, from three base units in mechanical systems, one can be eliminated by the other two when the law of gravitation is under consideration. This was already in effect in astronomy; for example, in *Treatise of Celestial Mechanics* by La Place [14] (Part1 Book 2 pp.27–28), where no gravitational constant appears in the presented differential equations of gravitational motion. Buckingham [13] made a valuable contribution to the study of physically similar systems by showing when the [LT] units only are appropriate and when not. The reduction to two dimensions is universally valid; however, it is practically troublesome owing to the low accuracy of the gravitational constant $\boldsymbol{G}$. The [MLT] system is adequate when gravitational effects are negligible; however, [M] can be regarded as a secondary unit (rather than superfluous), although not as the primary base unit. In 1933, Planck [15] pp.45–46, called the [L^3T^{-2}] unit 'astronomical mass,' and made the remark that 'dimensions of a physical quantity are not inherent in it, but constitute a conventional property conditioned by the choice of the system of measurement.' In his opinion, this was not often appreciated, causing 'unfruitful controversies.'

More recently, in 2002, Fiorentini, Okun, and Vysotski [16] analyzed possible ways to eliminate the base unit of mass. One identified method is to multiply each instance of the mass unit embedded in the dimensions of the physical quantities by $\boldsymbol{G}$ — a transformation that in their opinion results in units with no fundamental character. This conversion is similar to one of the transformations intended for the proposed system of units, as described in section 2.2. Fiorentini, Okun, and Vysotski [16] do not comment on electromagnetic units, as gravitation appears to be the main focus, and they also consider the possibility of replacing mass with its equivalent Schwarzschild radius. In 2007, Matsas *et al* [17] contributed to an ongoing discussion on the sufficient number of dimensional fundamental constants in nature and justified by dimensional analysis that all observables can be represented by [LT] units only, based on their argument that 'all we can directly measure are space and time intervals.' They presented a formal protocol wherein multiplying every instance of any physical quantity involving mass by $\boldsymbol{G}$ in the respective powers reduces the system to [LT]. They included no special protocol to eliminate the unit of charge because this was already adopted as a derived unit and included mass in the CGS system. The charge unit is automatically resolved by the protocol, similarly to any other physical quantity. In 2008, Veneziano [18] strongly supported two as the sufficient number of fundamental constants and, therefore, two base units, from the point of view of Quantum String theory. There seems to be a disagreement regarding what constitutes the fundamental units in [16] and [17]. Indeed, generally speaking, the term 'fundamental' has various interpretations. Weinberg [19] p.249 bluntly stated that 'the membership of a list of 'fundamental' constants necessarily depends on who is compiling the list.', Matsaset *et al* [17] presented a detailed argumentation concluding that 'the number of fundamental dimensional constants is two.' This implies that all observables can be expressed in terms of only two basic dimensional observables. Hence, the two dimensions [LT] are sufficient for describing physical processes in the current state of knowledge.

Apart from information published in peer-reviewed journals, there have also been attempts to build the alternative systems of units in [LT] dimensions by independent researches on the popular scientific exchange platform ResearchGate[1] based on the same foundations.

The main aim of this paper is to recognize the weight of arguments from multiple sources justifying length and time units as being sufficient and fundamental and to 'put them to the test' by exposing how the physical world would look like from this perspective.

[1] Richard Jowsey *The Geometry of Spacetime* http://doi.org/10.13140/RG.2.1.2404.2320; Randy Sorokowski *The Source of Interpretation Issues in Physics* https://researchgate.net/publication/341297059_The_Source_of_Interpretation_Issues_in_Physics

2. Rationalized metric system

In line with our argument above that two dimensions are sufficient to capture the basic reality of the physical world, in this section, we construct the Rationalized Metric System (RMS) to demonstrate the point by analyzing the impact this system has on dependencies among the constants of nature.

The RMS is intended as a platform for fundamental research to operate in two base units of length and time, but not as a replacement for the SI, which was perfected over a period of 60 years and currently provides a solid foundation for most applications in science, engineering, and everyday life.

2.1. Notation and symbols

To distinguish between the same physical quantities and constant symbols in SI, RMS, and UPNUS, the RMS symbols have dashed accents above, such that $\bar{m}$, $\bar{q}$, $\bar{e}$, and$\bar{F}$ are used for the mass, charge, elementary charge, and force, respectively. The same quantities in UPNUS are $\check{m}$, $\check{q}$, $\check{e}$,and$\check{F}$. Any other 'undecorated' physical quantity symbols are deemed to be in SI units unless otherwise indicated.

The meanings of the subscripts are as follows:

$_e$ – electron, as in electron radius r_e
$_P$ – Planck qualifier, as in Planck mass m_P
$_B$ – Boltzmann qualifier, as in Boltzmann constant k_B
$_S$ – Stoney unit qualifier, as in m_S
$_{pr}$ – proton, as in proton mass m_{pr}
$_E$ – 'electrical'
$_G$ – 'gravitational'
$_N$ – 'Newtonian'
$_{CGS}$ – CGS system of units
$_{SI}$ – International System of Units
$_{RMS}$ – RMS system of units.

Other non-listed subscripts are evident in the local context. Physical quantities are represented by traditionally accepted symbols. The generic electric charge symbol is q, whereas the elementary or electron charge is denoted by e.

Numerical data are often shown for constants of nature in three systems of units for information purposes only, to illustrate the magnitudes of interest. The SI constant values were sourced from CODATA 2018 [20] with reference's precision. Combinations of constants that involve the gravitational constant $\boldsymbol{G}$ have degraded precision owing to the low accuracy of $\boldsymbol{G}$ and are not validated as formal reference values, but may show only up to a maximum 6 digits precision after the decimal point following the CODATA convention applied to Planck length, time, and mass. However, the CODATA 2018 [20] gravitational constant is only certain to 5 digits $\boldsymbol{G} = 6.674\,30 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$. Recent publications claim a much higher accuracy for $\boldsymbol{G}$ (13 digits) and for the inverse of the fine structure constant α^{-1} (14 digits); for example, Geiger [21] pp. 1, 60.

2.2. Eliminating the kilogram

The author initially intended to reuse Newton's law, as deduced from the original description in *The Principia* [5] and prompted by Maxwell's suggestion (page 2), as in equation (1). However, considering Heaviside's remarks on the central source force fields, the $1/4\pi$ factor is introduced, as follows:

$$F_{GSI} = G\frac{M^2}{r^2} \rightarrow \bar{F}_G = \frac{1}{4\pi}\frac{\bar{M}^2}{r^2}. \tag{4}$$

This makes the inverse square law intuitive. The symbol $\bar{M}$ may be referred to as 'rationalized Newtonian mass.'

Acceleration is not affected by eliminating the kilogram; hence

$$g_{SI} = G\frac{M}{r^2} \quad and \quad g_{RMS} = g_{SI} = \frac{1}{4\pi}\frac{\bar{M}}{r^2}. \tag{5}$$

From the above, the conversion factor from SI units to RMS units is

$$C_{kg} = \frac{\bar{M}}{M} = 4\pi G \approx 8.38717 \times 10^{-10}\mathrm{m^3s^{-2}kg^{-1}} \tag{6}$$

2.3. Eliminating the coulomb

We begin with the SI variant of the Coulomb's law for use in unit conversion:

$$F_{ESI} = \frac{1}{4\pi\varepsilon_0}\frac{q_{SI}^2}{r^2}. \tag{7}$$

Following Heaviside's suggestions [12], equation (2) and Maxwell's dislike of additional factors, the chosen RMS representation of Coulomb's law can be expressed as

$$\bar{F}_E = \frac{1}{4\pi}\frac{\bar{q}^2}{r^2}, \tag{8}$$

as required by Heaviside [12]. Given that there is no electric permittivity of the vacuum constant ε_0 in RMS, the magnetic permeability of the vacuum must be $\mu_0 = c^{-2}$, as required by Maxwell's equations. Unlike in SI, the permeability of the vacuum in RMS is no longer contaminated by experimental error.

The gravitational-to-electric-force ratio of the two electrons in SI is given by the following equation:

$$\frac{F_{ESI}}{F_{GSI}} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}}{G\frac{m_e^2}{r^2}} \approx 4.16561\times10^{42}. \tag{9}$$

Assigning the symbol $\bar{e}$ to the elementary charge and $\bar{m}_e$ to its mass in the RMS, the same force ratio from equations (4) and (8) becomes

$$\frac{\bar{F}_E}{\bar{F}_G} = \frac{\frac{1}{4\pi}\frac{\bar{e}^2}{r^2}}{\frac{1}{4\pi}\frac{\bar{m}_e^2}{r^2}} = \frac{\bar{e}^2}{\bar{m}_e^2} \approx 4.16560 \times 10^{42}. \tag{10}$$

This is a squared charge-to-mass ratio, and the charge $\bar{e}$ dimensions are $[\mathrm{m^3 s^{-2}}]$.

From equations (9) and (10), it follows that the elementary charge in the RMS becomes

$$\bar{e} = e\sqrt{\frac{4\pi G}{\varepsilon_0}} \approx 1.55935 \times 10^{-18}\mathrm{m^3 s^{-2}}. \tag{11}$$

The SI to RMS conversion factor for the charge quantity is

$$C_c = \frac{\bar{q}}{q} = \sqrt{\frac{4\pi G}{\varepsilon_0}} \approx 9.73270\ \mathrm{m^3 s^{-2} C^{-1}}. \tag{12}$$

For electric current, the factor is given by

$$C_I = \frac{\bar{I}}{I} \approx 9.73270\ \mathrm{m^3 s^{-3} A^{-1}}. \tag{13}$$

2.4. Conversion from SI to RMS

For every dimensional constant value that has a kilogram unit in non-zero power in the associated dimensional expression, the constant must be multiplied by the value of C_{kg} from equation (6) to the power of the kilogram. Additionally, for every coulomb [As] at non-zero power, the constant must be multiplied by C_c from equation (12) to the power of the coulomb [As]. To transform to symbolic expressions instead of values, the constant expressions from equations (6) and (12) in respective powers should be used.

It is necessary to change the dimensions of the physical quantities when converting the physical equations or constants from SI to RMS. In SI, most quantities in mechanics and electromagnetism can be expressed in primary fundamental units[2] as $[\mathrm{Q_{SI}}] = [\mathrm{M}^{\alpha}\mathrm{L}^{\beta}\mathrm{T}^{\gamma}\mathrm{I}^{\delta}]$; for RMS, this is $[\mathrm{Q_{RMS}}] = [\mathrm{M}^{\varepsilon}\mathrm{L}^{\zeta}]$.

The conversion scheme is as follows:

$$[\mathrm{Q_{SI}}] = [\mathrm{kg}^{\alpha}\mathrm{m}^{\beta}\mathrm{s}^{\gamma}\mathrm{A}^{\delta}] \rightarrow [\mathrm{Q_{RMS}}] = [\mathrm{m}^{\beta+3(\alpha+\delta)}\mathrm{s}^{\gamma-2\alpha-3\delta}]$$

For example, for the charge density dimension:

$[\rho_{\mathrm{ESI}}] = [\mathrm{m^{-3}\,s\,A}] => [\rho_{\mathrm{ERMS}}] = [\mathrm{m}^{-3+3*(0+1)}\,\mathrm{s}^{1-2*0-3}] => [\rho_{\mathrm{ERMS}}] = [\mathrm{s^{-2}}]$, which is the correct dimension of density for both the mass and charge in the RMS.

In a more complex example of the vacuum electric permittivity constant ε_0,

$[\varepsilon_{0\mathrm{SI}}] = [\mathrm{kg^{-1}m^{-3}s^4A^2}] => [\varepsilon_{0\mathrm{RMS}}] = [\mathrm{m}^{-3+3*(-1+2)}\,\mathrm{s}^{4-2*(-1)-3*2}] => [\varepsilon_{0\mathrm{RMS}}] = [\mathrm{m^0 s^0}] = 1.$

[2] Kelvin and candela are omitted here. Kelvin remains unchanged in RMS after conversion, with only the Boltzmann constant changing the kilogram unit accordingly.

To convert the physical equations from the SI to the RMS form:

- Gravitational constant $\boldsymbol{G}$ is replaced by $1/4\pi$
- Vacuum magnetic permeability constant μ_0 is replaced by c^{-2}
- Vacuum electric permittivity constant ε_0 is removed.

The side effects of numerical conversions have an impact on high-accuracy calculations using RMS constants because the gravitational constant $\boldsymbol{G}$ degrades the original precision by many orders of magnitude. The measurements will improve in time, and possibly a new theory would provide better values for $\boldsymbol{G}$. Recent claims by Geiger [21] pp. 1, 60 of approximating the inverse fine structure constant α^{-1} at 14 digits precision and $\boldsymbol{G}$ at 13 digits precision would resolve the issue, if adopted by the CODATA.

2.5. The new perception of physical quantities in the RMS

Two significant consequences were observed after eliminating the base units of mass and charge. First, as described further below, this allows for the emergence of new relationships within the set of fundamental constants. Second, it logically necessitates the propagation of any changes made in RMS, to the base units on the Planck scale. The implications of this transformation are explained in detail in the following sections.

The unit system closest to the RMS is the Heaviside–Lorentz unit system (HLU; see [22] pp.22–24 for a summary), which constitutes one of the CGS unit subsystems (CGS-HLU). In HLU, the electromagnetic units are unified and rationalized; however, the creators stopped short of full unification by failing to modify the mechanical units, including the format of Newton's law of gravitation. The RMS bridges this gap.

The HLU has its own natural units representation, similar to the Planck system, known as the Heaviside–Lorentz natural unit system (with $c = \hbar = 1$). This is commonly used instead of the Planck system in Quantum Electrodynamics research; however, it shares the same problem of inconsistent rationalization.

2.5.1. Charge-to-mass ratio

The simplification of the electrical-to-gravitational force ratio formula is the first immediate effect of the RMS system that reduces the ratio to the squared charge-to-mass ratio. The large difference in strength between electric and gravitational fields has been the subject of discussion in elementary physics courses and numerous scientific publications, including those by Feynman [23], Dicke [24], and Wilczek [25–27]. Feynman [23] used the example of two electrons to illustrate the problem, stating that 'The gravitational attraction relative to the electrical repulsion between two electrons is 1 divided by 4.17×10^{42}! The question is, where does such a large number come from? It is not accidental […].'

For Dicke [24], 'The fact that the strength of the gravitational interaction is so different from other interactions might tend to strengthen the belief that it is completely different from other forces.' He justified this by pointing out the lack of success in 35 years of attempt to construct a unified field theory. Frank Wilczek also commented on the same subject, focusing on protons in his Nobel Lecture [25]. This is an interesting context; one that is described further below in section 2.6.

From the RMS perspective, Feynman's question can be precisely answered. The gravitational force is weak because protons or electrons have a much smaller mass than the charge, resulting in a disparity in the corresponding ratio of the acting forces. This explanation is straightforward, but still does not explain the underlying fundamental reason for the difference. In the SI, an explanation such as 'smaller mass than charge' does not make sense because of the different dimensions of mass and charge and the common belief that they are not comparable. Popular sources such as Wikipedia[3] even state that 'From the point of view of Planck units, this is comparing apples with oranges, because mass and electric charge are incommensurable quantities.'

However, the assertion of incommensurability between mass and charge arising only from unit conventions is now refuted by the RMS, which achieves seamless unification of the gravitational mass and electric charge without the oddities of the fractional powers of the base units, such as $[q]=[g^{1/2}cm^{3/2}s^{-1}]$, as found in the CGS-ESU subsystem (Everett [3] p.161).

2.5.2. Classical electron radius

The classical electron radius in SI is

$$r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2}, \tag{14}$$

[3] Wikipedia (2023, February). Planck Units [Online]. https://en.wikipedia.org/wiki/Planck_units

and in RMS, it is

$$r_e = \frac{\overline{e}^2}{4\pi \overline{m}_e c^2}. \tag{15}$$

The interpretation of equation (15) is straightforward: the radius can be calculated as the ratio of the square of the electron charge to its rest mass energy, obtained from the Einstein formula, and divided by 4π (from Heaviside rationalization). However, interpreting SI equation (14) without remembering the rationale behind its derivations is more challenging, because we need to explain the role of ε_0 beyond it being a mere unit conversion factor, which would then contradict its status as a constant of nature. Nonetheless, the situation is even clearer in the original CGS-ESU subsystem, where the same radius is represented as $r_e = e^2/m_e c^2$, with an obvious interpretation.

Thus, a particular algebraic manipulation result is worthy of further attention. The mass and charge are interchangeable in terms of the RMS units. We can substitute the electron mass $\bar{m}_e$ in equation (15) in place of its charge $\overline{e}$ and derive a type of radius, denoted r_{me}:

$$r_{me} = \frac{1}{4\pi}\frac{\bar{m}_e^2}{\bar{m}_e c^2} = \frac{1}{4\pi}\frac{\bar{m}_e}{c^2} \Rightarrow r_{me} = \frac{1}{4\pi}\frac{4\pi G m_e}{c^2} = \frac{G m_e}{c^2}. \tag{16}$$

As expected, r_{me} is related to the radius; in this case, it is the radius of the black hole of the electron mass. However, this is, only half of the Schwarzschild radius (SCR), which leads to the question of why?

The SCR represents the black hole size when the escape velocity of a receding object calculated along the radius is equal to the speed of light c. In contrast, r_{me} is the radius at which a hypothetical particle moving tangentially to a spherical surface[4] is forced onto a circular orbit at the speed of light c. This can be deduced from the following equation, using Newton's law:

$$a_r = \frac{G m_e}{r^2} \Rightarrow \omega^2 r = \frac{G m_e}{r^2} \Rightarrow \omega^2 r^2 = \frac{G m_e}{r} \Rightarrow c^2 = \frac{G m_e}{r} \Rightarrow r = \frac{G m_e}{c^2} = r_{me}, \tag{17}$$

where ω denotes the angular orbital velocity. The modified radius expression (16) used in RMS applies equally to masses of any size. Haug [28] demonstrated that 1/2 of the SCR results from limiting the escape velocity to c, by relativistic effects. However, in SI, substituting the charge with the mass to derive the radius is not a straightforward process. Meier [29] p.235 identified $r_g \equiv GM/c^2$ as 'gravitational radius' in the context of rotating black holes. However, this term is most frequently used as a synonym for the Schwarzschild radius (Ince [30] p.69). Landau and Lipshitz [31] pp.301, 309) followed the same conventions, and make a relevant remark on 'the basis for concluding that there must be a singularity of the space-time metric and that it is therefore impossible for bodies to exist that have a 'radius' (for a given mass) that is less than the gravitational radius,' stating that 'this conclusion would be wrong' (p.309).

Kittel *et al* [32] p.402 call r_g 'gravitational length' and also note that its derivation is arrived at via analogy with the classical electron radius expression, which is similar to what can be seen in equation (16), where this analogy emerges because of the commensurability of the RMS mass and charge units. The physical meaning of the appearance of r_g (or r_{me} in equation (16)) is not immediately clear. However, the clarification is attempted in section 5.

2.5.3. Planck constant and Planck-derived constants

The Planck constant h has now been assigned an exact value in the SI system, which in turn defines the values of other constants, such as the Planck time, length, mass, and temperature. In this study, these are collectively referred to as the 'Planck-derived constants.'

The RMS Planck constant h after straightforward conversion is $\overline{h} = h4\pi G$. Planck [33] introduced a temperature constant related to the Boltzmann constant, along with the time, length, and mass constants. The Boltzmann constant has the kelvin base unit embedded within it: $[k_B]=[\mathrm{kg\,m^2\,s^{-2}\,K^{-1}}]$.

In RMS, the Boltzmann constant can be expressed as

$$\bar{k}_B = k_B 4\pi G \approx 1.15797 \times 10^{-32} \mathrm{m^5 s^{-4} K^{-1}}. \tag{18}$$

The RMS Planck temperature can be derived using the following expressions:

$$T_P = \frac{1}{k_B}\sqrt{\frac{hc^5}{2\pi G}} \Rightarrow \overline{T}_P = \frac{1}{4\pi G k_B}\sqrt{\frac{4\pi G h c^5}{2\pi\left(\frac{1}{4\pi}\right)}} \Rightarrow$$

$$T_P = \overline{T}_P = \frac{1}{\bar{k}_B} c^2 \sqrt{2\overline{h}c} \approx 1.416784 \times 10^{32}, \tag{19}$$

[4] This is a classical mechanics approach.

while the meter, second, and kelvin base units in the RMS remain unchanged. With the elimination of the kilogram, the expressions used to calculate the Planck-derived constants have been modified using the procedure described above on section 2.4. Consequently, the *values* of all Planck-derived constants, except for the Planck mass, remained unchanged in the RMS.

Table 1 Presents the results obtained by applying RMS conversion rules to these constants.

The conversion of the Planck-derived constants to RMS yielded several findings:

Table 1. Conversion of the Planck constant and Planck-derived constants to RMS.

Name	SI formula	RMS formula / value	
Planck constant	Defined numerically	$\bar{h} = h4\pi G$	(20)
	$h = 6.626\,070\,15 \times 10^{-34}\mathrm{kgm^2}s^{-1}$	$\bar{h} \approx 5.557\,399 \times 10^{-43}\,\mathrm{m^5s^{-3}}$	
Planck time	$t_p = \sqrt{\frac{Gh}{2\pi c^5}}$	$t_p = \frac{1}{2\pi}\sqrt{\frac{\bar{h}}{2c^5}}$	(21)
	$t_p \approx 5.391\,247 \times 10^{-44}\mathrm{s}$	$t_p \approx 5.391\,247 \times 10^{-44}\mathrm{s}$	
Planck length	$l_p = \sqrt{\frac{Gh}{2\pi c^3}}$	$l_p = \frac{1}{2\pi}\sqrt{\frac{\bar{h}}{2c^3}}$	(22)
	$l_p \approx 1.616\,255 \times 10^{-35}\mathrm{m}$	$l_p \approx 1.616\,255 \times 10^{-35}\mathrm{m}$	
Planck mass	$m_P = \sqrt{\frac{ch}{2\pi G}}$	$\bar{m}_P = \sqrt{2c\bar{h}} = 4\pi\frac{l_P^3}{t_P^2}$	(23)
	$m_p \approx 2.176\,434 \times 10^{-8}\,\mathrm{kg}$	$\bar{m}_P \approx 1.825\,413 \times 10^{-17}\,\mathrm{m^3s^{-2}}$	
Planck temperature	$T_P = \frac{1}{k_B}\sqrt{\frac{hc^5}{2\pi G}}$	$T_P = \frac{1}{k_B}c^2\sqrt{2\bar{h}c} = \frac{1}{k_B}\bar{m}_P c^2$	(24)
	$T_p \approx 1.416\,784 \times 10^{32}\mathrm{K}$	$T_p \approx 1.416\,784 \times 10^{32}\mathrm{K}$	
Planck unit relations	$\frac{l_P^3}{t_P^2} = Gm_P$	$\frac{l_P^3}{t_P^2} = \frac{1}{4\pi}\bar{m}_P$	(25)
Fine structure constant	$\alpha = \frac{e^2}{2\pi\varepsilon_0 hc}$	$\alpha = \frac{\bar{e}^2}{2c\bar{h}} \Rightarrow \boldsymbol{\alpha = \frac{\bar{e}^2}{\bar{m}_P^2}}$	(26)
	$\alpha \approx 7.297\,352\,5693 \times 10^{-3}$	$\alpha \approx 7.297\,352\,5693 \times 10^{-3}$	

(1) The gravitational constant $\boldsymbol{G}$ is embedded directly only in the RMS value of the Planck constant $\bar{h}$, which affects the other constants, except for α where $\boldsymbol{G}$ cancels out.

(2) The Planck mass expression in equation (20) is remarkably simpler than that in SI.

(3) The Planck temperature expressions are straightforward and provide immediate insight into the meaning of temperature, which is obscured in the SI definition. Specifically, equation (24) indicates that the Planck temperature is equivalent to the total energy of the Planck rest mass converted to the absolute temperature scale. This relationship also holds in the SI: $\boldsymbol{T_p = m_P c^2/k_B}$.

(4) The fine structure constant α, expressed by equation (26), can be readily transformed into

$$\boldsymbol{\alpha = \frac{\bar{e}^2}{\bar{m}_P^2}}. \tag{27}$$

(5) This highlights the coupling between the elementary charge and the Planck mass, which is universally significant. Therefore, α can also be regarded as the charge-to-Planck-mass coupling constant in RMS, and it likely represents the minimum limit of the squared charge-to-mass ratio for all the charged particles.

(6) While elementary charge is a tangible physical property, Planck-derived constants have been occasionally criticized as mathematical artifacts arising from dimensional analysis, as noted by Haug [34]. Bridgman [35] p.101 commented on Planck's natural units as follows:

> Until some essential connection is discovered between the mechanisms which are accountable for the gravitational constant, the velocity of light, and the quantum, it would seem that no significance whatever should be attached to the particular size of the units defined in this way, beyond the fact that the size of such units is determined by phenomena of universal occurrence. [35] p.101.

In contrast, equation (27) demonstrates that the gravitational constant, velocity of light, quantum ($\bar{h}$), and elementary charge are intimately interconnected. Hence, the Planck mass is not an artifact, but a meaningful constant of nature that can alternatively be defined from equation (27) as

$$\bar{m}_P = \frac{\bar{e}}{\sqrt{\alpha}}. \tag{28}$$

Returning to the SI, the alternative definition of the Planck mass can be obtained as

$$4\pi G m_P = e\frac{\sqrt{\frac{4\pi G}{\varepsilon_0}}}{\sqrt{\alpha}} \Rightarrow m_P = \frac{e}{\sqrt{4\pi G\alpha\varepsilon_0}}. \tag{29}$$

2.6. Explaining why gravity is so feeble

In his Nobel Lecture, Frank Wilczek [25] argued that measuring the proton mass in Planck units leads to the following relationship:

$$m_{pr} \sim 10^{-18}\sqrt{\frac{hc}{G}},$$

and then in such case:

> [...] it makes no sense to ask 'Why is gravity so feeble?' Gravity, as the primary force, just is what it is. The right question is the one we confront here: 'Why is the proton so light?'

However, this explanation does not fully address Feynman's question regarding the weakness of the gravitational force of electrons, which has puzzled physicists for several decades. Prior to the Nobel Lecture, Wilczek shed some light on this issue in two different ways [26, 27], presenting a nondimensional constant X (or N in [27]) that could help explain the weakness of the gravitational force.

After expanding $\hbar$ in the original X formula, it takes the form:[5]

$$N \equiv X = \frac{Gm_{pr}^2}{hc/2\pi}$$

$$X \approx 6 \times 10^{-39}. \tag{30}$$

This is also known as the gravitational coupling constant $\boldsymbol{\alpha_g}$, as described by Silk [36] in 1977.

The magnitude of X is of a similar order as the gravitational-to-electrical force ratio; hence, it may be key to arrive at a complete explanation. In other words, the feebleness of gravity may be due not only to the mass of the proton being too small, but also to the smallness of the gravitational coupling constant itself. The significance of X is explained in as 'the fractional contribution of gravitational binding energy to the proton's rest mass,' which does not seem to directly address Feynman's question. However, by converting X to RMS units using the Planck mass formula from equation (23), we obtain the following relations:

$$X = \frac{Gm_{pr}^2}{hc/2\pi} \Rightarrow \bar{X} = \frac{4\pi G}{4\pi G}\frac{Gm_{pr}^2}{hc/2\pi}\frac{4\pi}{4\pi} \Rightarrow$$

$$\bar{X} = \frac{\bar{\boldsymbol{m}}_{\boldsymbol{pr}}^{\boldsymbol{2}}}{\boldsymbol{2\bar{h}c}} = \frac{\bar{\boldsymbol{m}}_{\boldsymbol{pr}}^{\boldsymbol{2}}}{\bar{\boldsymbol{m}}_{\boldsymbol{P}}^{\boldsymbol{2}}} = \frac{\boldsymbol{m}_{\boldsymbol{pr}}^{\boldsymbol{2}}}{\boldsymbol{m}_{\boldsymbol{P}}^{\boldsymbol{2}}} \approx 5.906149 \times 10^{-39}. \tag{31}$$

The identity of X and $\boldsymbol{\alpha_g}$ established above, $\mathrm{X} \equiv \boldsymbol{\alpha_g}$, reveals that X is equal to the squared ratio of the proton mass to the Planck mass, which now closely corresponds to the context presented in the Nobel Lecture, but this still does not directly answer Feynman's question. The squared mass ratio in equation (31) has already been recognized in the literature as a coupling constant of the same kind, as exemplified indirectly by Burrows and Ostriker [37] and by Haug [38], who presented an analogous expression for the electron, anticipating a deeper meaning of that constant. A satisfactory answer to this dilemma has been found through a new meaningful relation, by using equation (31), then applying the derived alternative definition of the Planck mass from equation (28):

$$\bar{X} = \alpha_g = \frac{\bar{m}_{pr}^2}{\bar{m}_p^2} = \alpha\frac{\bar{m}_{pr}^2}{\bar{e}^2} \approx 5.906149 \times 10^{-39}. \tag{32}$$

[5] Note: Wilczek used index p to refer to proton mass, but, to avoid confusion with the Planck mass index P, in this study, the index 'pr' is used instead.

In the SI, it becomes:

$$X \equiv \alpha_g = \frac{4\pi G\alpha\varepsilon_0 m_{pr}^2}{e^2} \approx 5.906149 \times 10^{-39}. \tag{33}$$

The simplicity of the equivalent relations among the constants of nature in the RMS is clearly demonstrated above. In conclusion:

1. X $\equiv \boldsymbol{\alpha_g}$ is, in fact, the gravitational mass-to-charge coupling constant, which is indeed significant in the context of Feynman's question, but still does not explain much more than the already derived equation (10).
2. Wilczek's intuition in recognizing the relevance of X was correct, overcoming the constraints imposed by the kilogram and ampere units in understanding the natural and simple relations among physical constants.
3. The weakness of the gravitational force of the proton is that it has a much smaller mass than charge, as seen in equation (32), which results in a disparity in forces. However, there is still no complete explanation for the magnitude of this difference because equation (32) is merely a statement of fact, and Feynman's question remains unanswered; thus, Wilczek's interpretation still holds.
4. The RMS relations among the fundamental constants are generally simpler than those in SI.

2.7. Einstein gravitational constant

The Einstein gravitational constant $\kappa = 8\pi G/c^4$, which appears in Einstein field equations, simply reduces to $\kappa = 2/c^4$, and general relativity can be pursued within space-time dimensions [LT] for all physical quantities involved.

2.8. Possible link between magnetic induction and gravitation

It is interesting to note that the RMS units of the magnetic field induction, denoted $\boldsymbol{B}$, transformed from [kg A^{-1} s^{-2}] in SI to [s^{-1}] in the RMS, whereas the units of the electric field, denoted $\boldsymbol{E}$, changed from [kg m s^{-1} A^{-1}] to [m s^{-2}]. Remarkably, the dimensions of $\overline{E}$ are equivalent to those of acceleration, which are exactly the same as those of gravitational field $\boldsymbol{g}$. However, $\overline{B}$ has frequency or angular velocity dimension. Two separate scenarios involving electrons should be considered to gain insights into these unusual unit associations. Scenario 1 is illustrated in figure 1.

Suppose that an electron moves at a scalar velocity v into a uniform magnetic field $\boldsymbol{B}$ in the absence of an external electric field. In this field, electromagnetic induction has a scalar value of B_e. The initial electron path is perpendicular to induction vector $\boldsymbol{B}$. At the point of entry, the Lorentz force $\boldsymbol{F_L}$ is assumed to be of scalar magnitude, as described by equation (34):

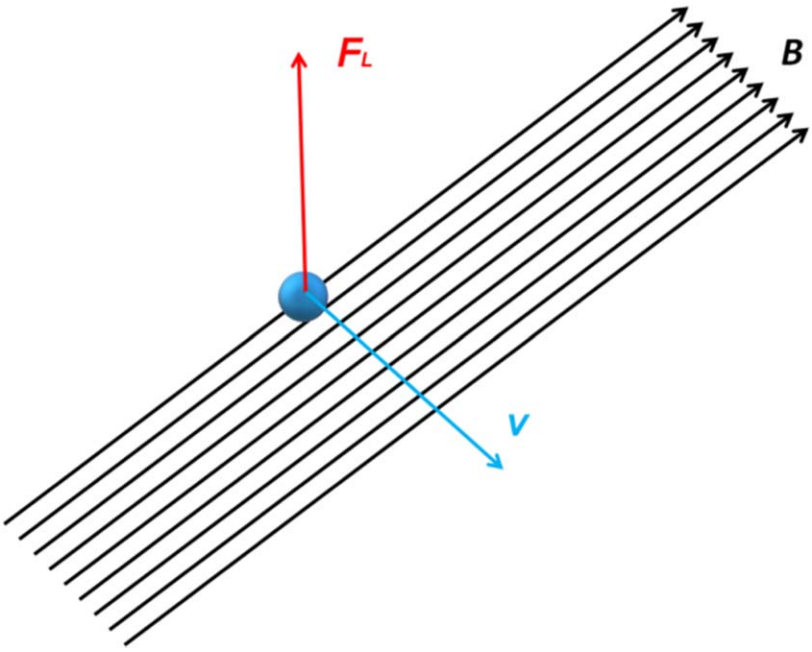


Figure 1. A free electron in the magnetic field generating Lorentz force upon entry.

$$F_L = evB_e \Rightarrow \bar{F}_L = \bar{e}v\bar{B}_e. \tag{34}$$

Let us now arbitrarily set the Lorentz force value equal to the repulsive force between an electron and an abstract elementary charge e, at the distance of the classical electron radius r_e. Therefore, in the RMS, we have

$$\bar{F}_L = \frac{1}{4\pi}\frac{\bar{e}^2}{r_e^2}. \tag{35}$$

Using the classical electron radius in the RMS from equation (15), the force equation becomes

$$\bar{F}_L = \frac{\bar{m}_e c^2}{r_e}. \tag{36}$$

The value of v can now be set to examine the value of $\bar{B}_e$, which is necessary for generating a force of magnitude $\bar{F}_L$. For simplicity let us assume that v=c. Combining equations (34) and (36) yields the following expression:

$$\frac{\bar{m}_e c^2}{r_e} = \bar{e}c\bar{B}_e \Rightarrow \bar{B}_e = \frac{\bar{m}_e c}{\bar{e}r_e} \approx 52.125432506340961 s^{-1}. \tag{37}$$

In independent scenario 2 shown in figure2, let us assume that an *uncharged* point mass equal to the mass of the electron is in gravitational orbit around the electron at radius $\boldsymbol{r_e}$ and calculate the angular velocity on the orbit.

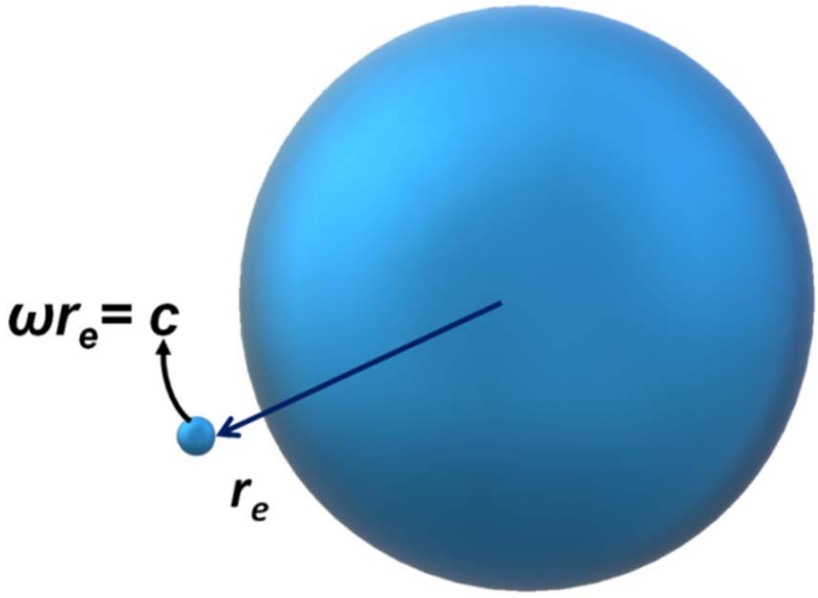


Figure 2. The point mass of the electron mass magnitude orbiting the electron fixed in space.

The central electron was assumed to be fixed in space to determine the angular velocity ω. For simplicity it is assumed that the central electron mass is stationary to avoid the complications of the two-body problem.

Applying Newton's law, ω_e can be calculated as

$$\omega_e^2 r_e = G\frac{m_e}{r_e^2} \Rightarrow \omega_e = \sqrt{G\frac{m_e}{r_e^3}} \approx 52.125432506295360 s^{-1}. \tag{38}$$

The coincidence of nearly identical numerical results can be easily noticed. The absolute difference was calculated as $\omega_e - \bar{B}_e = 5{,}471 \times 10^{-11}$; that is, $val(\omega_e) \approx val(\bar{B}_e)$. This numerical coincidence in the values of the two entirely different physical quantities, raises the question of why this has occurred—is this indeed a coincidence or could the expression be reduced to an identity? To investigate, we consider equations (37) and (38) and examine the nondimensional ratio that should reduce to 1 without contradiction:

$$\frac{\left(\frac{\bar{m}_e c}{\bar{e}\ r_e}\right)^2}{G\frac{m_e}{r_e^3}} \equiv 1. \tag{39}$$

It is easy to prove that equation (39) is true if the formula $r_e = \bar{e}^2/(4\pi\bar{m}_e c^2)$ is the RMS expression for the classical electron radius—which is indeed the case, as given by equation (15).

In short, the findings in this section give rise to two key questions: Are magnetic and electrical fields related to the gravitational field? Can the uncovered identity be a clue to gaining more understanding?

3. The universal planck natural unit system

In 1899, Max Planck proposed a new system of units that did not rely on arbitrary objects, which is now known as the Planck unit system. This system involves setting the values of certain fundamental constants to nondimensional 1, allowing them to disappear from the equations and from the concern of mathematicians. These constants include the following:

- The speed of light, $c = 1$
- The gravitational constant, $\boldsymbol{G} = 1$
- The Planck constant, $h = 1$
- The Boltzmann constant $k_B = 1$.

It is now widely accepted to use the reduced constant $\hbar = h/2\pi$ replacing the h originally defined by Planck.

The common practice of reducing physical constants to nondimensional 1 for convenience, thus making them disappear, is sometimes criticized. Wesson [39] p.117 argued that, in cosmological research, the use of units like $\boldsymbol{G} \equiv 1$ and $c \equiv 1$ should be avoided: although 'Mathematically it is an acceptable trick which saves labour. Physically it represents the loss of information and leads to confusion [...].' In addition to eliminating base units of mass and charge, one of the aims of UPNUS is to develop a Planck scale system that retains the dimensionality of all physical constants, regardless of their numerical values, thus avoiding the misleading side effects mentioned above (Wesson [39] p.117).

Notably, the *original* Planck unit system did not include a predefined elementary charge constant. However, the Stoney unit system [40], published in 1881, introduced the idea of setting the elementary 'unit quantity of electricity' value to 1, together with the speed of light and gravitational constant. (The Stoney unit system is discussed in the context of the elimination of the base unit of mass and charge in section 4). In UPNUS, the elementary charge naturally emerges at a specific value as a result of unit conversion, rather than by assumption.

UPNUS proposes changes to the Planck unit system such that, instead of preselecting and assigning unit values with no dimensions to physical constants, the base units for [LT] are defined only by the Planck time and length unit symbols. Additionally, the Planck temperature becomes another base unit, and all dimensional constants, other than the two fundamental length and time units, scale naturally.

It is worth noting that all Planck-derived constants can be directly measured using the principle of the Cavendish apparatus in a mode similar to the measurement of Earth's density by Cavendish [41], as described by Haug [34]. This implies that UPNUS does not necessarily depend on the measurement of the gravitational constant. For further details, refer to section 6.

For convenience, let us denote the Planck units for length, time, and temperature '*lpu*,' '*tpu*,' and '*Tpu*' respectively. The conversion factors from meters and seconds to the Planck units are then:

$$m2lpu = \frac{1}{l_P}[\text{lpu m}^{-1}]$$

$$s2tpu = \frac{1}{t_P}[\text{tpu s}^{-1}]$$

$$K2Tpu = \frac{1}{T_P}[\text{Tpu K}^{-1}], \tag{40}$$

where in SI:

$$l_P = \sqrt{\frac{Gh}{2\pi c^3}},\ t_P = \sqrt{\frac{Gh}{2\pi c^5}},\ T_P = \frac{1}{k_B}\sqrt{\frac{c^5 h}{2\pi G}}. \tag{41}$$

To convert a constant from RMS to UPNUS, every meter, second, or kelvin symbol in the expanded dimensional expression must be multiplied by the corresponding factor from the set of equations (40) in the respective powers, either symbolically or with substituted numeric values of the SI constants.

3.1. Effects of conversion to UPNUS

3.1.1. Speed of light

Similar to the original Planck unit system, the speed of light naturally emerges as $\check{c} = 1\ \mathrm{lpu\ tpu^{-1}}$.

3.1.2. Planck constant

For the Planck constant, which is currently defined as the exact value in SI, the RMS value is contaminated by the uncertainty of $\boldsymbol{G}$, as shown in the expression $\overline{h} = 4\pi Gh$. The formal conversion of $\overline{h}$ to UPNUS using the factors from equation (40) yields

$$\check{h} = 4\pi Gh\left(\frac{m2lpu^5}{s2tpu^3}\right) \Rightarrow \boldsymbol{\check{h} = 8\pi^2}\ \mathrm{lpu^5 tpu^{-3}}. \tag{42}$$

The value of $\check{h}$ is now the exact dimensional mathematical constant independent of $\boldsymbol{G}$. The Planck constant h is currently defined as exact in SI.

3.1.3. Planck mass

$$\overline{m}_P = 4\pi Gm_P \Rightarrow \check{m}_P = 4\pi Gm_P\left(\frac{m2lpu^3}{s2tpu^2}\right) \Rightarrow \boldsymbol{\check{m}_P = 4\pi}\ \mathrm{lpu^3 tpu^{-2}}. \tag{43}$$

3.1.4. Gravitational constant

The gravitational force equation form in UPNUS is

$$\check{F}_G = \frac{1}{4\pi}\frac{\check{m}\check{M}}{r^2}. \tag{44}$$

The constant $1/4\pi$ in the RMS is the spherical symmetry factor, following the Heaviside [12] rationalization concept; hence, $\check{G} = 1$, which is nondimensional.

3.1.5. Electron mass

The conversion process results in the following:

$$\overline{m}_e = 4\pi Gm_e \Rightarrow \check{m}_e = 4\pi Gm_e\left(\frac{m2lpu^3}{s2tpu^2}\right) \Rightarrow$$

$$\boldsymbol{\check{m}_e = \frac{4\pi m_e}{m_P}} \approx 5.259697 \times 10^{-22}\mathrm{lpu^3 tpu^{-2}}. \tag{45}$$

The accuracy of the value is affected by both the measurement errors of m_e and the low precision of $\boldsymbol{G}$ in m_P — the same problem as in the original Planck unit system.

3.1.6. Fine structure constant

From equation (27), the conversion of the fine structure constant from the RMS formula to UPNUS yields

$$\alpha = \frac{\overline{e}^2}{\overline{m}_P^2} = \frac{\check{e}^2}{\check{m}_P^2} = \frac{16\alpha\pi^2}{4\pi 4\pi} = \boldsymbol{\alpha}. \tag{46}$$

The fine structure constant emerges as an identity; therefore, it is an independent input parameter to the system. In other words, there is no definition of α in UPNUS matching the SI definition [1] p.128.

3.1.7. Elementary (electron) charge

Using equation (11) and performing formal conversion to UPNUS, taking the factors from equation (40) results in

$$\check{e} = e\sqrt{\frac{4\pi G}{\varepsilon_0}}\left(\frac{m2lpu^3}{s2tpu^2}\right) \Rightarrow \check{e} = \frac{e2\pi\sqrt{2}}{\sqrt{\varepsilon_0 ch}}. \tag{47}$$

The value of the elementary charge expression thus has dimension and is independent of $\boldsymbol{G}$. This can be further simplified as follows:

$$\check{e}^2 = \frac{e^2 8\pi^2}{\varepsilon_0 ch} = \frac{e^2}{2\varepsilon_0 ch}2 \times 8\pi^2 \Rightarrow \check{e}^2 = \alpha 16\pi^2 \mathrm{lpu^6 tpu^{-4}}. \tag{48}$$

The squared charge in the UPNUS expression has the fine structure constant α built into it. The elementary charge $\breve{e}$ in UPNUS is then reduced to

$$\breve{e} = 4\pi\sqrt{\alpha} \approx 1.073476469\text{lpu}^3\text{tpu}^{-2}, \tag{49}$$

which is as accurate as the measured fine structure constant. The important points to note are as follows:

1. In UPNUS, the elementary charge value and accuracy are determined solely by the fine structure constant.
2. There is an apparent logical conflict between the exact value of the elementary charge set to be exact in SI and the value of the elementary charge. In UPNUS, it depends on the accuracy of α. In contrast, the Planck constant is exact in SI and UPNUS, and no such conflict exists. However, this can be resolved in SI because the constants μ_0 and ε_0 are both functions of α and would have to be changed accordingly when a more accurate value of α is found, while keeping the elementary charge value unchanged in SI without inconsistencies in values dependent on charge.
3. While the fine structure constant α is nondimensional, the electric charge dimensions [lpu^3 tpu^{-2}] must be applied explicitly to the mathematical constant 4π, which then becomes identical numerically and dimensionally to the UPNUS Planck mass from equation (43). It is then possible to assume that the electron charge expression can be written as

$$\breve{\boldsymbol{e}} = \breve{\boldsymbol{m}}_{\boldsymbol{P}}\sqrt{\boldsymbol{\alpha}} \approx 1.073476469\text{lpu}^3\text{tpu}^{-2}. \tag{50}$$

It is risky to assign a physical entity to a coincidental numerical value, such as in equation (50), but this is algebraically proven by straightforward transformations of the equalities in equation (46). The elementary charge property of the real physical entity is then coupled by the universal constant $\sqrt{\alpha}$ with the Planck mass. The Planck mass among other Planck units were of 'no significance' for Bridgman [35] p101, which does not seem to be the case. The higher complexity of the equivalent and harder to interpret expression in SI, $e = \sqrt{2\alpha\varepsilon_0 hc}$, is evident.

3.1.8. Boltzmann constant and Planck temperature

Using equations (18), (24), and (40) yields

$$T_p = \frac{1}{\overline{k}_B}c^2\sqrt{2\overline{h}c} = \frac{1}{\overline{k}_B}c^2 m_P \Rightarrow$$

$$\breve{T}_p = \frac{1}{\overline{k}_B}c^2 m_P(K2Tpu) = 1Tpu. \tag{51}$$

Additionally, the Boltzmann constant transforms into

$$\overline{k}_B = k_B 4\pi G \Rightarrow \breve{k}_B = \overline{k}_B((K2Tpu)^{-1}\text{m2lpu}^5\text{s2tpu}^{-4}) \Rightarrow$$

$$\breve{k}_B = \mathbf{4\pi}\text{lpu}^5\text{tpu}^{-4}\text{Tpu}^{-1}. \tag{52}$$

4. The stoney unit system in the time-space domain

In 1874, George J Stoney delivered a presentation before section A. of the British Association at the Belfast Meeting [40] p.51, where, for the first time in the history of physics, he introduced the natural unit system. This was 25 years before Max Planck published a similar system based on the discovery of the quantum of action, known as the Planck constant *h*.

4.1. Stoney constants conversion to RMS

The expressions for the Stoney constants presented in the original publication [40] required conversion from the antiquated original notation and units to a more contemporary form in CGS, as can be found in the publication of Barrow [42] p.25. When converted to SI units, they become

$$t_S = \sqrt{\frac{Ge^2}{4\pi\varepsilon_0 c^6}}$$

$$l_S = \sqrt{\frac{Ge^2}{4\pi\varepsilon_0 c^4}}$$

$$m_S = \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}}. \tag{53}$$

The conversion of the above to RMS units yields

$$t_S = \sqrt{\frac{G}{4\pi\varepsilon_0}}\left(\frac{\bar{e}}{c^3}\right)\Big/\sqrt{\frac{4\pi G}{\varepsilon_0}} = \sqrt{\frac{1}{4\pi 4\pi}}\left(\frac{\bar{e}}{c^3}\right) = \frac{\mathbf{1}}{\mathbf{4\pi}}\frac{\bar{\mathbf{e}}}{\mathbf{c^3}}$$
$$l_S = \sqrt{\frac{G}{4\pi\varepsilon_0}}\left(\frac{\bar{e}}{c^2}\right)\Big/\sqrt{\frac{4\pi G}{\varepsilon_0}} = \sqrt{\frac{1}{4\pi 4\pi}}\left(\frac{\bar{e}}{c^2}\right) = \frac{\mathbf{1}}{\mathbf{4\pi}}\frac{\bar{\mathbf{e}}}{\mathbf{c^2}}$$

$$\bar{m}_S = \sqrt{\frac{1}{4\pi\varepsilon_0 G}}\, e\sqrt{4\pi G}\,\sqrt{4\pi G} = \sqrt{\frac{4\pi G}{\varepsilon_0}}\, e \equiv \bar{\mathbf{e}}. \tag{54}$$

Stoney units differ from Planck units by the constant ratio $\sqrt{\alpha}$ (Barrow [42] p.25); for example,

$$m_S/m_P = \bar{m}_S/\bar{m}_P = \sqrt{\alpha}. \tag{55}$$

Therefore, $m_s \ll m_P$, $l_S \ll l_P$, $t_S \ll t_P$.

4.2. The physical meaning of the Stoney mass

The original derivation of the Stoney mass was a dimensional-analysis-based operation (Barrow [42] p.25). A previously unforeseen assembly of the fundamental constants appeared to Stoney [40] as an entity in the unit of mass. Equation (54) in the RMS units is an interesting result, because it indicates that the mass unexpectedly becomes the elementary charge. Stoney mass has always been considered somewhat of a mystery and various opinions circulate about its physical meaning and significance. For instance, Buczyna *et al* [43] p.7 state that:

> It is interesting to note that the Stoney mass M_S[...] allows the description of the gravitational coupling constant as a fundamental electric charge to gravitational charge ratio $e/M_S = \sqrt{G}$. However, this is useful from a metrology point of view only if the Stoney mass can be identified with some fundamental mass, which is not the case today.[6]

The usefulness clause is critical. In another paper, Abdukadyrov [44] states that 'Stony mass m_s separates the macrocosm and microcosm; i.e., this mass is the minimum value for the masses of ordinary bodies and the maximum value for the masses of elementary particles.'

The conversion to RMS proves that the Stoney mass in equation (54) is nothing but 'the' elementary charge —not a mass with the magnitude of the charge in RMS units. The SI and RMS operate within the set of standard fundamental constants. If there were indeed another unknown genuine Stoney mass entity, it would have to appear naturally as another combination of known constants, as is the case in the RMS version of the Planck mass: $\bar{m}_P = \sqrt{2c\bar{h}}$. Unfortunately, the Stoney mass expression in equation (54) is reduced to only one constant $\bar{e}$ that is already known, and appears to be of the wrong kind. The charge quantity hidden in the CGS and SI representations of the Stoney mass emerges as a mass-equivalent form of the elementary charge. The confusion in the SI and CGS expressions in kilograms/grams is an inherent problem of these systems of units, owing to the incommensurability of charge and mass. This is a serious problem in contemporary metric systems when the mistaken identity of a physical constant is possible; one that has misled scientists for more than a century. It is difficult to accept that an algebraically valid combination of physical constants can yield an invalid result. There is no space in science for paradoxes and a rational explanation can and must be found.

The combination of the constants $m_S c^2$ is often called the Stoney energy. This can be converted into the RMS form of $\bar{e}c^2$. This expression has energy dimensions as is the case for the electron rest mass energy $\bar{m}c^2$. To become the elementary charge e, the elementary charge quantity should be converted from RMS back to SI in accordance with the charge conversion rule given by equation (11).

If the mass conversion factor was used instead of that for the charge, the Stoney mass would emerge in the SI mass units, and thus, a change in the quantity type would occur. The conversion of the standalone expression $\bar{e}c^2$ back to SI, formally yields ec^2, which has dimensions of [m^2As^{-1}], rather than energy in [$kg\,m^2\,s^{-2}$], which is wrong and of no use. This is because it is necessary for proper conversion to anticipate the type of converted quantity in the SI. If energy is expected, then the Stoney mass rest energy appears again, which allows us to propose the following conjecture:

> The RMS relation $\bar{E}_S = \bar{e}c^2$ represents the same physical amount of energy as the SI Stoney energy $E_S = m_S c^2$, and this is the finite electric field energy of the electron.

[6] These expressions are valid in the CGS units.

We prove below the equivalence of energies, but the fact that this is 'the finite electric field energy of the electron' remains to be confirmed by physical theory and experimental confirmation. The proof of the energy equivalence is as follows:

When converting a product of physical quantities, we may convert each quantity separately depending on its kind and algebraically combine all resulting dimensional expressions, or convert the entire product based on the combined dimensions in one step. Thus far, we have converted the Stoney mass quantity to

$$\bar{m}_S = 4\pi G m_S, \tag{56}$$

However, we can encapsulate $m_S c^2$ as the energy E_S, where $[E_S]=[\mathrm{kgm^{2}\cdot s^{-2}}]$. From the conversion rules described in section 2.4, it follows that only the kilogram must be converted. Hence,

$$\bar{E}_S = 4\pi G E_S \Rightarrow \bar{E}_S = 4\pi G m_s c^2 = \bar{m}_S c^2 = \bar{e}c^2 \Rightarrow [\bar{E}_S] = [m^5 s^{-4}] \; and$$
$$\bar{e}c^2 = E_{RMS} = E_{SI}\, 4\pi G \Rightarrow ec^2\sqrt{4\pi G/\varepsilon_0} = E_{SI}\, 4\pi G \Rightarrow E_{SI} = c^2\sqrt{e^2/4\pi G\varepsilon_0} = E_S \quad \text{Q.E.D.} \tag{57}$$

Thus, the energy equivalence of the Stoney Energy and field energy of the electron to and from the SI is proven. For the second part of the conjecture, we can state that:

1. There is currently no place for a Stoney particle in the Standard Model, and the most massive stable particle is the neutron.
2. Stoney energy in SI units $E_S{=}m_S c^2$ is converted to in RMS units, and
3. $\bar{e}c^2$ is an exclusive property of the electron charge, and
4. $\bar{e}$ is representative of the electric field surrounding the electron; therefore it is reasonable to expect that $\bar{e}c^2$ is the finite energy of the electron field, in contrast to the infinite energy alternative (resulting from the application of Coulomb's law to the electron as a point particle), the infinity which Dirac [45] pp.149,168 tried to avoid.
5. The energy of the electric field around an electron has never been proven to be infinite in any experiment.

Thus, it rests on the community of physicists to take action and to definitely prove or disprove the proposed conjecture on theoretical and experimental grounds. The conjecture resulted only from dimensional and metric system analyses, and cross-validation is required.

If the conjecture is validated, then we know that not only the energy of the electric field of the electron but also the fundamental status of the gravitational constant **G** is elevated to an unsuspected role of limiting the energy of the electric field of the electron to a finite level (although conveniently hidden inside RMS and UPNUS base units).

The statement by Buczyna *et al* [43] p.7 cited on page 15 can now be validated. The Stoney mass while not being a fundamental mass of a particle, is a handle to the electron's finite energy, and thereby reveals that the 'gravitational coupling constant' described as $\sqrt{\mathrm{G}} = \mathrm{e}/M_S$ in CGS units, plays a fundamental role in determining electron properties and cannot be dispensed with, even though it acts as a unit conversion constant between mass and charge.

4.3. The physical meaning of the Stoney length

The Stoney length l_s in the RMS equation (54) can be further transformed by multiplying the numerator and denominator of the *ls* expression by the symbol of elementary charge $\bar{e}$:

$$l_s = \frac{1}{4\pi}\frac{\bar{e}}{c^2} = \frac{1}{4\pi}\frac{\bar{e}^2}{\bar{e}c^2}. \tag{58}$$

The above equation has the same structure as equation (15) for the classical electron radius; however, the familiar factor, $\bar{e}c^2$, the Stoney energy, replaces the electron rest mass energy, $\bar{m}_e c^2$. Because equation (16) represents the electron's gravitational radius $r_g \equiv r_{me}$ equal to ½ of the SCR (as discussed on page 7), if one insists on interpreting $\bar{e}$ as the Stoney mass $\bar{m}_S$ having the elementary charge magnitude, then the *Stoney length could be called the 'gravitational radius' of the Stoney mass, being ½ of its SCR*. This is well known, but we did not derive it from the explicit application of the SCR expression. Rather, it unexpectedly emerged from the definition of the Stoney length *ls* in the classical electron radius-like expression. A particle of Stoney rest mass magnitude has not been identified, let alone has the elementary charge, yet according to Gorelik [46] referring to Weyl [47]: 'Having found a solution for an electrically charged point mass (also known as the Reissner—Nordström solution), Weyl [47] introduced the length $r_{gm} = Gm/c^2$(gravitational radius of mass *m*) and $r_{ge} = e\sqrt{G}/c^2$ (gravitational radius of the electric charge *e*).'[7] The r_{ge} is the Stoney length in CGS units, which is, 1/2 of the Stoney mass' SCR

[7] Gorelik's expressions are in CGS units.

under a different name. Thus, the general relativity approach of Weyl, Reissner, and Nordström, as stated in [46], seems relevant in studies of the electron model, which could be pursued in the RMS, UPNUS, or equivalent systems in the [LT] domain.

4.4. Stoney unit system in [LT] dimensions

The Stoney unit system is similar to the original Planck unit system, and the conversion process is the same as that for UPNUS, with the only difference being the magnitudes and expressions defining the units. Some highlights of these differences are presented below.

Stoney units differ from Planck units in that the ratio of their respective length, time, and mass units is $\sqrt{\alpha}$; for example, $l_S/l_P=\sqrt{\alpha}$. Therefore, $l_S \ll l_P$.The change in the units' expressions propagates to UPNUS-like expressions and constants according to the rules of algebra.

$$\check{h} = 8\pi^2 \Rightarrow \boldsymbol{\check{h}_S = 8\pi^2/\alpha}$$

$$\check{e} = \check{m}_P\sqrt{\alpha} \Rightarrow \boldsymbol{\check{e}_S \equiv \check{m}_{SS} = 4\pi}$$

$$\check{m}_e = 4\pi m_e/m_P \Rightarrow \boldsymbol{\check{m}_{eS} = 4\pi m_e/m_S}, \tag{59}$$

Where subscript S designates the Stoney scale quantities. The units are Stoney units of length and time, in place of the Planck units of UPNUS.

5. Electron radii predicted by special relativity?

In section 2.5.2, transforming the classical electron radius formula by substituting the electron charge with its mass in RMS units yielded ½ SCR of the electron, which is algebraically correct, yet unexpected. To clarify the physical meaning of such manipulated results, the origin of the SI expression (14) must first be clarified. The derivation of the classical electron radius is widely known and includes an assumption about the electron charge spatial distribution, which so far has not yet been validated. Due to the uncertainty, the derivation from the electron self-energy models resulted in an approximate, representative, 'truncated' (Jackson [48] p.589) form, by disregarding the coefficient A of the order of one, which would normally be in front of the classical expression (14) (Rohrlich [49] pp.124–125, Kittel *et al* [32] pp.278–279). The stripped classical r_e expression is claimed to be representative of the upper radius limit because the coefficient $0 < A < 1$ depends on the assumed charge distribution in the electron. One of the calculated coefficients, $A = {}^3/_5$, assumes a uniform distribution of charge in the volume of the spherical particle, but a uniform spherical surface distribution yields $A = ½$ (Rohrlich [49] pp.124–125). As a matter of convention, in the classical electron radius expression, $A = 1$. In the search for an explanation, given the absence of ideas about charge or mass distribution in an electron size particle, the motion of particles while they interact through their electric/gravitational fields was investigated for clues.

An abstract simplified model was considered. In this model, two point particles interact through their central fields. According to Newton's Shell Theorem, for a spherical surface distribution of mass, the field outside the sphere is equivalent to the field of the point mass at its center. This is also true for the electric fields. To prevent invoking complexity beyond the scope of this study, it is better to refer to the source and test particles with electron (or positron) mass and/or charge properties to avoid making statements about real particles. The abstraction is in the modeling of a thought experiment where

1. A source particle is fixed in a stationary inertial frame.
2. The initial test particle velocity $v = 0$ on an arbitrary $\boldsymbol{r}$ axis at r coordinate $r_0 > 0$ continues in accelerated motion toward the source owing to the attractive force of the source particle at $r = 0$.
3. Either particle is a point particle of mass $\bar{m}_e$ and may or may not have opposite charges of magnitude $\bar{e}$.
4. In the case of charged particles, the model does not include advanced features, such as magnetic field generation or emission of energy in the form of radiation.
5. The model includes an elementary special relativity extension consistent with Einstein's kinetic energy expression [50] p.170.
6. The charged particles interact according to the RMS Coulomb force expression, irrespective of the speed of the test charge.
7. The uncharged particles interact according to the RMS Newton force expression, irrespective of the speed of the test mass particle.

Approximate models of this type are available in literature. For example, Terzis [51] described a 'simple concise relativistic modification of the standard Bohr model for hydrogen-like atoms.' The key point of the approximation is that, while the accelerating particle gains relativistic inertia with increasing velocity taken into account, the Coulomb force expression in equation (8) remains unaffected. This is based on the assertion of French *et al* [52] p.232 that 'Coulomb's law correctly gives the force on the test charge, for any velocity of the test charge (however high), provided the source charge is at rest.' This principle has also been applied to a simplified model of gravitationally attracting uncharged particles.

Assuming that the Coulomb force and Newton force equations do not vary with fast particle motion,

$$\bar{F}(r) = -\frac{1}{4\pi}\frac{\bar{e}^2}{r^2}$$
$$\bar{F}(r) = -\frac{1}{4\pi}\frac{\bar{m}_e{}^2}{r^2}, \tag{60}$$

where $\bar{F}(r)$ is a suitable differential expression for the inertial force, r is the distance between the centers of mass of the particles, $\bar{e}$ is the elementary charge, and $\bar{m}_e$ i is the magnitude of the electron mass in RMS units. This problem can be analyzed by solving the equation of motion of the test particle in the scenario illustrated in figure 3 below.

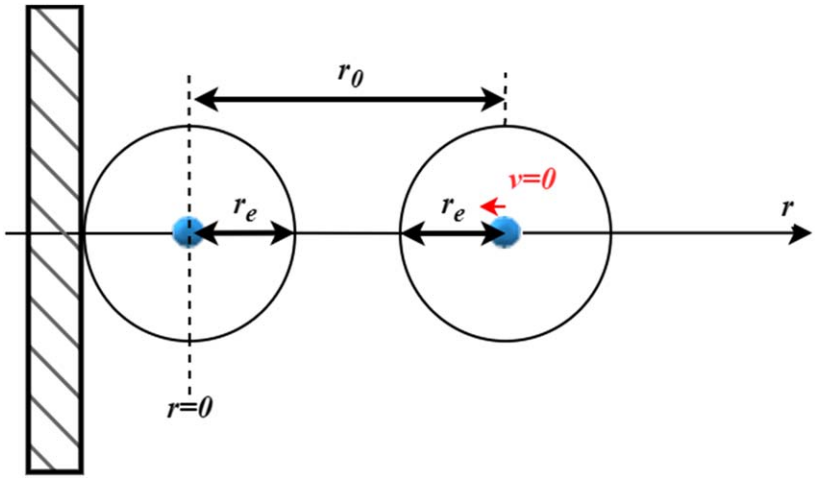


Figure 3. Test mass attracted by stationary source mass (the same applies to oppositely charged particles).

The following time-domain equations generalized for oppositely charged and uncharged particles describe the dynamics of the test particle in the special relativity context:

$$\gamma(v) = \frac{1}{1 - v^2/c^2}$$
$$\bar{F}(t) = \frac{\bar{m}_e d(\gamma(v(t))v(t))}{dt} \Rightarrow$$
$$\bar{F}(t) = \frac{\bar{m}_e c^3 \frac{dv(t)}{dt}}{(c^2 - v(t)^2)^{\frac{3}{2}}}. \tag{61}$$

We can change the variables in the above equation to the r domain using the well-known relation

$$\frac{dv(t)}{dt} = \frac{d^2r(t)}{dt^2} = v(r)\frac{dv(r)}{dr}, \tag{62}$$

and the differential formula (61) (after necessary algebraic simplifications) converts to

$$\bar{F}(r) = \frac{\bar{m}_e c^3 v(r)\frac{dv(r)}{dr}}{(c^2 - v(r)^2)^{3/2}}. \tag{63}$$

The equation of motion is now given as a static velocity distribution over the ***r*** axis of the test particle as follows:

$$\frac{\bar{m}_e c^3 v(r)\frac{dv(r)}{dr}}{(c^2 - v(r)^2)^{3/2}} = -\frac{1}{4\pi}\frac{\bar{\mu}_e^2}{r^2}, \tag{64}$$

where $\bar{\mu}_e$ is a placeholder for either $\bar{m}_e$ or $\bar{e}$, depending on the analyzed case, and has the following analytical solution in the implicit form:

$$\frac{4\pi\bar{m}_e c^3 r + \bar{\mu}_e^2(C_i r - 1)\sqrt{c^2 - v(r)^2}}{\bar{\mu}_e^2 r\sqrt{c^2 - v(r)^2}} = 0, \tag{65}$$

where C_i is an integration constant. Solving this equation with respect to $v(r)$ yields the following test particle velocity expressions:

$$\begin{aligned} v_1(r) &= \frac{\sqrt{-16\pi^2\bar{m}_e^2 c^4 r^2 + C_i^2\bar{\mu}_e^4 r^2 - 2C_i\bar{\mu}_e^4 r + \bar{\mu}_e^4}}{(C_i r - 1)\bar{\mu}_e^2}c \\ v_2(r) &= -\frac{\sqrt{-16\pi^2\bar{m}_e^2 c^4 r^2 + C_i^2\bar{\mu}_e^4 r^2 - 2C_i\bar{\mu}_e^4 r + \bar{\mu}_e^4}}{(C_i r - 1)\bar{\mu}_e^2}c \end{aligned} \tag{66}$$

The integration constant $C_i$can be calculated by setting the boundary condition to $r = r_0$ for $r_0 > 0$, where the velocity is set to zero. Substituting r_0 for r and equating $v_1(r_0) = 0$, or $v_2(r_0) = 0$ yields two identical sets of expressions for both v_1 and v_2:

$$\begin{aligned} C_{i1} &= \frac{4\pi\bar{m}_e c^2 r_0 + \bar{\mu}_e^2}{\bar{\mu}_e^2 r_0} \\ C_{i2} &= -\frac{4\pi\bar{m}_e c^2 r_0 - \bar{\mu}_e^2}{\bar{\mu}_e^2 r_0} \end{aligned} \tag{67}$$

Although C_{i1} is always positive for all $r_0 \geqslant 0$, the constant C_{i2} changes its sign and becomes zero at $r_0 = \bar{\mu}_e^2/(4\pi\bar{m}_e c^2)$.

Under the electric field attraction when $\bar{\mu}_e$ is set to $\bar{e}$, r_0 becomes the classical electron radius from equation (15).

Under gravitational attraction, when $\bar{\mu}_e$ is set to $\bar{m}_e$, the model works for abstract gravitating point particles with the electron mass, and $r_0 = \bar{m}_e/(4\pi c^2) = ½$ SCR of the electron mass, as shown in equation (16).

We have now exposed the classical electron radius expression without any explicit assumption regarding the structural charge volume or surface density, and have found ½ of the SCR for a mass under gravitational attraction without any deliberate manipulation, as in equation (16). The more significant consequences of this occurrence in the integration constant can be found in the complete valid solution presented below.

Considering the case of neutral particle attraction, which has the electron mass, by substituting C_{i1} and C_{i2} sequentially in the expressions for $v_1(r)$ and $v_2(r)$ from equation (66), we obtain two alternative forms of the velocity equation for each constant. Only two equations resulting from $v_1(r)$ meet the criteria for attraction from the positive r_0, such that $\lim_{r\to 0} v_{11}(r) = -c$ and $\lim_{r\to 0} v_{12}(r) = -c$. Only one of these two is a unique solution for the velocity, with no singularities in $r \in [0,\ r_0]$ for the widest possible range of positive r_0. The integration constant that does not violate the above constraints of interest when applied to $v_1(r)$, is C_{i2} from equation (67). Hence, focusing on gravitational attraction, with $\bar{\mu}_e$ set to $\bar{m}_e$, we obtain

$$v_{12}(r) = -\frac{\sqrt{\bar{m}_e}\sqrt{(r_0 - r)}\sqrt{8\pi c^2 r_0 r + \bar{m}_e(r_0 - r)}}{4\pi c^2 r_0 r + \bar{m}_e(r_0 - r)}c,$$

where:

$$\begin{cases} r \in [0,\ r_0] \cap r \leqslant r_0 \cap r_0 > 0 \\ \lim\limits_{r\to 0} v_{12}(r) = -c. \end{cases} \tag{68}$$

This is subject to the exclusion of singularities and discontinuities that are yet to be determined.

Two possible anomalous regions exist in the solution. One point singularity is due to the zero value of the denominator in equation (68), and the other is an interval or intervals of discontinuity when the expression under the rightmost square root is negative:

$$\begin{aligned} S_1\text{: } r_{sng} &= \frac{-\bar{m}_e r_0}{4\pi c^2 r_0 - \bar{m}_e} \cap r_0 > \frac{\bar{m}_e}{4\pi c^2} = 1/2SCR \\ D_1\text{: } r_{disc} &= \frac{-\bar{m}_e r_0}{8\pi c^2 r_0 - \bar{m}_e} \cap r_0 < \frac{\bar{m}_e}{8\pi c^2} = 1/4SCR \end{aligned} \tag{69}$$

The singularity S_1 for the velocity function $v_{12}(r)$ allows a vast valid nonsingular range of the linear trajectory for the initial point r_0: $r \in [0,\ r_0] \cap r_0 \in (1/2SCR,\ +\infty)$.

The region limited by the discontinuity condition D_1 is where the initial point r_0 is less than ¼ SCR away from $r = 0$ on the positive side of the axis. However, exploring this case is beyond the scope of the present study.

The ½ of the SCR plays a prominent limiting role in determining the point of singularity for the trajectory on the negative side of the $\boldsymbol{r}$ axis in the gravitational attraction model, which cannot be physically reached because it takes infinite time to arrive at $r = 0$, let alone to continue to $r < 0$. Additionally, the closest position of this singularity S_1 to the origin of the $\boldsymbol{r}$ axis at $r = 0$ is

$$\lim_{r0\to+\infty} r_{sng}(r_0) = -\frac{\bar{m}_e}{4\pi c^2} = -1/2SCR \tag{70}$$

The presented model can be easily applied to charged attracting particles; however, analysis of this interesting case is beyond the scope of this study. Although in the presented model the ½ SCR naturally emerges from relativistic effects, it fails to become the spatial limitation of the trajectory initiated at $r_0 \in (1/2SCR, +\infty)$, which would be characteristic of a spherical rigid boundary, such that particles would collide at some distance between the mass centers. This is most likely due to the simplistic nature of the proposed model; the general relativity approach would probably be better to make conclusive judgments.

The expression $r_{me} = \bar{m}_e/4\pi c^2$ from equation (16) developed by analogy is a legitimate gravitational radius for the electron mass, similar in principle but not in value to the classical electron radius. The 'mechanical' replacement in equation (16) is now justified. The physical significance of these findings for real particles is yet to be determined using more realistic models.

6. Metric systems without G

RMS and UPNUS are promising alternatives to SI for fundamental research. Using the derived units of charge and mass with dimensions [m^3s^{-2}], these systems reveal simplified dependencies among fundamental constants and new forms of relationships. To fully embrace this new approach, we must break with the tradition that a mass unit must be relative to a man-made artifact—an understanding that was problematic until the Planck constant was made exact in SI [1]. In fact, if the gravitational constant $\boldsymbol{G}$ had never been invented, we could have adopted an RMS-like system even in the 19th century, as discussed by Maxwell [7], via an experimental setup similar to that used by Cavendish [41] for Earth density measurements, which could have been used to determine the mass of a practical size standard in m^3s^{-2}. A simplified version of this experiment designed for educational purposes was described by Kurtis [53], demonstrating the underlying simple torsion balance principle illustrated in figure 4 below.

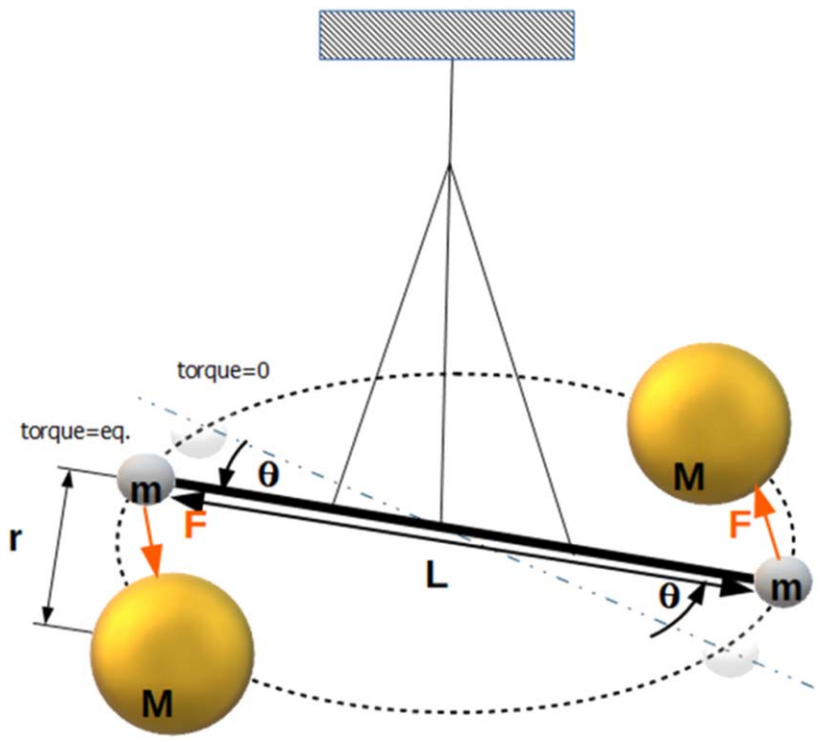


Figure 4. Illustration of torsion balance apparatus principle.

In this experiment, two large balls, M, simultaneously attract two small balls, m, which are connected by a bar of length L and suspended symmetrically on quartz fibers (or a wire). This opposes the torque generated by gravitational forces F over arm $L/2$, causing rotation of the bar with small balls m from the zero-torque position toward the large balls. At equilibrium, angle θ is reached after a series of damped oscillations of period T, and the small balls stop at a distance r from the large balls.

The formula for determining $\boldsymbol{G}$ from the setup shown in figure 4 given in [53] is

$$G = \frac{2L\pi^2 r^2\theta}{MT^2}. \tag{71}$$

In the RMS, equation (71) naturally yields the rationalized Newtonian mass as follows:

$$\frac{1}{4\pi} = \frac{2L\pi^2 r^2\theta}{\bar{M}T^2} \Rightarrow \bar{M} = \frac{8L\pi^3 r^2\theta}{T^2} \tag{72}$$

To directly measure Newtonian mass, Haug [8] p.184 used an equivalent formula (7) that does not account for the $1/4\pi$ factor not present in the original Newtonian law.

The masses used in the experiment must be equal pairwise to the highest possible accuracy. It is important to note that the unit of mass in the RMS is 1.192297×10^{9} kg, which is approximately equal to 1.9 million metric tons. For practical purposes, it may be necessary to preselect the desired practical standard weight for the larger balls. This is only an illustration of the principle of the measurement, which could be applied to more complex settings and devices.

7. Discussion

Several points highlight important features of the RMS and UPNUS worth some consideration.

1. The SI has been refined over several decades and is well-suited for practical purposes. However, the use of different systems is advantageous in fundamental research at the elementary particle level. This study introduces two systems of units that can serve as platforms for fundamental research: the RMS and the UPNUS. These systems no longer require base units of mass and charge; instead, they use derived mass and charge units with dimensions [L^3T^{-2}]. Through these systems, simplified dependencies among fundamental constants emerge, along with new forms of relationships.
2. The two developed systems of units, RMS and UPNUS, are rationalized in the sense defined by Heaviside [12], leading to a simplified electric-to-gravitational force ratio as a squared charge-to-mass nondimensional ratio. For an electron particle, this is

$$\frac{\bar{e}^2}{\bar{m}_e^2} \approx 4.165561 \times 10^{42}$$

3. With lpu, tpu, and Tpu symbols designating Planck scale units of length, time, and absolute temperature respectively, the values of the fundamental constants in UPNUS are
   a. Speed of light $\check{\boldsymbol{c}} = 1$ lpu tpu^{-1}
   b. Planck constant $\check{\boldsymbol{h}} = \boldsymbol{8\pi^2}\text{lpu}^5\text{tpu}^{-3}$
   c. Planck mass $\check{\boldsymbol{m}}_{\boldsymbol{P}} = \boldsymbol{4\pi}\text{lpu}^3\text{tpu}^{-2}$
   d. Elementary charge $\check{\boldsymbol{e}} = \check{\boldsymbol{m}}_{\boldsymbol{P}}\sqrt{\boldsymbol{\alpha}}\,\text{lpu}^3\text{tpu}^{-2}$
   e. Electron mass $\check{\boldsymbol{m}}_{\boldsymbol{e}} = \boldsymbol{4\pi m_e/m_P}\text{lpu}^3\text{tpu}^{-2}$
   f. Planck temperature $\check{\boldsymbol{T}}_{\boldsymbol{P}} = \boldsymbol{1}$ Tpu
   g. Boltzmann constant $\check{\boldsymbol{k}}_{\boldsymbol{B}} = \boldsymbol{4\pi}\text{lpu}^5\text{tpu}^{-4}\text{Tpu}^{-1}$.
4. Some interesting relations emerged in the RMS:
   a. A simpler expression for Planck mass

$$\bar{\boldsymbol{m}}_{\boldsymbol{P}} = \sqrt{2\boldsymbol{c}\bar{\boldsymbol{h}}}$$

b. The simplest disentangled RMS instance of the fine structure constant expression—which we might term the 'charge-to-Planck mass coupling constant'—was derived as

$$\alpha = \frac{\bar{e}^2}{\bar{m}_P^2},$$

as opposed to the SI-equivalent format

$$\alpha = \frac{e^2}{4\pi G \varepsilon_0 m_P^2}.$$

This can also be interpreted as the minimum limit of the charge-to-mass squared ratio within the set of elementary particles. From this, the simplest disentangled expression for the elementary charge in UPNUS emerges as

$$\check{e} = \check{m}_P \sqrt{\alpha} \approx 1.073476469 \text{lpu}^3\text{tpu}^{-2},$$

where the UPNUS Planck mass is $\check{m}_P = 4\pi \text{lpu}^3\text{tpu}^{-2}$.

The compact form of the above expressions appears to be the materialization of Newton's idea put forth in 'Rules Reasoning in Philosophy' stated in *The Principia* [5] p.384 Book III: 'Nature is pleased with simplicity, and affects not the pomp of superfluous causes.'

c. The above RMS expression can be used to provide an alternative definition of the Planck mass, which can be retrofitted to the SI. However, it is more complex than the RMS expression

$$\bar{m}_P = \frac{\bar{e}}{\sqrt{\alpha}} \Rightarrow m_P = \frac{1}{\sqrt{\alpha}} \sqrt{\frac{e^2}{4\pi G \varepsilon_0}}$$

Moreover, the SI form also shows an immediate relation to the Stoney mass, as described in section 4.1, and then the expression

$$m_P = \frac{m_s}{\sqrt{\alpha}},$$

becomes equally simple because the Stoney Mass expression naturally appears.

d. By utilizing the presented equations involving the Planck constant, it becomes possible to challenge certain criticisms of Planck units as arbitrary constructs that lack real significance. Bridgman [35] p.101 argued that 'No significance whatever should be attached to the particular size of the units defined.' However, by defining the Planck mass in terms of the established alternative universal constants of nature not present in the original definition, we can see its irreducible connection with the electron, and hence refute Bridgman's claims. The same cannot be said for the Stoney mass, as it converts to the elementary charge (see the paragraph below).

e. As described in section 4.2, the simplification power of the RMS allowed us to uncover the mistaken identity of the Stoney mass, which is proven to be nothing but a disguised elementary charge in SI units, while in RMS this is an identity: $\bar{m}_S \equiv \bar{e}$. In contrast, the SI Planck mass expression transforms into an irreducible and less complex fundamental combination $\bar{m}_P = \sqrt{2c\bar{h}}$.

However, the Stoney mass concept is of utmost importance because the identity $\bar{m}_S \equiv \bar{e}$ prompted the conjecture presented in section 4.2 on page 15 such that $\bar{m}_S c^2 \equiv \bar{e}c^2$ represents the finite energy of the electron field in the RMS units—which is exactly what is commonly called the Stoney energy. This should be the subject of future research.

8. Conclusions

1. Two time and length units [LT] are sufficient to describe any physical process currently described by [MLTI] units in the SI. The author believes that the two developed systems, RMS and UPNUS, provide a clear and logical perspective on physical quantities and constants and a useful platform for studying their dependencies. This is proven by the full explanation of the Stoney mass as nothing but the electron charge, misrepresented

in SI owing to the incommensurability of the mass and charge units. However, the role of the Stoney mass concept cannot be underestimated.

2. The Stoney mass related conjecture is proposed as a fitting topic for broader discussion. The Stoney energy appears to be equal to the energy of the electric field of the electron; however, much remains to be explored using RMS and UPNUS for other potential applications. One such example is the unexplained interplay between the magnetic induction and the gravitational force of the electron, as described in section 2.8. Further research and discussion in this area may provide valuable insights and pave the way for exciting discoveries.
3. Different perspectives on the same physical quantities could provide valuable insights and aid in comprehending the fundamental relationships in nature.
4. At this point in time, the highest-precision computations are not possible in RMS owing to the low precision of the gravitational constant; however, this may change in the future. The precision of the RMS, and some UPNUS, constants is only an issue of practicality.
5. The 'feebleness' of gravity discussed by Frank Wilczek in his Nobel Lecture [25] should not be interpreted as a reason to disregard its fundamental existence at small scales. Although no longer explicitly present in the physical equations, the gravitational constant G continues to play a critical role in the proposed Rationalized Metric System and remains an essential element that is indirectly present in most physical quantities, of which the kilogram was one of the base units in the SI. The reintroduction of early Newtonian gravitational mass as a derived unit renewed its significance in modern physics. Furthermore, gravitational properties are inherently linked not only to the hypothetical Stoney energy but also to the real energy of the physically tangible electric field surrounding the electron. The strength of the attractive properties of space-time and mass, as represented by $\boldsymbol{G}$, is critical in binding galaxies, elementary charges, and other particles as one united whole. $\boldsymbol{G}$ still remains a crucial component in our understanding of the natural world at both the largest and smallest scales of the universe.

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Data availability statement

There is no special data other than information included in the publication and refrenced literature. The data that support the findings of this study are available upon reasonable request from the authors.

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