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Synchrotron Radiation

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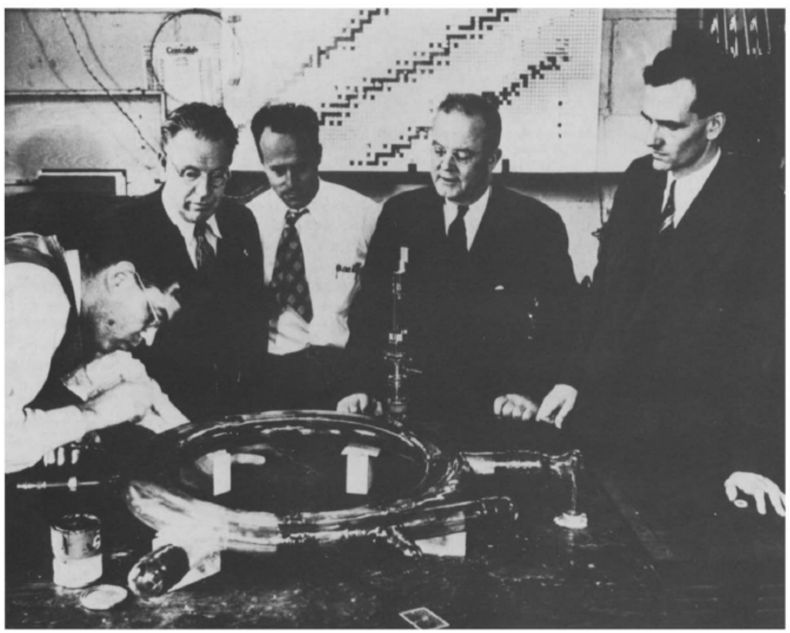
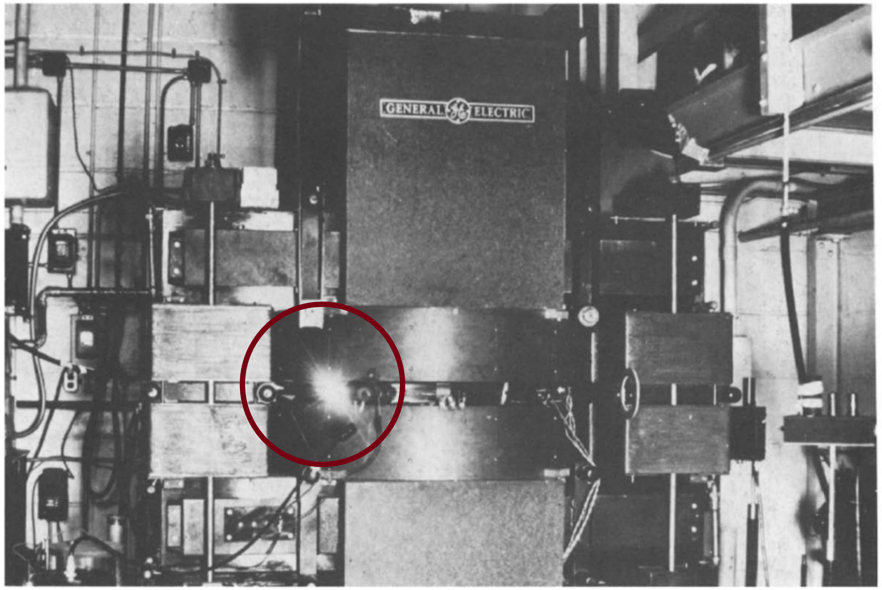
Key references

- D. A. Edwards and M. J. Syphers, An Introduction to the Physics of High Energy Accelerators, Wiley, Weinheim, Germany (1993).
  - DOI: 10.1002/9783527617272
- H. Wiedemann, Particle Accelerator Physics, 4^{th} ed., Springer, Heidelberg, Germany (2015).
  - DOI: 10.1007/978-3-319-18317-6
- E. J. N. Wilson, An Introduction to Particle Accelerators, Oxford University Press, Oxford, UK (2001).
  - DOI: 10.1093/acprof:oso/9780198508298.001.0001

Circular accelerators – observation of SR in 1947

"A small, very bright, bluish-white spot appeared at the side of the chamber where the beam was approaching the observer. At lower energies, the spot changed color."

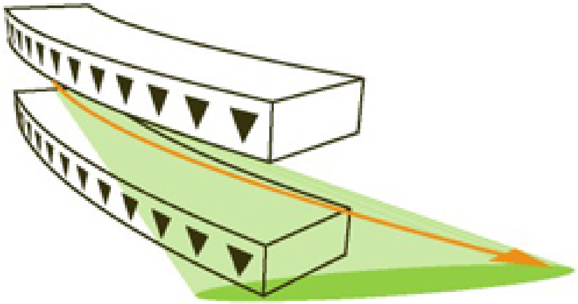




"The visible beam of "synchrotron radiation" was an immediate sensation. Charles E. Wilson, president of G.E. brought the whole Board of Directors to see it. During the next two years there were visits from six Nobel Prize winners."

J. P. Blewett, Nucl. Instrum. Methods Phys. Res., Sect. A, 266, 1 (1988).

Circular electron colliders – SR is a nuisance



Source: Australian Synchrotron

Source: CERN

- "… the radiative energy loss accompanying the circular motion."

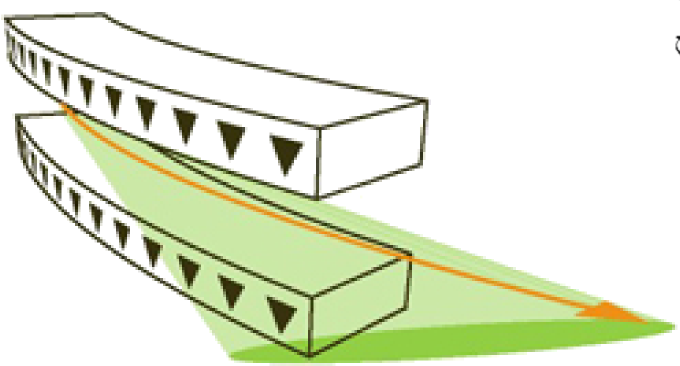
$$\delta E = \frac{4\pi}{3}\frac{e^2}{R}\gamma^4$$

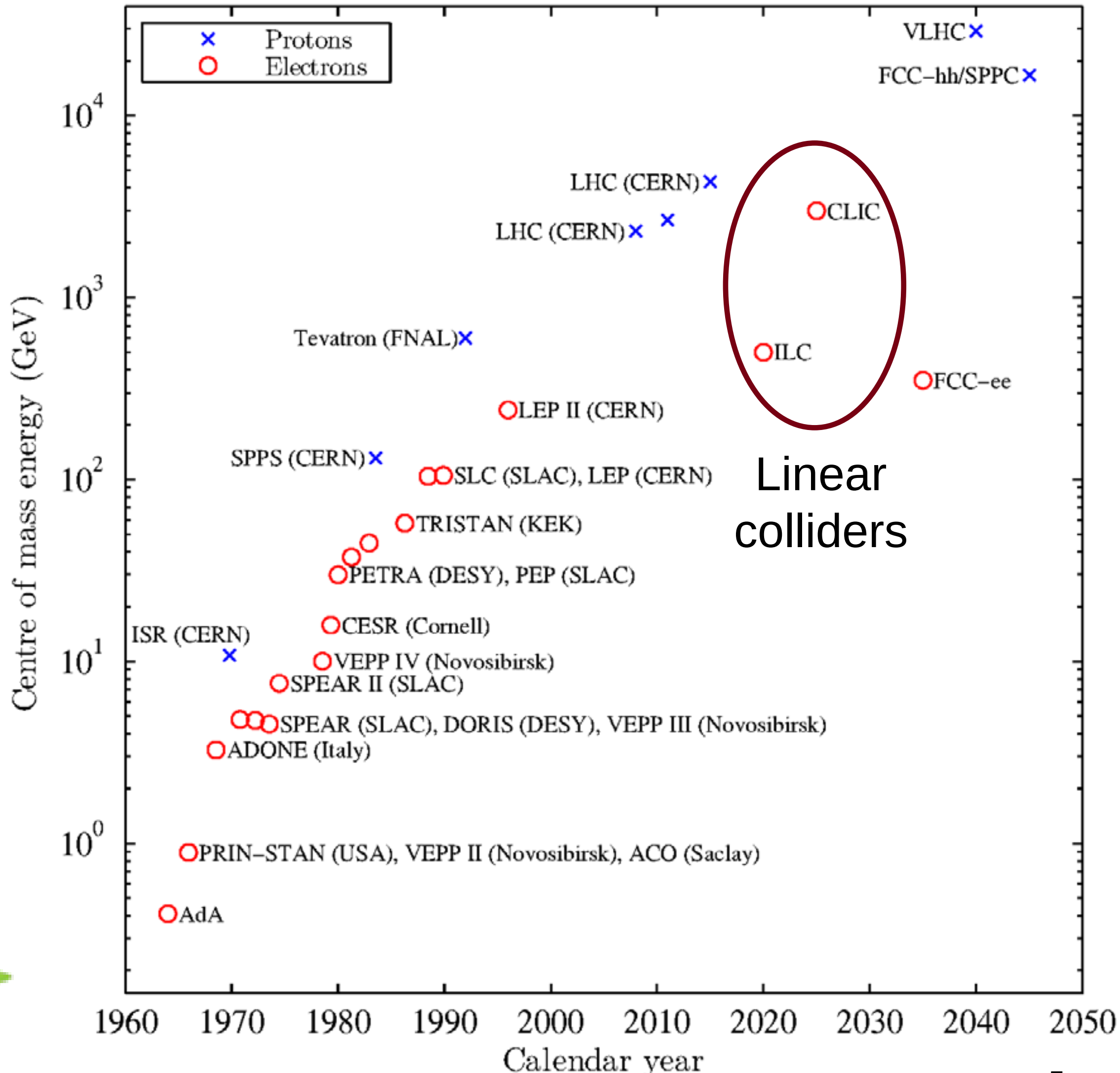
- Limits max energy
- Damps beam size

D. Iwanenko and I. Pomeranchuk, Phys. Rev., 65, 343 (1944).
J. Schwinger, Phys. Rev., 70 (9-10), 798 (1946).

‘Discovery machines’ and ‘Flavour factories’

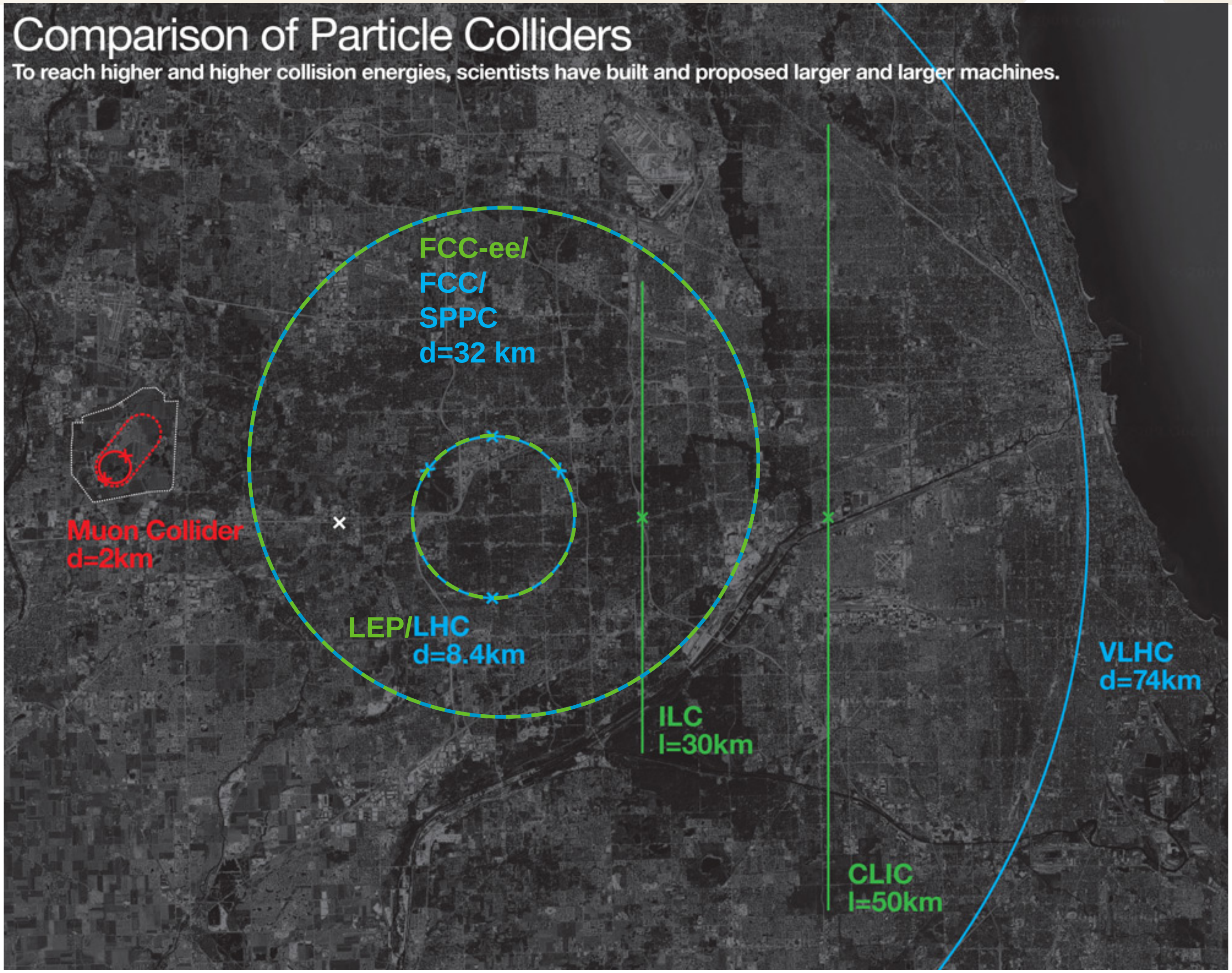
- Livingston plot
- Proton, electron colliders
- Protons, colliding partons (quarks)
- Linear colliders proposed





Source: Australian Synchrotron

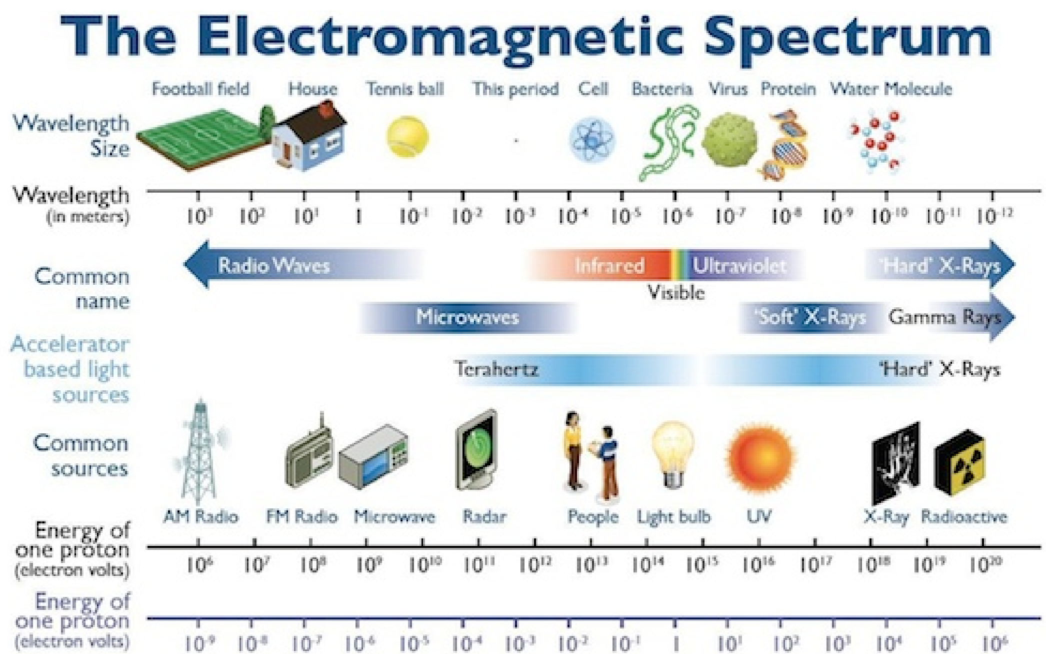
Future energy frontier colliders



Source: Fermilab

SR can be useful – accelerator light sources

- Unique source of electromagnetic radiation
- Wavelength-tunable
- High intensity
- Spatial coherence
- Polarised
- Pulsed



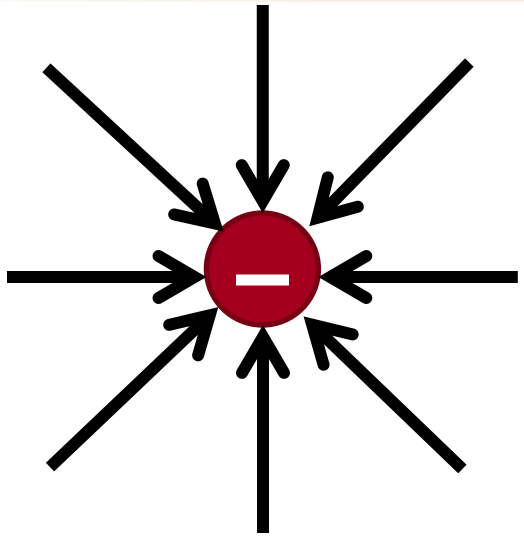
Source: lightsources.org

More in the next lecture – Overview of Light Sources

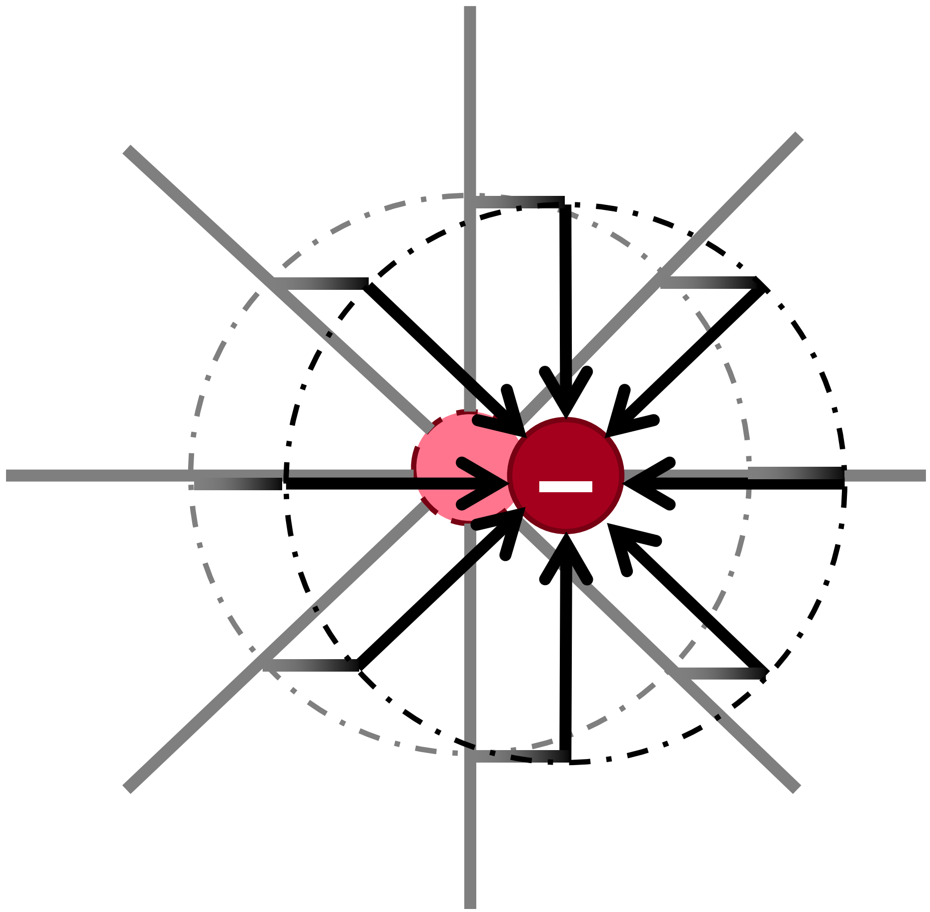
Outline – Synchrotron Radiation

- Physics of synchrotron radiation
- Characteristics of synchrotron radiation
  - Power, spectrum, flux
- Universal spectrum of bending magnet SR
  - Critical photon energy
- Accelerator physics implications
  - Damping rates
  - Equilibrium emittances

Electric field of an electric charge

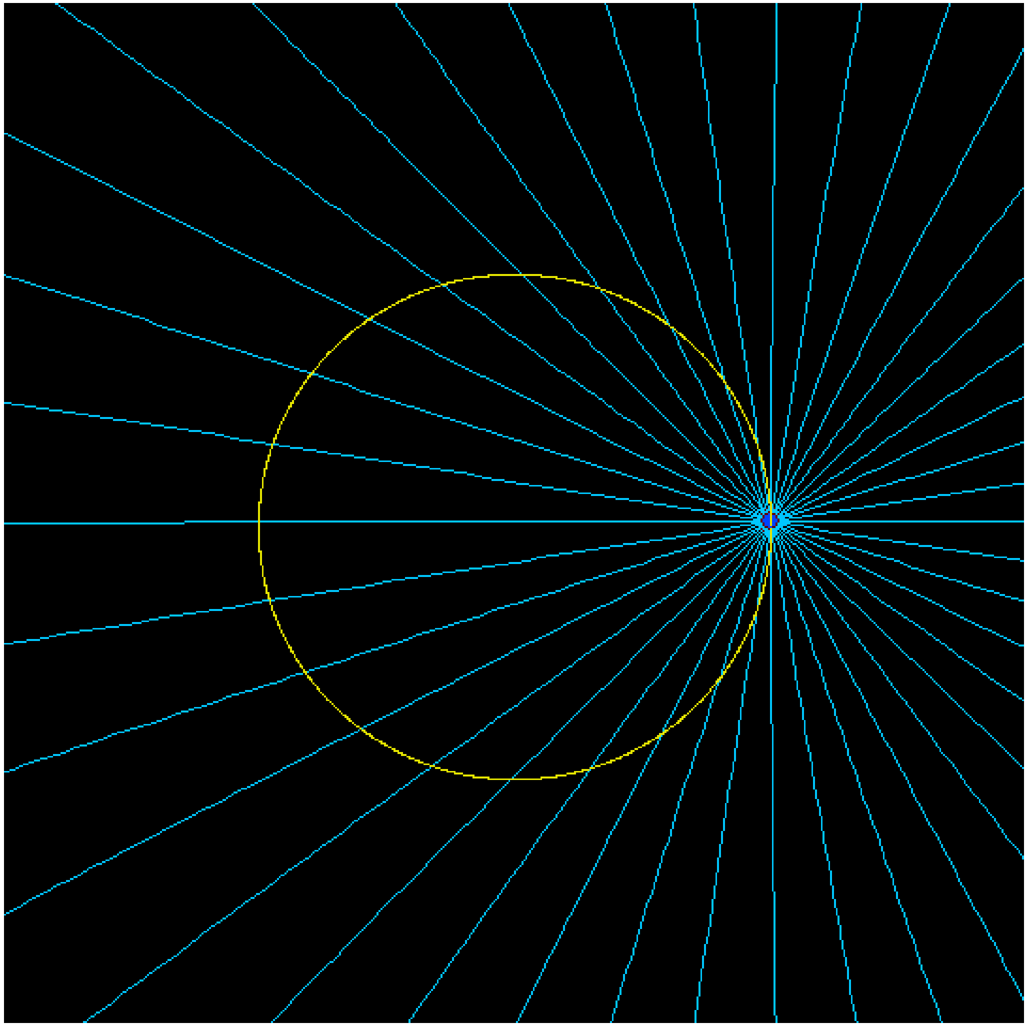


- Stationary charge
- Field lines extend radially outward



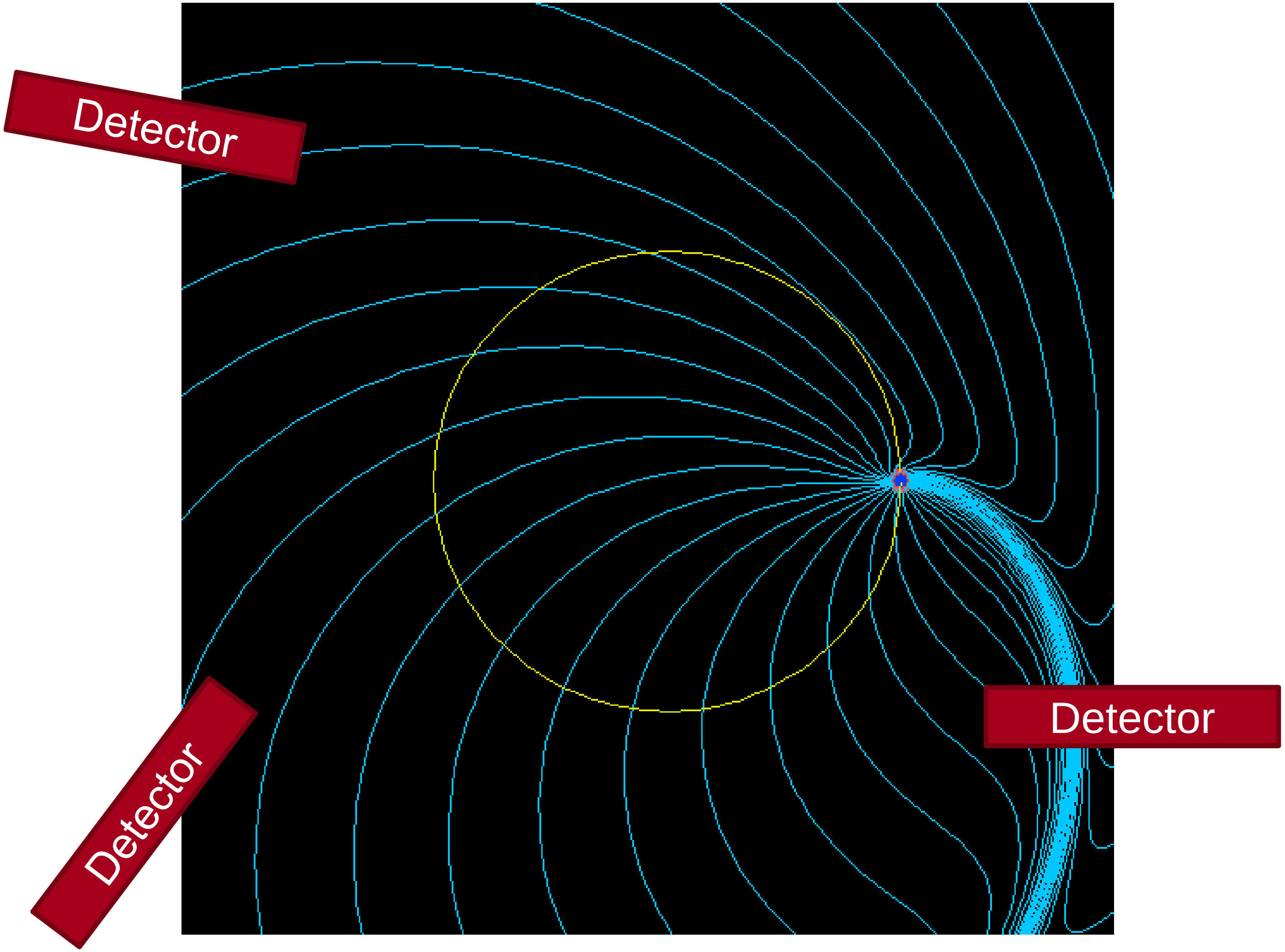
- Acceleration
- Finite speed of light, causality
- ‘Kinks’ in electric field distribution manifest as radiation

Electric field of an accelerated charge ($\beta = 0.1$)



http://www.shintakelab.com/en/enEducationalSoft.htm

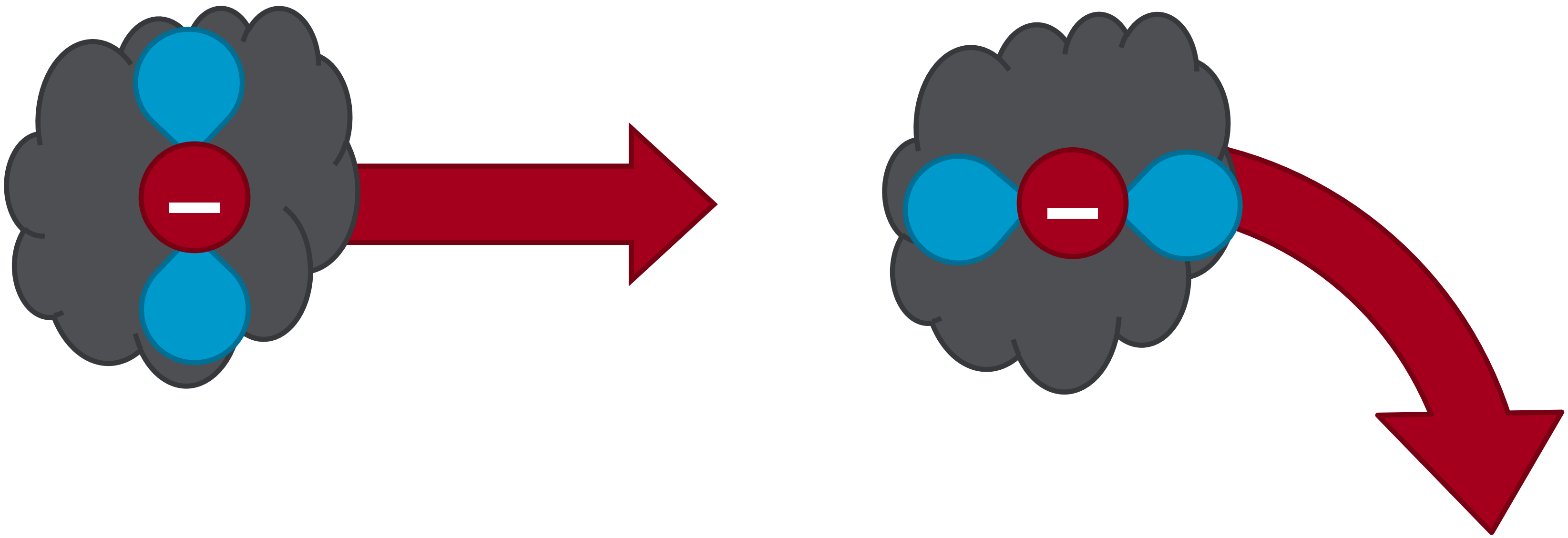
Electric field of an accelerated charge ($\beta = 0.9$)



http://www.shintakelab.com/en/enEducationalSoft.htm

Electric field, synchrotron radiation

- Acceleration of a relativistic electron



- Do virtual photons become real photons?
- How much more power from transverse acceleration?

Power – linear acceleration

$$P_R = \frac{1}{6\pi\epsilon_0}\frac{e^2 c}{(mc^2)^2}\boxed{a^2}$$ (charge rest frame)

$$P_L = \frac{1}{6\pi\epsilon_0}\frac{e^2 c}{(mc^2)^2}\boxed{\left[\left(\frac{\mathrm{d}p}{\mathrm{d}\tau}\right)^2 - \frac{1}{c^2}\left(\frac{\mathrm{d}E}{\mathrm{d}\tau}\right)^2\right]}$$ (lab frame)

Using $E^2 = (m_0 c^2)^2 + p^2 c^2$, $\mathrm{d}t \rightarrow \gamma \mathrm{d}\tau$ it can be shown that

$$P_{L\parallel} = \frac{1}{6\pi\epsilon_0}\frac{e^2 c}{(mc^2)^2}\left(\frac{\mathrm{d}p}{\gamma \mathrm{d}\tau}\right)^2 = \frac{1}{6\pi\epsilon_0}\frac{e^2 c}{(mc^2)^2}\left(\frac{\mathrm{d}p}{\mathrm{d}t}\right)^2$$ (lab)

Using $\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\mathrm{d}E}{\mathrm{d}s}$,

$$P_{L\parallel} = \frac{1}{6\pi\epsilon_0}\frac{e^2 c}{(mc^2)^2}\left(\frac{\mathrm{d}E}{\mathrm{d}s}\right)^2$$ (lab)

J. Schwinger, Phys. Rev., 75, 1912 (1949).

Power – transverse acceleration

$$P_L = \frac{1}{6\pi\epsilon_0}\frac{e^2c}{(mc^2)^2}\left[\left(\frac{\mathrm{d}p}{\mathrm{d}\tau}\right)^2 - \frac{1}{c^2}\left(\frac{\mathrm{d}\cancelto{0}{E}}{\mathrm{d}\tau}\right)^2\right] \text{ (lab)}$$

$$P_{L\perp} = \frac{1}{6\pi\epsilon_0}\frac{e^2c}{(mc^2)^2}\left(\frac{\mathrm{d}p}{\mathrm{d}\tau}\right)^2 \text{ (lab)}$$

Using $\mathrm{d}t \to \gamma\mathrm{d}\tau$,

$$P_{L\perp} = \frac{1}{6\pi\epsilon_0}\frac{e^2c}{(mc^2)^2}\gamma^2\left(\frac{\mathrm{d}p}{\mathrm{d}t}\right)^2 \text{ (lab)}$$

$$\boxed{P_{L\perp} = \gamma^2 P_{L\parallel}}$$

Using $\left(\frac{\mathrm{d}p}{\mathrm{d}t}\right)^2 = \beta^4\frac{E^2}{\rho^2}$,

$$P_{L\perp} = \frac{e^2c}{6\pi\epsilon_0}\beta^4\frac{\gamma^4}{\rho^2} \text{ (lab)}$$

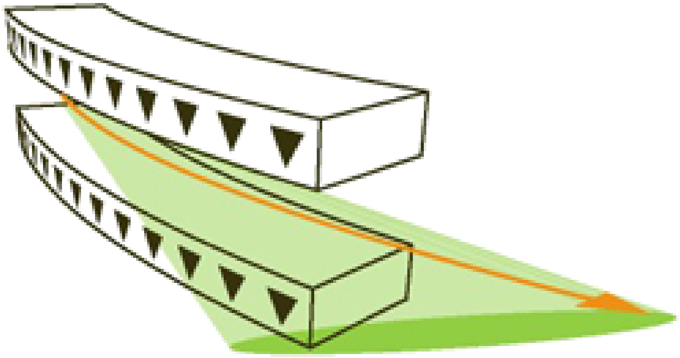
$$\boxed{P \propto \frac{\gamma^4}{\rho^2}}$$

J. Schwinger, Phys. Rev., 75, 1912 (1949).
J. Schwinger, 'On Radiation by Electrons in a Betatron', (2000), LBNL-39088.

Power – summary

$$P_{L\perp} = \frac{e^2 c}{6\pi\epsilon_0}\beta^4\frac{\gamma^4}{\rho^2}$$

- Transverse a factor of γ^2 larger than longitudinal
- $P \propto \gamma^4$
  - Factor of 10^{13} for e^-/p^+ (10^9 for e^-/μ^-)
  - Proton (or muon) circular colliders for high energy
- $P \propto \frac{1}{\rho^2}$, large bending radius beneficial



Source: Australian Synchrotron

Energy loss per turn

- Energy loss per turn

$$U_0 = \oint P_{L\perp}\ \mathrm{d}t$$

- Isomagnetic lattice (only one bending radius ρ), $\mathrm{d}t = \frac{2\pi\rho}{\beta c}$

$$\boxed{\begin{gathered} U_0 = \frac{e^2}{3\epsilon_0}\beta^3\frac{\gamma^4}{\rho} \\ U_0[\mathrm{GeV}] = C_\gamma \frac{E[\mathrm{GeV}]^4}{\rho[\mathrm{m}]}, \\ C_\gamma = 8.85 \times 10^{-5}\ \mathrm{m\ GeV}^{-3} \end{gathered}}$$

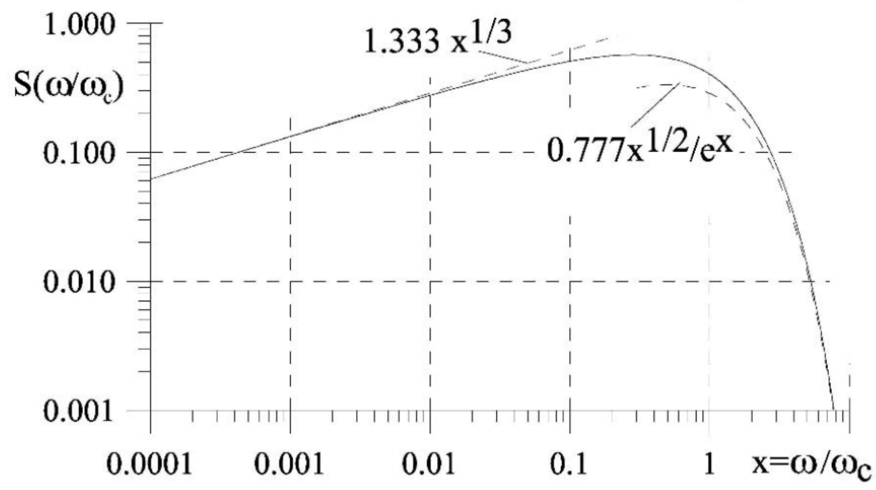
Spectrum – Universal function of synchrotron radiation

- Critical photon energy

$$\varepsilon_c = \frac{3}{2}\frac{\hbar c \gamma^3}{\rho} \equiv \hbar\omega_c$$

- For electrons

$$\varepsilon_c[\mathrm{keV}] = 2.218\frac{E[\mathrm{GeV}]^3}{\rho[\mathrm{m}]}$$
$$= 0.665 E[\mathrm{GeV}]^2 B[\mathrm{T}]$$



ω_c - half the power above, half below

- Synchrotron radiation spectrum

$$\frac{\mathrm{d}P_\gamma}{\mathrm{d}\omega} = \frac{P_\gamma}{\omega_c}\boxed{\frac{9\sqrt{3}}{8\pi}\frac{\omega}{\omega_c}\int_{x_0}^{\infty} K_{\frac{5}{3}}(x)\mathrm{d}x} = \frac{P_\gamma}{\omega_c}\boxed{S\left(\frac{\omega}{\omega_c}\right)}$$

- Universal function S

H. Wiedemann, (2007) Particle Accelerator Physics, Springer-Verlag, p. 823

Angular distribution

Spatial and spectral flux distribution of photon flux

$$\frac{\mathrm{d}^2 N_{\mathrm{ph}}}{\mathrm{d}\theta \mathrm{d}\psi} = C_\Omega E^2 I \frac{\Delta\omega}{\omega} \left(\frac{\omega}{\omega_c}\right)^2 \frac{\gamma}{(1+\gamma^2\theta^2)^{5/2}} K_{2/3}^2(\xi) F(\xi,\theta)$$

$$C_\Omega = \frac{3\alpha}{4\pi^2 e (m_e c^2)} = 1.3255 \times 10^{22} \left[\frac{\mathrm{photons}}{\mathrm{s\ rad^2 GeV^2 A}}\right]$$

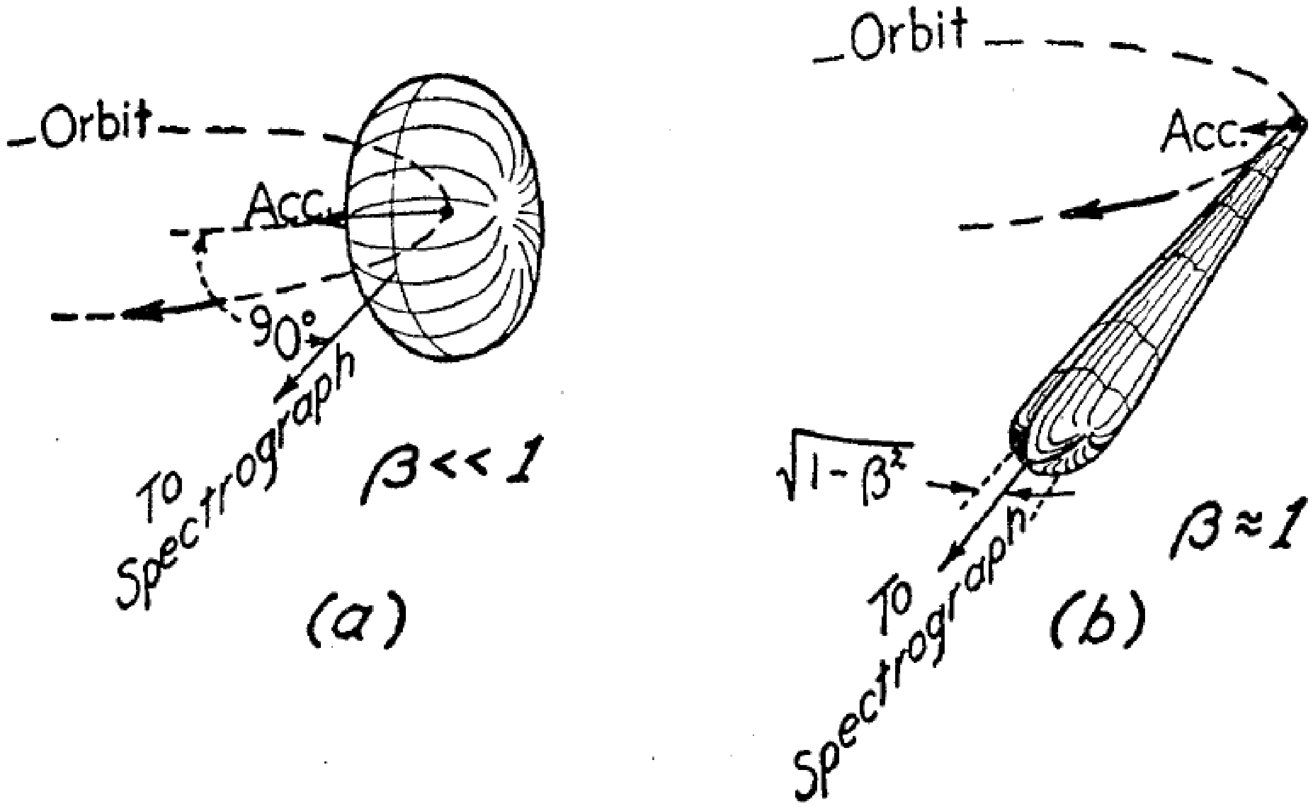
K is the modified Bessel's function,

$$F(\xi,\theta) = (1+\gamma^2\theta^2)^2 \left[1 + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2} \frac{K_{1/3}^2(\xi)}{K_{2/3}^2(\xi)}\right]$$

H. Wiedemann, Particle Accelerator Physics, Springer-Verlag, (2007).

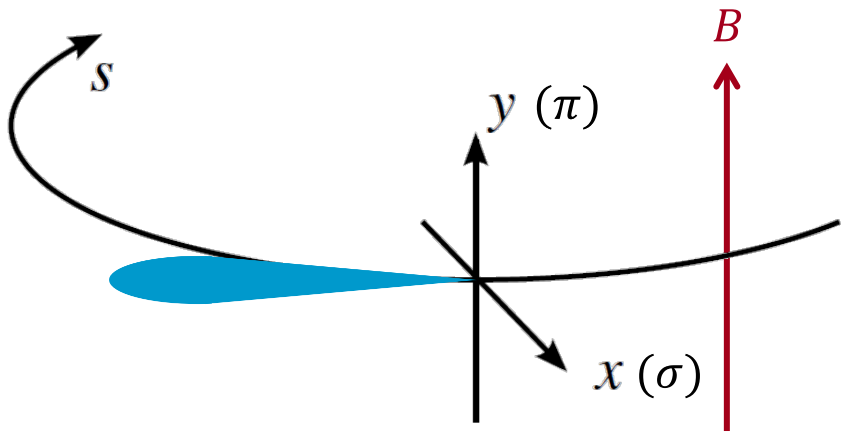
Angular distribution – relativistic acceleration

- Narrow cone of radiation $\left(\frac{1}{\gamma}\right)$
- Tangential to the direction of acceleration



D. H. Tomboulian and P. L. Hartman, Phys. Rev., 102, 1423 (1956).

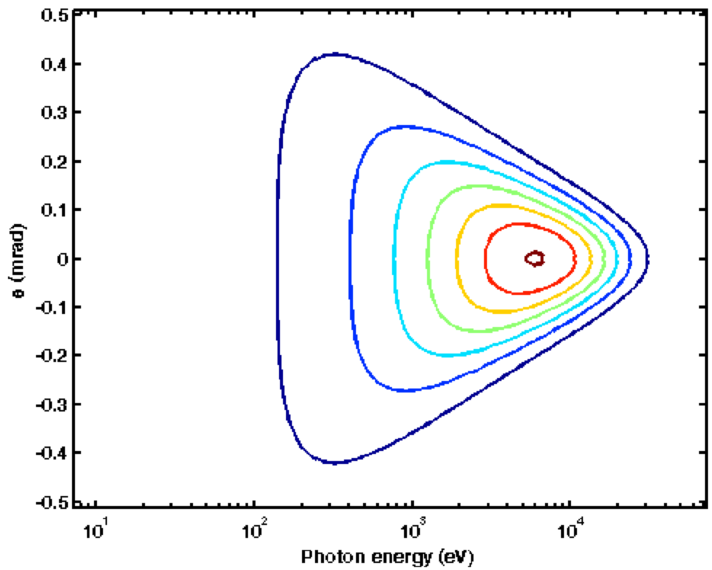
Polarisation



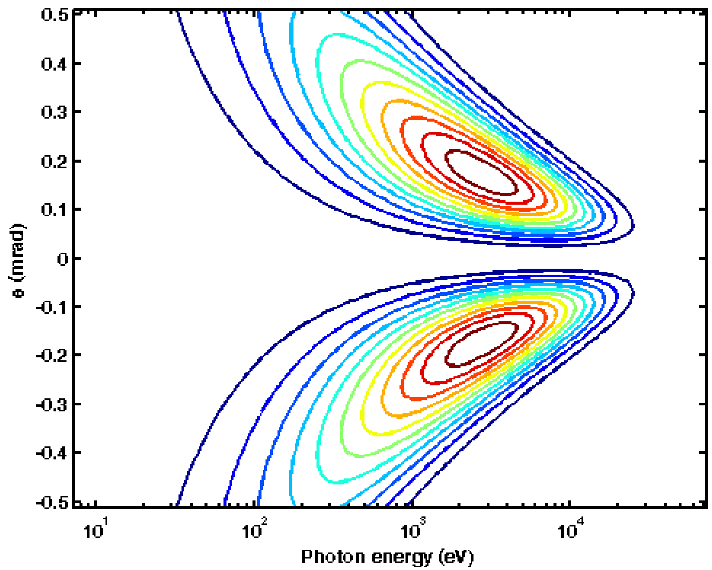
- Synchrotron radiation highly polarised
- Linear polarisations denoted with respect to magnetic field
  - Rings usually have a vertical main dipole field
  - σ – ‘senkrecht’, German for perpendicular (horizontal)
  - π – parallel (vertical)
- Power $P_\sigma = \frac{7}{8}P, \quad P_\pi = \frac{1}{8}P$

Polarisation – angular distribution

σ (horizontal)



π (vertical)



- Typical electron, $E = 3$ GeV, $B = 1.2$ T, $\varepsilon_c = 7.2$ keV
- Horizontal polarisation in orbit plane
- Vertical polarisation above and below the orbit plane

H. Wiedemann, (2007) Particle Accelerator Physics, Springer-Verlag, p. 814

Accelerator physics implications

Protons are like elephants…

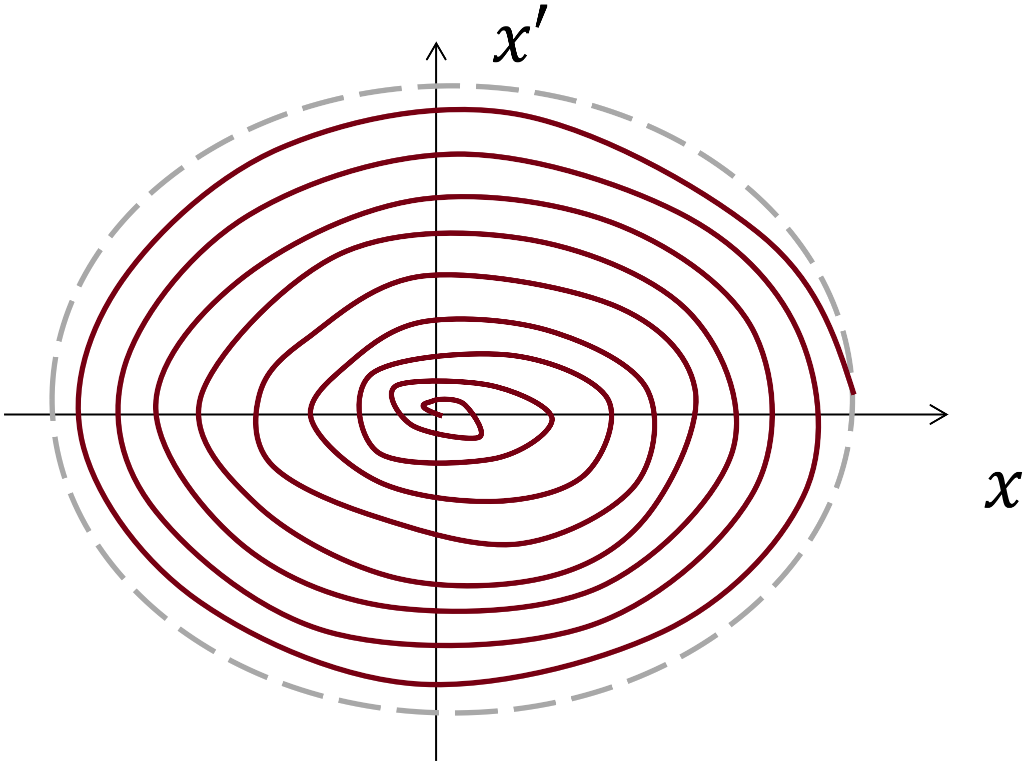
Source: http://www.mrwallpaper.com

… electrons (in rings) are like goldfish

Source: http://www.theresponsiblepet.com

Liouville's theorem

- Liouville's theorem
- Emittance conserved under acceleration
- SR introduces damping

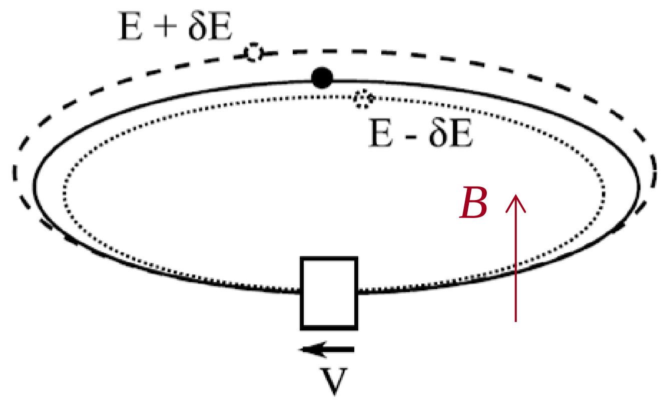


Accelerator physics implications

- Implications mostly for electron rings
- So far, described synchrotron radiation using statistical distributions
- Average energy loss per turn from SR results in damping
- SR photons are both waves and quanta
  - Random quantised SR emission results in random excitation of electrons
- Balance leads to equilibrium

L. Rivkin, 'Electron Dynamics with Synchrotron Radiation', CERN Accelerator School: Introduction to Accelerator Physics, 5th Sep 2014, Prague, Czech Republic (2014).

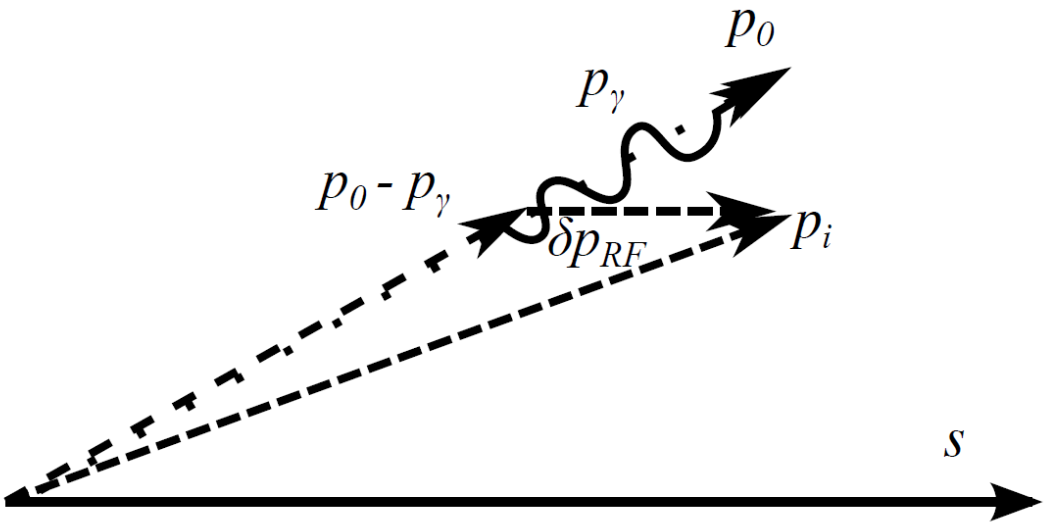
Damping rates



- Synchrotron radiation damping means that the amplitude of single particle oscillations (betatron, synchrotron oscillations) are damped
- Equilibrium determined by damping rates and lattice
- At the instantaneous rate, the time for an electron to lose all its energy through synchrotron radiation
- Damping time typically ~ms

SR damping – vertical

- Electron performing vertical betatron oscillations
- SR photons emitted in a very narrow cone, essentially parallel to electron trajectory with momentum p_0
- Photon with momentum p_γ carries away transverse momentum of electrons
- RF system replenishes only longitudinal momentum
- Transverse momentum asymptotically damps



Damping rate – vertical

- Consider betatron oscillation

$$y = A\cos\phi\,,\, y' = \frac{A}{\beta(s)}\sin\phi$$

- Oscillation amplitude

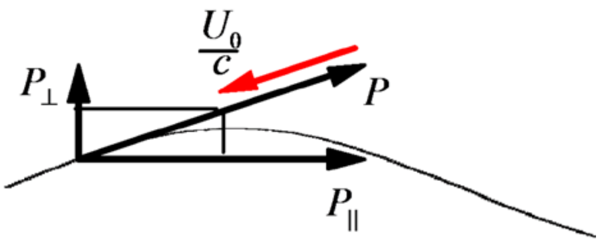
$$A^2 = y^2 + [\beta(s)y']^2$$

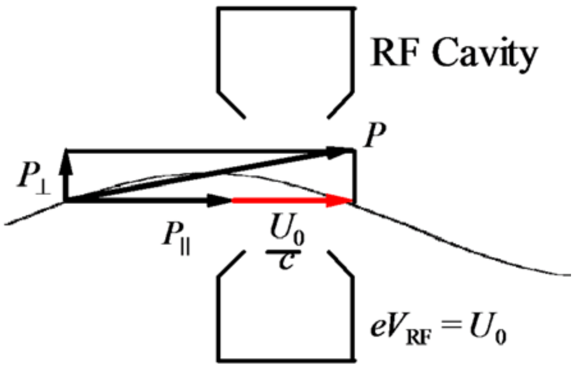
- Electron loses momentum to SR, gains only energy at cavity $\delta y' = -y'\frac{\delta E}{E}$
- Change of amplitude

$$2A\frac{dA}{dy'} = \beta^2(s)2y'$$

$$A\delta A = -\beta^2(s)y'^2\frac{\delta E}{E}$$

- Averaging over all betatron phases, $\langle\beta^2(s)y'^2\rangle = \frac{A^2}{2\pi}\int_0^{2\pi}\sin^2\phi\mathrm{d}\phi = \frac{A^2}{2}$





M. Sands, 'The Physics of Electron Storage Rings: An Introduction', Stanford Linear Accelerator Center, Menlo Park, CA, SLAC-R-121 (1970).

Damping rate – vertical

- Change in action $A\delta A = -\frac{A^2}{2}\frac{\delta E}{E}$
- Therefore $\frac{\delta A}{A} = -\frac{\delta E}{2E}$
- Summing all the energy losses around the ring must equal the total energy loss U_0, therefore for one turn T_0

$$\frac{\Delta A}{A} = -\frac{U_0}{2E} \rightarrow \frac{\Delta A}{AT_0} = \frac{1}{A}\frac{dA}{dt} = -\frac{U_0}{2ET_0}$$

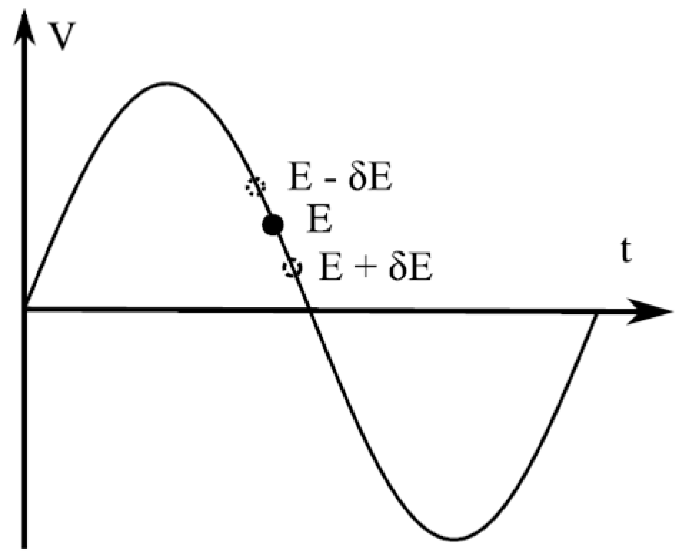
- Exponential damping $e^{-\zeta_y t}$ with damping coefficient

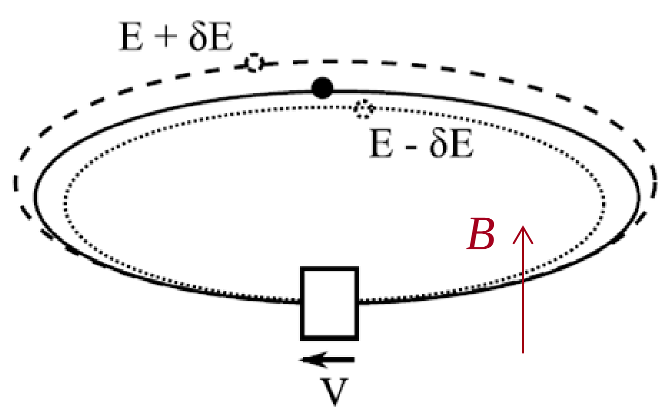
$$\boxed{\zeta_y = \frac{1}{\tau_y} = \frac{U_0}{2ET_0}}$$

M. Sands, 'The Physics of Electron Storage Rings: An Introduction', Stanford Linear Accelerator Center, Menlo Park, CA, SLAC-R-121 (1970).

SR damping – longitudinal

- SR photon emission carries away longitudinal momentum
- Electron rings need significant RF voltage to maintain beam
- SR emission is
  - Quantised – some electrons emit more, some less
  - Deterministic – high energy electrons emit more, low energy emit less
- Design for phase stability

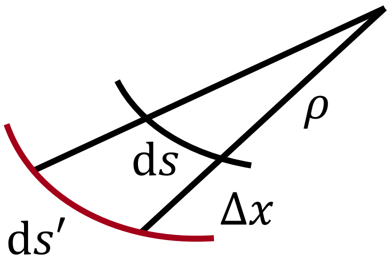




Damping rate – longitudinal

- An off-energy particle traverses a dispersive orbit

$$\mathrm{d}s' = \left(1 + \frac{\Delta x}{\rho}\right) ds = \left(1 + \frac{D_x}{\rho}\frac{\Delta E}{E}\right) ds$$



- This particle radiates energy U' over one turn

$$U' = \oint P\mathrm{d}t = \oint P\frac{1}{c}\mathrm{d}s' = \frac{1}{c}\oint P\left[1 + \frac{D_x}{\rho}\frac{\Delta E}{E}\right]\mathrm{d}s$$

- Differentiating with respect to energy

$$\frac{\mathrm{d}U}{\mathrm{d}E} = \frac{1}{c}\oint\left[\frac{\mathrm{d}P}{\mathrm{d}E} + \frac{D_x}{\rho}\left(\frac{\mathrm{d}P}{\mathrm{d}E}\frac{\Delta E}{E} + \frac{P}{E}\right)\right]\mathrm{d}s$$

- The average energy offset $\left(\frac{\Delta E}{E}\right)$ should be zero, therefore

$$\frac{\mathrm{d}U}{\mathrm{d}E} = \frac{1}{c}\oint\left[\frac{\mathrm{d}P}{\mathrm{d}E} + \frac{D_x}{\rho}\frac{P}{E}\right]\mathrm{d}s$$

K. Wille, The Physics of Particle Accelerators: An Introduction, Oxford University Press, Oxford, UK (2000).
M. Sands, ‘The Physics of Electron Storage Rings: An Introduction’, Stanford Linear Accelerator Center, Menlo Park, CA, SLAC-R-121 (1970).
Y. Papaphilippou, ‘Fundamentals of Storage Ring Design’, USPAS, Santa Cruz, CA, USA (2008).

Damping rate – longitudinal

- Using $P = C_\gamma E^2 B^2$, $\quad \frac{\mathrm{d}B}{\mathrm{d}E} = \frac{\mathrm{d}B}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}E} = \frac{\mathrm{d}B}{\mathrm{d}x}\frac{D_x}{E} = Bk\rho\frac{D_x}{E}$

$$\frac{\mathrm{d}P}{\mathrm{d}E} = 2\frac{P}{E}(1 + k\rho D_x)$$

- Substituting this in to the equation on the previous slide

$$\frac{\mathrm{d}U}{\mathrm{d}E} = \frac{2U_0}{E} + \frac{1}{cE}\oint P\frac{D_x}{\rho}\left(2k + \frac{1}{\rho^2}\right)\mathrm{d}s$$

- Using $\mathrm{P} = \frac{C_\gamma}{e^2c^2}\frac{E^4}{\rho^2}$, therefore $\mathrm{U}_0 = \frac{1}{c}\oint P\mathrm{d}s = \frac{C_\gamma E^4}{e^2c^3}\oint\frac{1}{\rho^2}\mathrm{d}s$

$$\frac{\mathrm{d}U}{\mathrm{d}E} = \frac{U_0}{E}\left[2 + \frac{\oint D_x\frac{1}{\rho}\left(2k+\frac{1}{\rho^2}\right)\mathrm{d}s}{\oint\frac{1}{\rho^2}\mathrm{d}s}\right]$$

- Energy oscillations exponentially damped $e^{-\zeta_s t}$ with the form

$$\boxed{\zeta_s = \frac{1}{2T_0}\frac{\mathrm{d}U}{\mathrm{d}E} = \frac{U_0}{2T_0E}[2 + \mathcal{D}], \quad \mathcal{D} = \frac{\oint D_x\frac{1}{\rho}\left(2k+\frac{1}{\rho^2}\right)\mathrm{d}s}{\oint\frac{1}{\rho^2}\mathrm{d}s}}$$

K. Wille, The Physics of Particle Accelerators: An Introduction, Oxford University Press, Oxford, UK (2000).
M. Sands, ‘The Physics of Electron Storage Rings: An Introduction’, Stanford Linear Accelerator Center, Menlo Park, CA, SLAC-R-121 (1970).
Y. Papaphilippou, ‘Fundamentals of Storage Ring Design’, USPAS, Santa Cruz, CA, USA (2008).

SR damping – horizontal

- It can be shown that …
- Horizontal oscillations exponentially damped $e^{-\zeta_x t}$ with the form

$$\boxed{\zeta_x = \frac{U_0}{2T_0 E}[1-\mathcal{D}], \quad \mathcal{D} = \frac{\oint D_x \frac{1}{\rho}\left(2k + \frac{1}{\rho^2}\right)\mathrm{d}s}{\oint \frac{1}{\rho^2}\mathrm{d}s}}$$

Robinson criterion and damping summary

$$\zeta_x = \frac{1}{\tau_x} = \frac{U_0}{2T_0 E}\mathcal{J}_x, \qquad \mathcal{J}_x = 1 - \mathcal{D}$$

$$\zeta_y = \frac{1}{\tau_y} = \frac{U_0}{2T_0 E}\mathcal{J}_y, \qquad \mathcal{J}_y = 1$$

$$\zeta_s = \frac{1}{\tau_s} = \frac{U_0}{2T_0 E}\mathcal{J}_s, \qquad \mathcal{J}_s = 2 + \mathcal{D}$$

$$\mathcal{D} = \frac{\oint D_x \frac{1}{\rho}\left(2k + \frac{1}{\rho^2}\right)\mathrm{d}s}{\oint \frac{1}{\rho^2}\mathrm{d}s}$$

$$\boxed{\mathcal{J}_x + \mathcal{J}_y + \mathcal{J}_s = 4}$$

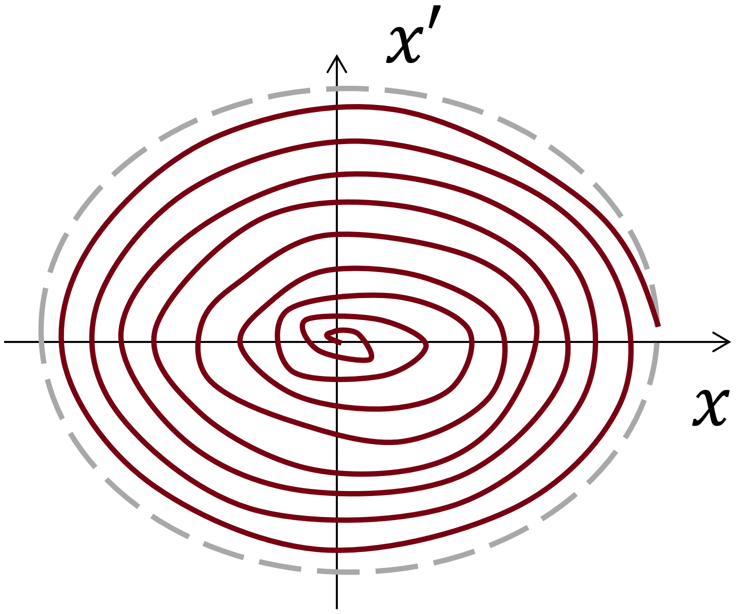
K.W. Robinson, Phys. Rev., 111, 373 (1958).

Consequences for electron rings

- If no vertical bending magnets, $\mathcal{J}_y = 1$
  - True for most rings, bending only in one plane
- If no quadrupole gradient in main bending magnets, $\mathcal{D} \to 0,\ \mathcal{J}_x \approx 1,\ \mathcal{J}_s \approx 2$
  - Most FODO synchrotron (collider) lattices are like this
  - Most new storage ring light sources are not

What happens to the emittance?

- Emittance damps to zero?
  - No!
- Average energy loss per turn from SR results in damping
- SR photons are both waves and quanta
  - Random quantised SR emission results in random excitation of electrons
- Balance leads to equilibrium



Equilibrium emittances – vertical

- Damps to almost zero
- Cone of synchrotron radiation, random vertical emission of photons
- Limit from opening angle of synchrotron radiation
  - Typically much larger, arising from uncorrected betatron coupling with horizontal plane
  - Emittance ratio $\boxed{\varepsilon_y = \kappa\varepsilon_x}$
- Arises from misalignment of quadrupole, sextupole centres on the order of ±20 μm.

Equilibrium emittances – energy spread, horizontal

- Damps to equilibrium energy spread

$$\boxed{\left(\frac{\sigma_E}{E}\right)^2 = C_q \frac{\gamma^2}{\mathcal{J}_s} \frac{\oint 1/\rho^3 \mathrm{d}s}{\oint 1/\rho^2 \mathrm{d}s}}$$

$$C_q = 3.84 \times 10^{-13}\ \mathrm{m}$$

- Damps to equilibrium horizontal emittance

$$\boxed{\varepsilon_x = C_q \frac{\gamma^2}{\mathcal{J}_x} \frac{\oint \mathcal{H}/\rho^3 \mathrm{d}s}{\oint 1/\rho^2 \mathrm{d}s}}$$

$$\mathcal{H} = \beta_x \eta'^2 + 2\alpha_x \eta\eta' + \gamma_x \eta^2$$

- Curly-H function? Next lecture!

Summary

- SR the main difference between electron, proton rings
- Significant only for acceleration of electrons transverse to their velocity
- Determined energy loss per turn, characteristics of SR
- Average energy loss per turn from SR results in damping
- Random quantised SR emission results in random excitation of electrons
- Balance results in an equilibrium emittance

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