Tests of the Standard Model and Lattice Simulations

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The NIC group Elementary Particle Physics carries out research in QCD, flavour physics, Higgs-Yukawa models as well as the development of new techniques and algorithms for lattice field theories. Here we describe aspects of the study of Higgs-Yukawa models and QCD and how they are connected.

1 Introduction

The standard model (SM) of particle physics comprises the electroweak and the strong interactions. The electroweak theory, which unifies the electromagnetic and the weak interactions within a chiral gauge theory, exploits the concept of spontaneous symmetry breaking, known as the Higgs mechanism to provide masses for all quarks, leptons and the weak gauge bosons (W, Z), postulating the existence of the Higgs boson.

In July 2012, the ATLAS and CMS experiments at CERN announced the discovery of a Higgs-like particle, with the mass being about 125 GeV. The observation of a Higgs-like state has been corroborated by many more experimental data and a rather precise value for the mass of this scalar boson (spin zero) has been given. This discovery appears to confirm the prediction of the SM and completes it.

The basic idea of the Higgs mechanism is illustrated in Fig. 1. The potential of the Higgs field develops a minimum at a non-zero value of the radial degree of freedom of the Higgs field providing thus a vacuum expectation value v for the Higgs field. It is through this vacuum expectation value that the particles acquire their masses, because a non-vanishing v induces mass-like terms in the Lagrangian, e.g. a term proportional to $v\bar{\Psi}\Psi$ for the quarks.

However, old questions about the special rôle of spin-zero fields remain. Scalar fields have much more severe divergences in a quantum field theory than the fermion fields and the gauge fields. Also new questions emerge from the observed Higgs mass, for instance concerning the stability of the vacuum. Another question is whether this field is in fact an elementary one or whether the Higgs boson rather is a bound state.

Attempting to answer these questions, a key issue is to have a precise theoretical control of the SM theory. This precision physics requires the accurate determination of its fundamental parameters as well as the theoretical study of the Higgs sector of the SM. For the determination of SM parameters the strong interactions, described by QCD, are the major challenge because they are intrinsically non-perturbative. But also in the Higgs sector, one needs to question what can be computed by the expansion in the couplings, i.e. by "perturbation theory", and how well this can be done.

These non-perturbative questions can be answered with techniques developed in lattice field theory where space and time are made discrete and a 4-dimensional finite grid

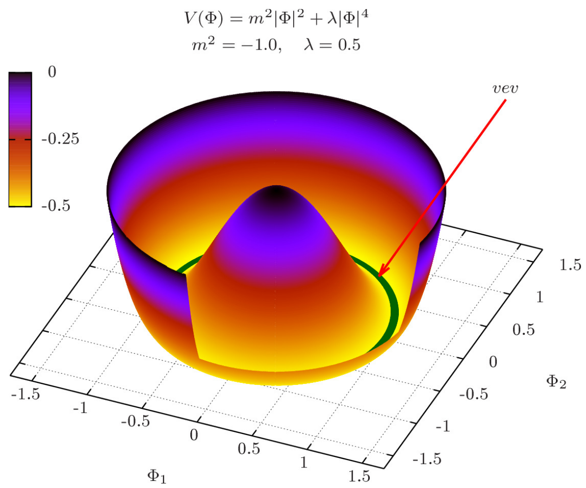


Figure 1. The classical potential of the Higgs field in the broken phase. Lattice units $a = 1$ are chosen.

with lattice spacing a is introduced. This setup allows for numerical simulations which in turn provide *ab initio* calculations that are not restricted to approximation methods such as perturbation theory. Of course, in the end the continuum limit has to be carried out numerically and one has to assure that symmetries broken by the discretisation are recovered in that limit.

2 Theoretical Studies of the Higgs Boson

The discovery of the Higgs boson and the subsequent measurements of the decay modes of the Higgs particle have led to the urgent need for understanding a list of theoretical issues which are relevant for the consistency of the SM and the possible extensions of it. Some of these theoretical problems are of non-perturbative nature, and these are the main targets of the lattice field theory approach to Higgs physics.

2.1 Non-Perturbative Studies of the Higgs-Yukawa Model

The upper and lower bounds of the Higgs boson mass are one main question of non-perturbative nature. For the lower bound, non-perturbative computations are highly desirable since the perturbative calculations rely on the instability of the effective potential, while it can be shown that the effective potential is, in fact, convex. Using a chirally-invariant lattice Higgs-Yukawa model the NIC group has performed non-perturbative computations to address these Higgs boson mass bounds. We refer to Refs. 1, 3 for reviews of lattice Higgs Yukawa models.

These calculations have been carried out for a standard model top quark mass of 175 GeV and also for heavier fermion masses up to 700 GeV in order to test the possibility of a fourth generation of quarks. As one important result of these previous computations[2,4] it was found that the suggested extension of the standard model by a heavy fourth fermion generation is not consistent with the lower Higgs boson mass bound values, given that the mass of such a fourth generation quark is at least 300 GeV as discussed in Ref. 7.

It is important to note that in the calculations a detailed study of finite size effects was performed. This becomes necessary since the model contains massless particles, the Goldstone bosons, and hence the finite size effects are algebraic and not exponentially suppressed. This led to simulations of lattice sizes of up to 40^4 which then finally allowed to carry out an infinite volume extrapolation of the Higgs boson and fermion masses, and of the vacuum expectation value v.

An another important ingredient has been that the numerical simulations have been complemented by analytical calculations, i.e. lattice perturbative calculations of the effective potential. This allowed to guide the simulations and helped e.g. in determining the cut-off dependence of the Higgs boson mass[5,6]. The next step of this research is to explore the influence of a higher dimensional ϕ^6 operator on the Higgs boson mass bounds. This is of very significant phenomenological impact, linking the discovery of the Higgs-like particle to physics beyond the SM. These above discussed Higgs boson mass bounds depend on the cut-off of the theory which represents the scale of some new, so far unknown, physics beyond the standard model. For a given value of the Higgs boson mass there will be a crossing point with one of these mass bounds.

In the SM and for a Higgs boson mass close to 125 GeV such a crossing will happen with the lower Higgs boson mass bound. As illustrated by Fig. 2 the crossing seems to happen at a very large value of the cut-off, a scenario which is much discussed in the literature. The interesting question is, whether the addition of a Φ^6 term in the action can substantially shift the lower Higgs boson mass bound. First results by the NIC group in this direction indicate that a Φ^6 term induces a rich phase structure of the model with first order phase transitions for certain combinations of the coupling parameters. The NIC group will carry out a detailed study of this situation with a combined effort of lattice perturbative and non-perturbative numerical computations. This effort is carried out in close collaboration with the group of Dr. D. Lin from Taiwan.

2.2 Vacuum Stability in Perturbation Theory

A quantitative shortcoming of the non-perturbative investigations in the Higgs-Yukawa model is that – at least for the time being – important interactions in the theory have to be dropped in the simulations. Due to various studies it is expected that in particular the strong interactions (QCD) can be important. Through the coupled renormalisation group equations, QCD influences the relationship between the renormalised Higgs mass, Higgs self-coupling and top-quark Yukawa coupling (at large renormalisation scales) to the observed Higgs, W-boson and top-quark mass. The recent experimental masses together with the inclusion of higher loop corrections into the perturbative renormalisation group equations have lead to a strong modification of the Higgs potential in Fig. 1. The so-computed effective potential is just stable, but close to being unstable[8]. One speaks of the stability/instability of the vacuum i.e. the ground state of the world. That the situation is really as sketched above cannot be considered as an established fact as yet since several assumptions enter into the logics. (i) the new boson observed at the LHC is interpreted to be the Standard model Higgs. (ii) the effective potential is assumed to be described accurately by perturbation theory in all couplings. (iii) the input parameters for the perturbation theory

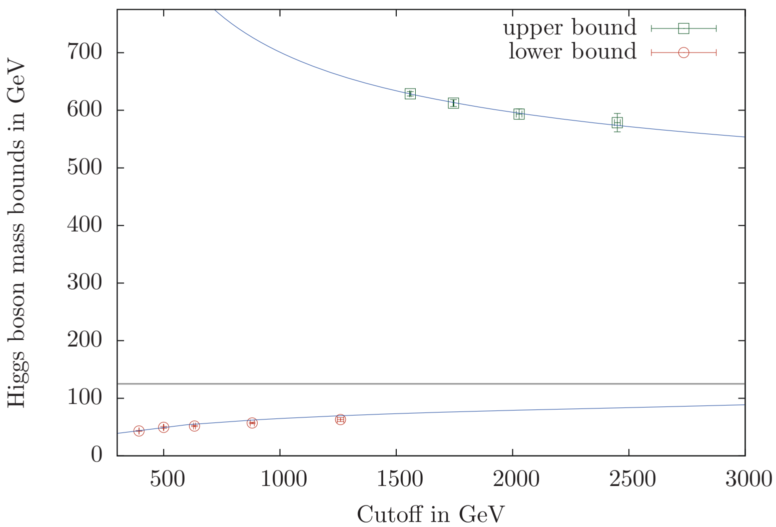


Figure 2. The upper and lower Higgs boson mass bounds for a top quark mass of 175 GeV as a function of the cut-off of the theory. The horizontal line represents a Higgs boson mass of 125 GeV. A crossing of this line with the bounds indicates the energy scale at which the standard model needs to be replaced by some new physics. The figure indicates that such a crossing point will first happen with the lower Higgs boson mass bound. The blue curve is a prediction of the constrained effective potential and is in good agreement with the lattice data.

are sufficiently well known. Assumption (ii) is put to a test in our study of the Higgs-Yukawa model. Concerning (iii) the most relevant uncertainties in the input parameters are the top-quark mass and the value of the strong coupling α_s at a high scale, say, of around 100 GeV.

3 The Strong Coupling

Apart from the above question of the stability of the electroweak vacuum, the value of the strong coupling is important in many high energy processes at the LHC. In particular, together with the so-called parton distribution functions, which describe how quarks and gluons are "distributed" inside the protons, α_s is the most important input parameter in the theoretical description of the production of the Higgs boson and other states from proton-proton collisions at the LHC.

The coupling is difficult to determine from experiments due to confinement. This property of QCD means that quarks and gluons exist only bound inside hadrons, such as the proton, as their constituents. One can therefore not directly determine the strength of a quark–gluon interaction in an experiment. Moreover, the question arises how one even defines the strength of the strong interactions.

3.1 Defining the Strength α of the Strong Interactions (QCD).

In quantum electro dynamics (QED), the coupling is usually defined by the low energy limit of the scattering of photons on (free) electrons, the Thomson cross section. For the aforementioned reasons, an analogous definition is not possible in QCD, but the potential energy between two static colour charges at large distances is very similar to Thomson scattering. This potential is nothing but the Coulomb potential governing all of atomic physics with quantum corrections included, such as multiple photon exchanges and virtual e^+e^- pair creation.

In QCD, the static potential is defined as the energy of a system with a static quark source at $\mathbf{x}$ and an anti-quark at $\mathbf{y}$. Static quarks have a divergent, unobservable, self energy in the quantum theory. One therefore discards in the potential the piece which is independent of the distance $r = |\mathbf{x} - \mathbf{y}|$ and considers the force,

$$F(r) \equiv \frac{\mathrm{d}}{\mathrm{d}r} V(r) = \frac{g_0^2}{4\pi} \frac{4}{3} \frac{1}{r^2} + \mathrm{O}(g_0^4)\,. \tag{1}$$

Here, we have also given the expansion of the force in terms of the bare coupling, g_0. This coupling appears in the Lagrangian of the theory – in exact analogy to the coupling between photons and electrons in QED. The bare coupling is again not observable; it suffers from the infamous divergences of the quantum field theory. In contrast,

$$\alpha_{\mathrm{qq}}(r) \equiv \frac{\bar{g}_{\mathrm{qq}}^2(r)}{4\pi} \equiv \frac{3}{4} r^2\, F(r)\,, \tag{2}$$

is finite since it is defined in terms of the force and it is related to the bare coupling by a perturbative series $\bar{g}_{\mathrm{qq}}^2(r) = g_0^2 + \mathrm{O}(g_0^4)$. In the weak coupling region, where $\bar{g}_{\mathrm{qq}}$ is small, it is connected to all kind of other definitions of the coupling in such a perturbative fashion; but it is also properly defined when it is large! By a proper definition we mean in particular that it is finite and invariant under gauge transformations which form the basic, defining, symmetry of QCD. The physical picture and Feynman graph behind Eq. 1 is that the leading term corresponds to the exchange of a single gluon between a (static) quark and an anti-quark. Apart from the factor $4/3$ originating from the SU(3) group of QCD, the leading term is equal to the one-photon exchange of the Coulomb potential.

3.2 Computing the Interaction Strength α

The r-dependence of $F(r)$ – and therefore of $\bar{g}_{\mathrm{qq}}(r)$ – can be computed self-consistently in perturbation theory when $\bar{g}_{\mathrm{qq}}(r)$ is small[9,10]:

$$r \frac{\mathrm{d}}{\mathrm{d}r} \bar{g}_{\mathrm{qq}}(r) \overset{r\to 0}{\sim} b_0 \bar{g}^3(r) \,+\, b_1 \bar{g}^5(r) + \ldots\ , \tag{3}$$

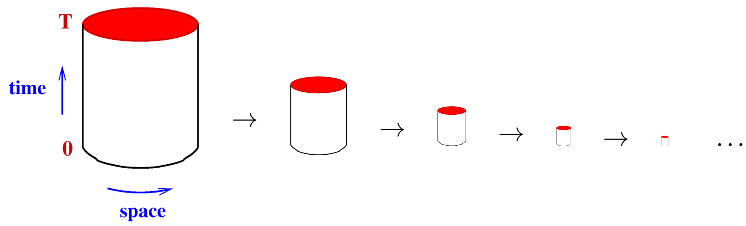
where $b_0 = (4\pi)^{-2}\,(11 - \frac{2}{3} N_{\mathrm{f}})$ and N_{f} denotes the number of quark flavours "active" at the scale r. Asymptotically all quarks count, i.e. $N_{\mathrm{f}} = 6$. Small coupling corresponds to small r, or equivalently to high momenta, via the relation

$$\bar{g}_{\mathrm{qq}}^2(r) \sim \frac{1}{-2 b_0 \log(\Lambda_{\mathrm{qq}} r)}\,. \tag{4}$$

The integration constant of the differential equation Eq. 3 is parameterised by Λ_{qq} and is *not* accessible to perturbation theory.

This property, called asymptotic freedom, allows to analytically compute the evolution of $\bar{g}_{\mathrm{qq}}(r)$ to arbitrarily small distances once a starting value is known which is small enough. We "only" have to enter the beginning of this asymptotic regime with a non-perturbative lattice simulation and connect the asymptotic region non-perturbatively to the low-energy properties of QCD, such as the proton mass. More precisely, one wants to express r in units of $m_{\rm proton}^{-1}$ for values of r where $\alpha_{\rm qq}$ is small, say around $\alpha_{\rm qq}(r) = 0.2$ or smaller. Here, $m_{\rm proton}$ has been chosen as a typical QCD scale. Others are possible and in fact a scale extracted from the weak decays of the lightest hadrons is technically advantageous[11]. In a straight-forward attempt to perform this computation, one would simulate an L^4-world with $L > 6$ fm (where finite size effects are known to be small) and a certain lattice spacing a. One then determines the proton mass in lattice units, $m_{\rm proton}a$, as well as $\alpha_{\rm qq} = f(r/a)$. Setting $r \equiv \rho/m_{\rm proton}$, one then knows $\alpha_{\rm qq}(r) = f\left(\rho/(am_{\rm proton})\right)$ up to effects due to the discretisation of the theory. Such a straightforward strategy is, however, not reliable since short distances of around $r \approx 0.1$fm, where $\alpha_{\rm qq}(r)$ is small, correspond to $r \approx L/60$. On the other hand, lattices with $L/a = 64$ are about at the limit of what can be simulated at present. In such a naive setup one would then determine $\alpha_{\rm qq}(r)$ at $r \approx a$, where the discretisation effects can be very large. Obviously a better strategy is needed, which allows for $r \gg a$ and a continuum limit.

In the strategy developed and continuously improved by the ALPHA-collaboration and in particular the NIC group, the scale problem is solved in the following way[12–14]. One introduces a definition[a] of the coupling $\bar{g}_{\rm SF}(L)$ which explicitly depends on the linear size L of the system, i.e. $\bar{g}_{\rm SF}(L)$ "runs" with L. For $L \approx 0.5$ fm to 1 fm, one connects L to $m_{\rm proton}^{-1}$. Then one performs so-called step scaling, namely one determines $\bar{g}^2_{\rm SF}(L)/\bar{g}^2_{\rm SF}(L/2)$ from two simulations and iterates according to the following scheme:



All of this is possible on lattices with moderate L/a and moderate cost[15,16], apart from the simulations to determine the physical scale in lattice units, $am_{\rm proton}$. For the latter, the "only" requirement is now $am_{\rm proton} \ll 1$ to get discretisation effects under control. In each step a continuum limit can be taken by a controlled extrapolation of the numerical results.

3.3 Improving the Precision

The strategy involves the simulation of many lattices and high precision on each one of them. In a simplified version of the theory with only two quarks, the full strategy was carried out, yielding a $\mathrm{O}(1\%)$ precision for the coupling at 100 GeV. For a realistic prediction relevant for LHC physics, the theory with at least three quarks needs to be considered and

[a]Due to space limitations, we can not describe here how this is done in detail. We just mention that the exact boundary conditions play an important rôle and that there are analogies to the electromagnetic Casimir effect.

one would also like to further improve the precision. The latter can now be achieved with a modified definition of the coupling. It is based on a flow equation[17,18], which evolves the fields in the path integral into fields which behave as classical fields in many ways. At leading order in the coupling, the flow equation is a heat equation and the quantum fields, $A_\mu(x)$, are related to the fields at flow time t by

$$B_{\mu,1}(x,t) = \int \mathrm{d}^4y \; K_t(x-y) \; A_\mu(y)\,, \qquad (5)$$

where $K_t(z) = (4\pi t)^{-2} \exp\left\{-z^2/(4t)\right\}$ is a heat kernel. Thus, the fields are smoothed over distances $\sqrt{8t}$. As a consequence the typical short distance singularities of the quantum field theory are removed[18] and completely new observables become available. Furthermore, also the fluctuations of these new observables in our simulations of the path integral, are tamed, leading to very high statistical accuracy.

The group has recently studied the application of these general ideas to a new definition of a QCD coupling in a finite volume[19]. Very good precision can indeed be reached[19,20]. The challenge of the overall calculation of $\alpha(100\ \mathrm{GeV})$ is then entirely reduced to the large volume computation (at small quark masses) of a scale such as am_{proton}. This part has just been started, within a collaboration of the NIC group with other partners in the so-called Coordinated Lattice Simulations initiative. The initial simulations are supported by two PRACE projects, one focusing on reaching the physical quark masses and the other one on the extrapolation to the continuum limit.

From the combination of these simulations with the step scaling for the new coupling we expect to obtain the QCD coupling with unprecedented precision, both systematic and statistical. It will help to further probe the Standard Model of Particle Physics.

4 Summary

In this contribution we have presented two important aspects of testing the SM by non-perturbative lattice simulations: through studies of the mass bounds and vacuum stability in the Higgs sector, and through the precise determination of the strong coupling constant in QCD.

An other research topic of the NIC research group “Elementary Particles” at DESY is heavy flavour physics, where the lattice computations provide the basis for many precision tests of the SM through the determination of the relevant hadronic matrix elements.

The progress of lattice computations for all these research directions strongly depends on developments and improvements on various frontiers. These include innovative theoretical approaches, efficient simulation algorithms, sophisticated data analysis methods, and of course the efficient use of the growing performance of the computer platforms.

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