TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

HEPHY
Institute of High Energy Physics

Dissertation

Precise predictions of Higgs boson decays including the full one-loop corrections in Supersymmetry

ausgeführt zum Zwecke der Erlangung des akademischen Grades eines Doktors der technischen Wissenschaften unter der Leitung von

Univ. Prof. Walter Majerotto
E 136
Institut für Theoretische Physik
und
Institut für Hochenergiephysik der ÖAW

eingereicht an der Technischen Universität Wien
Technisch–naturwissenschaftliche Fakultät

von

Dipl. Ing. Wolfgang Frisch
e0126153
Senefeldergasse 15/14, A-1100 Wien
frisch@hephy.oeaw.ac.at

Wien, am 7. Juni 2011

Kurzfassung

Das Standardmodell der Elementarteilchenphysik ist eine höchst erfolgreiche Theorie, welche die elektromagnetische, starke sowie die schwache Wechselwirkung von Materieteilchen bis zu Energieskalen von einigen hundert Giga Elektronvolt beschreibt. Trotz dieser großen Erfolge besteht kaum Zweifel darüber, dass es sich bei diesem Modell um eine effektive Theorie handelt, d.h. die Theorie verliert bei höheren Energien ihre Aussagekraft.

Daher muss das Standard Modell in geeigneter Art und Weise erweitert werden. Ein äußerst viel versprechendes Konzept für eine solche Erweiterung ist jenes der Supersymmetrie, in dem zu jedem bekannten Teilchen des Standardmodells ein oder mehrere supersymmetrische Partnerteilchen vorausgesagt werden. Die einfachste und attraktivste supersymmetrische Erweiterung des Standardmodells wird als minimal supersymmetrisches Standardmodell (MSSM) bezeichnet. Minimal verweist darauf, dass die Anzahl neuer Wechselwirkungen und Felder so gering wie möglich gehalten wird. Genau genommen besteht das MSSM aus den Feldern des Standardmodells und deren supersymmetrischen Partnern sowie einem zusätzlichen Higgsdublett. Die Anwesenheit dieses zusätzlichen Higgsdubletts führt zur Existenz von fünf physikalischen Higgsteilchen.

Die Suche nach diesen supersymmetrischen Teilchen sowie Higgsbosonen ist deshalb eines der wichtigsten Ziele des Large Hadron Collinders (LHC) am Teilchenforschungszentrum CERN, bei dem hinreichend hohe Kollisionsenergien für die Erzeugung von diesen neuen Teilchen zur Verfügung stehen, weshalb die Existenz von Supersymmetrie bestätigt werden könnte. Für die Entdeckung dieser neuen Teilchen sind genaue Vorhersagen ihrer Zerfallsbreiten sowie Verzweigungsverhältnisse unerlässlich. Um der Präzision am LHC und dem zukünftigen ILC gerecht zu werden, müssen Feynman Amplituden zumindest bis auf Einschleifenniveau berechnet werden. Da es bei diesen Rechnungen auf Einschleifenniveau zu sogenannten UV– und IR– Divergenzen kommt, wird eine Renormierungsprozedur, in der diese Divergenzen durch eine geeignete Wahl von Countertermen abgezogen werden, unerlässlich.

Ziel dieser Arbeit war die Entwicklung eines Computerprogramms *HFOLD*, das alle Zerfallsbreiten sowie Verzweigungsverhältnisse aller MSSM Higgsboson Zweikörperzerfälle auf vollem Einschleifenniveau berechnet. Bisher verfügbare Programme approximieren diese Korrekturen durch den Gebrauch von Renormierungsgruppen-Gleichungen. Aufgrund der Tatsache, dass bei der vollen Einschleifenrechnung dieser Prozesse fast alle Parameter des MSSM renormiert werden müssen, und daher eine Berechnung von einer entsprechend großen Anzahl von Feynmandiagrammen erforderlich ist, gestaltet sich diese Aufgabe als äußerst komplex.

Die erforderliche Renormierungsprozedur wurde in Übereinstimmung mit der SPA Konvention konsistent im $\overline{\mathrm{DR}}$ Schema durchgeführt. Besondere Aufmerksamkeit wurde auf die Unterscheidung zwischen den führenden SUSY-QCD und den elektroschwachen Korrekturen gelegt. Diese wurden im Detail herausgearbeitet und können getrennt ausgegeben werden.

Im Rahmen dieser Arbeit wurden dann mittels des Programms *HFOLD* Higgszerfälle in mehreren sogenannten high scale Szenarien untersucht, in denen die erforderlichen Modellparameter von einem Satz weniger Parameter auf der GUT Skala durch die Verwendung von Renormierungsgruppen-Gleichungen abgeleitet werden (z.B. minimal Supergravity (mSUGRA) oder Non Universal Higgs Mass (NUHM)). Für andere Szenarien dieser Art kann das Programm leicht adaptiert werden.

Dabei stellte sich wie zu erwarten heraus, dass die SUSY-QCD sowie die elektroschwachen Korrekturen für die Higgsbosonzerfälle in Quarks der dritten Generation unerlässlich sind. Sollten die supersymmetrischen Teilchen im Vergleich zu den Higgsbosonen jedoch relativ leicht sein, wie z.B. in NUHM Szenarien, aber auch in Teilen des mSUGRA Parameterraums, können die Zerfallskanäle in Charginos und Neutralinos die größten Verzweigungsverhältnisse aufweisen. Der Unterschied für diese führenden Zerfallsbreiten kann dann zwischen der niedrigsten Ordnung und der entsprechend vollen Einschleifenrechnung mehr als 10 Prozent betragen. Daher können diese Korrekturen für präzise Analysen nicht mehr vernachlässigt werden.

Abstract

The Standard Model of elementary particle physics is a highly successful theory, describing the electromagnetic, strong and weak interaction of matter particles up to energy scales to a few hundred giga electronvolt. Despite its great success in explaining experimental results correctly, there is hardly no doubt that the SM is an effective theory, which means that the theory loses its predictability at higher energies.

Therefore, the Standard Model has to be extended in a proper way to describe physics at higher energies. A most promising concept for the extension of the SM is those of Supersymmetry, where for each particle of the SM one or more superpartner particles are introduced. The simplest and most attractive extension of the SM is called Minimal Supersymmetric Standard Model (MSSM). Minimal refers to the additional field content, which is kept as low as possible. In fact the MSSM consists of the fields of the SM and their corresponding supersymmetric partner fields, as well as one additional Higgs doublet. The presence of this additional Higgs doublet leads to the existence of five physical Higgs bosons in the MSSM.

The search for supersymmetric particles and Higgs bosons is one of the primary goals of the Large Hadron Collider (LHC) at the CERN laboratory, producing collisions at sufficiently high energies to detect these particles. For the discovery of these new particles, precise predictions of the corresponding decay widths and branching rations are utmost mandatory. To contribute with the precision of the LHC and the future ILC, Feynman amplitudes should be calculated at least to one-loop order. Since these calculations lead to so called UV– and IR–divergences, it is essential to perform a renormalization procedure, where the divergences are subtracted by a proper definition of counterterms.

The goal of this work was to develop a program package, which calculates all MSSM two-body Higgs decay widths and corresponding branching ratios at full one-loop level. Up to now public available programs approximate these contributions using renormalization group improved born level amplitudes. Due the fact that the full one-loop calculation requires the renormalization of almost all parameters of the MSSM, a large number of Feynman diagrams has to be computed, which makes this task very complex and challenging.

The necessary renormalization procedure was performed consistently in the $\overline{\mathrm{DR}}$ scheme following the SPA convention. Special attention has been given to the distinction between the leading SUSY-QCD and the full electroweak contributions, which have been worked out in detail and can be output separately.

In this work we then used the program *HFOLD* to study Higgs decays in some so called high scale scenarios, where the necessary parameters are derived from a few parameters specified at the GUT scale using renormalization group equations (e.g. scenarios like minimal Supergravity (mSUGRA) or Non universal Higgs Mass (NUHM)). The program can be most easily adopted to any other scenario of this kind.

It was found as expected that the SUSY-QCD as well as the electroweak corrections for Higgs decays into third generation quarks are mandatory. If the Higgs bosons are relatively heavy compared to the supersymmetric particles like in NUHM scenarios, but also in parts of the mSUGRA parameter space, the decay channels into charginos and neutralinos can exhibit the largest branching ratios. The difference for these leading decay widths between the born level and the corresponding full one-loop calculation can be more than 10 percent. Therefore these contributions can not be neglected for precise analysis.

Danksagung

Ich danke Herrn Dr. Helmut Eberl für die Unterstützung und das freundliche Arbeitsklima in den letzten dreieinhalb Jahren. Des Weiteren möchte ich unserem Computerchef Herrn Dr. Gerhard Walzel für die schnelle und unkomplizierte Unterstützung bei allen Computerproblemen sowie für die Ausstattung mit überdimensionalen Monitoren danken. Auch möchte ich Herrn Dipl.-Physiker Moritz Nadler dafür danken, dass ich von seiner LaTeX Expertise profitieren konnte, dadurch blieben mir etliche Zusatzstunden bei der Formatierung meiner Doktorarbeit erspart. Zu guter Letzt danke ich meinem guten Freund Herrn DI Florian Teischinger für die Unterstützung bei diversen Compiler spezifischen Problemen sowie für etliche spannende Diskussionen über verschiedene Gebiete der Hochenergiephysik.

Contents

Chapter 1

Supersymmetry

The Standard Model (SM) of elementary particle physics [1, 2] is an extremely successful theory describing the electroweak and strong interactions of quarks and leptons. Quarks and leptons occur in three generations:

- **the leptons**: the electron, the muon, the tau and their corresponding neutrions. The left–handed doublets

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix}_L, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L,$$

  couple to $U(1)_Y$ and $SU(2)_L$, in contrast to the right–handed singlets

$$e_R,\ \mu_R,\ \tau_R\ ,$$

  only couple to $U(1)_Y$.

- **the quarks**: the up, down, charm, strange, top and bottom quark. The left–handed doublets

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L,$$

  couple to $U(1)_Y,\ SU(2)_L$ and $SU(3)_C$, the right–handed singlets

$$u_R,\ d_R,\ c_R\ , s_R\ , t_R\ , b_R\ ,$$

  couple only to $U(1)_Y$ and $SU(3)_C$.

The SM is based on local gauge invariance described by the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ in which the electroweak $SU(2)_L \otimes U(1)_Y$ symmetry is spontaneously broken down to the $U(1)_{EM}$ electromagnetic symmetry. In the SM the mechanism of spontaneous electroweak symmetry breaking (EWSB) [3] is based on a non vanishing vacuum expectation value v of a fundamental scalar field (*Higgs* field). This *Higgs* field gives masses to all particles which couple at tree–level to the Higgs boson, in particular to the $W^\pm$ and Z^0 vector bosons.
Although the SM describes almost all phenomena (except neutrino oscillations and neutrino masses) presently known at energies up to $\simeq$ 100 GeV, there are several fundamental questions that remain unanswered:

- **The hierarchy problem**
In the SM, radiative corrections to the squared Higgs mass m_H^2 depend quadratically on the ultraviolet cut-off scale Λ. If the scale Λ, where possibly new physics enters is high, the theory produces an unnatural large Higgs mass. To show the problem, consider the one–loop contributions to the Higgs mass, of a fermion f with a repetition number N_f and a Yukawa coupling given by $\lambda_f = \sqrt{2}m_f/v$. Assuming for simplicity that the fermion is very heavy so that one can neglect the external Higgs momentum, one obtains for the fermionic Higgs mass correction

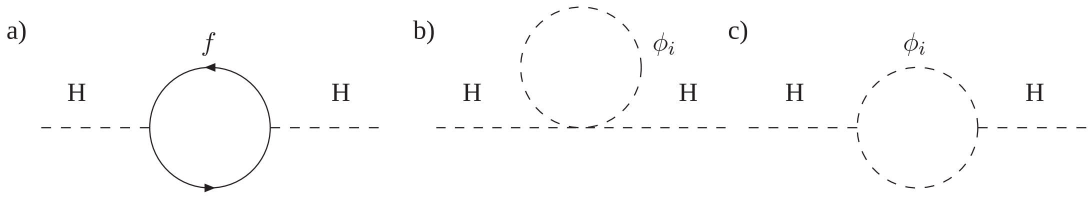


Figure 1.1: One-loop corrections to the Higgs mass m_H^2, (a) fermionic, (b) and (c) scalar .

$$\Delta m_H^2 = N_f \frac{\lambda_f^2}{8\pi^2}\left[-\Lambda^2 + 6m_f^2 \log\frac{\Lambda}{m_f} - 2m_f^2\right] + \mathcal{O}(1/\Lambda^2) \ ,$$

which shows the quadratically divergent behavior $\Delta m_H^2 \propto \Lambda^2$. If we chose the cut–off scale Λ to be the GUT scale, $m_{\rm GUT} \sim 10^{16}$ GeV, or the Planck scale, $M_P \sim 10^{18}$ GeV, the Higgs boson mass, which is supposed to lie in the range of the electroweak symmetry breaking scale $v \sim 250$ GeV, will become huge. For the SM Higgs boson to stay relatively light, at least $m_H \lesssim 1$ TeV, we need to add a counterterm to the mass squared and adjust it with a precision of $\mathcal{O}(10^{-30})$, which seems highly unnatural.

- **Electroweak symmetry breaking**
In the SM, the masses of fermions and gauge bosons are generated by the Higgs mechanism. A negative Higgs mass squared in the Higgs potential $V(\phi)$ leads to a non vanishing vacuum expectation value v. However the negative squared mass parameter μ^2 is introduced by hand and without any deeper justification, which is from the theoretical point of view unsatisfactory.

- **Gauge coupling unification and GUTs**
If the SM is part of a Grand Unified Theory (GUT) the fundamental forces should unify at a high scale. However, recent measurements of energy behavior of the couplings indicate that gauge unification is not possible with the particle content of the SM .

- **Baryogenesis**
One of the most fundamental open questions is the origin of the observed *baryon asymmetry* of the universe. Although the SM fulfills all the requirements for baryogenesis [5], the electroweak phase transition is too weak to preserve the generated baryon asymmetry. Therefore, baryon asymmetry generated at the electroweak phase transition claims for new physics at the electroweak scale.

- **Dark matter**
  The cosmic microwave background data strongly indicate that only about 5% of the total matter density of the universe consist of particles of the SM, while there is about five to six times more mass in the form of invisible *cold dark matter*. The SM does not provide a reliable candidate with the right properties to describe this kind of matter.

Therefore, the Standard Model has to be extended to describe physics also at higher energies. In the early 70's, J. Wess and B. Zumino found an attractive symmetry relating the two fundamental species of elementary particles, bosons and fermions, by a *supersymmetry* transformation given by

$$Q|\,\mathrm{Fermion}\rangle = |\,\mathrm{Boson}\rangle \qquad Q|\,\mathrm{Boson}\rangle = |\,\mathrm{Fermion}\rangle\,. \tag{1.1}$$

In such supersymmetric models for each particle a *superpartner* equal in mass and quantum numbers, but differerent in spin by 1/2 is introduced. Due to this boson $\leftrightarrow$ fermion symmetry the scalar masses are protected from quadratically divergent loop corrections, providing an elegant solution to the hierarchy problem.
Since none of the predicted supersymmetric particles have been observed up to now, SUSY must be a broken symmetry in nature. If this supersymmetry breaking is of a certain type known as *soft breaking* [6], it doesn't forfeit some of its advantages, e.g. it does not reintroduce quadratic divergences of scalar particle masses.
Even though theories including SUSY have to explain why the masses of the predicted superpartners are that high and up to now there is no direct evidence that the fundamental structure of nature is supersymmetric, such theories provide many remarkable features:

- **Hierarchy problem**
  One of the main reasons for introducing SUSY theories is their ability to solve the hierarchy problem [7]. By grouping fermions and bosons together in supermultiplets, the quadratically divergent radiative fermionic corrections to the Higgs boson mass are canceled by the corresponding bosonic loop contributions of opposite sign. Therefore SUSY stabilizes the hierarchy in the sense that the 'natural' mass of the Higgs boson lies in the range of electroweak symmetry breaking scale.

  To show the cancellation let us assume the existence of N_S scalar particles with masses m_S and with trilinear and quartic couplings to the Higgs boson given by $v\lambda_S$ and λ_S. They contribute to the Higgs boson self–energy via the two diagrams of Fig. 1.1b, which lead to a contribution to the Higgs boson mass squared

$$\Delta m_H^2 = \frac{\lambda_S N_S}{16\pi^2}\left[-\Lambda^2 + 2m_S^2 \log\left(\frac{\Lambda}{m_S}\right)\right] - \frac{\lambda_S^2 N_S}{16\pi^2} v^2 \left[-1 + 2\log\left(\frac{\Lambda}{m_S}\right)\right] + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)\,.$$

  Here again the quadratic divergences are present. However, if we make the assumption that the Higgs couplings of the scalar particles are related to the Higgs–fermion couplings in such a way that $\lambda_f^2 = 2m_f^2/v^2 = -\lambda_S$, and that the multiplicative factor for scalars is twice the one for fermions, $N_S = 2N_f$, we then obtain, once we add the two scalar and the fermionic contributions to the Higgs boson mass squared

$$\Delta m_H^2 = \frac{\lambda_f^2 N_f}{4\pi^2}\left[(m_f^2 - m_S^2)\log\left(\frac{\Lambda}{m_S}\right) + 3m_f^2 \log\left(\frac{m_S}{m_f}\right)\right] + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)\,.$$

As can be seen, the quadratic divergences have disappeared in the sum [7]. The logarithmic divergence is still present, but even for values $\Lambda \sim M_P$ of the cut–off, the contribution is rather small. This logarithmic divergence disappears also if, we assume that the fermion and the two scalars have the same mass $m_S = m_f$. In fact, in this case, the total correction to the Higgs boson mass squared vanishes altogether. However, if this symmetry is badly broken and the masses of the scalar particles are much larger than their corresponding fermionic superpartners, the hierarchy problem would be reintroduced again in the theory, since the radiative corrections to the Higgs mass $\propto (m_f^2 - m_S^2)\log(\Lambda/m_S)$ become large again. To keep the Higgs mass in the range of the electroweak symmetry breaking scale $m_H = \mathcal{O}(100\text{ GeV})$, we need the mass difference between the SM particles and their corresponding SUSY partners to be rather small. Therefore, the new particles should not be much heavier than the TeV scale $m_{S,F} = \mathcal{O}(1\text{ TeV})$.

- **Electroweak symmetry breaking**
  As already indicated, in the SM an effective Higgs potential $V(h) \propto \mu^2 h^2 + \lambda h^4$ with $\mu^2 < 0$ is introduced 'by hand' to trigger EWSB. In renormalizable (supersymmetric) theories, however, the mass parameters which enter in the Lagrangian are also scale–dependent, and renormalization-group equations can be used to evolve the parameters from the unification scale of the order of 10^{16} GeV down to the weak scale of order 10^2 GeV. Using the renormalization group equations the Higgs mass parameter μ is driven to a negative value due the top–Yukawa coupling, thus providing a plausible explanation of electroweak symmetry breaking (EWSB)[8, 9].

- **Gauge coupling unification**
  It can further be shown that in the minimal supersymmetric extension of the SM, the extrapolation of the low energy values of the gauge couplings unify at a scale $M_{\text{GUT}} \simeq 3 \times 10^{16}$ GeV [10], well in agreement with the limits on the proton lifetime.

- **Cold dark matter**
  As we have seen, supersymmetric models can solve many problems of the SM. However, without any additional structure, they can give rise to baryon and lepton number violation at unacceptable levels, e.g. proton decay can be mediated by the superpartners of quarks, i.e. $p \to \pi^0 e^+$. The non-observation of such decays has lead to the introduction of a discrete symmetry known as R–parity [11], to forbid such decays and to ensure baryon and lepton number conservation in an elegant way. As a consequence, the lightest supersymmetric particle (LSP) is absolutely stable, and, if electrically neutral it provides a nice cold dark matter candidate.

Chapter 2

The Minimal Supersymmetric Standard Model (MSSM)

2.0.1 The SUSY Algebra

The generators of SUSY must turn a bosonic state into a fermionic state, and vice versa. This implies that the generators themselves carry half–integer spin, i.e. are fermionic. This is in contrast with the generators of the Lorentz group, or with other gauge group generators, which are all bosonic. The simplest choice of SUSY generators is a two-component Weyl-spinor Q and its conjugate $\overline{Q}$. Since these generators are fermionic, their algebra can be mostly written in terms of anti-commutators

$$\{Q_\alpha, Q_\beta\} = 0 \,, \tag{2.1}$$
$$\left\{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\right\} = 0 \,, \tag{2.2}$$
$$\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \,, \tag{2.3}$$
$$[Q_\alpha, P_\mu] = 0 \,. \tag{2.4}$$

2.1 Construction of the MSSM Lagrangian

2.1.1 Superfields

To construct the MSSM Lagrangian we will need two kinds of superfields: the chiral superfields and the vector superfields. Each type of superfield is an irreducible representation of the SUSY algebra. Superfields can be understood as functions of Grassmann valuable 'fermionic' coordinates θ and $\bar\theta$ and the spacetime coordinates x_μ. An infinitesimal SUSY transformation can be written as

$$\delta_S(\alpha,\bar\alpha)\Phi(x,\theta,\bar\theta) = \left[\alpha\frac{\partial}{\partial\theta} + \bar\alpha\frac{\partial}{\partial\bar\theta} - i\left(\alpha\sigma_\mu\bar\theta - \theta\sigma_\mu\bar\alpha\right)\frac{\partial}{\partial x_\mu}\right]\Phi(x,\theta,\bar\theta) \,, \tag{2.5}$$

where Φ is a superfield and α, $\bar\alpha$ are again Grassmann variables. This corresponds to the following explicit representation of the SUSY generators

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu \,, \quad \bar Q_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu \,. \tag{2.6}$$

One can introduce SUSY-covariant derivatives, which anti–commute with the SUSY generators Q and $\bar{Q}$ by

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu \ , \qquad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu \ . \tag{2.7}$$

2.1.2 Chiral superfields

This name is derived from the fact that the SM fermions are chiral, because their left– and right–handed components transform differently under $SU(2)_L \times U(1)_Y$. Therefore, we need to introduce superfields with only two physical fermionic degrees of freedom, which can then describe the left– or right–handed component of a SM fermion. The same superfields will also contain their bosonic superpartners, the sfermions. Chiral superfields are defined by the following constraints

$$\bar{D}\Phi_L = 0 \quad (\Phi_L \text{ is left } - \text{chiral}) \ , \tag{2.8}$$
$$D\Phi_R = 0 \quad (\Phi_R \text{ is right} - \text{chiral}) \ . \tag{2.9}$$

Φ_L can be expanded in complex space ($y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$) by

$$\Phi_L(y,\theta) = \phi(y) + \sqrt{2}\theta^\alpha\psi_\alpha(y) + (\theta\theta)F(y) \ , \tag{2.10}$$

where the fields ϕ and F are complex scalars fields and ψ is a Weyl spinor.

2.1.3 Vector superfields

Vector superfields describe the spin–1 gauge bosons and their fermionic superpartners the gauginos. Vector superfields are constrained by

$$V(x,\theta,\bar{\theta}) \equiv V^\dagger(x,\theta,\bar{\theta}). \tag{2.11}$$

The vector superfield can be expanded (after using the Wess–Zumino gauge) by

$$V(x,\theta,\bar{\theta}) = (\theta\sigma^\mu\bar{\theta})A_\mu + i(\theta\theta)(\bar{\theta}\bar{\lambda}) - i(\bar{\theta}\bar{\theta})(\theta\lambda) + (\theta\theta)(\bar{\theta}\bar{\theta})d \ , \tag{2.12}$$

where A_μ is a real vector field, λ a compex Weyl spinor field and d denotes a real scalar field.

2.1.4 SUSY–invariant Lagrangian

By definition, the action is invariant under SUSY transformations

$$\delta_S \int d^4x\mathcal{L}(x) = 0 \ . \tag{2.13}$$

This is satisfied, if $\mathcal{L}$ itself transforms into a total derivative. The highest components of the superfields (those with the largest number of θ and $\bar{\theta}$ factors) satisfy this requirement, so they can therefore be used to construct a SUSY invariant Lagrangian. The action S can be written in more detail by

$$S = \int d^4x \left(\int d^2\theta\mathcal{L}_F + \int d^2\theta d^2\bar{\theta}\mathcal{L}_D \right) . \tag{2.14}$$

2.1.5 Yukawa interactions and mass terms

The product of two left–chiral superfields writes as

$$\begin{aligned}\Phi_{1,L}\Phi_{2,L} &= \left(\phi_1+\sqrt{2}\theta\psi_1+\theta\theta F_1\right)\left(\phi_2+\sqrt{2}\theta\psi_2+\theta\theta F_2\right) ,\\ &= \phi_1\phi_2+\sqrt{2}\theta\left(\psi_1\phi_2+\phi_1\psi_2\right)+\theta\theta\left(\phi_1F_2+\phi_2F_1-\psi_1\psi_2\right) ,\end{aligned} \tag{2.15}$$

performing the integration over the Grassmann variables the expression has the form (A detailed description of the integration over Grassmann variables can be found in the appendix B.)

$$\int d^2\theta\ \Phi_{1,L}\Phi_{2,L} = \left(\phi_1F_2+\phi_2F_1-\psi_1\psi_2\right) . \tag{2.16}$$

The product of two left–chiral superfields is again a left–chiral superfield, since it does not depend on $\bar{\theta}$, it is a candidate for a contribution to the $\mathcal{L}_F$ term in the action (2.14). The last term in eq. (2.16) gives rise to a fermion mass term. The product of three left– chiral superfields takes the following form after integration

$$\int d^2\ \theta\Phi_{1,L}\Phi_{2,L}\Phi_{3,L} = \phi_1\phi_2F_3+\phi_1F_2\phi_3+\phi_1\phi_2F_3\underbrace{-\psi_1\phi_2\psi_3-\phi_1\psi_2\psi_3-\psi_1\psi_2\phi_3}_{\text{Yukawa interactions}} , \tag{2.17}$$

and gives rise to Yukawa interactions. If ϕ_1 is the Higgs field, and ψ_2 and ψ_3 the left–and right-handed components of the top quark, eq. (2.17) will not only produce the desired Higgs–top–top interaction, but also interactions between a scalar top $\tilde{t}$, the fermionic "higgsino" $\widetilde{h}$, and the top quark, with equal strength. This is a first example of relations between couplings enforced by supersymmetry.

2.1.6 Kinematic terms

Let us now consider the product of a left-chiral superfield and its conjugate with

$$\begin{aligned}\Phi_L^\dagger(x,\theta) &= \phi^*-2i\theta\sigma_\mu\bar{\theta}\partial^\mu\phi^*-2\left(\theta\sigma_\mu\bar{\theta}\right)\left(\theta\sigma_\nu\bar{\theta}\right)\partial^\mu\partial^\nu\phi^*\\ &+ \sqrt{2}\bar{\theta}\bar{\psi}-2\sqrt{2}i\left(\theta\sigma_\mu\bar{\theta}\right)\partial^\mu\left(\bar{\theta}\bar{\psi}\right)+\bar{\theta}\bar{\theta}F^*.\end{aligned} \tag{2.18}$$

Obviously the product $\Phi_L\Phi_L^\dagger$ is self–conjugate and is therefore a vector superfield. It is a candidate contribution to the "D–terms" in the action (2.14)

$$\int d^2\theta d^2\bar{\theta}\Phi_L\Phi_L^\dagger = FF^*-\phi\partial_\mu\partial^\mu\phi^*-i\bar{\psi}\sigma_\mu\partial^\mu\psi. \tag{2.19}$$

As can be easily seen eq. (2.19) does not contain a kinetic terms for the field F, therefore this field does not propagate, it is an auxiliary field, which can be integrated out, using its algebraic equation of motion.

2.1.7 The superpotential

The interactions of a renormalizable supersymmetric theory can be generalized by the introduction of the superpotentail W. The superpotential W is defined by

$$W(\Phi_i) = \sum_i k_i \Phi_i + \frac{1}{2} \sum_{i,j} m_{ij} \Phi_i \Phi_j + \frac{1}{3} \sum_{i,j,k} g_{ijk} \Phi_i \Phi_j \Phi_k \ , \tag{2.20}$$

where the Φ_i are all left–chiral superfields, and the k_i, m_{ij} and g_{ijk} are constants with mass dimension 2, 1 and 0, respectively. The contributions to the Lagrangian that we have identified so far can be written compactly as

$$\begin{aligned} \mathcal{L} &= \sum_i \int d^2\theta d^2\bar{\theta} \Phi_i \Phi_i^\dagger + \left[\int d^2\theta W(\Phi_i) + h.c. \right] \\ &= \sum_i \left(F_i F_i^* + |\partial_\mu \phi|^2 - i\bar{\psi}_i \sigma_\mu \partial^\mu \psi_i \right) \\ &+ \left[\sum_j \frac{\partial W(\phi_i)}{\partial \phi_j} F_j - \frac{1}{2} \sum_{j,k} \frac{\partial^2 W(\phi_i)}{\partial \phi_j \partial \phi_k} \psi_j \psi_k + h.c. \right] . \end{aligned} \tag{2.21}$$

Note that in the last line of eq. (2.21), W is understood to be a function of the scalar fields ϕ_i, rather than of the superfields Φ_i. Using eq.(2.20) it is easy to convince oneself that the last line in eq.(2.21) indeed reproduces the previous results (2.16) and (2.17). Using the equation of motion one can integrate out the auxiliary fields F_j. Their equation of motions are simply given by

$$\frac{\partial \mathcal{L}}{\partial F_j} = 0 \rightarrow F_j = - \left[\frac{\partial W(\phi_i)}{\partial \phi_j} \right]^* \ , \tag{2.22}$$

reinserting this back into eq. (2.21) gives

$$\mathcal{L}_{int} = - \left[\sum_{j,k} \frac{\partial^2 W(\phi_i)}{\partial \phi_j \partial \phi_k} \psi_j \psi_k + h.c. \right] - \sum_j \left| \frac{\partial W(\phi_i)}{\partial \phi_j} \right|^2 . \tag{2.23}$$

The Lagrangian (2.23) describes fermion masses and Yukawa interactions, while the last term describes scalar mass terms and scalar interactions. Since both terms are determined by the single function W, there are clearly many relations between coupling constants.

2.1.8 Gauge interactions

The coupling of the gauge (super)fields to the (chiral) matter (super)fields is accomplished by a SUSY version of the familiar "minimal coupling" by

$$\begin{aligned} \int d^2\theta d^2\bar{\theta} \Phi^\dagger \Phi \quad &\longrightarrow \quad \int d^2\theta d^2\bar{\theta} \Phi^\dagger e^{2gV} \Phi \\ &= \quad |D_\mu \phi|^2 - i\bar{\psi}\sigma_\mu D^\mu \psi + g\phi^* D\phi + ig\sqrt{2} \left(\phi^* \lambda \psi - \bar{\lambda}\bar{\psi}\phi \right) + |F|^2 \ , \end{aligned} \tag{2.24}$$

in the second step the W–Z gauge has been used. We introduce the usual gauge–covariant derivative $D_\mu = \partial_\mu + igA_\mu^a T_a$, where the T_a are group generators. This part of the Lagrangian does not only describes the interactions of the matter fields (both fermions and scalars) with the gauge fields, but also contains gauge–strength Yukawa–interactions between fermions ψ, sfermions ϕ, and gauginos λ. Finally, the kinetic energy terms of the gauge fields can be described with the help of the superfield by

$$W_\alpha = \left(\bar{D}_{\dot{\alpha}}\bar{D}_{\dot{\beta}}\epsilon^{\dot{\alpha}\dot{\beta}}\right) e^{-gV} D_\alpha e^{gV} , \tag{2.25}$$

where D and $\bar{D}$ denote the SUSY–covariant derivatives, which carry spinor subscripts. For abelian symmetries, this reduces to $W_\alpha = \left(\bar{D}_{\dot{\alpha}}\bar{D}_{\dot{\beta}}\epsilon^{\dot{\alpha}\dot{\beta}}\right) D_\alpha V$. Since $\bar{D}_{\dot{\alpha}}\bar{D}_{\dot{\alpha}} = 0$ and $\bar{D}_{\dot{\alpha}}W_\alpha = 0$, W_α is a left–chiral superfield. One can show that the product $W_\alpha W^\alpha$ is gauge invariant, it is also a left–chiral superfield, therefore its $\theta\theta$ component can be used the Lagrangian

$$\begin{aligned}\frac{1}{32g^2}W_\alpha W^\alpha &= -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \frac{1}{2}D_a D^a \\ &+ \left(-\frac{i}{2}\lambda^a\sigma_\mu\partial^\mu\bar{\lambda}_a + \frac{1}{2}gf^{abc}\lambda_a\sigma_\mu A^\mu_b\bar{\lambda}_c + h.c.\right) .\end{aligned} \tag{2.26}$$

In addition to the familiar kinetic energy term for the gauge fields, this also contains a kinetic energy terms for the gauginos λ_a, as well as the canonical coupling of the gauginos to the gauge fields, which is determined by the group structure constants f^{abc}. Equation (2.26) does not contain a kinetic energy term for the D_a fields. Therefore these fields have just auxiliary nature, and can again easily be integrated out. Equations (2.24) and (2.26) show that their equation of motion is given by

$$D_a = -g\sum_{i,j}\phi_i^* T_a^{ij}\phi_j, \tag{2.27}$$

where the group indices have been written explicitly. The third term in the second line in eq. (2.24) and the second term in eq. (2.26) then combine to give a contribution

$$-V_D = -\frac{1}{2}\sum_a\left|\sum_{i,j} g\phi_i^* T^a_{ij}\phi_j\right|^2 , \tag{2.28}$$

to the scalar interactions in the Lagrangian. These interactions are completely fixed by the gauge couplings. This completes the construction of the Lagrangian for a renormalizable supersymmetric field theory.

2.2 Field content of the MSSM

The simplest and most attractive extension of the Standard Model is the *Minimal Supersymmetric Standard Model* (MSSM).Minimal refers to the additional field content, which is kept as low as possible. In particular, the field content of the MSSM consists only of the SM fields and their supersymmetric partners, and one additional Higgs doublet.

- **Gauge fields**
  In order to respect the $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$ gauge symmetry of the SM, the spin–1

gauge bosons are described by the corresponding vector superfields. In particular, the eight gluons of QCD, G^a_μ, get eight spin–$\frac{1}{2}$ partners $\tilde{G}^a$ called *gluinos*, the $SU(2)$ gauge bosons W^i_μ get three *winos* $\tilde{W}^i$ as partners and the $U(1)$ gauge boson B_μ gets a *bino* $\tilde{B}$. Note that since $SU(2)_L \times U(1)_Y$ is broken in the SM, the winos and the bino do not form mass eigenstates but mix with fields with the same charge but different $SU(2)_L \otimes U(1)_Y$ quantum numbers.

Superfield	spin–1	spin–1/2	$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$	Names
$\hat{V}^a_s$	G^a_μ	$\tilde{G}^a$	$(\mathbf{8},\, \mathbf{1},\, 0)$	gluons, gluinos
$\hat{V}^i$	W^i_μ	$\tilde{W}^i$	$(\mathbf{1},\, \mathbf{3},\, 0)$	W-bosons, winos
$\hat{V}'$	B_μ	$\tilde{B}$	$(\mathbf{1},\, \mathbf{1},\, 0)$	B-boson, bino

Table 2.1: Gauge supermultiplet fields in the MSSM.

- **Matter fields**
  The matter content of the SM is described by three generations of leptons and quarks, i. e. for each generation two $SU(2)_L$ fermion doublets and three singlets for the right-handed fermions,

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad E = e^c_R\,, \quad Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad D = d^c_R\,, \quad U = u^c_R\,. \tag{2.29}$$

  Therefore, one generation of the SM is represented by five left-chiral superfields which contain the leptons and quarks given above plus their supersymmetric partners, the *sleptons* and *squarks*:

$$\tilde{L} = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}, \quad \tilde{E} = \tilde{e}^*_R\,, \quad \tilde{Q} = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}, \quad \tilde{D} = \tilde{d}^*_R\,, \quad \tilde{U} = \tilde{u}^*_R\,. \tag{2.30}$$

- **Higgs sector**
  Contrary to the SM, two chiral superfield doublets with hypercharges ± 1 are required to break $SU(2)_L \times U(1)_Y$ invariance and to give masses to both up– and down–type fermions. One reason is that if there would be only one single chiral superfield doublet, the gauge symmetry would suffer a fermion triangle gauge anomaly. This can easily be seen from the conditions for anomaly cancellation, $\mathrm{Tr}[Y^3] = \mathrm{Tr}[T_3^2 Y] = 0$, where T_3 and Y denote the third component of the isospin and the weak hypercharge, respectively, and the electric charge given by $Q = T_3 + Y/2$. In the SM these conditions are satisfied by a complete generation of SM fermions. To cancel the contribution from one superfield doublet, one needs a second doublet to get a consistent quantum theory. The two Higgs

Superfield	spin–1/2	spin–0	$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$	Names
$\hat{Q}$	(u_L, d_L)	$(\tilde{u}_L, \tilde{d}_L)$	$(\mathbf{3},\, \mathbf{2},\, \frac{1}{3})$	quarks, squarks
$\hat{U}^c$	$\bar{u}_R$	$\tilde{u}^*_R$	$(\bar{\mathbf{3}},\, \mathbf{1},\, -\frac{4}{3})$	(× 3 families)
$\hat{D}^c$	$\bar{d}_R$	$\tilde{d}^*_R$	$(\bar{\mathbf{3}},\, \mathbf{1},\, \frac{2}{3})$	
$\hat{L}$	(ν_L, e_L)	$(\tilde{\nu}_L, \tilde{e}_L)$	$(\mathbf{1},\, \mathbf{2},\, \text{-}1)$	leptons, sleptons
$\hat{E}^c$	$\bar{e}_R$	$\tilde{e}^*_R$	$(\mathbf{1},\, \mathbf{1},\, 2)$	(× 3 families)
$\hat{H}_1$	$(\tilde{H}^1_1, \tilde{H}^2_1)$	(H^1_1, H^2_1)	$(\mathbf{1},\, \mathbf{2},\, \text{-}1)$	higgsinos, Higgs
$\hat{H}_2$	$\tilde{H}^1_2, \tilde{H}^2_2$	H^1_2, H^2_2	$(\mathbf{1},\, \mathbf{2},\, 1)$	

Table 2.2: Chiral supermultiplet fields in the MSSM.

doublets and their superpartners, the *higgsinos* $\tilde{H}^j_i$, are given as follows:

$$
\begin{aligned}
H_1 &= \begin{pmatrix} H^1_1 \\ H^2_1 \end{pmatrix}, \qquad H_2 = \begin{pmatrix} H^1_2 \\ H^2_2 \end{pmatrix}, \\
\tilde{H}_1 &= \begin{pmatrix} \tilde{H}^1_1 \\ \tilde{H}^2_1 \end{pmatrix}, \qquad \tilde{H}_2 = \begin{pmatrix} \tilde{H}^1_2 \\ \tilde{H}^2_2 \end{pmatrix}.
\end{aligned} \tag{2.31}
$$

In general, interactions in the MSSM have two different sources:

- **Superpotential**
  The superpotential for the MSSM is given by

$$
W = \varepsilon_{ij} \left[h_e \hat{H}^i_1 \hat{L}^j \hat{E}^c + h_d \hat{H}^i_1 \hat{Q}^j \hat{D}^c + h_u \hat{H}^j_2 \hat{Q}^i \hat{U}^c - \mu \hat{H}^i_1 \hat{H}^j_2 \right], \tag{2.32}
$$

  where the hatted quantities $\hat{H}^i_j, \hat{Q}^i, \hat{L}^j, \hat{U}, \hat{D}, \hat{E}$ are the chiral superfields given in Table 2.2. Due to better readability we have suppressed all colour, weak isospin and generation indices.

  The superpotential determines two kinds of interactions mentioned in eq. (2.40). Firstly, the Yukawa potential V_Y can be obtained by replacing two superfields in the superpotential by the corresponding fermionic fields and the remaining superfield by its scalar representative,

$$
\begin{aligned}
V_Y &= \varepsilon_{ij} \left[h_e H^i_1 L^j E^c + h_d H^i_1 Q^j D^c + h_u H^j_2 Q^i U^c - \mu \tilde{H}^i_1 \tilde{H}^j_2 \right] \\
&+ \varepsilon_{ij} \left[h_e \tilde{H}^i_1 L^j \tilde{E}^c + h_d \tilde{H}^i_1 Q^j \tilde{D}^c + h_u \tilde{H}^j_2 Q^i \tilde{U}^c \right] \\
&+ \varepsilon_{ij} \left[h_e H^i_1 \tilde{L}^j E^c + h_d H^i_1 \tilde{Q}^j D^c + h_u H^j_2 \tilde{Q}^i U^c \right] \\
&+ \text{h.c.}
\end{aligned} \tag{2.33}
$$

The F–term potential V_F originates from using the equations of motion for the auxiliary fields F_i,

$$V_F = \sum_i F^{*i} F_i = \sum_i \left| \frac{\delta W(\varphi)}{\delta \varphi_i} \right|^2 \tag{2.34}$$

where the sum is taken over all scalar components φ_i of the superfields.

- **Gauge Symmetry**
  The MSSM contains the usual gauge interactions, but there are also gauge–strength interactions involving neither gauge bosons nor gauginos arising from the D-term potential in eq.(2.28).

  In fact, there exists one additional kind of interaction allowed by gauge invariance involving the gaugino fields. The corresponding potential $V_{\tilde{G}\psi\tilde{\psi}}$ is given by

$$V_{\tilde{G}\psi\tilde{\psi}} = i\sqrt{2} g_a \varphi_k \bar{\lambda}^a (T^a)_{kl} \bar{\psi}_l + \text{h.c.}\,, \tag{2.35}$$

  where (φ, ψ) are the spin–0 and spin spin–$\frac{1}{2}$ components of the chiral superfield, respectively, and λ^a denoting the gaugino field.

- **SUSY breaking sector**
  Sine no SUSY particle has been observed up to now, SUSY must be broken to be in agreement with experiment. Therefore it is necessary to introduce explicit SUSY breaking terms in the Lagrangian. The last term in the full Lagrangian of the MSSM $\mathcal{L}_{\text{soft}}$ involves the soft SUSY–breaking terms and can be written as

  - Mass terms for the gluinos, winos and binos:

$$-\mathcal{L}_{\text{gaugino}} = \frac{1}{2}\left[M_1 \tilde{B}\tilde{B} + M_2 \sum_{a=1}^{3} \tilde{W}^a \tilde{W}_a + M_3 \sum_{a=1}^{8} \tilde{G}^a \tilde{G}_a \ + \ \text{h.c.} \right] \tag{2.36}$$

  - Mass terms for the scalar fermions:

$$-\mathcal{L}_{\text{sfermions}} = \sum_{i=gen} m^2_{\tilde{Q}_i} \tilde{Q}_i^\dagger \tilde{Q}_i + m^2_{\tilde{L}_i} \tilde{L}_i^\dagger \tilde{L}_i + m^2_{\tilde{u}_i} |\tilde{u}_{R_i}|^2 + m^2_{\tilde{d}_i} |\tilde{d}_{R_i}|^2 + m^2_{\tilde{\ell}_i} |\tilde{\ell}_{R_i}|^2 \tag{2.37}$$

  - Mass and bilinear terms for the Higgs bosons:

$$-\mathcal{L}_{\text{Higgs}} = m^2_{H_2} H_2^\dagger H_2 + m^2_{H_1} H_1^\dagger H_1 + B\mu (H_2 \cdot H_1 + \text{h.c.}) \tag{2.38}$$

  - Trilinear couplings between sfermions and Higgs bosons

$$-\mathcal{L}_{\text{tril.}} = \sum_{i,j=gen} \left[A^u_{ij} Y^u_{ij} \tilde{u}^*_{R_i} H_2 \cdot \tilde{Q}_j + A^d_{ij} Y^d_{ij} \tilde{d}^*_{R_i} H_1 \cdot \tilde{Q}_j + A^l_{ij} Y^\ell_{ij} \tilde{\ell}^*_{R_i} H_1 \cdot \tilde{L}_j \ + \ \text{h.c.} \right] . \tag{2.39}$$

The complete Lagrangian of the MSSM can be written as

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{kinetic}} - V_Y - V_F - V_D - V_{\tilde{G}\psi\tilde{\psi}} + \mathcal{L}_{\text{soft}}\,, \tag{2.40}$$

where $\mathcal{L}_{\text{kinetic}}$ stands for both the standard kinetic terms for each particle and their interactions with the gauge bosons. The interactions are described by the potentials V_X and the last term $\mathcal{L}_{\text{soft}}$ includes the SUSY–breaking terms.

2.3 MSSM spectrum

2.3.1 Higgs sector

As stated before, two complex Higgs doublets (eight degrees of freedom) are required to keep the MSSM anomaly free. Three of the eight degrees of freedom become the longitudinal modes of the vector bosons Z^0 and $W^\pm$, therefore these bosons become massive. The remaining five degrees of freedom, correspond to three neutral Higgs bosons h^0, H^0, A^0 and the two charged ones H^+ and H^-. In the MSSM the scalar Higgs potential V_H has three different sources :
i) The D terms containing the quartic Higgs interactions, eq. (2.28). For the two Higgs fields H_1 and H_2 with $Y = -1$ and $+1$, these terms are given by

$$\mathrm{U(1)_Y} \quad : \quad V_D^1 = \frac{1}{2}\left[\frac{g'}{2}(|H_2|^2 - |H_1|^2)\right]^2 , \tag{2.41}$$

$$\mathrm{SU(2)_L} \quad : \quad V_D^2 = \frac{1}{2}\left[\frac{g}{2}(H_1^{i*}\tau^a_{ij}H_1^j + H_2^{i*}\tau^a_{ij}H_2^j)\right]^2 , \tag{2.42}$$

with $\tau^a = 2T^a$. Using the SU(2) identity $\tau^a_{ij}\tau^a_{kl} = 2\delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl}$, one obtains the potential

$$V_D = \frac{g^2}{8}\left[4|H_1^\dagger \cdot H_2|^2 - 2|H_1|^2|H_2|^2 + (|H_1|^2)^2 + (|H_2|^2)^2\right] + \frac{g'^2}{8}(|H_2|^2 - |H_1|^2)^2 . \tag{2.43}$$

ii) The F term of the Superpotential eq. (2.32) can be written as $V_F = \sum_i |\partial W(\phi_j)/\partial\phi_i|^2$. From the term $W \sim \mu\hat{H}_1\cdot\hat{H}_2$, one obtains the component

$$V_F = \mu^2(|H_1|^2 + |H_2|^2) . \tag{2.44}$$

iii) Finally, there is a part originating from the soft SUSY–breaking scalar Higgs mass terms and the bilinear term

$$V_{\text{soft}} = m_{H_1}^2 H_1^\dagger H_1 + m_{H_2}^2 H_2^\dagger H_2 + B\mu(H_2\cdot H_1 + \text{h.c.}) . \tag{2.45}$$

The full scalar potential involving the Higgs fields, is the sum of these three terms

$$\begin{aligned} V_H &= (|\mu|^2 + m_{H_1}^2)|H_1|^2 + (|\mu|^2 + m_{H_2}^2)|H_2|^2 - \mu B\epsilon_{ij}(H_1^i H_2^j + \text{h.c.}) \\ &+ \frac{g^2 + g'^2}{8}(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2}g^2|H_1^\dagger H_2|^2 , \end{aligned}$$

using the abbreviations $m_i^2 = m_{H_i}^2 + |\mu|^2$ and $m_{12}^2 = \mu B$, the scalar Higgs potential can be expressed in a more compact form by

$$\begin{aligned} V_H &= m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_{12}^2(H_1H_2 + H_1^\dagger H_2^\dagger) \\ &\quad + \frac{1}{8}(g^2 + g'^2)(|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2}|H_1^\dagger H_2|^2 . \end{aligned} \tag{2.46}$$

In the MSSM Higgs sector we have three free parameters: m_1^2, m_2^2 and m_{12}^2. The two combinations $m_{H_1,H_2}^2 + |\mu|^2$ are real, only $B\mu$ can be complex. However, any phase in $B\mu$ can be absorbed into the phases of the fields H_1 and H_2. Therefore the scalar potential of the MSSM is CP conserving at tree–level. In contrast to the SM, where the strength of the Higgs self–interaction is an unknown free parameter, the quartic interactions in the MSSM are completely determined by the electroweak gauge couplings g' and g.

Electroweak symmetry breaking (EWSB)

To break electroweak symmetry, we demand that the minimum of the potential V_H breaks the $\mathrm{SU(2)_L \times U(1)_Y}$ group down to the electromagnetic symmetry $\mathrm{U(1)_Q}$. At the minimum of the potential $V_H^{\rm min}$ we can always choose the vacuum expectation value of the field H_1^- to be zero $\langle H_1^- \rangle = 0$, this can be done due to the SU(2) symmetry without loss of generality. At $\partial V/\partial H_1^-$=0, one then automatically obtains $\langle H_2^+ \rangle$=0. Since both charged components of the Higgs scalars remain unaffected, electromagnetism is not spontaneously broken, which is in agreement with experiment. Therefore, only the neutral Higgs boson fields acquire a non-vanishing VEV, i.e.

$$\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix} , \qquad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix} . \tag{2.47}$$

To have electroweak symmetry breaking, one needs a combination of the H_1^0 and H_2^0 fields to have a negative squared mass term. This occurs only if

$$m_{12}^2 > m_1^2 m_2^2 , \tag{2.48}$$

if not, V_H will have a stable minimum and there is no EWSB.In the direction $|H_1^0|$=$|H_2^0|$, there is no quartic term. V_H is bounded from below for large values of the field H_i only if

$$m_1^2 + m_2^2 > 2|m_{12}^2| \tag{2.49}$$

is satisfied. To have explicit electroweak symmetry breaking and, thus, a negative squared term in the Lagrangian, the potential at the minimum should have a saddle point and therefore

$$\mathrm{Det}\left(\frac{\partial^2 V_H}{\partial H_i^0 \partial H_j^0}\right) < 0 \Rightarrow m_1^2\, m_2^2 < m_{12}^4 . \tag{2.50}$$

The two conditions above on the masses m_i are not satisfied if $m_1^2 = m_2^2$ and, thus, we must have non–vanishing soft SUSY–breaking scalar masses m_{H_1} and m_{H_2}

$$m_1^2 \neq m_2^2 \Rightarrow m_{H_1}^2 \neq m_{H_2}^2 \tag{2.51}$$

Therefore, to break the electroweak symmetry, we need also to break SUSY. This provides a close connection between gauge symmetry breaking and SUSY–breaking. In constrained models such as mSUGRA, the soft SUSY–breaking scalar Higgs masses have equal values at a high–energy scale ($m_{H_1}^2 = m_{H_1}^2 > 0$). However, in the running to lower energy scales via RGEs, one obtains $m_{H_2}^2 < 0$ or $m_{H_2}^2 \ll m_{H_1}^2$ which thus triggers EWSB: this is the radiative breaking of the symmetry [103]. Thus, electroweak symmetry breaking is more natural and elegant in the MSSM than in the SM since, we do not need to make the ad hoc choice $\mu^2 < 0$, in the MSSM this comes simply from radiative corrections.

2.3.2 MSSM Higgs mass matrices

For the two Higgs doublets we choose the following common parameterization:

$$H_1 \equiv \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix} , \qquad Y_{H_1} = -1 , \tag{2.52}$$

$$H_2 \equiv \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix} , \qquad Y_{H_2} = +1 . \tag{2.53}$$

The minimum of the Higgs potential can now be obtained easily by solving the equations

$$\left.\frac{\partial V}{\partial H_1^0}\right|_{\langle H_n^0\rangle=v_n} = \left.\frac{\partial V}{\partial H_2^0}\right|_{\langle H_n^0\rangle=v_n} = 0\,, \tag{2.54}$$

resulting in the two minimization conditions

$$m_1^2 v_1 = -m_{12}^2 v_2 - \frac{1}{4}(g^2+g'^2)(v_1^2-v_2^2)\,, \tag{2.55}$$

$$m_2^2 v_2 = -m_{12}^2 v_1 + \frac{1}{4}(g^2+g'^2)(v_1^2-v_2^2)\,. \tag{2.56}$$

The weak boson masses can be expressed with the VEVs v_1 and v_2 by

$$m_Z^2 = \frac{g^2+g'^2}{2}(v_1^2+v_2^2)\,, \qquad m_W^2 = \frac{g^2}{2}(v_1^2+v_2^2)\,, \tag{2.57}$$

and hence

$$v^2 \equiv (v_1^2+v_2^2) = \frac{2m_Z^2}{g^2+g'^2} \approx (174\ \text{GeV})^2\,, \tag{2.58}$$

is very well known from experiment, we can express both VEVs in terms of one single parameter,

$$\tan\beta \equiv \frac{v_2}{v_1} \geq 0\,, \qquad 0 \leq \beta \leq \frac{\pi}{2}\,. \tag{2.59}$$

Eqs. (2.55) and (2.56) may now be written as

$$m_1^2 = -m_{12}^2 \tan\beta - \frac{1}{2}m_Z^2\cos 2\beta\,, \tag{2.60}$$

$$m_2^2 = -m_{12}^2 \cot\beta + \frac{1}{2}m_Z^2\cos 2\beta\,. \tag{2.61}$$

Therefore, the Higgs sector at tree level only depends on two free parameters.
The Higgs mass spectrum is obtained by evaluating the second derivatives of the Higgs potential, taken at its minimum,

$$M_{ij}^{2,\text{Higgs}} = \frac{1}{2}\left.\frac{\partial^2 V_H}{\partial H_i \partial H_j}\right|_{\langle H_n^0\rangle=v_n}\,. \tag{2.62}$$

At tree level, $M_{ij}^{2,\text{Higgs}}$ splits into four independent 2×2 mass matrices which can be separately diagonalized. In terms of the original gauge eigenstate fields, the mass eigenstates are given by

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}\,, \tag{2.63}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} -\cos\beta & \sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix}\,, \tag{2.64}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} -\cos\beta & \sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}\,. \tag{2.65}$$

The Goldstone bosons G^0 and $G^\pm$ are 'eaten' by the longitudinal components of the massive vector bosons Z^0 and $W^\pm$, respectively. The five remaining physical Higgs bosons form two CP–even states (h^0, H^0), one CP–odd state A^0 and the two charged Higgs bosons $H^\pm$. As already mentioned above, the two free parameters in the Higgs sector are conventionally chosen to be the mass of the pseudo–scalar Higgs boson A^0 and the ratio of the two VEVs,

$$m_{A^0} \qquad \text{and} \qquad \tan\beta\,. \tag{2.66}$$

The remaining parameters such as the masses and the mixing angle α can be expressed using these free parameters as

$$m^2_{h^0,H^0} = \frac{1}{2}\left[m^2_{A^0} + m^2_Z \mp \sqrt{(m^2_{A^0}+m^2_Z)^2 - 4m^2_{A^0}m^2_Z\cos^2\beta}\right], \tag{2.67}$$

$$m^2_{H^\pm} = m^2_{A^0} + m^2_W\,, \tag{2.68}$$

$$\tan 2\alpha = \tan 2\beta\frac{m^2_{A^0}+m^2_Z}{m^2_{A^0}-m^2_Z}\,, \qquad -\frac{\pi}{2} \le \alpha \le 0\,. \tag{2.69}$$

2.3.3 Sfermion sector

The sfermion mixing is described by the sfermion mass matrix in the left–right basis $(\tilde f_L, \tilde f_R)$, and in the mass basis $(\tilde f_1, \tilde f_2)$, $\tilde f = \tilde t, \tilde b$ or $\tilde\tau$,

$$\mathcal{M}^2_{\tilde f} = \begin{pmatrix} m^2_{\tilde f_L} & a_f\, m_f \\ a_f\, m_f & m^2_{\tilde f_R}\end{pmatrix} = \left(R^{\tilde f}\right)^\dagger \begin{pmatrix} m^2_{\tilde f_1} & 0 \\ 0 & m^2_{\tilde f_2}\end{pmatrix} R^{\tilde f}\,, \tag{2.70}$$

where $R^{\tilde f}_{i\alpha}$ is a 2×2 rotation matrix with rotation angle $\theta_{\tilde f}$,

$$R^{\tilde f}_{ij} = \begin{pmatrix} \cos\theta_{\tilde f} & \sin\theta_{\tilde f} \\ -\sin\theta_{\tilde f} & \cos\theta_{\tilde f}\end{pmatrix}, \tag{2.71}$$

which relates the mass eigenstates $\tilde f_i$, $i = 1, 2$, $(m_{\tilde f_1} < m_{\tilde f_2})$ to the gauge eigenstates $\tilde f_\alpha$, $\alpha = L, R$, by $\tilde f_i = R^{\tilde f}_{i\alpha}\tilde f_\alpha$ and

$$m^2_{\tilde f_L} = M^2_{\{\tilde Q,\,\tilde L\}} + (I^{3L}_f - e_f\sin^2\theta_W)\cos 2\beta\, m^2_Z + m^2_f\,, \tag{2.72}$$

$$m^2_{\tilde f_R} = M^2_{\{\tilde u,\,\tilde d,\,\tilde e\}} + e_f\sin^2\theta_W\cos 2\beta\, m^2_Z + m^2_f\,, \tag{2.73}$$

$$a_f = A_f - \mu\,(\tan\beta)^{-2I^{3L}_f}\,. \tag{2.74}$$

$M_{\tilde Q}$, $M_{\tilde L}$, $M_{\tilde u}$, $M_{\tilde d}$ and $M_{\tilde e}$ are soft SUSY–breaking masses, A_f is the trilinear scalar coupling parameter, μ the higgsino mass parameter, I^{3L}_f denotes the third component of the weak isospin of the fermion f, e_f the electric charge in terms of the elementary charge e_0 and θ_W denotes the Weinberg angle.
The mass eigenvalues and the mixing angle in terms of primary parameters are

$$m^2_{\tilde f_{1,2}} = \frac{1}{2}\left(m^2_{\tilde f_L} + m^2_{\tilde f_R} \mp \sqrt{(m^2_{\tilde f_L} - m^2_{\tilde f_R})^2 + 4a^2_f m^2_f}\right), \tag{2.75}$$

$$\cos\theta_{\tilde f} = \frac{-a_f\, m_f}{\sqrt{(m^2_{\tilde f_L} - m^2_{\tilde f_1})^2 + a^2_f m^2_f}} \qquad (0 \le \theta_{\tilde f} < \pi)\,, \tag{2.76}$$

and the trilinear breaking parameter A_f can be written as

$$m_f A_f \;=\; \frac{1}{2}(m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}) \sin 2\theta_{\tilde{f}} + m_f\, \mu\, (\tan\beta)^{-2I^{3L}_f} \,. \tag{2.77}$$

The mass of the sneutrino $\tilde{\nu}_\tau$ is given by

$$m^2_{\tilde{\nu}_\tau} = M^2_{\tilde{L}} + \frac{1}{2}\, m^2_Z \cos 2\beta \,. \tag{2.78}$$

2.3.4 Chargino and Neutralino sector

The fermionic superpartners of the gauge bosons, the gauginos, and the superpartners of the Higgs bosons, the higgsinos, mix to form mass eigenstates called charginos and neutralinos. The charginos are therefore the superpartners of the gauge bosons $W^\pm$ and the charged Higgs bosons $H^\pm$. In the Weyl representation, the chargino fields [12]

$$\psi^+ = (-i\tilde{W}^+, \tilde{H}^+_2)\,, \tag{2.79}$$
$$\psi^- = (-i\tilde{W}^-, \tilde{H}^-_1)\,, \tag{2.80}$$

enter in the mass term of the Lagrangian in the following form:

$$\mathcal{L} = -\frac{1}{2}\left(\psi^+, \psi^-\right) \cdot \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \cdot \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{h.c.}\,, \tag{2.81}$$

with

$$X = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix} . \tag{2.82}$$

Since we work in the CP–conserving MSSM, the mass matrix X can be diagonalized by two *real* 2×2 matrices U and V according to

$$U X V^{-1} = \begin{pmatrix} m_{\tilde{\chi}^\pm_1} & 0 \\ 0 & m_{\tilde{\chi}^\pm_1} \end{pmatrix} , \qquad |m_{\tilde{\chi}^\pm_1}| \leq |m_{\tilde{\chi}^\pm_2}| \,. \tag{2.83}$$

In the Dirac representation, the mass eigenstates are related to the gauge eigenstates by

$$\tilde{\chi}^+_i \;\equiv\; \begin{pmatrix} V_{ij}\, \psi^+_j \\ U_{ij}\, \bar{\psi}^-_j \end{pmatrix} . \tag{2.84}$$

As these matrices are only of rank 2, the mass eigenvalues can be given analytically:

$$m^2_{\tilde{\chi}^\pm_{1,2}} \;=\; \frac{1}{2}\left[M^2_2 + \mu^2 + 2m^2_W \mp \sqrt{(M^2_2 + \mu^2 + 2m^2_W)^2 - 4(m^2_W \sin 2\beta - \mu M_2)^2} \right] \tag{2.85}$$

The superpartners of the neutral gauge bosons, $\tilde{B}_\mu$ and $\tilde{W}^3_\mu$, and of the neutral Higgs bosons,

$\tilde{H}_1^0$ and $\tilde{H}_2^0$, mix to form four neutral mass eigenstates called *neutralinos*. In the interaction base one can combine the four Weyl states as

$$\psi_j^0 = (-i\tilde{B}, -i\tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)\,. \tag{2.86}$$

In terms of the vector ψ^0 the neutralino mass terms in the Lagrangian are

$$\mathcal{L} \;=\; -\frac{1}{2}\left(\psi^0\right)^T Y\,\psi^0 + h.c.\,, \tag{2.87}$$

where we used the neutralino mass matrix defined as

$$Y = \begin{pmatrix} M_1 & 0 & -m_Z s_W \cos\beta & m_Z s_W \sin\beta \\ 0 & M_2 & m_Z c_W \cos\beta & -m_Z c_W \sin\beta \\ -m_Z s_W \cos\beta & m_Z c_W \cos\beta & 0 & -\mu \\ m_Z s_W \sin\beta & -m_Z c_W \sin\beta & -\mu & 0 \end{pmatrix}. \tag{2.88}$$

We use the short forms s_W and c_W for the sine and the cosine of the Weinberg angle.
Due to the Majorana nature of the neutralinos, the matrix can be diagonalized using only one single rotation matrix,

$$ZYZ^{-1} = \mathrm{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}), \qquad |m_{\tilde{\chi}_1^0}| \le |m_{\tilde{\chi}_2^0}| \le |m_{\tilde{\chi}_3^0}| \le |m_{\tilde{\chi}_4^0}|\,, \tag{2.89}$$

where we again assume the mixing matrix to be real and allow the eigenvalues to be negative. The 4–component Majorana spinors for the neutralino fields can be constructed as

$$\tilde{\chi}_i^0 \;\equiv\; Z_{ij}\tilde{\psi}_j^0\,, \tag{2.90}$$

with the corresponding mass term Lagrangian

$$\mathcal{L} \;=\; -\frac{1}{2}\sum_{i=1}^{4} m_{\tilde{\chi}_i^0}\,\bar{\tilde{\chi}}_i^0\,\tilde{\chi}_i^0\,. \tag{2.91}$$

2.4 The unconstrained and constrained MSSMs

In the general unconstrained MSSM, where one allows intergenerational mixing and complex phases, the soft SUSY–breaking terms introduce a huge number (105) of unknown parameters, in addition to the 19 parameters of the SM [13]. This large number of parameters makes any phenomenological analysis in the MSSM very complicated. Many sets of these parameters are excluded by various phenomenological constraints. A phenomenologically viable MSSM can be defined by making the following assumptions:

- All the soft SUSY–breaking parameters are real and as a consequence there is no new source of CP–violation, in addition to the one from the CKM matrix.

- The matrices for the sfermion masses and for the trilinear couplings are all diagonal, implying the absence of FCNCs at tree–level.

- The soft SUSY–breaking masses and trilinear couplings of the first and second sfermion generations are the same at low energy to fulfill the severe constraints from K^0–$\bar{K}^0$ mixing.

Using these three assumptions leads to only 22 input parameters:

- $\tan\beta$: the ratio of the vevs of the two–Higgs doublet fields
- $m^2_{H_1}, m^2_{H_2}$: the Higgs mass parameters squared
- M_1, M_2, M_3: the bino, wino and gluino mass parameters
- $m_{\tilde{q}}, m_{\tilde{u}_R}, m_{\tilde{d}_R}, m_{\tilde{l}}, m_{\tilde{e}_R}$: the first/second generation sfermion mass parameters
- A_u, A_d, A_e: the first/second generation trilinear couplings
- $m_{\tilde{Q}}, m_{\tilde{t}_R}, m_{\tilde{b}_R}, m_{\tilde{L}}, m_{\tilde{\tau}_R}$: the third generation sfermion mass parameters
- A_t, A_b, A_τ: the third generation trilinear couplings

The Higgs–higgsino mass parameter $|\mu|$ (up to a sign) and the soft SUSY–breaking bilinear Higgs term B are determined, given the parameters above, through the electroweak symmetry breaking conditions.
Alternatively, one can express the values of $m^2_{H_1}$ and $m^2_{H_2}$ with the "more physical" pseudoscalar Higgs boson mass m_{A^0} and the parameter μ. Since the trilinear sfermion couplings will be always multiplied by the fermion masses, they are in general important only in the case of the third generation; there are, however, a few exceptions such as the electric and magnetic dipole moments for instance.

Such a model, with this relatively moderate number of parameters has much more predictability and is much easier to investigate phenomenologically, compared to the unconstrained MSSM, given the fact that in general only a small subset appears when one looks at a given sector of the model. One can refer to this 22 free input parameters model as the "phenomenological" MSSM or pMSSM [14].

Almost all problems of the general or unconstrained MSSM are solved at once if the soft SUSY–breaking parameters obey a set of universal boundary conditions at the GUT scale. If one takes these parameters to be real, this solves all potential problems with CP violation as well. The underlying assumption is that SUSY–breaking occurs in a hidden sector which communicates with the visible sector only through gravitational–strength interactions, as specified by Supergravity. Universal soft breaking terms then emerge, if these Supergravity interactions are "flavor–blind". This is assumed to be the case in the constrained MSSM or minimal Supergravity (mSUGRA) model [15, 16].

Besides the unification of the gauge coupling constants $g^{'}$, g, g_s which is verified given the experimental results from LEP1 [17] and which can be viewed as fixing the Grand Unification scale, $M_U \sim 2 \cdot 10^{16}$ GeV, the unification conditions in mSUGRA, are as follows [15]:

- Unification of the gaugino [bino, wino and gluino] masses

$$M_1(M_U) = M_2(M_U) = M_3(M_U) \equiv m_{1/2} \ . \tag{2.92}$$

- Universal scalar [i.e. sfermion and Higgs boson] masses [i denotes the generation index]:

$$\begin{aligned} m_{\tilde{Q}_i}(M_U) &= m_{\tilde{u}_{Ri}}(M_U) = m_{\tilde{d}_{Ri}}(M_U) = m_{\tilde{L}_i}(M_U) = m_{\tilde{\ell}_{Ri}}(M_U) \ , \\ &= m_{H_1}(M_U) = m_{H_2}(M_U) \equiv m_0 \ . \end{aligned} \tag{2.93}$$

- Universal trilinear couplings:

$$A^u_{ij}(M_U) = A^d_{ij}(M_U) = A^\ell_{ij}(M_U) \equiv A_0\, \delta_{ij} \ . \tag{2.94}$$

Besides the three parameters $m_{1/2}, m_0$ and A_0, the supersymmetric sector is described at the GUT scale by the bilinear coupling B and the supersymmetric Higgs(ino) mass parameter μ. However, one has to require that EWSB takes place at some low energy scale. This results in two necessary minimization conditions of the two–Higgs doublet scalar potential which fix the values μ^2 and $B\mu$ with the sign of μ not determined. Therefore, in this model, one is left with only four continuous free parameters, and an unknown sign

$$m_0 \ , \ m_{1/2} \ , \tan\beta \ , \ \mathrm{sign}(\mu) \ , \ A_0 \ . \tag{2.95}$$

All soft SUSY–breaking parameters at the weak scale are derived via RGEs [18, 19, 20]. There are also other constrained MSSM scenarios like anomaly mediated SUSY–breaking (AMSB) models [21, 22] and gauge mediated SUSY–breaking [27, 28, 29] models.

2.4.1 Non Universal Higgs Mass (NUHM) scenario

In the more general MSSM with non–universal Higgs masses (NUHM), the soft SUSY–breaking scalar masses for the Higgs multiplets m_1 and m_2 or alternatively the pairs μ and m_{A^0} become free again [23, 24, 25]. Thus one may use the parameters $(m_0,\ m_{1/2},\ \tan\beta,\ \mathrm{sign}(\mu),\ A_0,\ m_1,\ m_2)$ to parametrize this more general NUHM. In the NUHM1 scenario m_1 and m_2 have the same value, in NUHM2 they have different values at the GUT scale.

2.5 The Higgs sector in the decoupling regime

The decoupling limit is reached for $m_{A^0} \gg m_Z$. For the discussion of the other parameter regimes in the MSSM Higgs sector we refer to [26]. When the pseudoscalar Higgs mass becomes large compared to m_Z, the lighter CP–even Higgs boson h^0 approaches its maximal mass value, when the dominant radiative corrections are taken into account. In this limit the masses of the heavier CP–even Higg boson H^0 and the charged Higgs bosons $m^2_{H^\pm} = m^2_{A^0} + m^2_W$, become very close to m_{A^0}. The existence of only one light Higgs boson is one important aspect of the decoupling regime in the MSSM. The other Higgs particles are very heavy and degenerate in mass $m_{H^0} \simeq m_{H^\pm} \simeq m_{A^0}$.

2.5.1 Higgs couplings to fermions

Expressing the couplings of the CP–even Higgs bosons to isospin $\frac{1}{2}$ and $-\frac{1}{2}$ fermions in terms of $\cos(\beta-\alpha)$ with the latter given by eq. (2.98) in the decoupling limit one obtains the following results $m_{A^0} \gg m_{Z^0}$ [104]

$$
\begin{aligned}
g_{h^0uu}: &\overset{m_{A^0}\gg m_Z}{\rightarrow} 1+\frac{m_Z^2}{2m_{A^0}^2}\frac{\sin 4\beta}{\tan\beta} \overset{\tan\beta\gg 1}{\rightarrow} 1-\frac{2m_Z^2}{m_{A^0}^2\tan^2\beta} \rightarrow 1\ , \\
g_{h^0dd}: &\overset{m_{A^0}\gg m_Z}{\rightarrow} 1-\frac{m_Z^2}{2m_{A^0}^2}\sin 4\beta\tan\beta \overset{\tan\beta\gg 1}{\rightarrow} 1+\frac{2m_Z^2}{m_{A^0}^2} \rightarrow 1\ , \\
g_{H^0uu}: &\overset{m_{A^0}\gg m_Z}{\rightarrow} -\cot\beta+\frac{m_Z^2}{2m_{A^0}^2}\sin 4\beta \overset{\tan\beta\gg 1}{\rightarrow} -\cot\beta\left(1+\frac{2m_Z^2}{m_{A^0}^2}\right) \rightarrow -\cot\beta\ , \\
g_{H^0dd}: &\overset{m_{A^0}\gg m_Z}{\rightarrow} \tan\beta+\frac{m_Z^2}{2m_{A^0}^2}\sin 4\beta \overset{\tan\beta\gg 1}{\rightarrow} \tan\beta\left(1-\frac{2m_Z^2}{m_{A^0}^2\tan^2\beta}\right) \rightarrow \tan\beta\ .
\end{aligned}
\tag{2.96}
$$

The couplings of the h^0 boson become SM like, $g_{h^0uu} = g_{h^0dd} = 1$, while the couplings of the H^0 boson reduce, up to a sign, to those of the pseudoscalar Higgs boson, $g_{H^0uu} \simeq g_{A^0uu} = \cot\beta$ and $g_{H^0dd} \simeq g_{A^0dd} = \tan\beta$. The H^0 boson coupling to down type fermions becomes enhanced by $\tan\beta$ which is phenomenologically very important. Again, as a result of the presence of the $\tan\beta$ factors in the denominators of the expansion terms in eq. (2.96), these limits are reached more quickly for large values of $\tan\beta$, except for g_{h^0dd} and g_{H^0uu}. These results are not significantly altered by the inclusion of the radiative corrections in general. For large values of $\tan\beta$, the decoupling limit is already reached for $m_{A^0} \gtrsim m_Z$ at tree–level, but the inclusion of the radiative corrections shifts this value to $m_{A^0} \gtrsim m_{h^0}^{\max}$.

2.5.2 Higgs couplings to vector bosons

CP–invariance does not allow tree–level couplings of the pseudoscalar and charged Higgs bosons to two gauge bosons. The couplings of the CP–even h^0 and H^0 bosons to WW and ZZ states are suppressed by mixing angle factors but are complementary, the sum of their squares being the square of the $h_{\rm SM}VV$ coupling. For large values of m_{A^0}, one can expand these couplings in powers of m_Z/m_{A^0} to obtain at tree–level

$$
\begin{aligned}
g_{H^0VV} = \cos(\beta-\alpha) &\overset{m_{A^0}\gg m_Z}{\rightarrow} \frac{m_Z^2}{2m_{A^0}^2}\sin 4\beta \overset{\tan\beta\gg 1}{\rightarrow} -\frac{2m_Z^2}{m_{A^0}^2\tan\beta} \rightarrow 0\ , \\
g_{h^0VV} = \sin(\beta-\alpha) &\overset{m_{A^0}\gg m_Z}{\rightarrow} 1-\frac{m_Z^4}{8m_{A^0}^4}\sin^2 4\beta \overset{\tan\beta\gg 1}{\rightarrow} 1-\frac{2m_Z^4}{m_{A^0}^4\tan^2\beta} \rightarrow 1\ ,
\end{aligned}
\tag{2.97}
$$

where we have also displayed the limits for large values of $\tan\beta$ using the relation

$$
\sin 4\beta = 4\tan\beta(1-\tan^2\beta)(1+\tan^2\beta)^{-2} \overset{\tan\beta\gg 1}{\rightarrow} -4\cot\beta\ . \tag{2.98}
$$

For $m_{A^0} \gg m_Z$, g_{H^0VV} vanishes while g_{h^0VV} reaches 1, the SM value. This occurs more quickly if $\tan\beta$ is large, since the first term of the expansion involves this parameter in the denominator.

This statement can be generalized to the couplings of two Higgs bosons and one gauge boson and to the quartic couplings between two Higgs and two gauge bosons, which are proportional to either $\cos(\beta-\alpha)$ or $\sin(\beta-\alpha)$ [there are also several angle independent couplings, such as the γH^+H^-, ZH^+H^- and $W^\pm H^\mp A$ couplings and those involving two identical gauge and Higgs bosons as well as the $H^\pm A$ states]. In particular, all couplings involving at least one gauge boson and exactly one non–minimal Higgs particle $A^0, H^0, H^\pm$ vanish for $m_{A^0} \gg m_Z$, while all the couplings involving no other Higgs boson than the lighter h^0 boson reduce to their SM values.

2.5.3 Trilinear Higgs couplings

In the case of the trilinear Higgs couplings, one has to take the radiative corrections into account, as without these contributions, many of the tree–level couplings would vanish. Using the abbreviations

$$x_0 = M_h^2/m_Z^2, x_1 = \sqrt{(x_0 - \epsilon_Z \sin^2\beta)(1 - x_0 + \epsilon_Z \sin^2\beta)} \tag{2.99}$$

$$\epsilon_Z = \epsilon/m_Z^2 \ , \tag{2.100}$$

one obtains for the self–couplings among the neutral Higgs bosons in the ϵ approach [108]

$$\lambda_{hhh} \overset{m_{A^0}\gg m_Z}{\rightarrow} 3x_0 \ , \tag{2.101}$$

$$\lambda_{hHH} \overset{m_{A^0}\gg m_Z}{\rightarrow} 2 - 3(x_0 - \epsilon_Z) \ , \tag{2.102}$$

$$\lambda_{hAA} \overset{m_{A^0}\gg m_Z}{\rightarrow} -(x_0 - \epsilon_Z) \ , \tag{2.103}$$

$$\lambda_{Hhh} \overset{m_{A^0}\gg m_Z}{\rightarrow} -3x_1 - 3\epsilon_Z \sin\beta\cos\beta \ , \tag{2.104}$$

$$\lambda_{HHH} \sim \frac{1}{3}\lambda_{HAA} \overset{m_{A^0}\gg m_Z}{\rightarrow} 3x_1 - 3\epsilon_Z \cot\beta\cos^2\beta \ .$$

At high $\tan\beta$, the expressions simplify to

$$\lambda_{hhh} \simeq 3M_h^2/m_Z^2 \ , \ \lambda_{hHH} \simeq \lambda_{hAA} = -1 \ , \ \lambda_{Hhh} \simeq \lambda_{HHH} \simeq \lambda_{HAA} \simeq 0 \ . \tag{2.105}$$

To summarize:

- The lighter CP–even Higgs boson h^0 approaches its maximal mass value $m_{h^0} \lesssim 140$ GeV.
- The other Higgs bosons are very heavy and degenerate in mass, $m_H \simeq m_{H^\pm} \simeq m_{A^0}$
- H^0 coupling to WW and ZZ becomes supressed
- h^0 coupling to WW and ZZ becomes SM like
- h^0 couplings to up and down type fermions become SM like
- H^0 and A^0 couplings to down type fermions becomes $\tan\beta$ enhanced
- H^0 and A^0 couplings to down type fermions becomes $\tan\beta$ supressed

Chapter 3

Renormalization

Amplitudes beyond–tree level can suffer so called UV divergences originating from integrals with infinite loop momenta. Therefore these integrals have to be treated in a proper way. In order to give such expressions a physical meaning, the divergences have to be absorbed into redefinitions of the fields and parameters of the Lagrangian. However, before we can start with the renormalization procedure, we have to regularize the integrals. Therefore, we will briefly discuss the technique of dimensional regularization and reduction.

3.1 Dimensional regularization (DREG)

In the dimensional regularization approach, the integration over the loop momentum q is taken from 4 to D dimensions with $D = 4 - x\varepsilon$ (with $x = 1$, 2 depending on the literature)

$$\int dq^4 \to \mu^{4-D} \int dq^D \ , \tag{3.1}$$

where μ denotes an arbitrary reference mass. The introduction of the mass μ in D dimensions is necessary in order to keep the dimensions of the integrals the same as in $D = 4$ dimensions. The coupling constant g is replaced by

$$g^2 \to g^2 \mu^{4-D} \ , \tag{3.2}$$

once again this is necessary to keep the dimensions of the Green's function unchanged. The Lorentz covariants $g^{\mu\nu}$ and γ^μ are simply extended to D dimensions by

$$\begin{aligned} g^{\mu\nu} g_{\mu\nu} &= D \ , & (3.3) \\ \{\gamma^\mu, \gamma^\nu\} &= 2 g^{\mu\nu} \mathbb{1} \ , & (3.4) \\ \mathrm{Tr}(\mathbb{1}) &= 4 \quad \text{(per definition)}, & (3.5) \\ \gamma^\mu \gamma_\mu &= D \mathbb{1} \ . & (3.6) \end{aligned}$$

Problems arise with the definition of the γ^5 matrix in D dimensions, because of the unrestricted number of dimensions, it is not possible to construct the equivalent 4-dimensional object $\gamma_5^{(4)}$ as a product of all the γ–matrices. One can either take the product $\bar{\gamma}_5 \propto \gamma_1\gamma_2\gamma_3\gamma_4$

or construct a total anticommuting object γ_5 [102]. The matrix $\bar{\gamma}_5$ will not be totally anti-commuting, and γ_5 is not the product of all γ–matrices. DREG violates SUSY, because fermion and gauge fields have different degrees of freedom in D dimensions. For example a massless gauge field has $(D-2)$ degrees of freedom in D dimensions, but a fermion field has still 2 degrees of freedom in D dimensions. This violates SUSY, since the fermion and the gauge field are located in the same supermultiplet. Therefore DREG is not applicable for MSSM calculations.

3.1.1 Dimensional reduction (DRED)

In dimensional reduction, the loop integration is still performed in D dimensions, but all other tensors connected to vector fields are kept 4–dimensional. In DRED it will be necessary to introduce two different metric tensors $g^{\mu\nu}$ and $\hat{g}^{\mu\nu}$ which fulfill the conditions

$$g^{\mu\nu}\hat{g}_{\nu\rho} = \hat{g}^{\mu}_{\rho}\,, \qquad \hat{g}_{\nu\rho}\hat{g}^{\nu\rho} < D. \tag{3.7}$$

Since DRED keeps the degrees of freedom of the components of the super–multiplets equal in D dimension, it is the appropriate regularization scheme for MSSM calculations at least at one–loop level. Analogously to the $\overline{\text{MS}}$ scheme, one can define a minimal–subtraction scheme in DRED which is then called $\overline{\text{DR}}$.

3.1.2 Multiplicative Renormalization

We will make use of the so–called *multiplicative renormalization*. In this scheme all *bare* parameters and fields entering in the original Lagrangian are replaced by their corresponding *renormalized* ones, which are obtained by the multiplication with appropriate renormalization constants:

$$g_0 \;\to\; Z_g\, g \;=\; \Big(1 + \frac{\delta g}{g}\Big) g\,, \tag{3.8}$$

$$\phi_0 \;\to\; Z_\phi^{1/2}\phi \;=\; \Big(1 + \frac{1}{2}\delta Z_\phi\Big)\phi\,. \tag{3.9}$$

Expanding the renormalization constants Z_g and $Z_\phi^{1/2}$ around the value 1, the original Lagrangian splits into a renormalized Lagrangian and a part containing the *counter terms* δg and δZ_ϕ, i.e.

$$\mathcal{L}\big(g_0, \phi_0\big) \;=\; \mathcal{L}\big(g, \phi\big) + \delta\mathcal{L}\big(g, \phi, \delta g, \delta Z_\phi\big)\,. \tag{3.10}$$

In order to absorb the divergences and to give the parameters a well–defined meaning, these counter terms have to fulfill several requirements depending on the chosen renormalization scheme.

In the program package *HFOLD*, we use the $\overline{\text{DR}}$ renormalization scheme. In the following we will review some results from the renormalization of the Standard Model and discuss the renormalization of two–point–functions. Furthermore, we give all renormalization conditions of the parameters needed for the explicit calculation. In our approach the tree-level couplings are defined in the $\overline{\text{DR}}$ scheme at the scale Q. This implies that there are no coupling CTs at all. The $\overline{\text{DR}}$ scheme is defined by setting the UV divergence parameter $\Delta = 0$. However, in our calculations we work with $\Delta \neq 0$ and take for the coupling CTs only the divergent part $\propto \Delta$. This is very helpful because having a UV finite renormalized amplitude is equivalent to the RG invariance proof of the ordinary $\overline{\text{DR}}$ scheme.

3.2 SM gauge sector

The gauge sector of the Standard Model is not affected by its minimal extension, the MSSM. Thus, the treatment of the electroweak gauge sector is identical to the renormalization procedure in the SM, which is discussed in detail in [31, 32]. For the gauge fields the renormalization constants are given by

$$W_\mu^\pm \;\to\; \left(1 + \tfrac{1}{2}\delta Z_W\right) W_\mu^\pm\,, \tag{3.11}$$

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} \;\to\; \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{AA} & \frac{1}{2}\delta Z_{AZ} \\ \frac{1}{2}\delta Z_{ZA} & 1 + \frac{1}{2}\delta Z_{ZZ} \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}. \tag{3.12}$$

Since the photon stays massless also after renormalization, only the weak gauge bosons Z^0 and $W^\pm$ receive mass corrections, i.e.

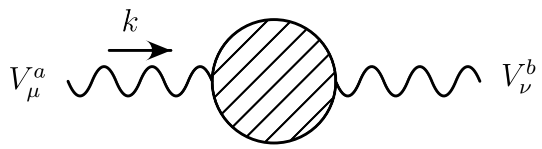
$$m_W^2 \to m_W^2 + \delta m_W^2\,, \qquad m_Z^2 \to m_Z^2 + \delta m_Z^2. \tag{3.13}$$

Decomposing the vector two–point–functions and the associated self–energies into their transverse and longitudinal parts,

$$\Gamma^W_{\mu\nu}(k) \;=\; -ig_{\mu\nu}\big(k^2 - m_W^2\big) - i\Big(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\Big)\Pi^W_T(k^2) - i\frac{k_\mu k_\nu}{k^2}\Pi^W_L(k^2)\,, \tag{3.14}$$

$$\Gamma^{ab}_{\mu\nu}(k) \;=\; -ig_{\mu\nu}\big(k^2 - m_a^2\big)\delta_{ab} - i\Big(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\Big)\Pi^{ab}_T(k^2) - i\frac{k_\mu k_\nu}{k^2}\Pi^{ab}_L(k^2)\,, \tag{3.15}$$

with $a, b = \{A, Z\}$, the corresponding renormalized self–energies in



$$\mathcal{M} = i\,\varepsilon^\mu(k)\,\hat{\Gamma}^{ab}_{\mu\nu}(k)\,\varepsilon^{*\nu}(k)$$

can be written as

$$\hat{\Pi}^W(k^2) \;=\; \Pi^W(k^2) + \left(k^2 - m_W^2\right)\delta Z^W - \delta m_W^2\,, \tag{3.16}$$

$$\hat{\Pi}^{ab}(k^2) \;=\; \Pi^{ab}(k^2) + \tfrac{1}{2}\left(k^2 - m_a^2\right)\delta Z^{ab} + \tfrac{1}{2}\left(k^2 - m_b^2\right)\delta Z^{ba} - \delta_{ab}\delta m_a^2\,, \tag{3.17}$$

valid for both the transverse and longitudinal parts with $m_A^2 = \delta m_A^2 = 0$. Applying the *on–shell renormalization conditions* to $\hat{\Gamma}^{ab}_{\mu\nu}(k)$,

$$\mathrm{Re}\,\hat{\Gamma}^{ab}_{\mu\nu}(k)\,\varepsilon^\nu(k)\Big|_{k^2=m_a^2} \;=\; 0, \qquad \lim_{k^2\to m_a^2}\frac{1}{k^2 - m_a^2}\mathrm{Re}\,\hat{\Gamma}^{ab}_{\mu\nu}(k)\,\varepsilon^\nu(k) \;=\; -\,\varepsilon^\mu(k)\,, \tag{3.18}$$

which means that the poles of the propagators are determined by the physical (*pole*) masses and the residua are set to 1 on shell ($k^2 = m_a^2$), the counter terms are given by

$$\delta Z^{aa} = -\mathrm{Re}\,\dot{\Pi}_T^{aa}(m_a^2)\,, \qquad \delta Z^W = -\mathrm{Re}\,\dot{\Pi}_T^W(m_W^2)\,, \tag{3.19}$$

$$\delta Z^{ab} = \frac{2}{m_a^2 - m_b^2}\,\mathrm{Re}\,\Pi_T^{ab}(m_b^2)\,, \qquad a \neq b\,, \tag{3.20}$$

$$\delta m_Z^2 = \mathrm{Re}\,\Pi_T^{ZZ}(m_Z^2)\,, \qquad \delta m_W^2 = \mathrm{Re}\,\Pi_T^{WW}(m_W^2)\,, \tag{3.21}$$

with $\dot{\Pi}(m^2) = \frac{\partial}{\partial k^2}\Pi(k^2)\big|_{k^2=m^2}$.

Since the weak mixing angle is a derived quantity in the on–shell scheme, determined by the condition $m_W = m_Z\, c_W$ ($c_W \equiv \cos\theta_W$) [30], its renormalization constant can be expressed in terms of the mass counter terms of the weak gauge bosons

$$\frac{\delta c_W^2}{c_W^2} = \frac{\delta m_W^2}{m_W^2} - \frac{\delta m_Z^2}{m_Z^2}\,, \qquad \frac{\delta s_W^2}{s_W^2} = -\frac{c_W^2}{s_W^2}\frac{\delta c_W^2}{c_W^2}\,. \tag{3.22}$$

3.3 Electric charge

For the renormalization of the electric charge one only has to renormalize one single vertex, for which usually the electron–positron–photon vertex is taken. In requiring for the renormalized elementary charge to describe the electromagnetic coupling in the Thomson limit, i.e. for on–shell external particles and vanishing photon momentum,

$$\bar{u}(p)\,\hat{\Gamma}_\mu^{ee\gamma}(p,p)\,u(p)\Big|_{p^2=m_e^2} = ie\,\bar{u}(p)\gamma_\mu u(p), \tag{3.23}$$

the counter term for the electric charge in $e_0 = e + \delta e$ is given by

$$\frac{\delta e}{e} = -\frac{1}{2}\delta Z^{AA} + \frac{s_W}{c_W}\frac{1}{2}\delta Z^{ZA} = \frac{1}{2}\,\dot{\Pi}_T^{AA}(0) + \frac{s_W}{c_W}\frac{\Pi_T^{AZ}(0)}{m_Z^2}\,. \tag{3.24}$$

However, the scale of high energy processes lies in the range of hundreds of GeV and thus far away from the Thomson limit. In addition, contributions of light hadrons in $\dot{\Pi}_T^{AA}(0)$ lead to large theoretical uncertainties [33, 32]. To avoid this problem, we use as input an effective $\overline{\mathrm{MS}}$ running coupling at $Q = m_Z$, where the contributions from light fermions are already absorbed [34, 35],

$$\alpha_{\overline{\mathrm{MS}}}^{\mathrm{eff}}(m_Z^2) = \frac{\alpha}{1 - \Delta\alpha_{\overline{\mathrm{MS}}}^{\mathrm{eff}}(m_Z^2)} \simeq \frac{1}{127.7}\,. \tag{3.25}$$

Here, α is the fine structure constant given in the Thomson limit, $\alpha = 1/137.036$, and

$$\Delta\alpha_{\overline{\mathrm{MS}}}^{\mathrm{eff}}(m_Z^2) = \frac{\alpha}{\pi}\left(\frac{5}{3} + \frac{55}{27}\Big(1 + \frac{\alpha}{\pi}\Big)\right) + \Delta\alpha_{\mathrm{lep}}(m_Z^2) + \Delta\alpha_{\mathrm{had}}^{(5)}(m_Z^2)\,, \tag{3.26}$$

where $\Delta\alpha_{\text{lep}}(m_Z^2) \simeq 0.031497687$ are the leptonic and $\Delta\alpha^{(5)}_{\text{had}}(m_Z^2) = 0.02769 \pm 0.00035$ are the hadronic contributions [36]. The counter term for the electric charge δe is then given by

$$\begin{aligned}\frac{\delta e}{e} \;=\; \frac{1}{(4\pi)^2}\,\frac{e^2}{6}\Bigg[& 4\sum_f N_C^f\, e_f^2\left(\Delta + \log\frac{Q^2}{x_f^2}\right) + \sum_{\tilde f}\sum_{m=1}^{2} N_C^f\, e_f^2\left(\Delta + \log\frac{Q^2}{m^2_{\tilde f_m}}\right)\\ &+4\sum_{k=1}^{2}\left(\Delta + \log\frac{Q^2}{m^2_{\tilde\chi^+_k}}\right) + \sum_{k=1}^{2}\left(\Delta + \log\frac{Q^2}{m^2_{H^+_k}}\right) - 22\left(\Delta + \log\frac{Q^2}{m^2_W}\right)\Bigg]\,, \end{aligned} \tag{3.27}$$

with $x_f = m_Z \;\forall\; m_f < m_Z$ and $x_t = m_t$. N_C^f is the colour factor, $N_C^f = 1, 3$ for (s)leptons and (s)quarks, respectively. Δ denotes the UV divergence factor, $\Delta = 2/\epsilon - \gamma + \log 4\pi$, with γ being the Euler–Mascheroni constant $\gamma = \lim\limits_{m\to\infty}\left(\sum_{k=1}^{m}\frac{1}{k} - \log m\right) \sim 0.577216$.

3.4 Renormalization of two–point functions

Before starting with the renormalization of the remaining parameters and fields of the MSSM, which are necessary for our calculations, i.e. the ones of the Higgs and sfermion sector, we will have a short look on the subject of renormalizing two–point functions, as they are the basic building blocks for calculating higher order corrections.

3.4.1 Scalar particles with mixing

According to multiplicative renormalization, the unrenormalized fields $\phi_{0,i}$ and mass parameters $m_{0,i}$ in the bare Lagrangian

$$\mathcal{L}_0 = -\phi^*_{0,i}\,\delta_{ij}\left(\partial_\mu\partial^\mu + m^2_{0,i}\right)\phi_{0,j} \tag{3.28}$$

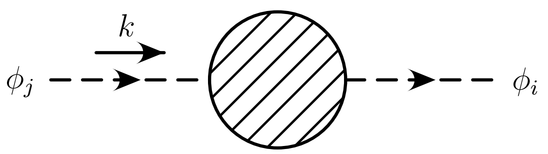
are replaced by the corresponding renormalized ones, i.e.

$$\mathcal{L} \;=\; -\phi^*_i\,\delta_{ij}\left(\partial_\mu\partial^\mu + m_i^2\right)\phi_j\,, \tag{3.29}$$

$$\phi_{0,j} \;=\; \sqrt{Z_{jk}}\,\phi_k = \left(\delta_{jk} + \tfrac{1}{2}\,\delta Z_{jk}\right)\phi_k + \mathcal{O}(\delta Z^2)\,, \tag{3.30}$$

$$(m_{0,i})^2 \;=\; m_i^2 + \delta m_i^2\,. \tag{3.31}$$

For the full renormalized two–point–function



$$\mathcal{M} = i\,\hat\Gamma_{ij}(k^2) = i\delta_{ij}(k^2 - m_i^2) + i\hat\Pi_{ij}(k^2)$$

we demand the on–shell renormalization conditions

$$\text{Re}\,\hat\Gamma_{ij}(k^2)\Big|_{k^2=m_j^2} \;=\; 0\,, \qquad \lim_{k^2\to m_i^2}\frac{1}{k^2 - m_i^2}\,\text{Re}\hat\Gamma_{ii}(k^2) \;=\; 1\,. \tag{3.32}$$

Inserting the renormalized self–energy $\hat{\Pi}_{ij}(k^2) = \Pi_{ij}(k^2) + \Pi^{(c)}_{ij}(k^2)$ with the self–energy counter–term

$$\Pi^{(c)}_{ij}(k^2) \;=\; -\delta_{ij}\;\delta m_i^2 + \tfrac{1}{2}\left(k^2 - m_i^2\right)\delta Z_{ij} + \tfrac{1}{2}\left(k^2 - m_j^2\right)\delta Z^*_{ji} \qquad (3.33)$$

into eq. (3.32) leads to

$$\delta m_i^2 \;=\; \mathrm{Re}\,\Pi_{ii}(m_i^2)\,, \qquad (3.34)$$

$$\delta Z_{ij} \;=\; \frac{2}{m_i^2 - m_j^2}\,\mathrm{Re}\,\Pi_{ij}(m_j^2) \qquad i \neq j\,, \qquad (3.35)$$

$$\delta Z_{ii} \;=\; \delta Z^*_{ii} = -\mathrm{Re}\,\dot{\Pi}_{ii}(m_i^2)\,. \qquad (3.36)$$

3.4.2 Fermionic particles with mixing

Like in the previous chapter we have the same structure for the physical as well as for the bare Lagrangian:

$$\mathcal{L} \;=\; \bar{\psi}_j\,\delta_{ij}\,(i\,\partial\!\!\!/ - m_i)\,\psi_i\,, \qquad (3.37)$$

$$\mathcal{L}_0 \;=\; \bar{\psi}_{0,j}\,\delta_{ij}\,(i\,\partial\!\!\!/ - m_{0,i})\,\psi_{0,i}\,. \qquad (3.38)$$

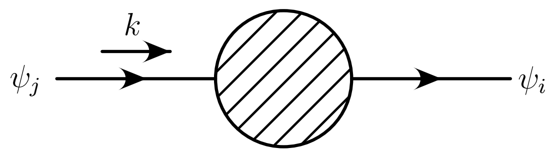
The relation between the unrenormalized and the renormalized quantities is given by attaching multiplicative renormalization constants to the unrenormalized fermion fields $\psi_{0,i}$ and the mass parameter $m_{0,i}$, i.e.

$$\psi_{0,j} \;=\; \left(\delta_{jk} + \tfrac{1}{2}\delta Z^L_{jk} P_L + \tfrac{1}{2}\delta Z^R_{jk} P_R\right)\psi_k\,, \qquad (3.39)$$

$$\bar{\psi}_{0,i} \;=\; \bar{\psi}_l\left(\delta_{il} + \tfrac{1}{2}\delta Z^{R\dagger}_{il} P_L + \tfrac{1}{2}\delta Z^{L\dagger}_{il} P_R\right)\,, \qquad (3.40)$$

$$m_{0,i} \;=\; m_i + \delta m_i\,, \qquad (3.41)$$

where the 'dagger' † in $\delta Z^{L,R\dagger}_{il}$ indicates hermitian conjugation with regard to the spinor indices. For the renormalized one particle irreducible (1PI) two–point–function



$$\mathcal{M} = i\,\bar{u}_i(k)\hat{\Gamma}_{ij}(k)u_j(k)$$

$$\hat{\Gamma}_{ij}(k) = \delta_{ij}(k\!\!\!/ - m_i) + \hat{\Pi}_{ij}(k)$$

with the renormalized self–energy

$$\hat{\Pi}_{ij}(k) \;=\; k\!\!\!/\,P_L\,\hat{\Pi}^L_{ij}(k) + k\!\!\!/\,P_R\,\hat{\Pi}^R_{ij}(k) + \hat{\Pi}^{S,L}_{ij}(k)P_L + \hat{\Pi}^{S,R}_{ij}(k)P_R \qquad (3.42)$$

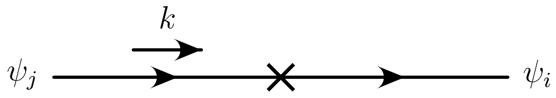
we require the on–shell renormalization conditions

$$\mathrm{Re}\,\hat{\Gamma}_{ij}(k)\,u_j(k)\Big|_{k^2=m_j^2} \;=\; 0\,, \qquad \lim_{k^2\to m_i^2}\frac{1}{k\!\!\!/ - m_i}\,\mathrm{Re}\,\hat{\Gamma}_{ii}(k)u_i(k) \;=\; u_i(k)\,. \qquad (3.43)$$

Inserting the counter–term Lagrangian

$$\delta\mathcal{L} \;=\; \bar{\psi}_i\left(k\!\!\!/ P_L C^L_{ij} + k\!\!\!/ P_R C^R_{ij} - C^{S,L}_{ij} P_L - C^{S,R}_{ij} P_R\right)\psi_j\,, \qquad (3.44)$$

into $\mathcal{L}_0 = \mathcal{L} + \delta\mathcal{L}$,



$$\mathcal{M} = i\big(\not{k} P_L C^L_{ij} + \not{k} P_R C^R_{ij} - C^{S,L}_{ij} P_L - C^{S,R}_{ij} P_R\big)\,,$$

we get for the coefficients $C^{\cdots}_{ij}$

$$C^L_{ij} = \tfrac{1}{2}\big(\delta Z^L_{ij} + \delta Z^{L\dagger}_{ji}\big)\,, \tag{3.45}$$

$$C^R_{ij} = \tfrac{1}{2}\big(\delta Z^R_{ij} + \delta Z^{R\dagger}_{ji}\big)\,, \tag{3.46}$$

$$C^{S,L}_{ij} = \tfrac{1}{2}\big(m_i\,\delta Z^L_{ij} + m_j\,\delta Z^{R\dagger}_{ji}\big) + \delta_{ij}\,\delta m_i\,, \tag{3.47}$$

$$C^{S,R}_{ij} = \tfrac{1}{2}\big(m_i\,\delta Z^R_{ij} + m_j\,\delta Z^{L\dagger}_{ji}\big) + \delta_{ij}\,\delta m_i\,. \tag{3.48}$$

Thus the renormalized self–energies can be written as

$$\hat{\Pi}^L_{ij} = \Pi^L_{ij} + \tfrac{1}{2}\big(\delta Z^L_{ij} + \delta Z^{L\dagger}_{ji}\big)\,, \tag{3.49}$$

$$\hat{\Pi}^R_{ij} = \Pi^R_{ij} + \tfrac{1}{2}\big(\delta Z^R_{ij} + \delta Z^{R\dagger}_{ji}\big)\,, \tag{3.50}$$

$$\hat{\Pi}^{S,L}_{ij} = \Pi^{S,L}_{ij} - \tfrac{1}{2}\big(m_i\,\delta Z^L_{ij} + m_j\,\delta Z^{R\dagger}_{ji}\big) - \delta_{ij}\,\delta m_i\,, \tag{3.51}$$

$$\hat{\Pi}^{S,R}_{ij} = \Pi^{S,R}_{ij} - \tfrac{1}{2}\big(m_i\,\delta Z^R_{ij} + m_j\,\delta Z^{L\dagger}_{ji}\big) - \delta_{ij}\,\delta m_i\,. \tag{3.52}$$

Taking the renormalization conditions in eq. (3.43) into account, we obtain the counter terms for the mass parameter and the wave–function corrections

$$\delta m_i = \frac{1}{2}\,\mathrm{Re}\Big[m_i\Big(\Pi^L_{ii}(m_i) + \Pi^R_{ii}(m_i)\Big) + \Pi^{S,L}_{ii}(m_i) + \Pi^{S,R}_{ii}(m_i)\Big]\,. \tag{3.53}$$

$$\delta Z^L_{ij} = \frac{2}{m_i^2 - m_j^2}\,\mathrm{Re}\Big[m_j^2\,\Pi^L_{ij}(m_j) + m_i m_j\,\Pi^R_{ij}(m_j) + m_i\,\Pi^{S,L}_{ij} + m_j\,\Pi^{S,R}_{ij}\Big]\,, \tag{3.54}$$

$$\begin{aligned}\delta Z^L_{ii} &= -\Pi^L_{ii}(m_i) + \frac{1}{2m_i}\Big[\Pi^{S,L}_{ii}(m_i) - \Pi^{S,R}_{ii}(m_i)\Big]\\ &\quad - m_i\frac{\partial}{\partial k^2}\Big[m_i\big(\Pi^L_{ii}(k) + \Pi^R_{ii}(k)\big) + \Pi^{S,L}_{ii}(k) + \Pi^{S,R}_{ii}(k)\Big]\Big|_{k^2=m_i^2}\,,\end{aligned} \tag{3.55}$$

and the corresponding right–handed terms, $\delta Z^{(S),R}_{ij} = \delta Z^{(S),L}_{ij}(L \leftrightarrow R)$.

3.5 Sfermion sector

According to the results of section 3.4, where we have derived the mass corrections and the wave–function renormalization constants in terms of self–energies, these counter terms are given in the sfermion sector by

$$\delta m^2_{\tilde{f}_i} = \mathrm{Re}\,\Pi^{\tilde{f}}_{ii}(m^2_{\tilde{f}_i}) \tag{3.56}$$

and

$$\delta Z_{ii}^{\tilde{f}} = -\operatorname{Re}\dot{\Pi}_{ii}^{\tilde{f}}(m_{\tilde{f}_i}^2)\,, \qquad i = (1,2)\,, \tag{3.57}$$

$$\delta Z_{ij}^{\tilde{f}} = \frac{2}{m_{\tilde{f}_i}^2 - m_{\tilde{f}_j}^2}\operatorname{Re}\Pi_{ij}^{\tilde{f}}(m_{\tilde{f}_j}^2)\,, \qquad i \neq j\,. \tag{3.58}$$

3.5.1 Renormalization of mixing angles

In order to renormalize the parameters entering in the sfermion mass matrix $\mathcal{M}_{\tilde{f}}^2$ (see eq. (2.70)), we have to look at its counter term $\delta\mathcal{M}_{\tilde{f}}^2$,

$$\delta\mathcal{M}_{\tilde{f}}^2 = \left(\delta R^{\tilde{f}}\right)^\dagger \begin{pmatrix} m_{\tilde{f}_1}^2 & 0 \\ 0 & m_{\tilde{f}_2}^2 \end{pmatrix} R^{\tilde{f}} + \left(R^{\tilde{f}}\right)^\dagger \begin{pmatrix} \delta m_{\tilde{f}_1}^2 & 0 \\ 0 & \delta m_{\tilde{f}_2}^2 \end{pmatrix} R^{\tilde{f}} + \left(R^{\tilde{f}}\right)^\dagger \begin{pmatrix} m_{\tilde{f}_1}^2 & 0 \\ 0 & m_{\tilde{f}_2}^2 \end{pmatrix} \delta R^{\tilde{f}}, \tag{3.59}$$

with

$$\delta R^{\tilde{f}} = -\begin{pmatrix} \sin\theta_{\tilde{f}} & -\cos\theta_{\tilde{f}} \\ \cos\theta_{\tilde{f}} & \sin\theta_{\tilde{f}} \end{pmatrix}\delta\theta_{\tilde{f}}\,. \tag{3.60}$$

The renormalization constant of the rotation matrix $R_{ij}^{\tilde{f}}$ is determined such as to cancel the anti–hermitian part of the sfermion wave–function corrections,

$$\delta R_{ij}^{\tilde{f}} = \sum_{k=1}^{2}\frac{1}{4}\left(\delta Z_{ik}^{\tilde{f}} - \delta Z_{ki}^{\tilde{f}}\right)R_{kj}^{\tilde{f}}\,. \tag{3.61}$$

Therefore, the counter term for the sfermion mixing angle $\theta_{\tilde{f}}$ is given by [38, 39]

$$\delta\theta_{\tilde{f}} = \frac{1}{4}\left(\delta Z_{12}^{\tilde{f}} - \delta Z_{21}^{\tilde{f}}\right) = \frac{1}{2\left(m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2\right)}\operatorname{Re}\left(\Pi_{12}^{\tilde{f}}(m_{\tilde{f}_2}^2) + \Pi_{21}^{\tilde{f}}(m_{\tilde{f}_1}^2)\right)\,, \tag{3.62}$$

and thus

$$\delta\left(\mathcal{M}_{\tilde{f}}^2\right)_{ij} = \frac{1}{2}\sum_{k,l=1}^{2}\left(R_{ik}^{\tilde{f}}\right)^\dagger\operatorname{Re}\left[\Pi_{kl}^{\tilde{f}}(m_{\tilde{f}_l}^2) + \Pi_{lk}^{\tilde{f}}(m_{\tilde{f}_k}^2)\right]R_{lj}^{\tilde{f}}\,. \tag{3.63}$$

3.6 Higgs sector

The renormalization of the Higgs mixing angle α is treated in a similar way as the sfermion mixing angle $\theta_{\tilde{f}}$. Consider the mass matrix of the CP even Higgs bosons h^0 and H^0 (cf. section 2.3.1),

$$\begin{aligned} M^2(H^0,h^0) &= \begin{pmatrix} \sin^2\beta\, m_{A^0}^2 + \cos^2\beta\, m_Z^2 & -\sin\beta\cos\beta(m_{A^0}^2 + m_Z^2) \\ -\sin\beta\cos\beta(m_{A^0}^2 + m_Z^2) & \cos^2\beta\, m_{A^0}^2 + \sin^2\beta\, m_Z^2 \end{pmatrix} \\ &= \left(R^{H^0}\right)^T \cdot \begin{pmatrix} m_{H^0}^2 & 0 \\ 0 & m_{h^0}^2 \end{pmatrix} \cdot R^{H^0}\,, \qquad m_{h^0} < m_{H^0} \end{aligned} \tag{3.64}$$

with the rotation matrix

$$R_{ij}^{H^0} \quad \equiv \quad R_{ij}(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}_{ij} . \tag{3.65}$$

Analogously to the case of the sfermion mixing angle, the renormalization constant of the rotation matrix $R_{ij}^{H^0}$ is determined such as to cancel the anti–hermitian part of the Higgs wave–function corrections,

$$\delta R_{ij}^{H^0} \quad = \quad \sum_{k=1}^{2} \frac{1}{4}\big(\delta Z_{ki}^{H^0} - \delta Z_{ik}^{H^0}\big) R_{kj}^{H^0} , \tag{3.66}$$

which leads to the counter term

$$\delta\alpha \quad = \quad \frac{1}{4}\Big(\delta Z_{21}^{H} - \delta Z_{12}^{H}\Big) = \frac{1}{2\big(m_{H^0}^2 - m_{h^0}^2\big)} \,\mathrm{Re}\Big(\Pi_{12}^{H}(m_{H^0}^2) + \Pi_{21}^{H}(m_{h^0}^2)\Big) . \tag{3.67}$$

Note that the indices the wave–function renormalization constants $\delta Z_{ij}^{H^0}$ are interchanged due to the conventional nomenclature labelling the light Higgs boson by an index 1 and the heavy one by an index 2.
For the correct definition of the wavefunction counterterms in the Higgs sector, we have to be careful of the contribution originating from the tadpoles. We will discuss this issue in the next section.

3.7 Renormalization of tan $\boldsymbol{\beta}$

Due its close connection to spontaneous symmetry breaking SSB, $\tan\beta$ enters almost all sectors of the MSSM. As a consequence, it has a major effect on allmost all MSSM observables. The problem with the renormalization of $\tan\beta$ is, that it does not correspond to direct measurable observable. Several renormalization schemes for $\tan\beta$ suffer from specific disadvantages, leading either to gauge dependences or numerical instabilities [42].

Dabelstein-Chankowski-Pokorski-Rosiek Scheme (DCPR) [79, 37]

This scheme is based on an on-shell renormalisation scheme in the Higgs sector working in the usual linear gauge. The definition of $\tan\beta$ however is difficult to connect with an on-shell quantity that represents a direct interpretation in terms of a physical observable. One first introduces a wavefunction renormalisation constant δZ_{H_i}, for each Higgs doublet H_i, i.e. before rotation

$$H_i \to (1 + \frac{1}{2}\delta Z_{H_i})H_i \;, \qquad\qquad i = 1, 2 \;. \tag{3.68}$$

The vacuum expectation values are also shifted such that the counter term for each v_i writes as

$$v_i \to v_i(1 - \frac{\delta v_i}{v_i} + \frac{1}{2}\delta Z_{H_i})H_i \;, \tag{3.69}$$

obtaining

$$\frac{\delta\tan\beta}{\tan\beta} = \frac{\delta v_1}{v_1} - \frac{\delta v_2}{v_2} + \frac{1}{2}(\delta Z_{H_2} - \delta Z_{H_1}) \;. \tag{3.70}$$

In the DCPR scheme one takes $\frac{\delta v_1}{v_1} = \frac{\delta v_2}{v_2}$ [99], leading to the expression

$$\frac{\delta \tan\beta}{\tan\beta} = \frac{1}{2}(\delta Z_{H_2} - \delta Z_{H_1}) \; . \tag{3.71}$$

The counter term $\delta \tan\beta$ can be now defined by requiring that the renormalized $A^0 - Z^0$ transition vanishes at $p^2 = m^2_{A^0}$

$$\mathrm{Re}\, \hat{\Sigma}_{A^0 Z^0}(m^2_{A^0}) = 0 \; , \tag{3.72}$$

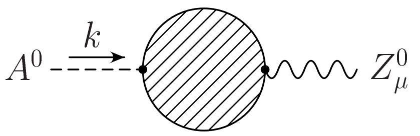
with

$$\hat{\Sigma}_{A^0 Z^0}(m^2_{A^0}) = \Sigma_{A^0 Z^0}(m^2_{A^0}) + i\frac{m_Z}{4} s_{2\beta}\,(\delta Z_{H_2} - \delta Z_{H_1}) = 0 \; , \tag{3.73}$$

one then obtains

$$\frac{\delta \tan\beta}{\tan\beta} = \frac{i}{m_Z s_{2\beta}} \mathrm{Re}\, \Sigma_{A^0 Z^0}(m^2_{A^0}) \; . \tag{3.74}$$

This definition is obviously not related to an observable. Moreover $\delta \tan\beta$ is expressed in terms of unphysical wavefunction renormalization constants.



$$\mathcal{M} = -i\, k^\mu\, \Pi_{AZ}(k^2)\, \epsilon^*_\mu(k)$$

Figure 3.1: $A^0 Z^0$ mixing self–energy relevant for the renormalization of $\tan\beta$.

3.7.1 $\overline{\mathrm{DR}}$ scheme

In the $\overline{\mathrm{DR}}$ scheme the counter term for $\tan\beta$ is simpliy defined by

$$\frac{\delta \tan\beta}{\tan\beta} \stackrel{\varepsilon}{=} \frac{i}{m_Z s_{2\beta}} \mathrm{Re}\, \Sigma_{A^0 Z^0}(m^2_{A^0}) \; , \tag{3.75}$$

where $\stackrel{\varepsilon}{=}$ takes only terms proportional to $\frac{1}{\varepsilon}$ into account.

3.8 Tadpole contributions

To treat correctly the tadpole contributions, we have to look at the mass matrices and the minimization condition of the Higgs potential. We will use the scheme proposed by Pierce and Papadopoulos [100], where the terms linear in h^0 and H^0 are to be thought as counterterms for the tadpole contributions. To each order in the loop expansion, we require that the tadpole contribution vanishes.

3.8.1 The CP-even Higgs mass matrix

The mass matrix of the CP-even Higgs system has the following form

$$M_H^2 = \begin{pmatrix} \frac{3g^2v_1^2}{8} - \frac{g^2v_2^2}{8} + \frac{3g'^2v_1^2}{8} - \frac{g'^2v_2^2}{8} + m_1^2 & -\frac{1}{4}g^2v_1v_2 - \frac{1}{4}g'^2v_1v_2 - m_{12}^2 \\ -\frac{1}{4}g^2v_1v_2 - \frac{1}{4}g'^2v_1v_2 - m_{12}^2 & -\frac{g^2v_1^2}{8} + \frac{3g^2v_2^2}{8} - \frac{g'^2v_1^2}{8} + \frac{3g'^2v_2^2}{8} + m_2^2 \end{pmatrix} . \quad (3.76)$$

We can reexpress g, g', v_1, v_2 in terms of M_Z, c_β and s_β as

$$M_H^2 = \begin{pmatrix} \frac{3c_\beta^2m_Z^2}{2} + m_1^2 - \frac{m_Z^2s_\beta^2}{2} & -c_\beta m_Z^2 s_\beta - m_{12}^2 \\ -c_\beta m_Z^2 s_\beta - m_{12}^2 & -\frac{c_\beta^2m_Z^2}{2} + m_2^2 + \frac{3m_Z^2s_\beta^2}{2} \end{pmatrix} . \quad (3.77)$$

The tadpoles T_i correspond to the terms linear in the fields ϕ_1^0 and ϕ_2^0 in the Higgs potential V_H. Due the minimization condition of the Higgs potential T_1 and T_2 are zero, therefore they relate m_1^2, m_2^2 and m_{12}^2 by

$$\begin{aligned} m_1^2 &= -\frac{c_\beta^2m_Z^2}{2} + \frac{m_{12}^2s_\beta}{c_\beta} + \frac{m_Z^2s_\beta^2}{2} + \frac{T_1}{v_1} , \\ m_2^2 &= \frac{c_\beta^2m_Z^2}{2} + \frac{c_\beta m_{12}^2}{s_\beta} - \frac{m_Z^2s_\beta^2}{2} + \frac{T_2}{v_2} . \end{aligned} \quad (3.78)$$

It is illuminating to express the mass matrix in terms of the tadpole parameters T_i

$$M_H^2 = \begin{pmatrix} c_\beta^2m_Z^2 + m_{A^0}^2s_\beta^2 + \frac{T_1}{v_1} & -c_\beta m_{A^0}^2s_\beta - c_\beta m_Z^2 s_\beta \\ -c_\beta m_{A^0}^2s_\beta - c_\beta m_Z^2s_\beta & c_\beta^2m_{A^0}^2 + m_Z^2s_\beta^2 + \frac{T_2}{v_2} \end{pmatrix} . \quad (3.79)$$

In the next step we express the tadpoles T_1 and T_2 in terms of the tadpoles of the physical fields T_{h^0} and T_{H^0} by

$$\begin{pmatrix} T_{H^0} \\ T_{h^0} \end{pmatrix} = O(\alpha) \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} \Rightarrow \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha T_{H^0} - \sin\alpha T_{h^0} \\ \sin\alpha T_{H^0} + \cos\alpha T_{h^0} \end{pmatrix} . \quad (3.80)$$

We then introduce the tadpole matrices

$$\begin{aligned} \begin{pmatrix} t_{H^0H^0} & t_{H^0h^0} \\ t_{h^0H^0} & t_{h^0h^0} \end{pmatrix} &= O(\alpha) \begin{pmatrix} \frac{T_1}{v_1} & 0 \\ 0 & \frac{T_2}{v_2} \end{pmatrix} O(\alpha)^T , \\ \begin{pmatrix} t_{G^0G^0} & t_{G^0A^0} \\ t_{A^0G^0} & t_{A^0A^0} \end{pmatrix} &= O(\beta) \begin{pmatrix} \frac{T_1}{v_1} & 0 \\ 0 & \frac{T_2}{v_2} \end{pmatrix} O(\beta)^T = \begin{pmatrix} t_{G^\pm G^\pm} & t_{G^\pm H^\pm} \\ t_{H^\pm G^\pm} & t_{H^\pm H^\pm} \end{pmatrix} , \end{aligned} \quad (3.81)$$

with the help of relations $v_1 = v\cos\beta, v_2 = v\sin\beta, gv = 2m_W$ and $e = gs_w$ we find that

$$\begin{aligned} \begin{pmatrix} t_{H^0H^0} & t_{H^0h^0} \\ t_{h^0H^0} & t_{h^0h^0} \end{pmatrix} &= \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \frac{T_1}{v_1} & 0 \\ 0 & \frac{T_2}{v_2} \end{pmatrix} \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} \Rightarrow \\ t_{H^0H^0} &= \frac{e}{2m_Ws_w} \left[T_{h^0} \left(\frac{s_\alpha^2c_\alpha}{s_\beta} - \frac{c_\alpha^2s_\alpha}{c_\beta} \right) + T_{H^0} \left(\frac{c_\alpha^3}{c_\beta} + \frac{s_\alpha^3}{s_\beta} \right) \right] , \\ t_{H^0h^0} &= \frac{e}{2m_Ws_w} \left[T_{h^0} \left(\frac{s_\alpha^2c_\alpha}{c_\beta} + \frac{c_\alpha^2s_\alpha}{s_\beta} \right) + T_{H^0} \left(-\frac{c_\alpha^2s_\alpha}{c_\beta} + \frac{s_\alpha^2c_\alpha}{s_\beta} \right) \right] , \\ t_{h^0h^0} &= \frac{e}{2m_Ws_w} \left[T_{h^0} \left(\frac{c_\alpha^3}{s_\beta} - \frac{s_\alpha^3}{c_\beta} \right) + T_{H^0} \left(\frac{s_\alpha^2c_\alpha}{c_\beta} + \frac{c_\alpha^2s_\alpha}{s_\beta} \right) \right] . \end{aligned} \quad (3.82)$$

Thus, we can express the Higgs potential up to terms linear and bilinear in the Higgs fields

$$
\begin{aligned}
V_H &= V_0 + H^0 T_{H^0} + h^0 T_{h^0} + \frac{1}{2}\begin{pmatrix} H^0 & h^0 \end{pmatrix}\begin{pmatrix} m_{H^0}^2 + t_{H^0H^0} & t_{H^0h^0} \\ t_{h^0H^0} & m_{h^0}^2 + t_{h^0h^0} \end{pmatrix}\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} \\
&+ \frac{1}{2}\begin{pmatrix} G^0 & A^0 \end{pmatrix}\begin{pmatrix} t_{G^0G^0} & t_{G^0A^0} \\ t_{A^0G^0} & m_{A^0}^2 + t_{A^0A^0} \end{pmatrix}\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \\
&+ \frac{1}{2}\begin{pmatrix} G^- & H^- \end{pmatrix}^\dagger \begin{pmatrix} t_{G^-G^-} & t_{G^\pm H^\pm} \\ t_{H^\pm G^\pm} & m_{H^\pm}^2 + t_{H^\pm H^\pm} \end{pmatrix}\begin{pmatrix} G^- \\ H^- \end{pmatrix} + \ldots\,.
\end{aligned} \tag{3.83}
$$

From the minimalization condition it follows that the terms linear in the fields h^0 and H^0 must vanish at tree–level. This condition can be satisfied by setting the tadpole parameters zero ($T_{h^0} = 0$, $T_{H^0} = 0$), which leads to the tree–level mass matrices. However, beyond tree–level the tadpole parameters T_{h^0} and T_{H^0} receive loop-corrections from tadpole diagrams τ_{H^0} and τ_{h^0}. Therefore we introduce the following renormalization condition

$$
\delta T_i + \tau_i = 0\,, \tag{3.84}
$$

where the tadpole contributions are exactly canceled by their corresponding counterterms. Then the counterterm for the mass matrix M_H^2 reads as

$$
\delta M_H^2 = \begin{pmatrix} \delta c_\beta^2 m_Z^2 + c_\beta^2 \delta m_Z^2 + \delta m_{A^0}^2 s_\beta^2 + m_{A^0}^2 \delta s_\beta^2 + \frac{\delta T_1}{v_1} & -\delta s_{2\beta}(m_{A^0}^2 + m_Z^2) - s_{2\beta}(\delta m_{A^0}^2 + \delta m_Z^2) \\ -\delta s_{2\beta}(m_{A^0}^2 + m_Z^2) - s_{2\beta}(\delta m_{A^0}^2 + \delta m_Z^2) & \delta c_\beta^2 m_{A^0}^2 + c_\beta^2 \delta m_{A^0}^2 + \delta m_Z^2 s_\beta^2 + m_Z^2 \delta s_\beta^2 + \frac{\delta T_2}{v_2} \end{pmatrix}.
$$

After rotating to mass eigenstates the counterterm for the H^0h^0–system reads as

$$
\delta M_{H^0h^0}^2 = \begin{pmatrix} \delta M_{H^0}^2 + \delta t_{H^0H^0} & \delta t_{H^0h^0} \\ \delta t_{h^0H^0} & \delta M_{h^0}^2 + \delta t_{h^0h^0} \end{pmatrix}\,, \tag{3.85}
$$

with

$$
\delta t_{H^0h^0} = \delta t_{h^0H^0} = -\frac{1}{v}\left[\tau_{h^0}\left(\frac{s_\alpha^2 c_\alpha}{c_\beta} + \frac{c_\alpha^2 s_\alpha}{s_\beta}\right) + \tau_{H^0}\left(-\frac{c_\alpha^2 s_\alpha}{c_\beta} + \frac{s_\alpha^2 c_\alpha}{s_\beta}\right)\right]\,. \tag{3.86}
$$

Therfore we have to take the tadpole contributions in the wavefunction renormalization constants into account by

$$
\begin{aligned}
\delta Z_{h^0h^0} &= -\mathrm{Re}\,\dot{\Pi}_{h^0h^0}(m_{h^0}^2)\,, & (3.87)\\
\delta Z_{H^0H^0} &= -\mathrm{Re}\,\dot{\Pi}_{H^0H^0}(m_{H^0}^2)\,, & (3.88)\\
\delta Z_{h^0H^0} &= \frac{2}{m_{h^0}^2 - m_{H^0}^2}\left(\mathrm{Re}\,\Pi_{h^0,H^0}(m_{h^0}^2) - \delta t_{h^0H^0}\right)\,, & (3.89)\\
\delta Z_{H^0h^0} &= \frac{2}{m_{H^0}^2 - m_{h^0}^2}\left(\mathrm{Re}\,\Pi_{H^0h^0}(m_{H^0}^2) - \delta t_{H^0h^0}\right)\,. & (3.90)
\end{aligned}
$$

3.8.2 The CP-odd Higgs mass matrix

The mass matrix of the CP-odd Higgs system has the following form

$$M_P^2 = \begin{pmatrix} \frac{g^2v_1^2}{8} - \frac{g^2v_2^2}{8} + \frac{g'^2v_1^2}{8} - \frac{g'^2v_2^2}{8} + m_1^2 & m_{12}^2 \\ m_{12}^2 & -\frac{g^2v_1^2}{8} + \frac{g^2v_2^2}{8} - \frac{g'^2v_1^2}{8} + \frac{g'^2v_2^2}{8} + m_2^2 \end{pmatrix} . \quad (3.91)$$

We can reexpress g, g', v_1, v_2 in terms of M_Z, c_β and s_β by

$$M_P^2 = \begin{pmatrix} \frac{c_\beta^2 m_Z^2}{2} + m_1^2 - \frac{m_Z^2 s_\beta^2}{2} & m_{12}^2 \\ m_{12}^2 & -\frac{c_\beta^2 m_Z^2}{2} + m_2^2 + \frac{m_Z^2 s_\beta^2}{2} \end{pmatrix} . \quad (3.92)$$

We can then express m_1^2 and m_2^2 with the tadpole parameters T_1 and T_2 using eqs. (3.78) as

$$M_P^2 = \begin{pmatrix} \frac{T_1}{v_1} + m_{A^0}^2 s_\beta^2 & c_\beta m_{A^0}^2 s_\beta \\ c_\beta m_{A^0}^2 s_\beta & c_\beta^2 m_{A^0}^2 + \frac{T_2}{v_2} \end{pmatrix} , \quad (3.93)$$

then the counterterm of the matrix M_P^2 reads

$$\delta M_P^2 = \begin{pmatrix} \delta m_{A^0}^2 s_\beta^2 + m_{A^0}^2 \delta s_\beta^2 + \frac{\delta T_1}{v_1} & \frac{1}{2}\delta s_{2\beta} m_{A^0}^2 + \frac{1}{2} s_{2\beta} \delta m_{A^0}^2 \\ \frac{1}{2}\delta s_{2\beta} m_{A^0}^2 + \frac{1}{2} s_{2\beta} \delta m_{A^0}^2 & \delta c_\beta^2 m_{A^0}^2 + c_\beta^2 \delta m_{A^0}^2 + \frac{\delta T_2}{v_2} \end{pmatrix} . \quad (3.94)$$

Again we introduce the tadpole matrices for the $A_0 G_0$–system

$$\begin{aligned}
\begin{pmatrix} t_{G^0G^0} & t_{G^0A^0} \\ t_{A^0G^0} & t_{A^0A^0} \end{pmatrix} &= \begin{pmatrix} -c_\beta & s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \frac{T_1}{v_1} & 0 \\ 0 & \frac{T_2}{v_2} \end{pmatrix} \begin{pmatrix} -c_\beta & s_\beta \\ s_\beta & c_\beta \end{pmatrix} \Rightarrow \\
t_{G^0G^0} &= \frac{e}{2m_W s_w} \left[T_{h^0} \left(-s_\alpha c_\beta + s_\beta c_\alpha\right) + T_{H^0} \left(c_\beta c_\alpha + s_\beta s_\alpha\right)\right] , \\
t_{G^0A^0} &= \frac{e}{2m_W s_w} \left[T_{h^0} \left(s_\alpha s_\beta + c_\beta c_\alpha\right) + T_{H^0} \left(-s_\beta c_\alpha + c_\beta s_\alpha\right)\right] , \\
t_{A^0A^0} &= \frac{e}{2m_W s_w} \left[T_{h^0} \left(\frac{-s_\beta^2 s_\alpha}{c_\beta} + \frac{c_\beta^2 c_\alpha}{s_\beta}\right) + T_{H^0} \left(\frac{s_\beta^2 s_\alpha}{c_\beta} + \frac{c_\beta^2 s_\alpha}{s_\beta}\right)\right] . \quad (3.95)
\end{aligned}$$

After rotation to mass eigenstates the counterterm for the G^0A^0–system reads as

$$\delta M_{G^0A^0}^2 = \begin{pmatrix} \frac{c_\beta \delta T_1}{v} + \frac{s_\beta \delta T_2}{v} & \frac{c_\beta \delta T_2}{v} - \frac{s_\beta \delta T_1}{v} \\ \frac{c_\beta \delta T_2}{v} - \frac{s_\beta \delta T_1}{v} & \delta m_{A^0}^2 + \frac{\delta T_2 c_\beta^2}{s_\beta v} + \frac{s_\beta^2 \delta T_1}{v c_\beta} \end{pmatrix} . \quad (3.96)$$

Reexpressing the tadpoles T_1 and T_2 in terms of T_{h^0} and T_{H^0} leads to

$$\delta M_{G^0A^0}^2 = \begin{pmatrix} \delta t_{G^0G^0} & \delta t_{A^0G^0} \\ \delta t_{G^0A^0} & \delta m_{A^0}^2 + \delta t_{A^0A^0} \end{pmatrix} , \quad (3.97)$$

with

$$\begin{aligned}
\delta t_{A^0G^0} &= \frac{c_\beta \delta T_2}{v} - \frac{s_\beta \delta T_1}{v} & (3.98) \\
&= -\frac{1}{2} \left[(c_\beta s_\alpha - s_\beta c_\alpha) T_{H^0} + (c_\beta c_\alpha + s_\beta s_\alpha) T_{h^0} \right] . & (3.99)
\end{aligned}$$

with $\delta T_{H^0} = -\tau_{H^0}$ and $\delta T_{h^0} = -\tau_{h^0}$.
The wavefunction renormalization constants for the G^0A^0–system are then given by

$$\begin{aligned}
\delta Z_{A^0A^0} &= -\mathrm{Re}\,\dot{\Pi}_{A^0A^0}(m^2_{A^0})\ , && (3.100)\\
\delta Z_{G^0G^0} &= -\mathrm{Re}\,\dot{\Pi}_{G^0G^0}(m^2_{G^0})\ , && (3.101)\\
\delta Z_{A^0G^0} &= \frac{2}{m^2_{A^0} - m^2_{G^0}}\left(\mathrm{Re}\,\Pi_{A^0G^0}(m^2_{A^0}) - \delta t_{A^0G^0}\right)\ , && (3.102)\\
\delta Z_{G^0A^0} &= \frac{2}{m^2_{G^0} - m^2_{A^0}}\left(\mathrm{Re}\,\Pi_{G^0A^0}(m^2_{G^0}) - \delta t_{A^0G^0}\right)\ . && (3.103)
\end{aligned}$$

3.8.3 The charged Higgs mass matrix

The mass matrix of the charged Higgs system is given by

$$M^2_C = \begin{pmatrix} s^2_\beta(m^2_{A^0} + m^2_W) + \frac{T_1}{v_1} & \frac{s_{2\beta}}{2}(m^2_{A^0} + m^2_W) \\ \frac{s_{2\beta}}{2}(m^2_{A^0} + m^2_W) & c^2_\beta(m^2_{A^0} + m^2_W) + \frac{T_2}{v_2} \end{pmatrix}\ . \tag{3.104}$$

The corresponding mass counterterm can be written as

$$\delta M^2_C = \begin{pmatrix} \delta s^2_\beta(m^2_{A^0} + m^2_W) + s^2_\beta(\delta m^2_{A^0} + \delta m^2_W) + \frac{\delta T_1}{v_1} & \frac{\delta s_{2\beta}}{2}(m^2_{A^0} + m^2_W) + 2s_{2\beta}(\delta m^2_{A^0} + \delta m^2_W) \\ \frac{\delta s_{2\beta}}{2}(m^2_{A^0} + m^2_W) + 2s_{2\beta}(\delta m^2_{A^0} + \delta m^2_W) & \delta c^2_\beta(m^2_{A^0} + m^2_W) + c^2_\beta(\delta m^2_{A^0} + \delta m^2_W) + \frac{\delta T_2}{v_2} \end{pmatrix}\ . \tag{3.105}$$

Performing the rotation to mass eigenstates the counterterm reads as

$$\delta M^2_{G^\pm} = \begin{pmatrix} \delta t_{G^0G^0} & \delta t_{G^0A^0} \\ \delta t_{A^0G^0} & \delta m^2_{H^\pm} + \delta t_{A^0A^0} \end{pmatrix}\ , \tag{3.106}$$

with $\delta m^2_{H^\pm} = \delta m^2_{A^0} + \delta m^2_W$. The wavefunction renormalization constants for the charge Higgs system $G^\pm H^\pm$ can then be expressed as

$$\begin{aligned}
\delta Z_{H^+H^+} &= -\mathrm{Re}\,\dot{\Pi}_{H^+H^+}(m^2_{H^+})\ , && (3.107)\\
\delta Z_{G^+G^+} &= -\mathrm{Re}\,\dot{\Pi}_{G^+G^+}(m^2_{G^+})\ , && (3.108)\\
\delta Z_{H^+G^+} &= \frac{2}{m^2_{H^+} - m^2_{G^+}}\left(\mathrm{Re}\,\Pi_{H^+G^+}(m^2_{H^+}) - \delta t_{H^+G^+}\right)\ , && (3.109)\\
\delta Z_{G^+H^+} &= \frac{2}{m^2_{G^+} - m^2_{H^+}}\left(\mathrm{Re}\,\Pi_{G^+H^+}(m^2_{G^+}) - \delta t_{H^+G^+}\right)\ . && (3.110)
\end{aligned}$$

Chapter 4

MSSM two–body Higgs decays at full one-loop level

4.1 Introduction

The search for the Higgs boson(s) is one of the main goals of the LHC. The Standard Model predicts one Higgs boson, with the present lower bound of its mass $m_H \geq 114.4$ GeV (at 95% confidence level) [43]. Extensions of the SM allow the existence of more than one Higgs bosons. As already mentioned, the Minimal Supersymmetric Standard Model (MSSM) contains five physical Higgs bosons: two neutral CP–even (h^0 and H^0), one neutral CP–odd (A^0) and two charged ones ($H^\pm$) [44, 45]. The existence of a charged Higgs boson or a CP–odd neutral one would give clear evidence for physics beyond the SM. For a MSSM Higgs discovery precise predictions for its decay modes and branching ratios are mandatory. The goal of this thesis was to develop a program package, that calculates all two–body MSSM Higgs boson decays at full one–loop level. Therefore the program code *HFOLD* (Higgs Full One Loop Decays) was developed. In the following section we will discuss the MSSM Higgs decay modes, the renormalization used in *HFOLD* and details of the program code. Since there are many decay modes, we will keep the discussion on a quite general level.

4.1.1 Decay patterns

As fermion number is conserved we only have four possibilities of Feynman graphs (at any loop level) for a two-body decay of a scalar: the decay into two scalars, into two fermions, into a scalar and a vector boson, and into two vector bosons, see Fig. (4.1). The following

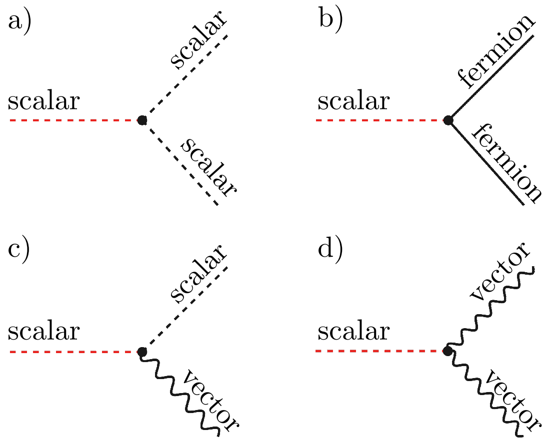


Figure 4.1: Four possibilities of two-body decays of a scalar particle

MSSM Higgs boson decays have been calculated:

$$\begin{array}{rrcl}
\text{Fig. 4.1a}: & \phi & \to & \tilde{f}_i\,\tilde{f}_j^*\;(i,j=1,2), \\
 & H^+ & \to & \tilde{f}_i\,\tilde{f}_j^{\prime *}\,, \\
 & H^0 & \to & h^0\,h^0\,,\;A^0\,A^0\,,\;A^0\,Z^0\,, \\
\text{Fig. 4.1b}: & \phi & \to & f\,\bar{f}\,, \\
 & \phi & \to & \tilde{\chi}^0_k\,\tilde{\chi}^0_l\;(k,l=1,\ldots,4)\,, \\
 & \phi & \to & \tilde{\chi}^+_r\,\tilde{\chi}^-_s\;(r,s=1,2)\,, \\
 & H^+ & \to & f\,\bar{f}'\,, \\
 & H^+ & \to & \tilde{\chi}^0_k\,\tilde{\chi}^+_s\,, \\
\text{Fig. 4.1c}: & A^0 & \to & h^0\,Z^0\,,H^0\,Z^0\,, \\
 & H^+ & \to & h^0\,W^+\,,\;H^0\,W^+\,, \\
\text{Fig. 4.1d}: & H^0 & \to & Z^0\,Z^0,\;W^+\,W^-\,, \\
 & \phi & \to & \gamma\gamma,\;gg,\;\gamma Z^0\quad\text{(loop induced)}\,,
\end{array}$$

$\phi = h^0$, H^0, A^0 and $f = \nu_l,\ e,\ \mu,\ \tau,\ u,\ d,\ c,\ s,\ t,\ b$, f' denotes the isospin partner to f, e.g. $f = t,\ f' = b$, $\tilde{f}_i$ and $\tilde{f}'_j$ denote the SUSY partners of f and f', $\tilde{\chi}^0_k$ and $\tilde{\chi}^\pm_s$ are the neutralinos and charginos, respectively. The Higgs bosons couple to fermions via their Yukawa couplings. Therefore, the branching ratios (BRs) into top quark(s) are large, if they are kinematically allowed. The BRs of $h^0 \to \bar{b}\,b$ and to $\tau^+\tau^-$ are dominant, especially for large $\tan\beta$. The decays into the third generation sfermions may become dominant when they are kinematically possible. The decays into quarks and squarks can have large one-loop SUSY–QCD corrections. The decays into charginos and/or neutralinos can have significant one-loop contributions from the third generation (s)fermions depending on the gaugino/higgsino mixing.

4.2 Calculation at full one-loop level

Beyond tree level one has to deal with so called ultraviolet (UV) and infrared (IR) divergences. The UV divergences originate from the infinite momentum in loops. These divergences have to be subtracted in a consistent way to obtain finite amplitudes at one–loop level. The IR divergencies arise form photons or gluons in loops. Since these particles are massless, they cause divergences for zero momentum. These IR divergences can be cured by adding the corresponding three-body process decay with one additional photon/gluon in the final state.
At full one-loop level one has to deal with hundreds of Feynman diagrams in the MSSM, this makes a correct calculation per hand almost impossible. Therefore tools for generating the amplitudes and for their tensorial reduction are necessary. We have generated all amplitudes with the tool FeynArts 3.3 (FA) and the corresponding Fortran code was produced with the help of FormCalc 5.4 (FC)[91]. It was also necessary to work out all counterterms for the whole MSSM. Since there are many decay channels it was worthwhile to develop tools at Mathematica level, that perform the renormalization automatically.

4.2.1 Renormalization of two–body decays

The definition of the counterterms has been given in the previous section. Now we want to discuss the renormalization of two–body decay processes. The renormalized finite one-loop amplitude $\mathcal{M}^1$ is the sum of all vertex diagrams, the amplitudes arising from the coupling counterterms $\mathcal{M}^{CT}$ and the amplitudes arising from the wavefunction renormalization constants $\mathcal{M}^{WFR}$,

$$\mathcal{M}^1 = \mathcal{M}^{vertex} + \mathcal{M}^{CT} + \mathcal{M}^{WFR} \,. \tag{4.1}$$

Since the renormalization in *HFOLD* is performed in the $\overline{\text{DR}}$ scheme, the counterterms contain only UV divergent parts. We first calculated the counterterms in the on–shell scheme, where we checked the UV and IR convergence of our amplitudes. This was a good check for the correctness of the calculated amplitudes. Then we modified the counterterm file so that the CTs contain only the UV divergent part. However, to maintain IR convergence of our amplitudes we take into account the finite parts of the wavefunction renormalization constants. The formulae for the wavefunction and mass counterterms (CTs) for sfermions, fermions and vector bosons in the on-shell scheme and the corresponding renormalization conditions can be found e.g. in [88, 89, 90]. The results for the $\overline{\text{DR}}$ scheme are then simply the UV divergent parts of the on-shell CTs. The renormalized one-loop amplitude is the sum of the tree-level amplitude and the one-loop contributions, see Fig E.4.
The vertex corrections and all selfenergy contributions can be directly calculated with FA/FC.
The total two-body Higgs decay width can be written in one-loop approximation as

$$\begin{aligned} \Gamma &= N_C \times kin \times \Big(|\mathcal{M}_0|^2 + 2\,\mathrm{Re}(\mathcal{M}_0^\dagger \mathcal{M}_1) \Big) \,, \\ kin &= \frac{\kappa(m_0^2, m_1^2, m_2^2)}{16\pi m_0^3} \,, \end{aligned}$$

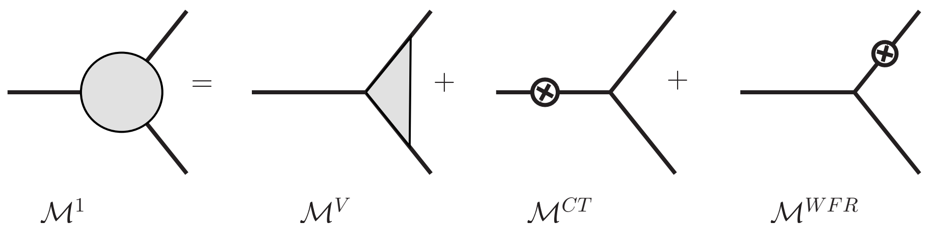


Figure 4.2: One-loop renormalization procedure of a 1 to 2 process schematically

with the colour factor $N_C = 3$ for decays into quarks and squarks and $N_C = 1$ for decays into other particles, respectively.

4.3 Setup for automatic renormalization

4.3.1 Coupling renormalization

For the coupling renormalization we shift every inpendent coupling by $g^0 \to g + \delta g$, where g denotes the renormalized parameter and δg denotes the corresponding counterterm. Then we only take into account terms in linear in δg. This procedure has been implemented at the Mathematica level. For example: the charged Higgs decay $H^+ \to t\bar{b}$ is described at tree–level by the matrix element

$$\mathcal{M}_{H^+\to t\bar{b}} = \frac{h_b s_\beta}{t_\beta} P_L + h_t c_\beta t_\beta P_R \ . \tag{4.2}$$

Now we perform the shift in the parameters:

$$\begin{aligned} s_\beta &\to s_\beta + \delta s_\beta \ , \\ c_\beta &\to c_\beta + \delta c_\beta \ , \\ t_\beta &\to t_\beta + \delta t_\beta \ , \\ h_b &\to h_b + \delta h_b \ , \\ h_t &\to h_t + \delta h_t \ . \end{aligned}$$

Plugging this back into eq. (4.2) and taking only terms linear in δg, we obtain the coupling counterterm matrix element

$$\mathcal{M}^{CT}_{H^+\to t\bar{b}} = -\frac{1}{t_\beta t_{\beta^2}} \left(\delta c_\beta P_R h_b t_\beta^2 + \delta s_\beta P_L h_t t_\beta^2 + c_\beta \delta t_\beta P_R t_\beta h_b \tan\beta^2 - \delta t_\beta P_L s_\beta t_\beta h_t\right) \ . \tag{4.3}$$

4.3.2 Wavefunction renormalization

We have also automatized the necessary procedure of wavefunction renormalization at the Mathematica level with the use of generic formulae. In our renormalization approach the one–loop induced particle mixing has been taken into account at the stage of wavefunction renormalization for all the Higgs systems (H^0h^0, G^0A^0, $G^\pm H^\pm$), the chargino system $\tilde{\chi}_i^\pm$ and the neutralino system $\tilde{\chi}_i^0$. Therefore we need the Lagrangians, where one external particle of the original configuration is exchanged by its corresponding mixing partner due wavefunction renormalization. The four generic Lagrangians for the decay modes are given by

$$\begin{aligned}
\mathcal{L}_{SFF} &= \bar{\psi}_1(g_L P_L + g_R P_R)\psi_2\phi_0 \ , &(4.4)\\
\mathcal{L}_{SSS} &= g\phi_0\phi_1\phi_2 \ , &(4.5)\\
\mathcal{L}_{SVV} &= g\phi_0 V_1 V_2 \ , &(4.6)\\
\mathcal{L}_{SSV} &= g\phi_0\phi_1 V_2 \ . &(4.7)
\end{aligned}$$

(We skipped Lorentz indices in this section, since they don't provide further insight.) We will perform the explicit calculation for a scalar decay into two fermions. Replacing successively every external field in eq. (4.4) by its corresponding mixing partner leads to three Lagrangians given by

$$\begin{aligned}
\mathcal{L}^{mI}_{SFF} &= \bar{\psi}_1(g^{m0}_L P_L + g^{m0}_R P_R)\psi_2\phi^m_0 \ , &(4.8)\\
\mathcal{L}^{mII}_{SFF} &= \bar{\psi}_{m1}(g^{m1}_L P_L + g^{m1}_R P_R)\psi_2\phi_0 \ , &(4.9)\\
\mathcal{L}^{mIII}_{SFF} &= \bar{\psi}_1(g^{m2}_L P_L + g^{m2}_R P_R)\psi_{m2}\phi_0 \ . &(4.10)
\end{aligned}$$

Where the mixing partner fields will be denoted with an upper index m. Now we peform the wavefunction renormalization of the field ϕ^m_0 (The wavefunction renmormalization matrix can of course be larger than 2×2.)

$$\begin{pmatrix}\phi_0\\ \phi^m_0\end{pmatrix} = \begin{pmatrix}1+\frac{1}{2}\delta Z_{11} & \frac{1}{2}\delta Z_{12}\\ \frac{1}{2}\delta Z_{21} & 1+\frac{1}{2}\delta Z_{22}\end{pmatrix}\begin{pmatrix}\phi_0\\ \phi^m_0\end{pmatrix} \ , \tag{4.11}$$

we only need the term

$$\phi^m_0 = \frac{1}{2}\delta Z_{21}\phi_0 + (1+\frac{1}{2}\delta Z_{22})\phi^m_0 \ , \tag{4.12}$$

plugging this back into eq. (4.8), we obtain the following expression

$$\mathcal{L}^{mI}_{SFF} = \bar{\psi}_1(g^{m0}_L P_L + g^{m0}_R P_R)\psi_2(\frac{1}{2}\delta Z_{21}\phi_0 + (1+\frac{1}{2}\delta Z_{22})\phi^m_0) \ . \tag{4.13}$$

As can be seen the resulting term gives a contribution to the ϕ^0 decay, neglecting the term proportional to ϕ^m_0 in eq. (4.13) leads to

$$\mathcal{L}^{mI}_{SFF} = \bar{\psi}_1(g^{m0}_L P_L + g^{m0}_R P_R)\psi_2\frac{1}{2}\delta Z_{21}\phi_0 \ . \tag{4.14}$$

The rest of the calculation with the two remaining external fermions is straightforward. Therefore we will just give the results for all four generic structures

$$\begin{aligned}
\mathcal{M}^{WFR}_{SFF} &= \frac{1}{2}g_L P_L\delta Z^\dagger_{\phi_0} + \frac{1}{2}g_R P_R\delta Z^\dagger_{\phi_0} + \frac{1}{2}g^{m0}_L P_L\delta Z^\dagger_{\phi^m_0} + \frac{1}{2}g^{m0}_R P_R\delta Z^\dagger_{\phi^m_0} &(4.15)\\
&+ \frac{1}{2}g_R P_R(\delta Z^L_{\psi_1})^\dagger + \frac{1}{2}g_L P_L(\delta Z^R_{\psi_1})^\dagger + \frac{1}{2}g^{m1}_R P_R(\delta Z^L_{\psi^m_1})^\dagger + \frac{1}{2}g^{m1}_L P_L(\delta Z^R_{\psi^m_1})^\dagger\\
&+ \frac{1}{2}g_L P_L\delta Z^L_{\psi_2} + \frac{1}{2}g_R P_R\delta Z^R_{\psi_2} + \frac{1}{2}g^{m2}_L P_L\delta Z^L_{\psi^m_2} + \frac{1}{2}g^{m2}_R P_R\delta Z^R_{\psi^m_2} \ ,
\end{aligned}$$

$$\mathcal{M}^{WFR}_{SSS} = \frac{1}{2} g \delta Z^{\dagger}_{\phi_0} + \frac{1}{2} g^m_0 \delta Z^{\dagger}_{\phi^m_0} + \frac{1}{2} g \delta Z^{\dagger}_{\phi_1} + \frac{1}{2} g^m_1 \delta Z^{\dagger}_{\phi^m_1} + \frac{1}{2} g \delta Z_{\phi_2} + \frac{1}{2} g^m_2 \delta Z_{\phi^m_2} \ , \tag{4.16}$$

$$\mathcal{M}^{WFR}_{SSV} = \frac{1}{2} g \delta Z_{V_2} + \frac{1}{2} g^m_2 \delta Z_{V^m_2} + \frac{1}{2} g \delta Z^{\dagger}_{\phi_0} + \frac{1}{2} g^m_0 \delta Z^{\dagger}_{\phi^m_0} + \frac{1}{2} g \delta Z^{\dagger}_{\phi_1} + \frac{1}{2} g^m_1 \delta Z^{\dagger}_{\phi^m_1} \ , \tag{4.17}$$

$$\mathcal{M}^{WFR}_{SVV} = \frac{1}{2} g \delta Z_{V_2} + \frac{1}{2} g^m_2 \delta Z_{V^m_2} + \frac{1}{2} g \delta Z^{\dagger}_{\phi_0} + \frac{1}{2} g^m_0 \delta Z^{\dagger}_{\phi^m_0} + \frac{1}{2} g \delta Z^{\dagger}_{\phi_1} + \frac{1}{2} g^m_1 \delta Z^{\dagger}_{\phi^m_1} \ . \tag{4.18}$$

Example: $H^+ \to t\bar{b}$, the top and bottom quark don't mix at one–loop level with other particles, therefore we have to choose:

$$\begin{aligned} \delta Z^L_{\psi^m_1} &= 0 \ , & \delta Z^R_{\psi^m_1} &= 0 \ , & (4.19)\\ \delta Z^L_{\psi^m_2} &= 0 \ , & \delta Z^R_{\psi^m_2} &= 0 \ . & (4.20) \end{aligned}$$

The charged Higgs boson H^+ mixes with the charged Goldstone boson G^+ therefore we have to choose

$$\delta Z_{\phi^m_0} = \delta Z_{H^+G^+} \ . \tag{4.21}$$

Choosing the other wavefunction renormalization constants by

$$\begin{aligned} \delta Z^L_{\psi_1} &= \delta Z^L_t \ , & \delta Z^R_{\psi_1} &= \delta Z^R_t \ , & (4.22)\\ \delta Z^L_{\psi_2} &= \delta Z^L_b \ , & \delta Z^R_{\psi_2} &= \delta Z^R_b \ , \\ \delta Z_{\phi_0} &= \delta Z_{H^+H^+} \ , \end{aligned}$$

plugging eqs. (4.21–4.22) in the generic formlua in eq. (4.15), we finally obtain the matrix element originating from the wavefunction renormalization by

$$\begin{aligned} \mathcal{M}^{WFR}_{H^+\to t\bar{b}} &= \left(-\frac{s_\beta h_t \delta Z^{R\dagger}_t}{2t_\beta} - \frac{s_\beta \delta Z^{\dagger}_{H^+H^+} h_t}{2t_\beta} - \frac{1}{2} s_\beta \delta Z^{\dagger}_{H^+G^+} h_t - \frac{s_\beta \delta Z^L_b h_t}{2t_\beta} \right) P_L & (4.23)\\ &+ \left(-\frac{1}{2} c_\beta t_\beta h_b \delta Z^{L\dagger}_t - \frac{1}{2} c_\beta t_\beta \delta Z^{\dagger}_{H^+H^+} h_b + \frac{1}{2} c_\beta \delta Z^{\dagger}_{H^+G^+} h_b - \frac{1}{2} c_\beta t_\beta \delta Z^R_b h_b \right) P_R \ . \end{aligned}$$

4.3.3 Types of vertex diagrams

There are four different topological types of vertex diagrams shown in fig. (4.3-4.4). For the complete discussion of these diagrams with gerneric couplings and fields in terms of Passarino Veltman integrals, we refer to e.g. [101].

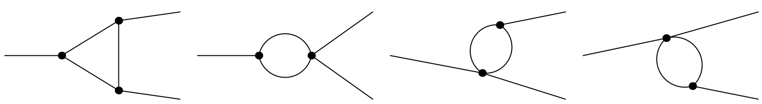


Figure 4.3: different types of vertex topologies

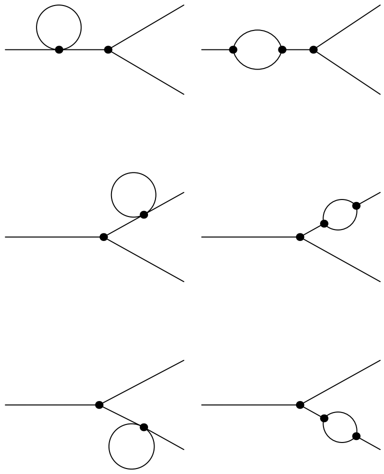


Figure 4.4: diagrams with a transition

4.3.4 SUSY Parameter Analysis (SPA) convention

The definition of the parameters is not unique beyond the leading order and depends on the renormalization scheme. Therefore, a well-defined theoretical framework was proposed within the so-called SPA (SUSY Parameter Analysis) project [87]. The "SPA convention" provides a clear base for calculating masses, mixing angles, decay widths and production processes. It also provides a clear definition of the fundamental parameters using the $\overline{\mathrm{DR}}$ renormalization scheme which allows one to extract them from future data.

- Masses of SUSY particles and Higgs bosons are defined as pole masses.
- All SUSY Lagrangian parameters are defined in the $\overline{\mathrm{DR}}$ scheme at the scale Q = 1 TeV.
- All elements in mass matrices, rotation matrices and corresponding mixing angles are defined in the $\overline{\mathrm{DR}}$ scheme at the scale Q = 1 TeV, except $\alpha_{H^0h^0}$ the $(h^0 - H^0)$ mixing angle, which is defined on-shell at $p = m_{h^0}$.
- SM input parameters are: $G_{Fermi}, \alpha, m_Z, \alpha_s(m_Z)$ and the fermion masses.

4.4 Input parameters

HFOLD is designed to be applied to SUSY models like mSUGRA, NUHM, AMSB etc. where the low energy model parameters are given at some scale Q. The low energy spectrum is derived from a few parameters defined at a high scale using renormalization group equations. At the program start, *HFOLD* reads the spectrum, where the Yukawa couplings, the gauge couplings $g_{\prime}$, g, g_s, the soft breaking terms, the VEVs, $m_{A^0}, \tan\beta$, μ and the on-shell Higgs masses are taken as input parameters. The input parameters are understood as running parameters in the $\overline{\mathrm{DR}}$ scheme at the scale Q. In loops we are free to use $\overline{\mathrm{DR}}$ masses because the difference is of higher order in perturbation theory. Since our renormalization is done in the $\overline{\mathrm{DR}}$ scheme the coupling counterterms contain only UV-divergent parts. Therefore we do not fix δm_W with G_{Fermi} as input parameter. In the Higgs sector we use m_{A^0} and the running $\tan\beta$ as inputs. We can then simply derive the $\overline{\mathrm{DR}}$ running Higgs mixing angle α at the scale Q. We do not take α_{eff} as input parameter because we consider our calculation a self consistent one–loop expansion.

Gauge sector :	$g^{'},\ g,\ g_s$
Higgs sector :	$\tan\beta,\ v,\ \mu,\ m_{A^0}$
	$m_{h^0}^{\mathrm{pole}},\ m_{H^0}^{\mathrm{pole}}\ ,m_{A^0}^{\mathrm{pole}}\ m_{H^\pm}^{\mathrm{pole}}$
Yukawa–couplings :	$y_u,\ y_d,\ y_l$
Sfermion–masses :	$M_{\tilde{L}},\ M_{\tilde{E}},\ M_{\tilde{U}},\ M_{\tilde{D}}$
Soft–SUSY-breaking–terms :	$A_e,\ A_u,\ A_d$
Gaugino masses :	$M_1,\ M_2\ ,M_3$

Table 4.1: Input parameters for *HFOLD* (generation and sfermion indices are suppressed)

4.5 Resummation of $\tan\beta$

The down-type fermions couple to the up-type Higgs doublet with radiative corrections,

$$-\,y_b H_d^0 \bar{b} b - y_b \Delta_b \cot\beta H_u^0 \bar{b} b\,. \tag{4.24}$$

The selfenergy Δ_b is proportional to $\tan\beta$ and can be enhanced for large values of $\tan\beta$. This term can be resummed (in the effective potential approach) by replacing the bottom Yukawa coupling [92]

$$y_b \to \frac{y_b}{1+\Delta_b}. \tag{4.25}$$

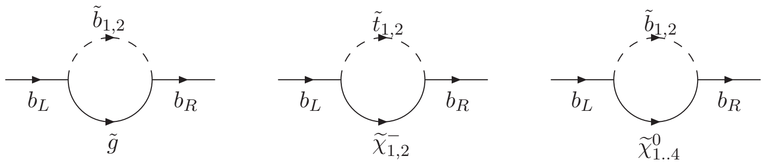


Figure 4.5: $\tan\beta$-enhanced selfenergy diagrams with gluinos, charginos and neutralinos.

The resummation can also be performed in the diagrammatic approach [93]. Different renormalization schemes correspond to different choices of counterterms. Therefore the analytic form of the $\tan\beta$ enhanced corrections depend on the chosen renormalization scheme. In the on-shell scheme one takes the measured bottom mass as input parameter. The choice of δm_b fixes δy_b by

$$y_b = \frac{m_b}{v_d} \to \delta y_b = \frac{\delta m_b}{v_d}\,. \tag{4.26}$$

The quark mass counterterm δm_b is a source of $\tan\beta$-enhanced corrections. The selfenergy $\Sigma^{RL}(m_b)$ contains terms proportional to $y_b \sin\beta$ and is therefore $\tan\beta$ enhanced,

$$\Sigma^{RL} = m_b \Delta_b \,, \tag{4.27}$$

$$\Delta_b = \Delta_b^{\tilde{g}} + \Delta_b^{\tilde{\chi}^{\pm}} + \Delta_b^{\tilde{\chi}^0} \,. \tag{4.28}$$

In leading order this means : $\delta m_b = -\Sigma_b^{RL} = -m_b \epsilon_b \tan\beta$. We write the bare Yukawa couplings as $y_b^{(0)} = y_b + \delta y_b$, where y_b is the renormalized coupling and δy_b is the counterterm. The choice of δm_b fixes δy_b through

$$\delta y_b = \frac{\delta m_b}{v_d} = -y_b \epsilon_b \tan\beta. \tag{4.29}$$

The supersymmetric loop effects encoded in ϵ_b enter physical observables only through δy_b. Choosing e.g. a minimal subtraction scheme like the $\overline{\text{DR}}$ scheme for δm_b removes the $\tan\beta$-enhanced terms and there is nothing to resum anymore. Since we do not use the measured bottom mass as input, the resummation of $\tan\beta$ is absent in our approach. However, the resummation eq. (4.25) is implemented in the code and can be turned on.

4.5.1 Gauge used

The code of *HFOLD* is derived in the SPA convention in the general linear R_ξ gauge for the $W^\pm$ and Z^0-boson. The gauge fixing Lagrangian in the general linear R_ξ gauge is

$$\mathcal{L}^{GF} = -\frac{1}{\xi_W} F^+ F^- \frac{1}{\xi_A} |F^A|^2 \,, \quad A = Z, \gamma, g \,,$$

with $F^+ = \partial_\mu W^{\mu +} + i \xi_W m_W G^+$, $F^Z = \partial_\mu Z^\mu + \xi_Z m_Z G^0$, $F^\gamma = \gamma_\mu A^\mu$, and $F^g = \gamma_\mu G^{a\mu}$.

The Higgs-ghost propagators are $i/(q^2 - \xi_V m_V^2)$ and the vector-boson propagator in the R_ξ gauge reads

$$D_V^{\mu\nu} = \frac{-i \left(g^{\mu\nu} - (1-\xi_V) \frac{q^\mu q^\nu}{q^2 - \xi m_V^2} \right)}{q^2 - m_V^2} \,.$$

The ξ-dependent part is a product of two propagators leading to a (n+1)-point loop integral. Performing an expansion into partial fractions, it can be split into a form with single propagators only,

$$D_V^{\mu\nu} = \frac{-i\, g^{\mu\nu}}{q^2 - m_V^2} + \frac{i}{m_V^2} \left(\frac{q^\mu q^\nu}{(q^2 - m_V^2)} - \frac{q^\mu q^\nu}{(q^2 - \xi m_V^2)} \right)$$

We have implemented this second form into FA in order to check gauge independence for W and Z. For the massless particles γ and gluon we get derivatives of loop integrals. In these cases it is possible to proof gauge invariance analytically.

Chapter 5

Numerical results

5.1 Comparison at mSUGRA–benchmark point SPS1a'

In the following section we will compare *HFOLD* with existing programs at the mSUGRA point SPS1a', which was proposed in the SPA project [87] (in the meantime already experimentally excluded). Our comparison with other programs is based on the same input file with the MSSM spectrum given in the SUSY Les Houches accord form [86] created by `SPheno3.0beta50` [95].
A list of available decay programs is given at `http://home.fnal.gov/ skands/slha/`. In the following tables the Higgs bosons partial and total decay widths are compared to *HDECAY* 3.53[97] and *FeynHiggs* 2.7.4[96]. In *FeynHiggs* 2.7.4 the Higgs decays in fermions

	m_0	$m_{1/2}$	$\tan\beta$	sign(μ)	A_0
SPS1a'	250 [GeV]	70 [GeV]	10	+	-300 [GeV]

are evaluated at full one-loop level. *HDECAY* 3.53 has implemented higher order QCD and some EW corrections. Most of these corrections are incorporated into running masses.
In table (5.1) the SM input parameters for *SPheno* are shown, the tables (5.2,5.3) exhibit the relevant $\overline{\text{DR}}$ parameters and the SUSY breaking parameters at the scale of 1 TeV. Table (5.4) shows the numerical values of the SUSY particle masses calculated with *HFOLD* and *SPheno*.
The total widths of the MSSM Higgs bosons calculated with various programs is shown in the tables (5.5,5.6,5.7,5.8). The partial decay widths are given in the tables (5.9,5.10,5.11,5.12). To avoid confusion the column SQCD (=SUSY–QCD) in the tables exhibiting the partial decay widths, denote the corrections arising form the gluons and the gluino, these corrections only exist at one–loop level, if the external particles have color charge. Therefore the SUSY–QCD corrections to e.g. $H^0 \to \tilde{\chi}_1^0\tilde{\chi}_1^0$ are zero, since the neutralinos have no color charge and do not couple to SUSY–QCD at one–loop level. Thus the SUSY–QCD corrections have the same numerical value as the tree–level calculation for Higgs decays into uncolored particles.

At the SPS1a' point, Higgs decays into bottom quarks have the largest BRs for all neutral Higgs bosons. Decays into the lightest neutralinos, charginos and sleptons are also kinematically allowed. These decays can receive sizeable corrections up to 20 percent for the neutral Higgs bosons. For H^+ the difference between the tree–level and full one–loop calculation for $H^+ \to \tilde{\chi}_1^+ \tilde{\chi}_1^0$ is 12 percent. However, the corresponding branching rations are small. Since the main focus of this work is on the evaluation of the decay widths and corresponding BRs of the heavy Higgs bosons at full one-loop level, we have implemented only these corrections for h^0. For precise predictions of h^0 decays it is necessary to consider also below treshold decays as well as leading two–loop contributions. This explains the deviation for the total widths of h^0 compared to the other programs exhibited in table (5.5).

Details of the comparison

For our comparison with the other programs the bremsstrahlung in *HFOLD* is calculated using the hard bremsstrahlung approach, the on–shell masses of the SUSY particles are calculated with the corresponding one–loop selfenergy shifts from the $\overline{\text{DR}}$ masses ($m^{pole} = m^{\overline{\text{DR}}} - \Sigma^{self}(m^{\overline{\text{DR}}})$). For sfermions we also take the off–diagonal elements in the one–loop corrected mass matrix into account. The flags for the generation of the particle spectrum using *SPheno* can be found in the appendix D.

SM input	value
$\boldsymbol{\alpha_{em}^{-1\overline{\text{MS}}}(m_Z)}$	127.932
$\boldsymbol{G_\mu}$	1.167 10^{-5} [GeV^{-2}]
$\boldsymbol{\alpha_s^{\overline{\text{MS}}}(m_Z)}$	0.118
$\boldsymbol{m_Z^{\text{pole}}}$	91.188 [GeV]
$\boldsymbol{m_b^{\overline{\text{MS}}}(m_b)}$	4.2 [GeV]
$\boldsymbol{m_t^{\text{pole}}}$	171.2 [GeV]
$\boldsymbol{m_{tau}^{\text{pole}}}$	1.777 [GeV]

Table 5.1: SM input parameters for *SPheno*

Parameter	value	Parameter	value
g'	0.364	y_t	0.862
g	0.647	y_b	0.137
g_3	1.080	y_τ	0.104

Table 5.2: $\overline{\text{DR}}$ parameters at the scale Q = 1 TeV

Parameter	value	Parameter	value
M_1	103.589	M_{L1}	180.817
M_2	193.499	M_{L2}	180.811
M_3	568.361	M_{L3}	179.057
M_{E1}	115.632	M_{Q1}	522.271
M_{E2}	115.612	M_{Q2}	522.269
M_{E3}	109.940	M_{Q3}	468.245
M_{U1}	503.598	M_{D1}	501.395
M_{U2}	503.596	M_{D2}	501.393
M_{U3}	384.882	M_{D3}	497.174
A_t	-565.868	m_d^2	25352.31
A_b	-937.403	m_u^2	-140313.122
A_τ	-444.542		

Table 5.3: SUSY breaking parameters in GeV at the scale Q = 1 TeV

Particle	*SPheno* mass	*HFOLD* mass	Particle	SPheno mass	*HFOLD* mass
h^0	110.699	110.699	$\tilde{g}$	611.445	608.299
H^0	421.139	421.139	$\tilde{d}_2$	566.228	562.645
A^0	420.982	420.982	$\tilde{d}_1$	542.859	539.647
$H^\pm$	428.942	428.942	$\tilde{u}_2$	560.749	557.135
χ_1^0	97.973	96.235	$\tilde{u}_1$	543.163	539.863
χ_2^0	183.869	183.846	$\tilde{s}_2$	566.228	562.645
χ_3^0	396.727	396.742	$\tilde{s}_1$	542.854	539.643
χ_4^0	410.499	410.471	$\tilde{c}_2$	560.761	557.148
χ_1^+	183.631	183.598	$\tilde{c}_1$	543.146	539.847
χ_2^+	411.93	411.949	$\tilde{b}_1$	502.661	499.933
$\tilde{e}_1$	125.253	125.212	$\tilde{b}_2$	541.697	538.623
$\tilde{e}_2$	189.659	189.625	$\tilde{t}_1$	361.071	360.615
$\tilde{\nu}_{eL}$	172.308	172.301	$\tilde{t}_2$	583.088	579.591
$\tilde{\mu}_2$	189.681	189.647	$\tilde{\tau}_1$	108.001	107.96
$\tilde{\mu}_1$	125.19	125.15	$\tilde{\tau}_2$	194.65	194.65
$\tilde{\nu}_{\mu L}$	172.301	172.294	$\tilde{\nu}_{\tau L}$	170.253	170.26

Table 5.4: *SPheno* vs. *HFOLD* pole-masses in GeV at SPS1a'

h^0	HF-tree	HF-SQCD	HF-full	FH 2.7.4	HD 3.53
Γ^{total}	1.9	3.0	2.8	3.2	3.7

Table 5.5: Comparison of the total decay widths of the CP-even Higgs boson h^0 (in MeV)

H^0	HF-tree	HF-SQCD	HF-full	FH 2.7.4	HD 3.53
Γ^{total}	0.8389	1.0171	1.0274	0.9890	1.0495

Table 5.6: Comparison of the total decay widths of the CP-even Higgs boson H^0

A^0	HF-tree	HF-SQCD	HF-full	FH 2.7.4	HD 3.53
Γ^{total}	1.2471	1.4405	1.5256	1.4183	1.4139

Table 5.7: Comparison of the total decay widths of the CP-odd Higgs boson A^0

H^+	HF-tree	HF-SQCD	HF-full	FH 2.7.4	HDECAY
Γ^{total}	0.7534	0.9057	0.8948	0.7875	0.9671

Table 5.8: Comparison of the total decay widths of the charged Higgs boson H^+

h^0	BR-tree	HF-tree	HF-SQCD	HF-full	1-SQCD/full	FH 2.7.4	HD 3.53
$b\bar{b}$	0.8044	0.0015	0.0026	0.0024	-0.102	0.0025	0.0029
$\tau\bar{\tau}$	0.1544	0.0003	0.0003	0.0003	-0.076	0.0003	0.0003
$c\bar{c}$	0.0403	0.0001	0.0001	0.0001	-0.002	0.0001	0.0001

Table 5.9: Comparison of the partial decay widths of the CP-even Higgs boson h^0

H^0	BR-tree	HF-tree	HF-SQCD	HF-full	1-SQCD/full	FH 2.7.4	HD 3.53
$b\bar{b}$	0.5546	0.4652	0.6262	0.6216	-0.007	0.6283	0.6466
$\tau\bar{\tau}$	0.1058	0.0887	0.0887	0.0914	0.029	0.0983	0.0909
$t\bar{t}$	0.0549	0.0460	0.0631	0.0564	-0.119	0.0607	0.0937
$\tilde{\chi}_1^0\tilde{\chi}_2^0$	0.0539	0.0452	0.0452	0.0465	0.047	0.0429	0.0442
$\tilde{\chi}_1^+\tilde{\chi}_1^-$	0.0515	0.0432	0.0432	0.0527	0.181	0.0528	0.0568
$\tilde{\tau}_1\tilde{\tau}_1$	0.0212	0.0177	0.0177	0.0184	0.037	0.0183	0.0095
$\tilde{\tau}_1\tilde{\tau}_2$	0.0206	0.0173	0.0173	0.0191	0.093	0.0183	0.0262
$\tilde{\chi}_2^0\tilde{\chi}_2^0$	0.0205	0.0172	0.0172	0.0206	0.168	0.0210	0.0225
$\tilde{\chi}_1^0\tilde{\chi}_1^0$	0.0172	0.0144	0.0144	0.0140	-0.03	0.0122	0.0127

Table 5.10: Comparison of the partial decay widths of the CP-even Higgs boson H^0

A^0	BR-tree	HF-tree	HF-SQCD	HF-full	1-SQCD/full	FH 2.7.4	HD 3.53
$b\bar{b}$	0.3741	0.4665	0.6282	0.6250	-0.005	0.6269	0.6439
$\tilde{\chi}_1^+\tilde{\chi}_1^-$	0.1800	0.2245	0.2245	0.2862	0.216	0.2395	0.2389
$t\bar{t}$	0.0862	0.1074	0.1389	0.1289	-0.078	0.1881	0.1815
$\tilde{\chi}_1^0\tilde{\chi}_2^0$	0.0755	0.0942	0.0942	0.0972	0.097	0.0890	0.0871
$\tilde{\chi}_2^0\tilde{\chi}_2^0$	0.0729	0.0909	0.0909	0.1166	0.22	0.0975	0.0955
$\tau\bar{\tau}$	0.0713	0.0889	0.0889	0.0919	0.032	0.0980	0.0911
$\tilde{\tau}_1\tilde{\tau}_2$	0.0225	0.0280	0.0280	0.0297	0.055	0.0292	0.0272
$\tilde{\chi}_1^0\tilde{\chi}_1^0$	0.0170	0.0212	0.0212	0.0205	-0.03	0.0181	0.0183

Table 5.11: Comparison of the partial decay widths of the CP-odd Higgs boson A^0

H^+	BR-tree	HF-tree	HF-sqcd	HF-full	1-SQCD/full	FH 2.7.4	HD 3.53
$t\bar{b}$	0.6171	0.4649	0.6170	0.5989	-0.03	0.5060	0.6850
$\tilde{\chi}_1^+\tilde{\chi}_1^0$	0.1712	0.1290	0.1290	0.1306	0.012	0.1194	0.1228
$\tau\nu_\tau$	0.1203	0.0906	0.0906	0.0944	0.04	0.0922	0.0927
$\tilde{\nu}_\tau\tilde{\tau}_1$	0.0809	0.0610	0.0610	0.0643	0.052	0.0630	0.0581

Table 5.12: Comparison of the partial decay widths of the charged Higgs boson H^+

5.1.1 Checks of IR and UV correctness

Processes generated with FA/FC provide a good opportunity for checking their correctness. The UV divergence parameter Δ and the photon/gluon mass Λ are implemented in the code. The one–loop corrected widths should be independent of the numerical value of these parameters. To check the correctness of our calculated processes, we set $\Delta = 10^9$ and then $\lambda = 10^{160}$. To perform the checks of UV and IR convergence, it is necessary to set the *HFOLD* flags *osmasses* and *susymasses* to zero.

	$\boldsymbol{\Gamma^{tot}\ (\Delta = 0\ , \Lambda = 0)}$	$\boldsymbol{\Gamma^{tot}(\Delta = 10^9\ , \Lambda = 0)}$	$\boldsymbol{\Gamma^{tot}(\Delta = 0\ , \Lambda = 10^{160})}$
$\boldsymbol{h^0}$	0.00153052817	0.00153052817	0.00153052817
$\boldsymbol{H^0}$	0.665873399	0.665873399	0.665873399
$\boldsymbol{A^0}$	1.09668832	1.09668832	1.09668832
$\boldsymbol{H^+}$	0.77892842	0.778929415	0.77892842

Table 5.13: Dependence of the total withs on the UV and IR parameters Δ and Λ

As can be seen in table (5.13), the difference between the various total widths is tiny, which indicates that our calculated amplitudes are IR and UV convergent.

5.2 High scale scenarios

Studying the radiative corrections for MSSM Higgs boson decays for the complete mSUGRA parameter space would be beyond the scope of this work. However, we have created three benchmark scenarios to show important trends of these corrections. Since the program is public available it can be most easily adopted to any other high scale scenario.

- **scenario mSUGRA1** :
  $\boldsymbol{(m_0,\ m_{1/2},\ \tan\beta,\ \mathrm{sign}(\mu),\ A_0) = (700 - 1300, 200, 3, +, 0)}$
- **scenario mSUGRA2** :
  $\boldsymbol{(m_0,\ m_{1/2},\ \tan\beta,\ \mathrm{sign}(\mu),\ A_0) = (700 - 1300, 200, 10, +, 0)}$
- **scenario NUHM** :
  $\boldsymbol{(m_0,\ m_{1/2},\ \tan\beta,\ \mathrm{sign}(\mu),\ A_0,\ m_1,\ m_2) = (250, 250, 10, +, 0, 550{-}1650, 550)}$

The parameters m_0, $m_{1/2}$, m_1, m_2 and A_0 are given in GeV.

5.2.1 Scenario mSUGRA1

In scenario mSUGRA1 we will vary the universal scalar mass m_0 from 700 to 1300 GeV, keeping the other parameters fixed. This area in mSUGRA space is in agreement with recent CMS and ATLAS measurements e.g. [106]. Figure (5.1) exhibits the m_0 dependence of the mass spectrum. As can be seen, the Higgs sector is in the decoupling regime. The heavy Higgs boson masses are highly degenerate in mass around 900 GeV, the mass of light CP–even Higgs boson h^0 remains almost constant about 100 GeV. The neutralino masses $m_{\tilde{\chi}_3^0}$, $m_{\tilde{\chi}_4^0}$ and the chargino mass $m_{\tilde{\chi}_2^+}$ show a strong strong increasing behavior with m_0. This can be explained by looking at the approximation formulae for the chargino and neutralino masses. In the limit of large $|\mu|$ values, $|\mu| \gg M_{1,2} \gg M_Z$ the neutralino masses simplify to [104]

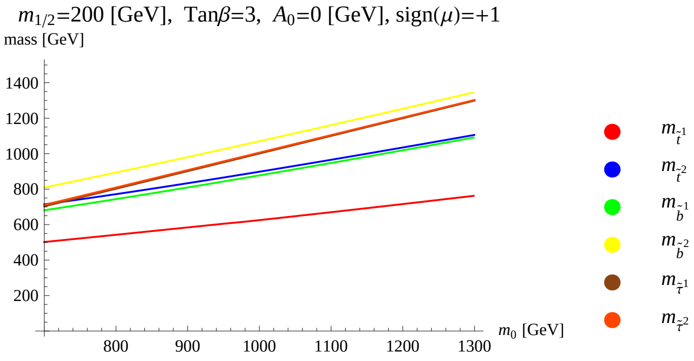
$$\begin{aligned} m_{\tilde{\chi}_1^0} &\simeq M_1 - \frac{M_Z^2}{\mu^2}\left(M_1 + \mu s_{2\beta}\right) s_W^2 \ , \qquad (5.1) \\ m_{\tilde{\chi}_2^0} &\simeq M_2 - \frac{M_Z^2}{\mu^2}\left(M_2 + \mu s_{2\beta}\right) c_W^2 \ , \\ m_{\tilde{\chi}_{3/4}^0} &\simeq |\mu| + \frac{1}{2}\frac{M_Z^2}{\mu^2}\epsilon_\mu (1 \mp s_{2\beta})\left(\mu \pm M_2 s_W^2 \mp M_1 c_W^2\right) \ , \qquad (5.2) \end{aligned}$$

where $\epsilon_\mu = \mu/|\mu|$ is the sign of μ. The chargino masses reduce in the limit of $|\mu| \gg M_2, M_W$ to

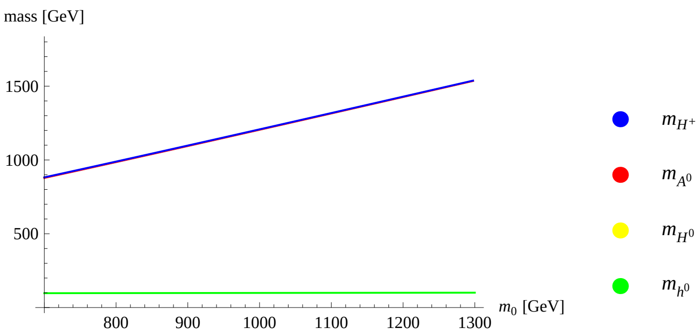
$$\begin{aligned} m_{\tilde{\chi}_1^\pm} &\simeq M_2 - M_W^2 \mu^{-2}\left(M_2 + \mu s_{2\beta}\right) \ , \qquad (5.3) \\ m_{\tilde{\chi}_2^\pm} &\simeq |\mu| + M_W^2 \mu^{-2} \epsilon_\mu \left(M_2 s_{2\beta} + \mu\right) \ . \end{aligned}$$

The heavier neutralinos and chargino masses show a linear dependence on μ without any suppression factor, which explains their increasing behavior. Figure (5.2) shows the BRs in the tree–level and full one–loop approximation for H^0, A^0 and H^+. The dominant BRs are in both cases $H^0/A^0 \to t\bar{t}$ (since the $\tan\beta$ – enhancement for Higgs decays into down type fermions is not active for $\tan\beta = 3$), which also receive sizeable full one–loop corrections. Figure (5.3) and (5.4) exhibit the relevant partial decay widths for the Higgs bosons H^0 and A^0. The stop loops entering the Higgs wavefunction corrections cause a pseudo–threshold for $H^0 \to t\bar{t}$ and $H^0 \to \tilde{\chi}_1^+\tilde{\chi}_2^-$ at $m_0 \sim 1080$ GeV.
In both cases, the difference between the leading SUSY–QCD and the full electroweak calculation for $H^0/A^0 \to t\bar{t}$ is about 10 percent. Therefore these contributions are mandatory for precise predictions. The second largest BR in both cases is $H^0/A^0 \to \tilde{\chi}_1^+\tilde{\chi}_2^-$, where the difference between the tree–level and the full one–loop calculation for the partial decay widths is rather small and shows a decreasing behavior for increasing values of m_0 down to two percent.
The charged Higgs boson H^+ has the largest BR into $H^+ \to t\bar{b}$ shown in fig. (5.2), followed by the decays into a neutralino and chargino. The BR $H^+ \to t\bar{b}$ receives sizeable corrections, the BRs into neutralinos and charginos have corrections up to a few percent.

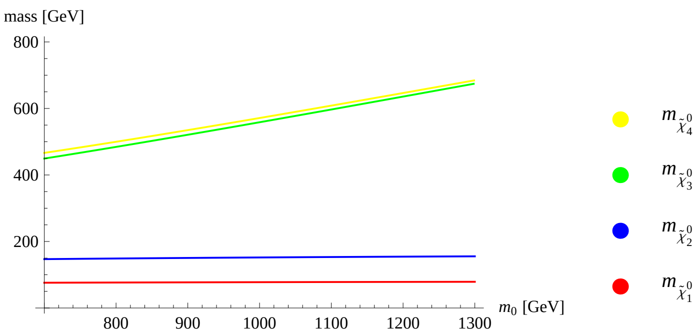
The partial decay widths of the charged Higgs boson are exhibited in fig. (5.5). The difference between the leading SUSY–QCD calculation and the calculation including the full electroweak contributions is about 10 percent. Again these contributions are necessary for precise predictions. The partial decay widths into neutralinos and charginos have corrections about three percent. Therefore the tree–level calculation for Higgs decays into neutralinos and charginos is a sufficient approximation in this area of mSUGRA space.
Finally figure (5.6) shows the total decay widths for H^0, A^0 and H^+ in the tree–level, SUSY–QCD and full one–loop approximation. As can be seen, the difference between the tree–level, SUSY–QCD and full one–loop calculation becomes larger with increasing values of m_0. This behavior can be explained, since with increasing values of m_0, more decay channels into pairs of neutralinos and charginos become kinematically available. These new decay channels receive only electroweak corrections at one–loop level, therefore they cause a larger deviation from the SUSY–QCD and tree–level calculation.



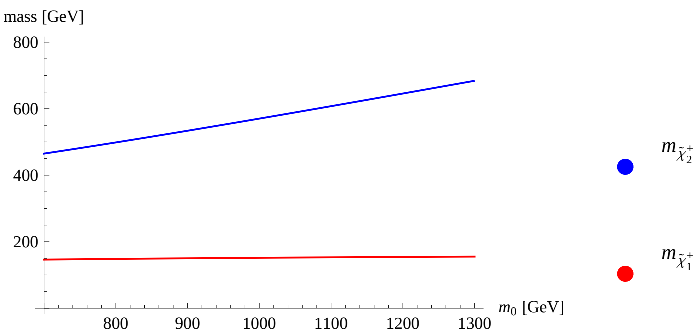
(a) Sfermion masses



(c) MSSM Higgs masses



(e) Neutralino masses



(g) Chargino masses

Figure 5.1: Mass spectrum in scenario mSUGRA1

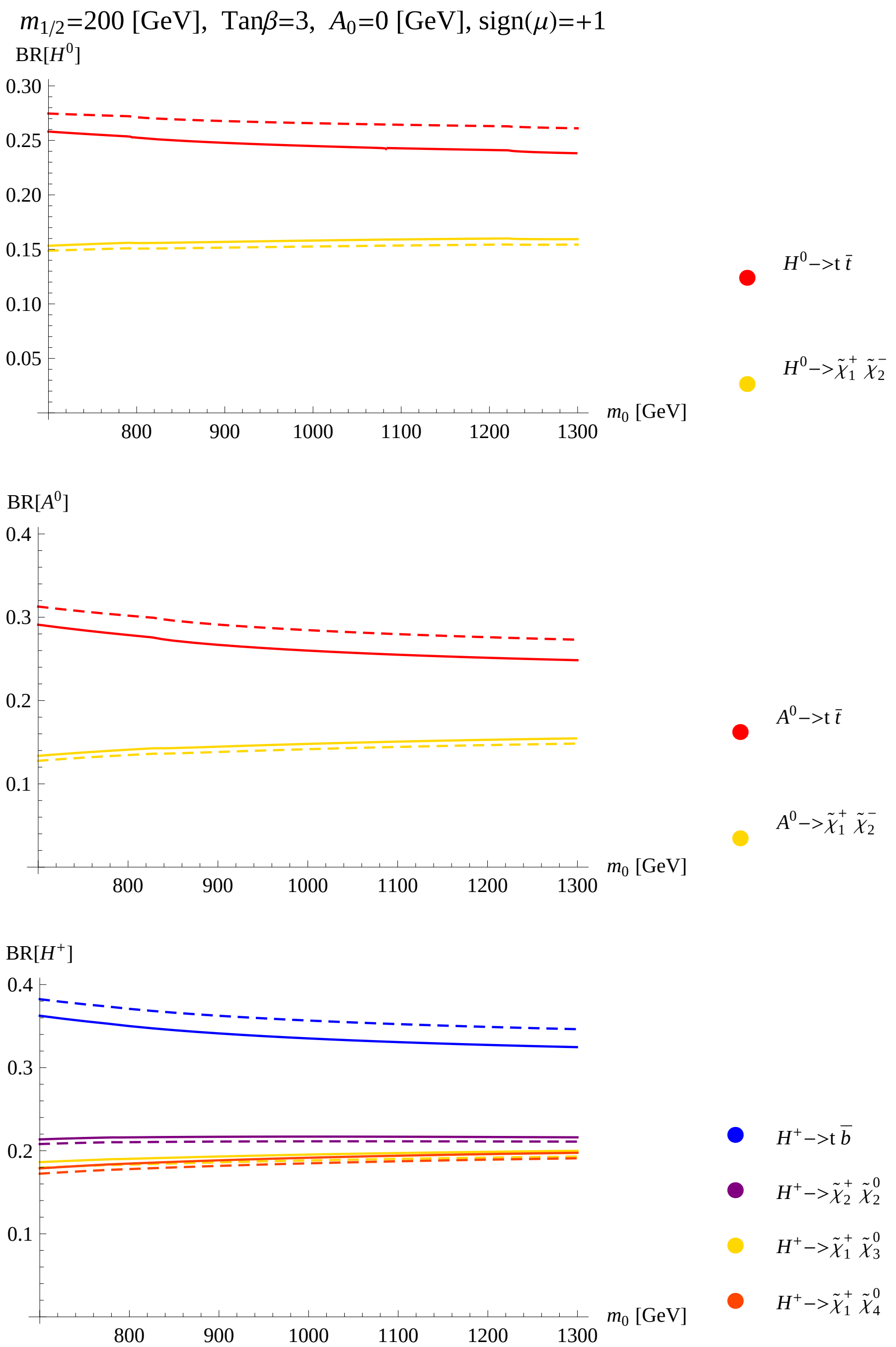


Figure 5.2: H^0, A^0 and H^+ branching ratios (solid = full one–loop, dashed = tree-level)

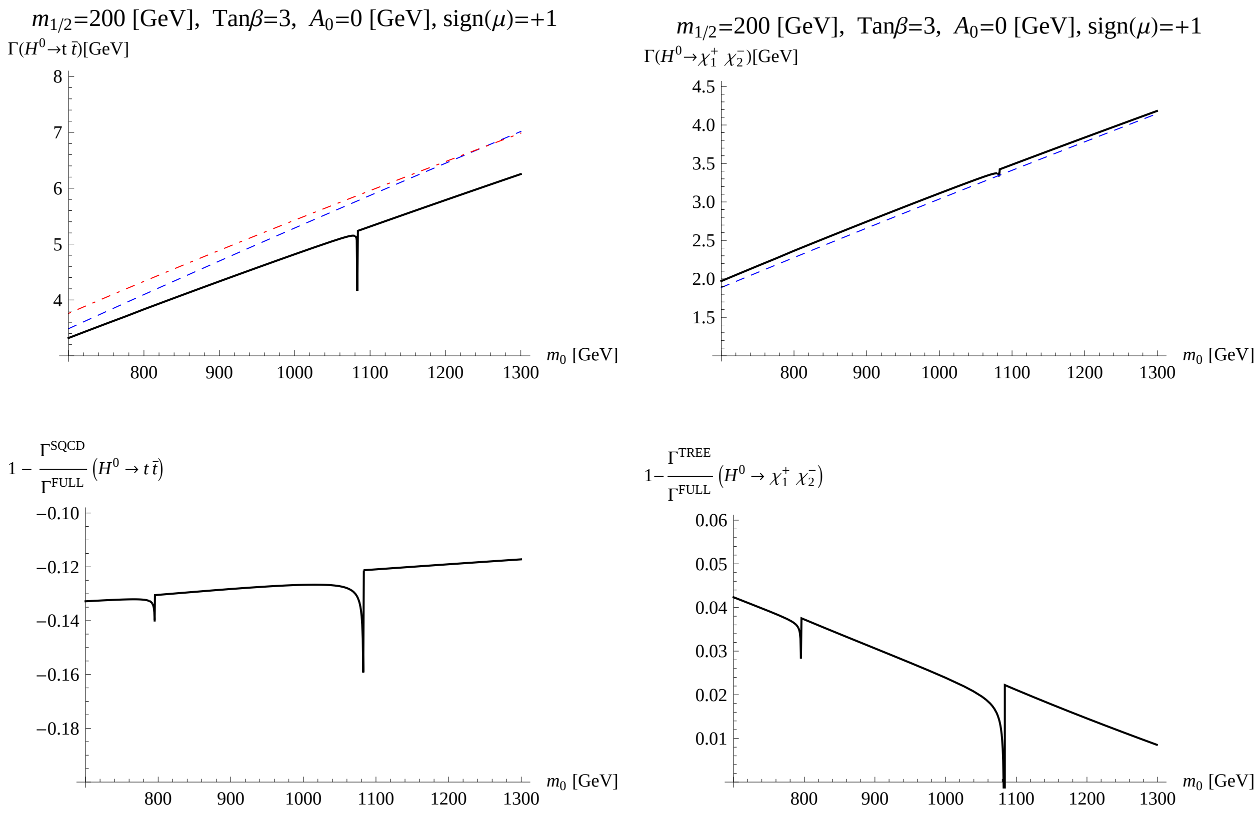


Figure 5.3: H^0 decay widths in GeV (blue=tree, red=SUSY–QCD, solid = full one–loop)

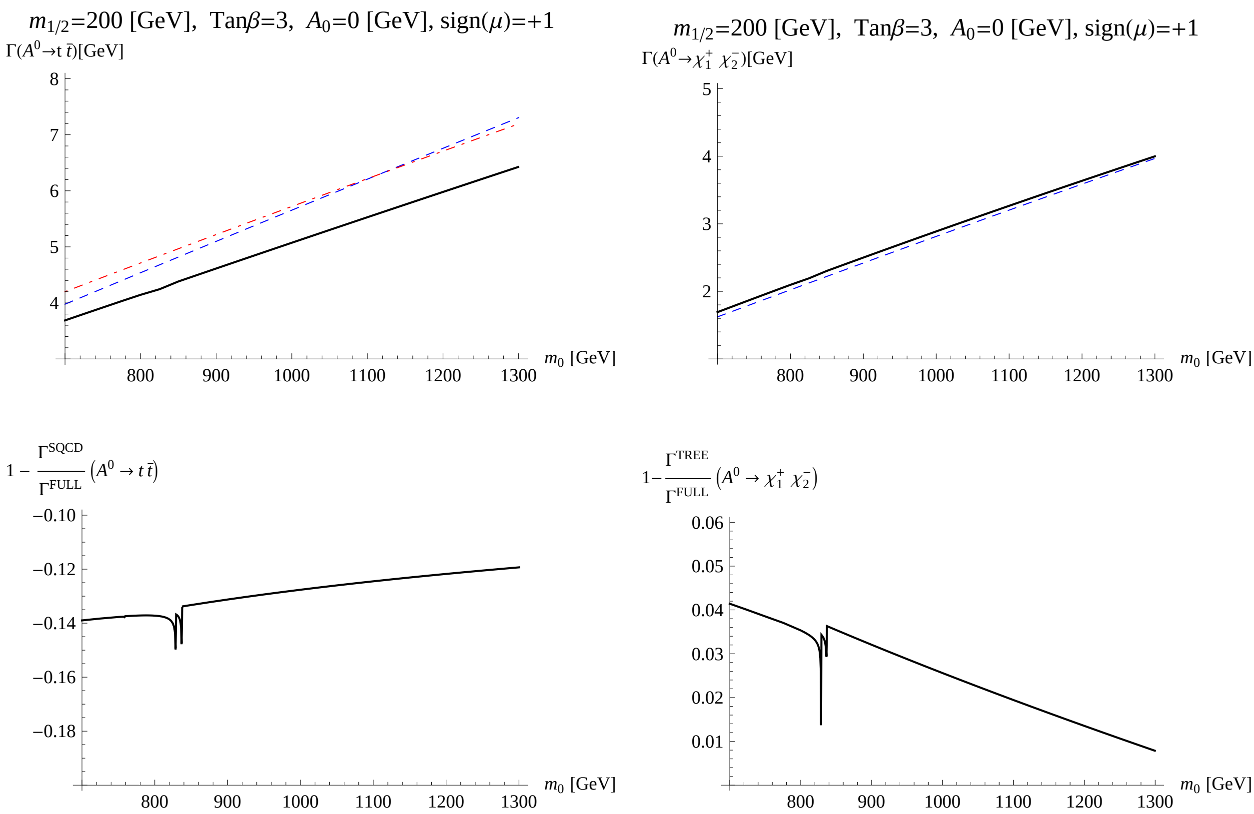


Figure 5.4: A^0 decay widths in GeV (blue=tree, red=SUSY–QCD, solid = full one–loop)

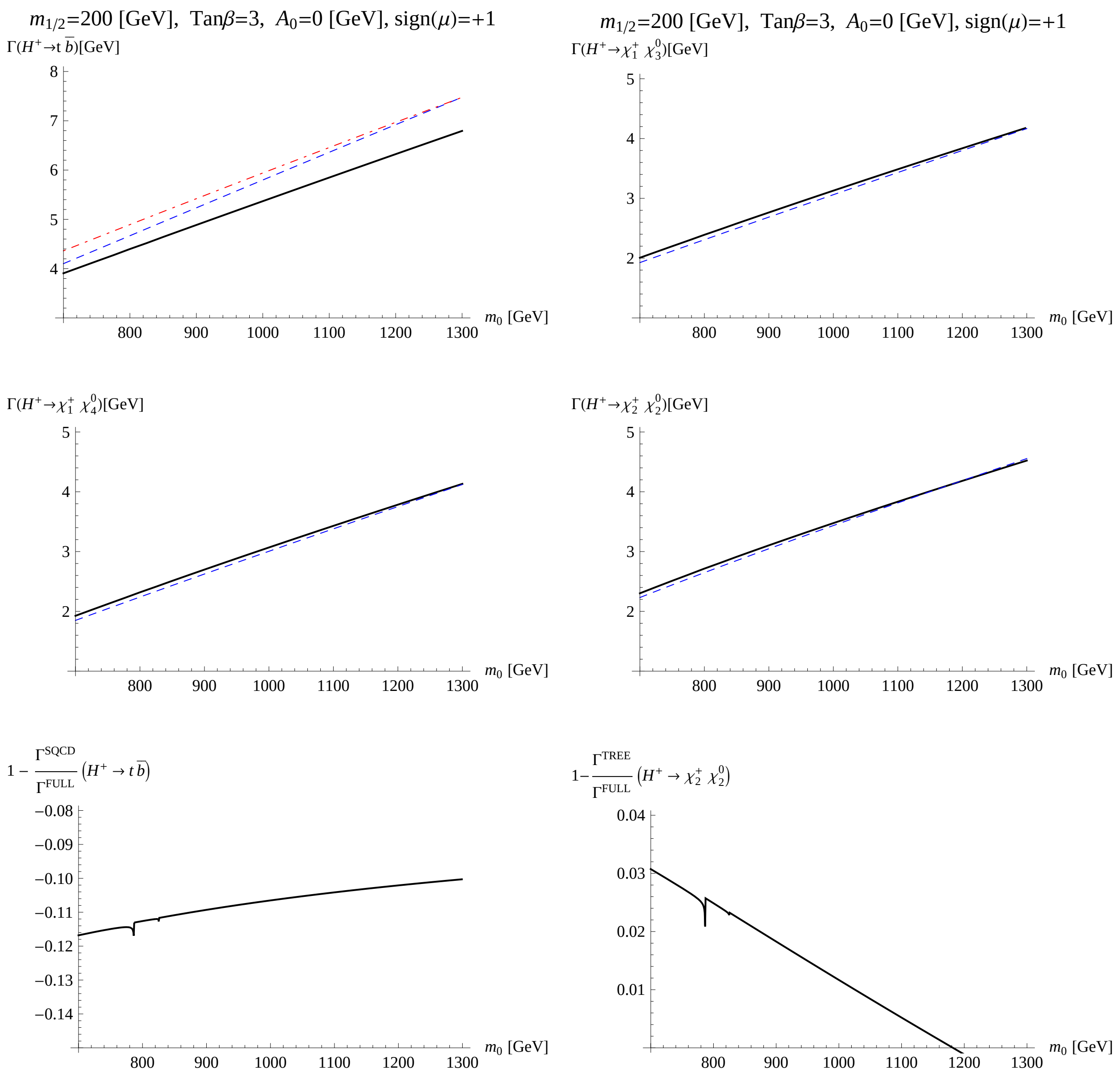


Figure 5.5: H^+ decay widths in GeV (blue=tree, red=SUSY–QCD, solid = full one–loop)

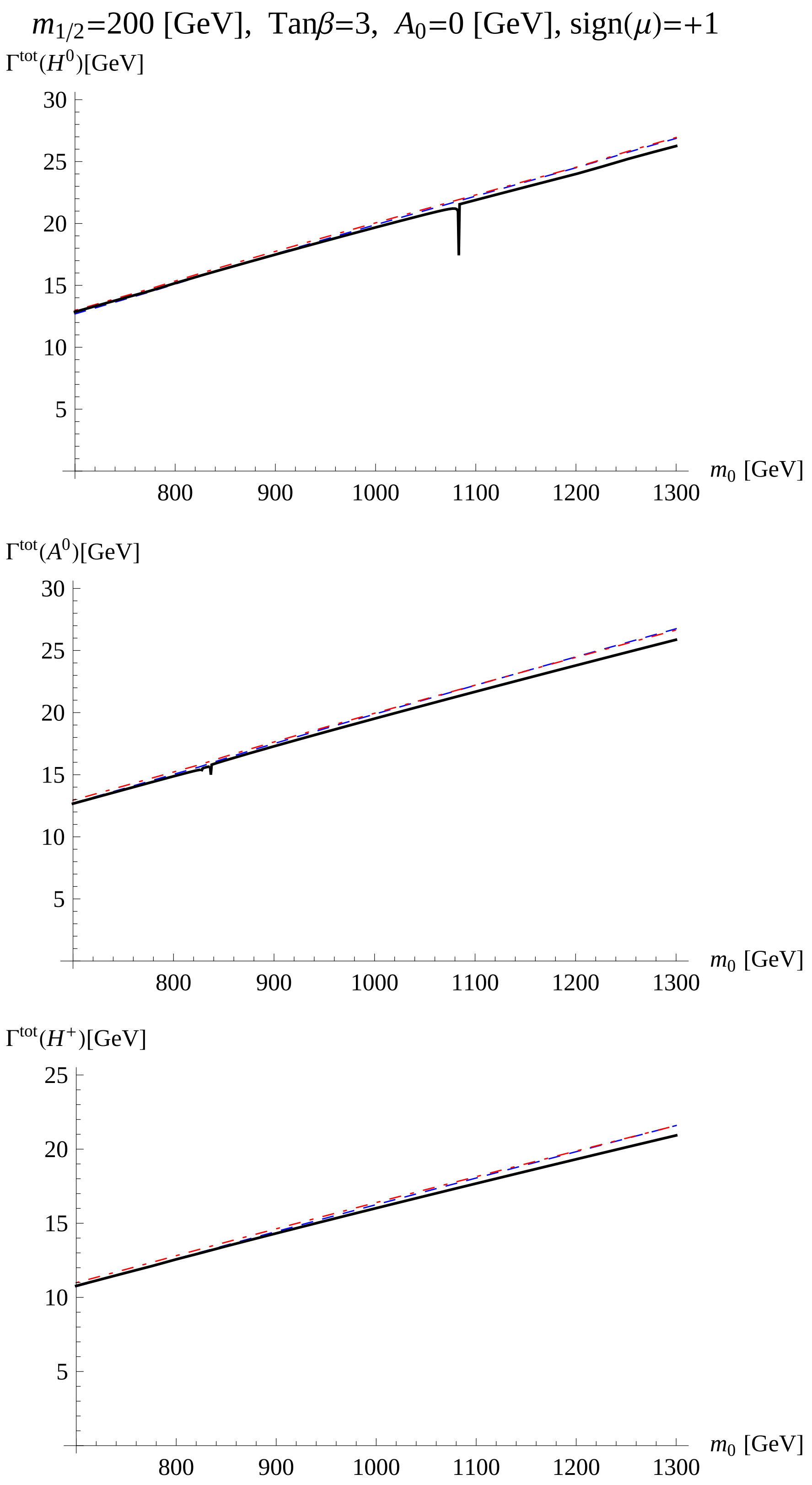
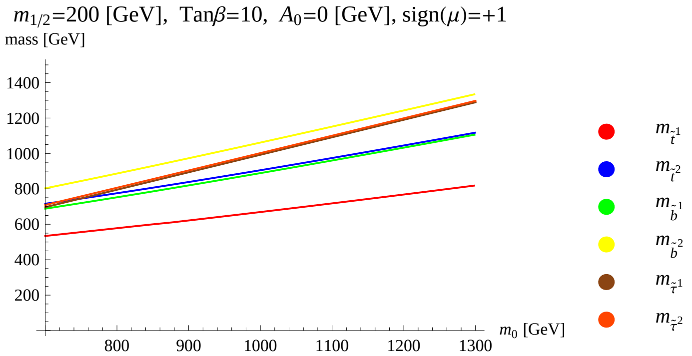


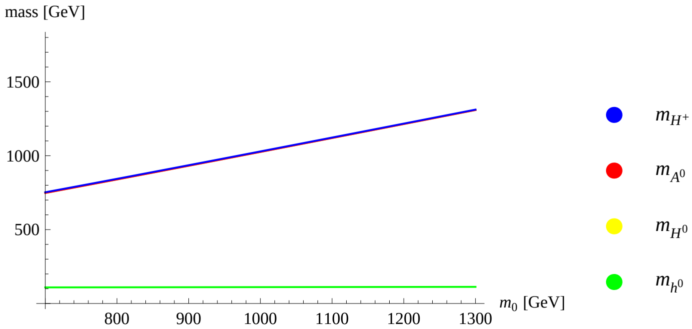
Figure 5.6: Total widths in GeV in scenario mSUGRA1

5.2.2 Scenario mSUGRA2

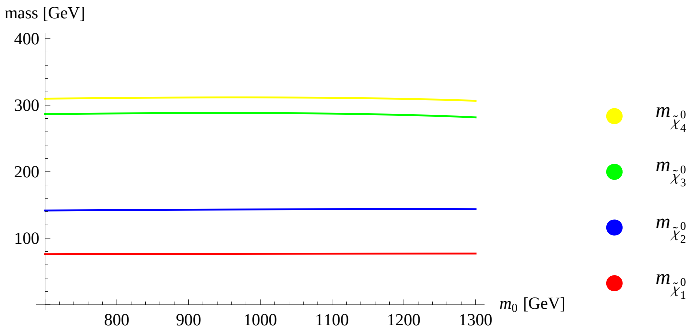
Our next mSUGRA scenario is the same as mSGURA1, except for $\tan\beta$. We now take $\tan\beta = 10$. This has important phenomenological consequences, the leading BRs of neutral Higgs decays into pairs of top quarks are now suppressed from ~ 30 percent (scenario mSUGRA1) down to approximately two percent in scenario mSUGRA2. Therefore decays into pairs of charginos and/or neutralinos have the largest BRs for the whole range of m_0. The BRs of H^0, A^0 and $H^\pm$ calculated in the tree–level and full one–loop approximation are exhibited in fig. (5.8). One might suspect that the BR $A^0 \to \chi_1^+\chi_1^-$ is larger than $A^0 \to \chi_1^+\chi_2^-$, since there is more phase space available. However, this behavior can be explained by the tree–level coupling of $A^0 \to \chi_1^+\chi_2^-$ which is larger than $A^0 \to \chi_1^+\chi_1^-$ in this parameter set. The radiative corrections for the leading chargino decay modes are rather small, only the BRs of neutral Higgs decays into pairs of bottom quarks receive corrections of the magnitude of a few percent.
The relevant partial decay widths of H^0, A^0 and $H^\pm$ are shown in fig. (5.9, 5.10) and (5.11). As can be seen the leading decay widths into pairs of charginios and/or neutralinos receive corrections up to four percent. The difference between the leading SUSY–QCD and full one–loop widths for neutral Higgs decays into pairs of bottom quarks are increasing with m_0 up to three percent, for the charged Higgs the difference between both calculations is five percent at $m_0 = 1300$ GeV .
Finally figure (5.12) shows the total decay widths for H^0, A^0 and H^+ in the tree–level, SUSY–QCD and full one–loop approach. Since the corrections for the relevant partial decay widths are of the magnitude of a few percent, the difference for the corresponding total decay widths between the three approximations is small, e.g. the difference for $\Gamma^{tot}(H^0)$ at tree–level and at full one–loop level is two percent.



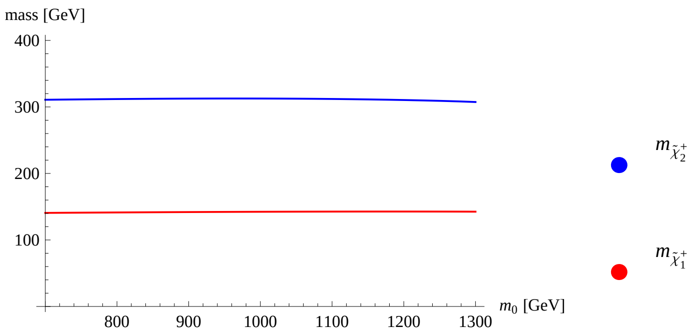
(a) Sfermion masses



(c) MSSM Higgs masses



(e) Neutralino masses



(g) Chargino masses

Figure 5.7: Mass spectrum in scenario mSUGRA2

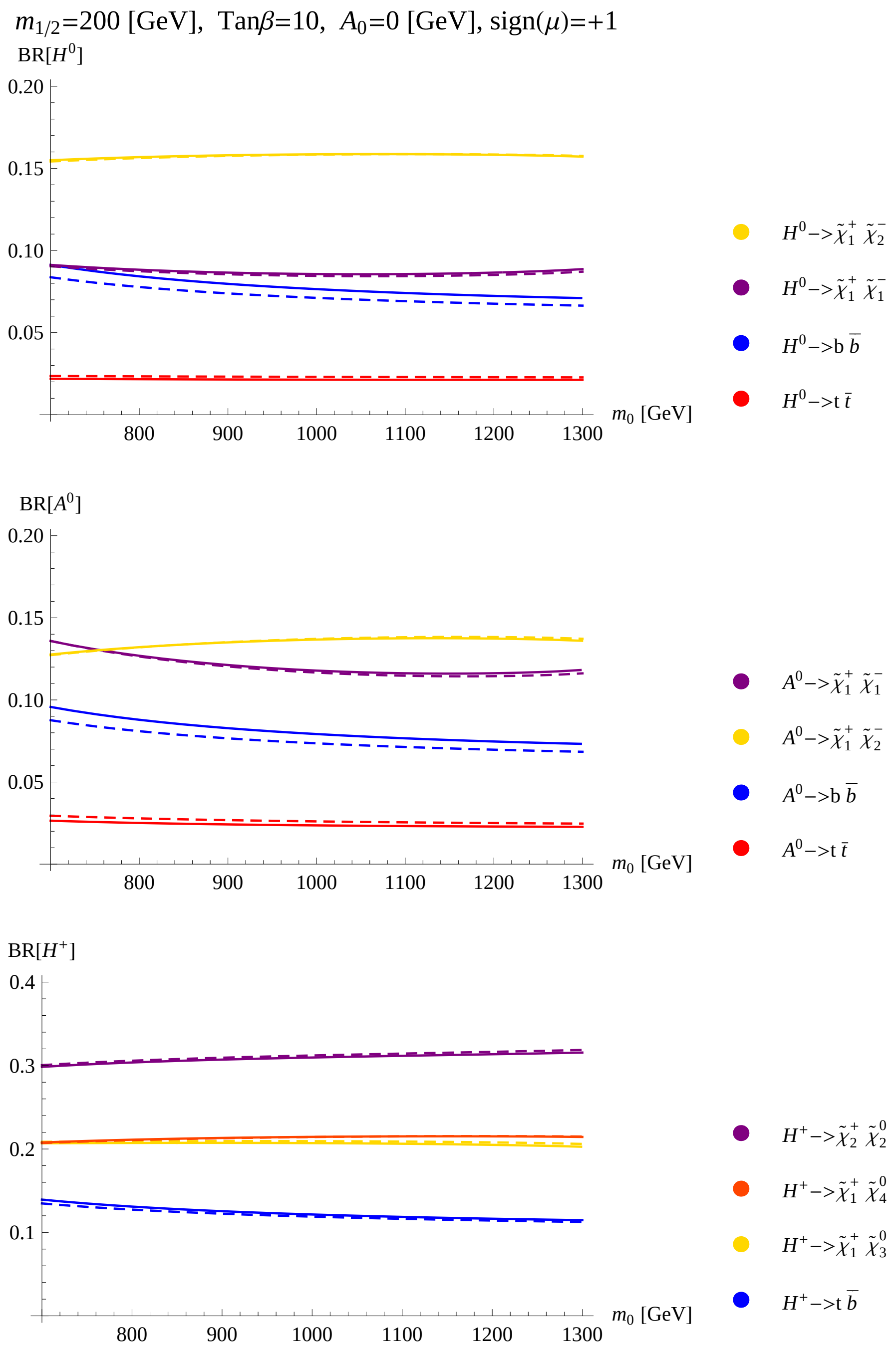


Figure 5.8: H^0, A^0 and H^+ branching ratios (solid = full one–loop, dashed = tree-level)

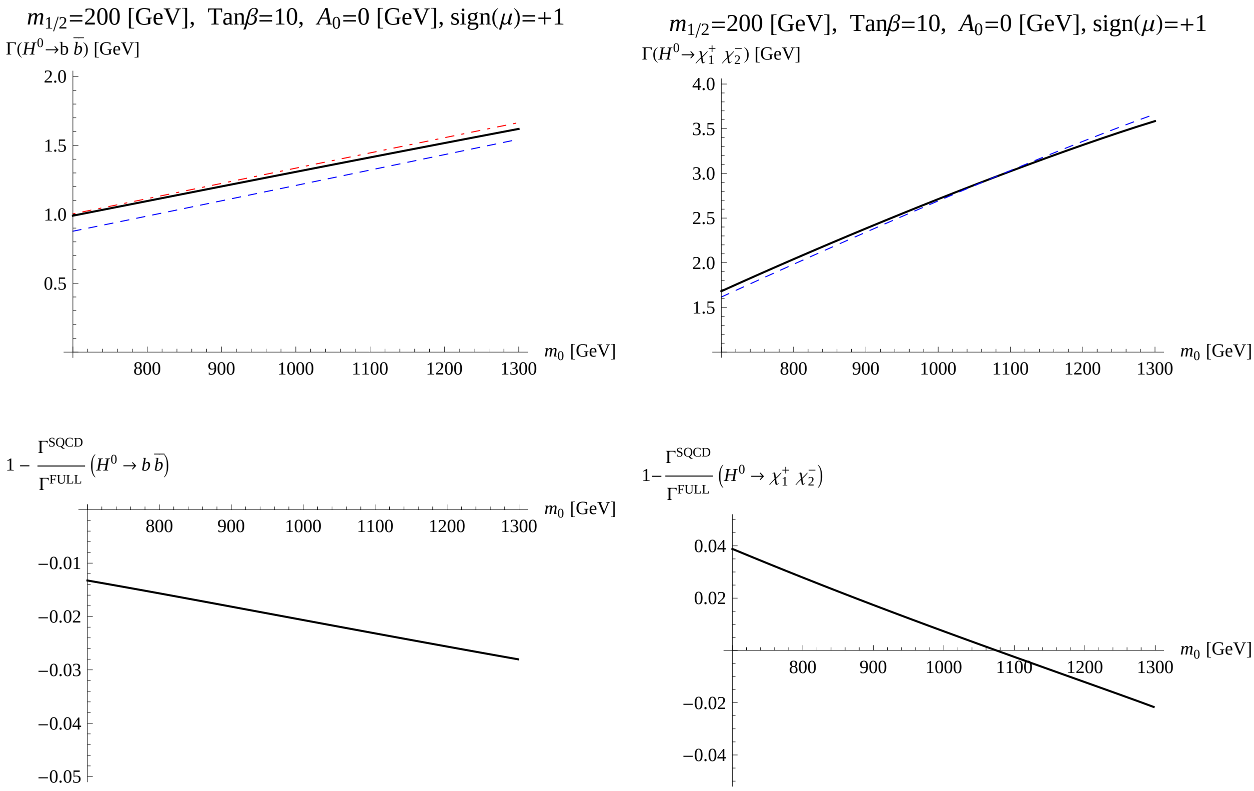


Figure 5.9: H^0 decay widths in GeV (blue=tree, red=SUSY–QCD, solid = full one–loop)

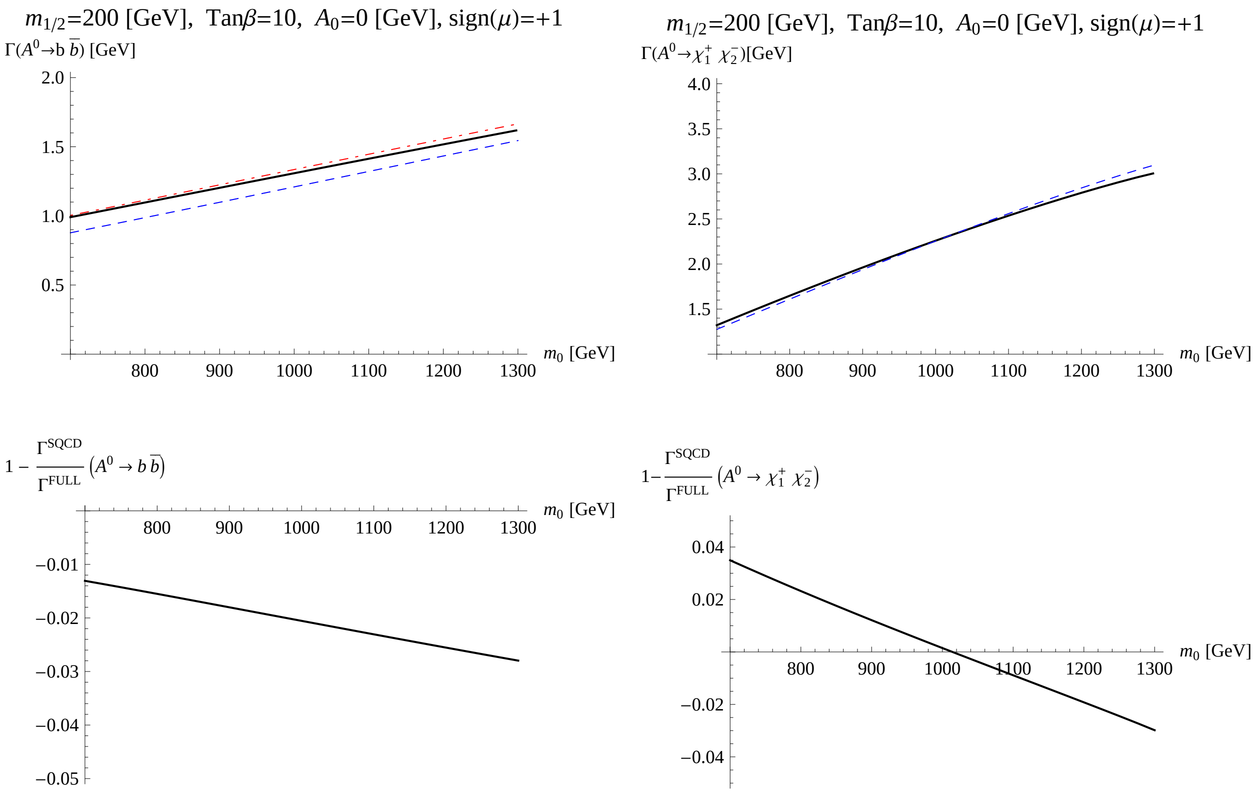


Figure 5.10: A^0 decay widths in GeV (blue=tree, red=SUSY–QCD, solid = full one–loop)

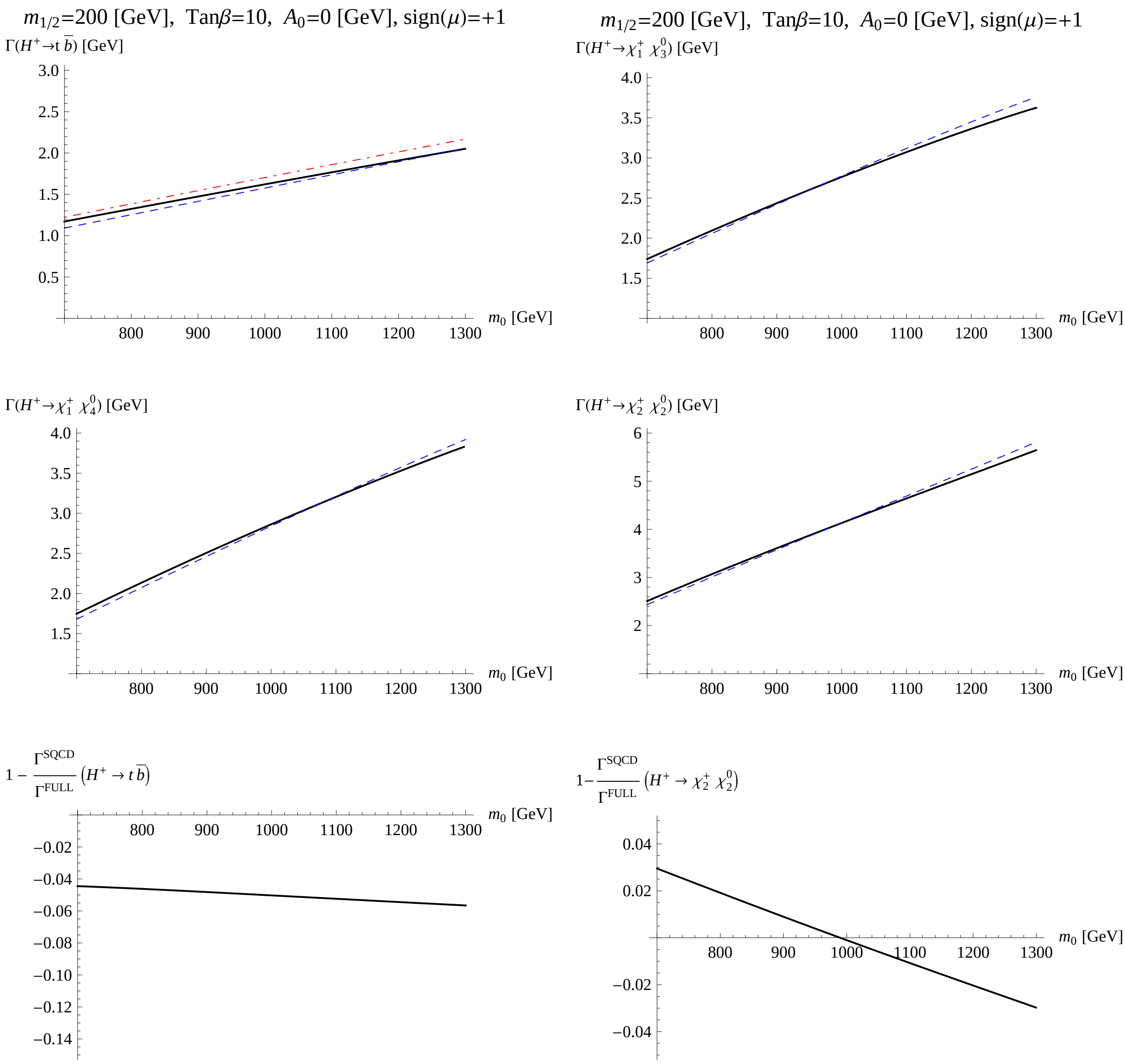


Figure 5.11: H^+ decays widths in GeV (blue=tree, red=SUSY–QCD, solid = full one–loop)

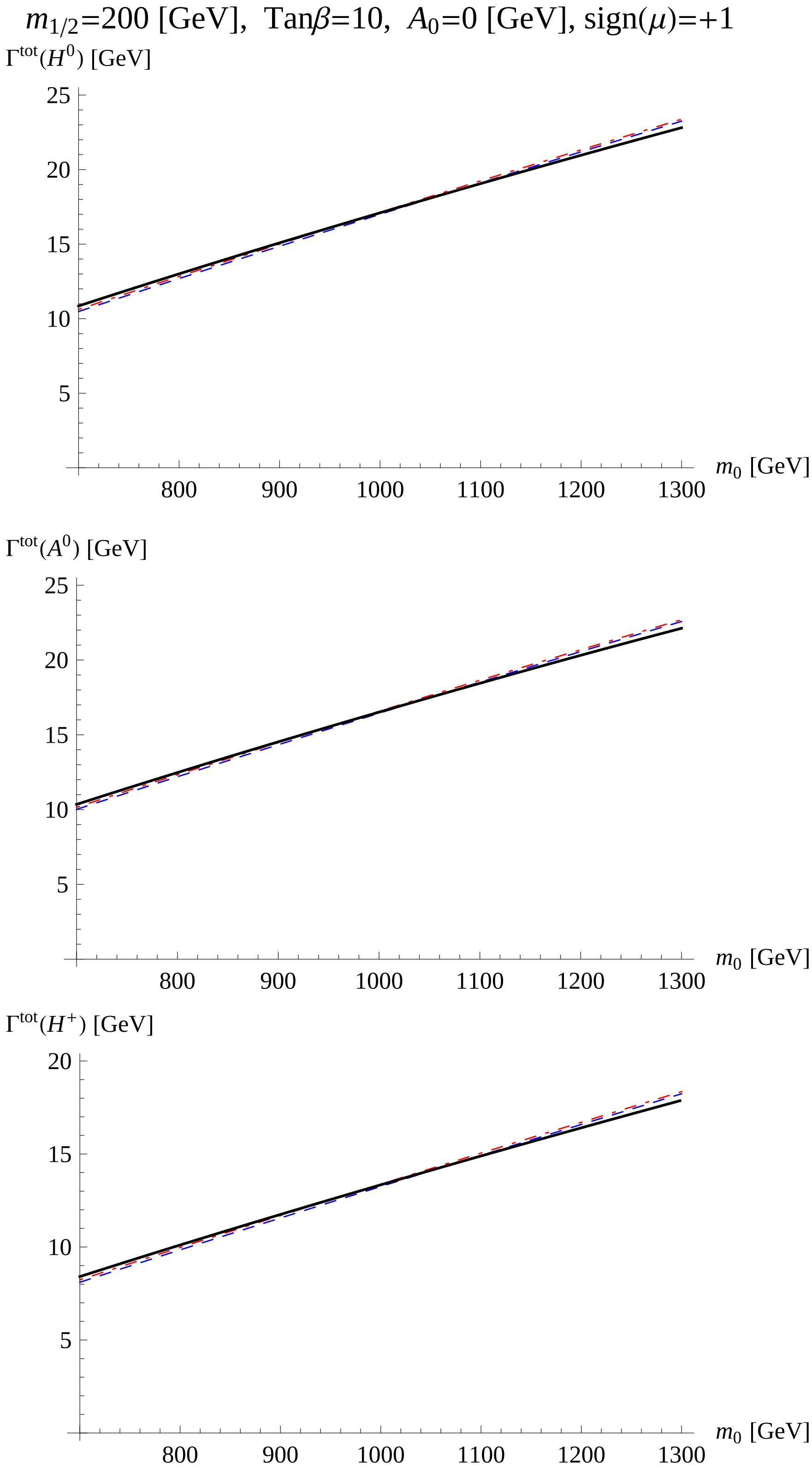


Figure 5.12: Total widths in GeV in scenario mSUGRA2

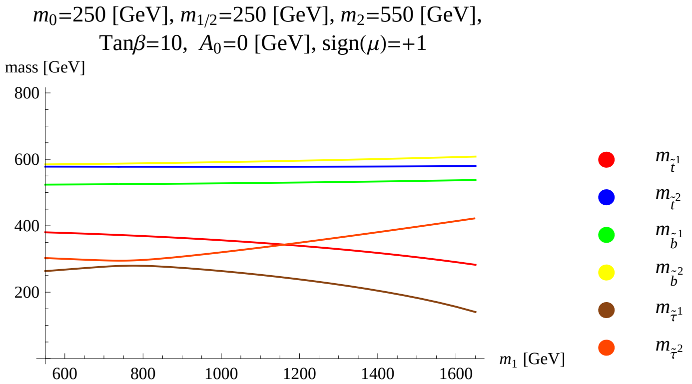
5.2.3 Scenario NUHM

In this Non Universal Higgs Mass (NUHM) scenario, we will vary the Higgs mass parameter m_1 given at the GUT scale, keeping the other parameters fixed. As a consequence we will obtain heavy Higgs bosons, while the SUSY particles will remain relatively light. The corresponding mass spectrum is shown in fig. (5.18).Thus, there will be more kinematically allowed Higgs decays into SUSY particles compared to usual mSUGRA scenarios.
Figure (5.14) shows the most important BRs for H^0, A^0 and H^+. The BRs are calculated in the tree–level and the full one–loop approximation. This scenario is almost dominated by decays into charginos and neutralinos. This behavior can be explained since the Higgs couplings to top quarks are suppressed while the enhancement of the coupling to bottom quarks is not strong yet. The domination of the chargino and neutralino decay modes become even more pronounced with increasing values of m_1. The following table exhibits the partial widths of neutralino/chargino decays of the heavier Higgs bosons H^0, A^0 and H^+ in units of $G_\mu M_W^2 M_{H_k}/(4\sqrt{2}\pi)$ in the limit of $M_A \gg |\mu| \gg M_2$. Thus in this limit the neutral Higgs decay into charginos are equal to 1, while the neutral Higgs decays into pairs of neutralinos are suppressed by a factor 1/2.

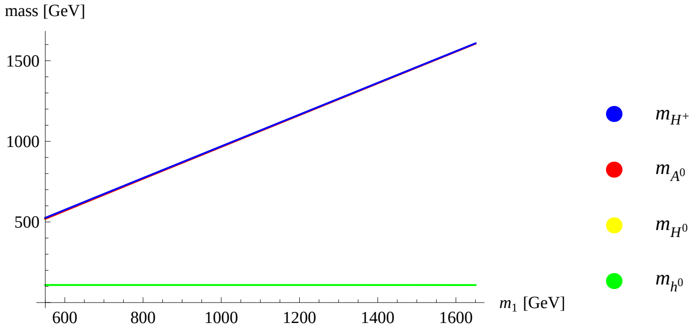
	$\Gamma(H \to \chi\chi)$	$\Gamma(A \to \chi\chi)$		$\Gamma(H^\pm \to \chi\chi)$
$\chi_1^0\chi_3^0$	$\frac{1}{2}\tan^2\theta_W(1+\sin 2\beta)$	$\frac{1}{2}\tan^2\theta_W(1-\sin 2\beta)$	$\chi_1^\pm\chi_3^0$	1
$\chi_1^0\chi_4^0$	$\frac{1}{2}\tan^2\theta_W(1-\sin 2\beta)$	$\frac{1}{2}\tan^2\theta_W(1+\sin 2\beta)$	$\chi_1^\pm\chi_4^0$	1
$\chi_2^0\chi_3^0$	$\frac{1}{2}(1+\sin 2\beta)$	$\frac{1}{2}(1-\sin 2\beta)$	$\chi_2^\pm\chi_1^0$	$\tan^2\theta_W$
$\chi_2^0\chi_4^0$	$\frac{1}{2}(1-\sin 2\beta)$	$\frac{1}{2}(1+\sin 2\beta)$	$\chi_2^\pm\chi_2^0$	1
$\chi_1^\pm\chi_2^\mp$	1	1	–	–

The difference for the BRs between the two calculations is about a few percent and becomes even smaller with increasing values of m_1. However, the partial decay widths exhibited in fig. (5.15) show a different behavior: For the partial decay width $H^0 \to b\bar{b}$ the difference between the leading SUSY–QCD and the full one–loop calculation does not become larger than five percent. The leading partial decay widths into pairs of charginos receive corrections up to 10 percent between the tree–level and the full one–loop calculation. Therefore these contributions are mandatory for precise predictions.
The situation for the pseudoscalar Higgs A^0 boson is similar to CP–even Higgs boson H^0. Only the BR $A^0 \to \tilde{\chi}_1^+\tilde{\chi}_1^-$ shown in fig. (5.14) remains longer dominant for a wider range of m_1. Again the partial decay width into pairs of charginos show a difference up to 10 percent between the tree–level and the corresponding full–loop calculation.
The largest BR for the charged Higgs boson is $H^+ \to \tilde{\chi}_2^+\tilde{\chi}_2^0$. Here the BRs receive corrections up to a few percent. The difference between the tree–level and full one–loop approximation for the BR becomes smaller with increasing values of m_1. However, the leading partial decay widths into pairs of charginos and neutralinos, show a different behavior. These decay widths receive corrections more than 10 percent.

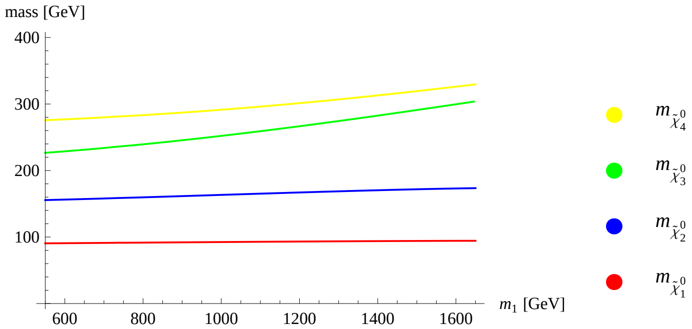
One might suspect that the Higgs decays into squarks e.g. $H^0 \to \tilde{t}_1\tilde{t}_1$ have the largest BR, since there would be enough phase space available. (In detail the BR $H^0 \to \tilde{t}_1\tilde{t}_1$ at $m_1 =$ 1650 GeV is only one percent in the full one–loop approximation.) However, these decays are inversely proportional to the Higgs masses, therefore the sfermion decays are suppressed for large Higgs masses compared to the decays into fermions and chargions/neutralinos which increase with the Higgs mass. In the asymptotic regime, $m_{H^0,A^0,H^+} \gg m_{\tilde{f}}$, the decay widths of the H^0, A^0 and H^+ bosons into sfermions are proportional to $\sin^2 2\beta m_Z^2/m_{H^0,A^0,H^+}$ and can be significant only for low values of $\tan\beta$ where $\sin^2 2\beta \sim 1$.
The total decay widths of H^0, A^0 and H^+ are exhibited in fig. (5.18). For values of m_1 between 550 and 800 GeV the SUSY–QCD or even the tree–level calculation is a sufficient approximation for the total decay widths. Then the deviation between the SUSY–QCD and the full electroweak calculation becomes larger. This behavior can be explained similarly as in scenario mSUGRA1. New decay channels into pairs of neutralinos and charginos become kinematically possible and with increasing Higgs masses more phase space becomes available for these decays. At $m_1 \sim 1650$ GeV, the difference for the total width of H^0 between the SUSY–QCD and full one–loop approximation is about 10 percent.



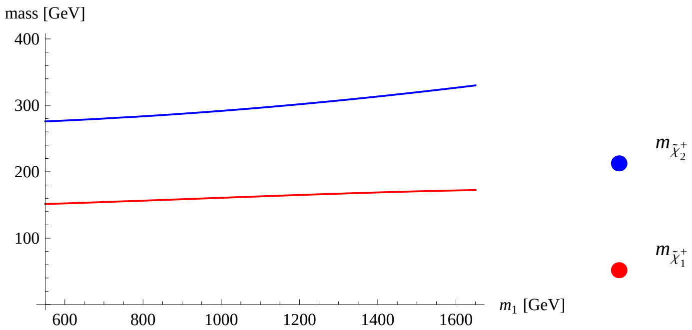
(a) Sfermion masses



(c) MSSM Higgs masses



(e) Neutralino masses



(g) Chargino masses

Figure 5.13: Mass spectrum in scenario NUHM

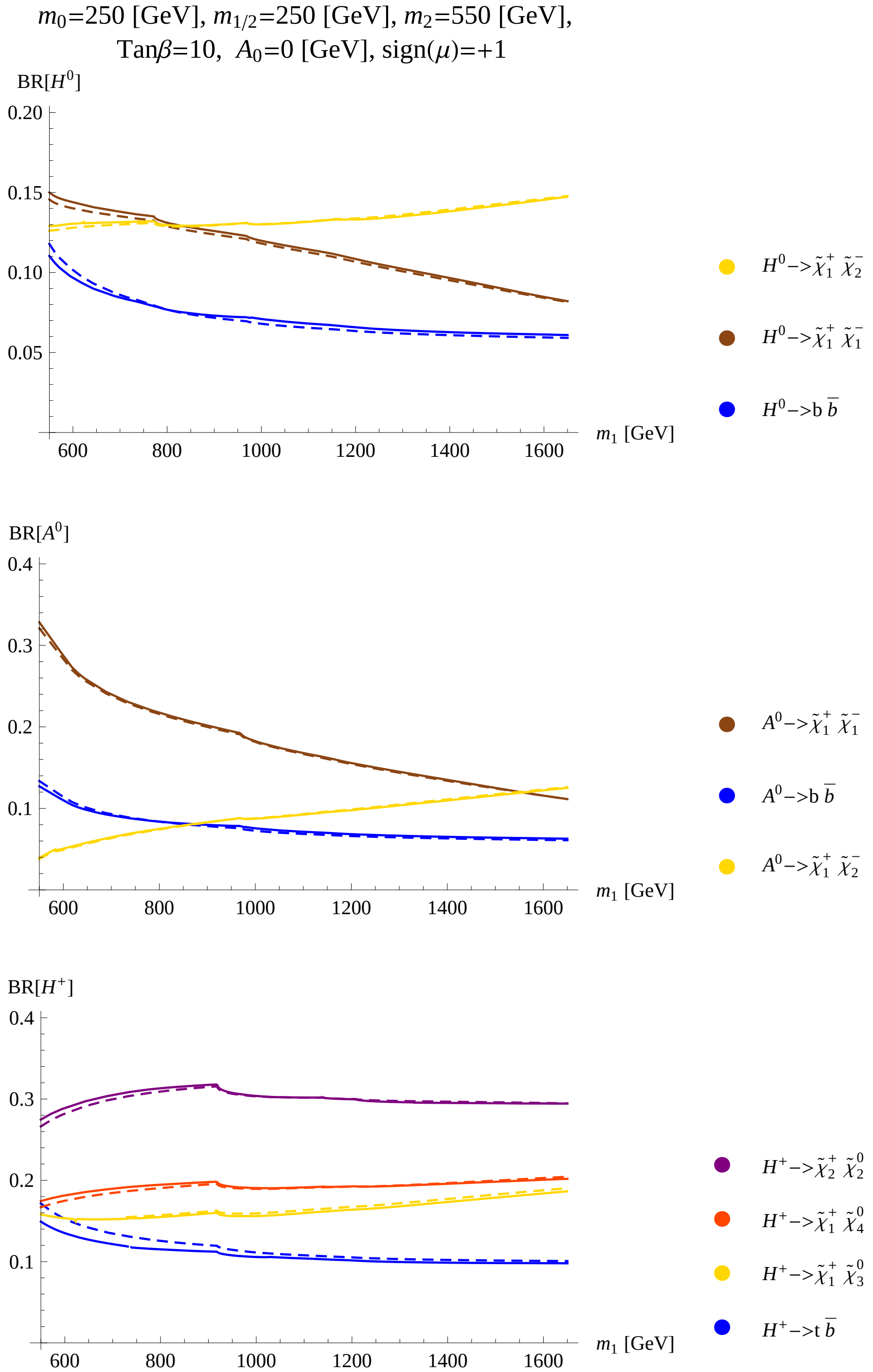


Figure 5.14: H^0, A^0 and H^+ branching ratios (solid = full one–loop, dashed = tree-level)

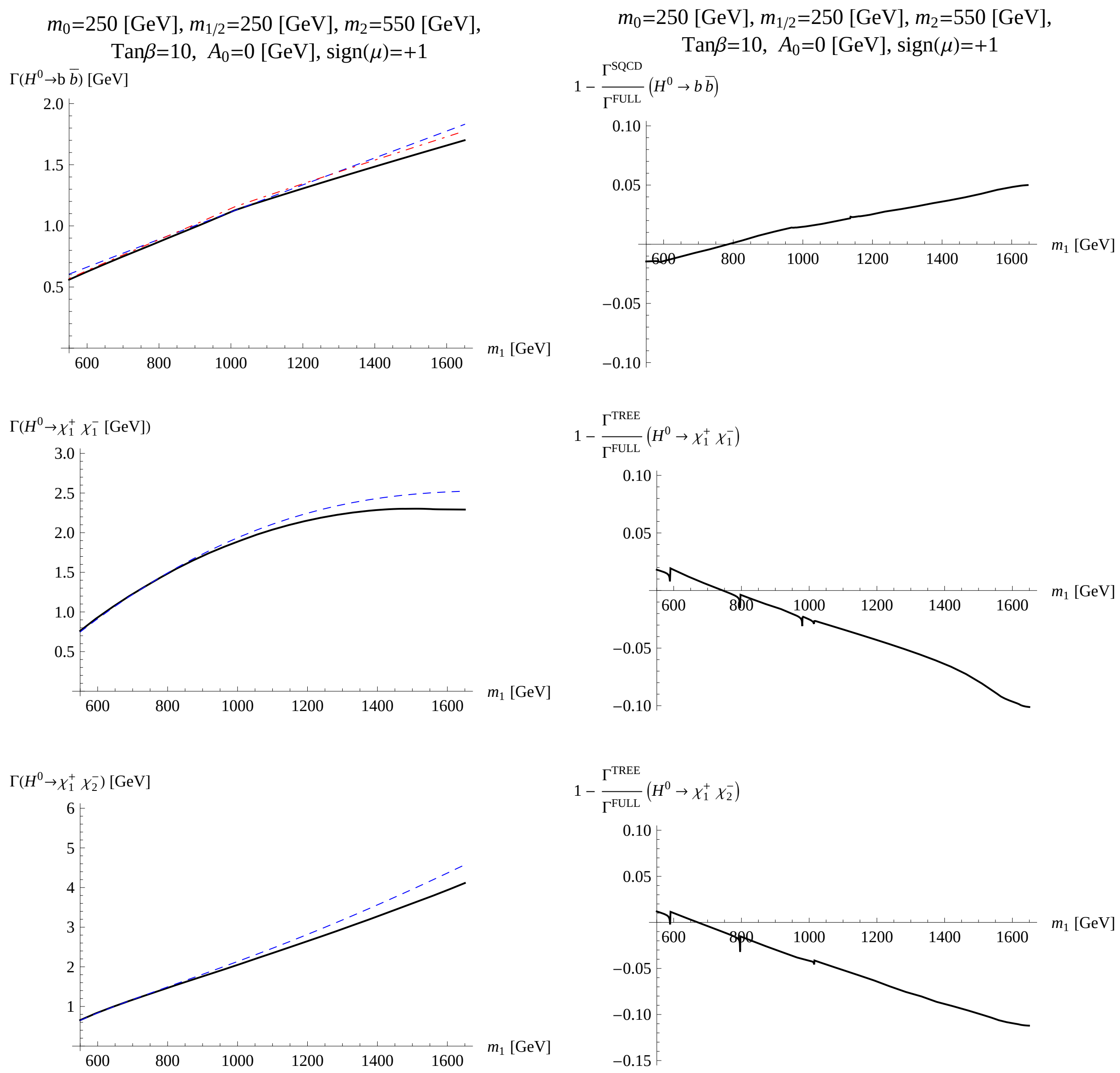


Figure 5.15: H^0 partial decay widths in GeV (blue=tree, solid = full one–loop)

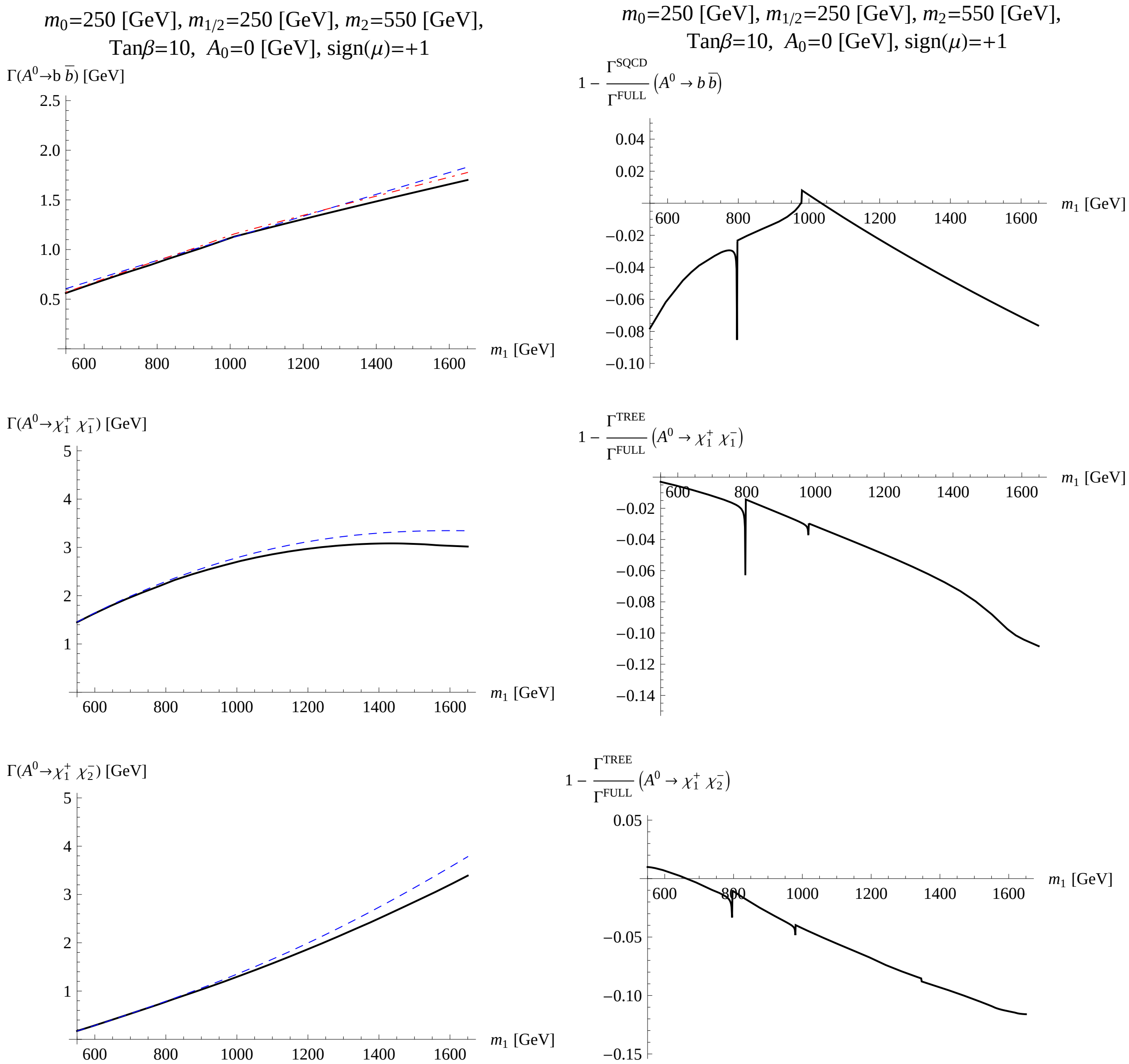


Figure 5.16: A^0 partial decay widths in GeV (blue=tree, red=susy–qcd, solid = full one–loop)

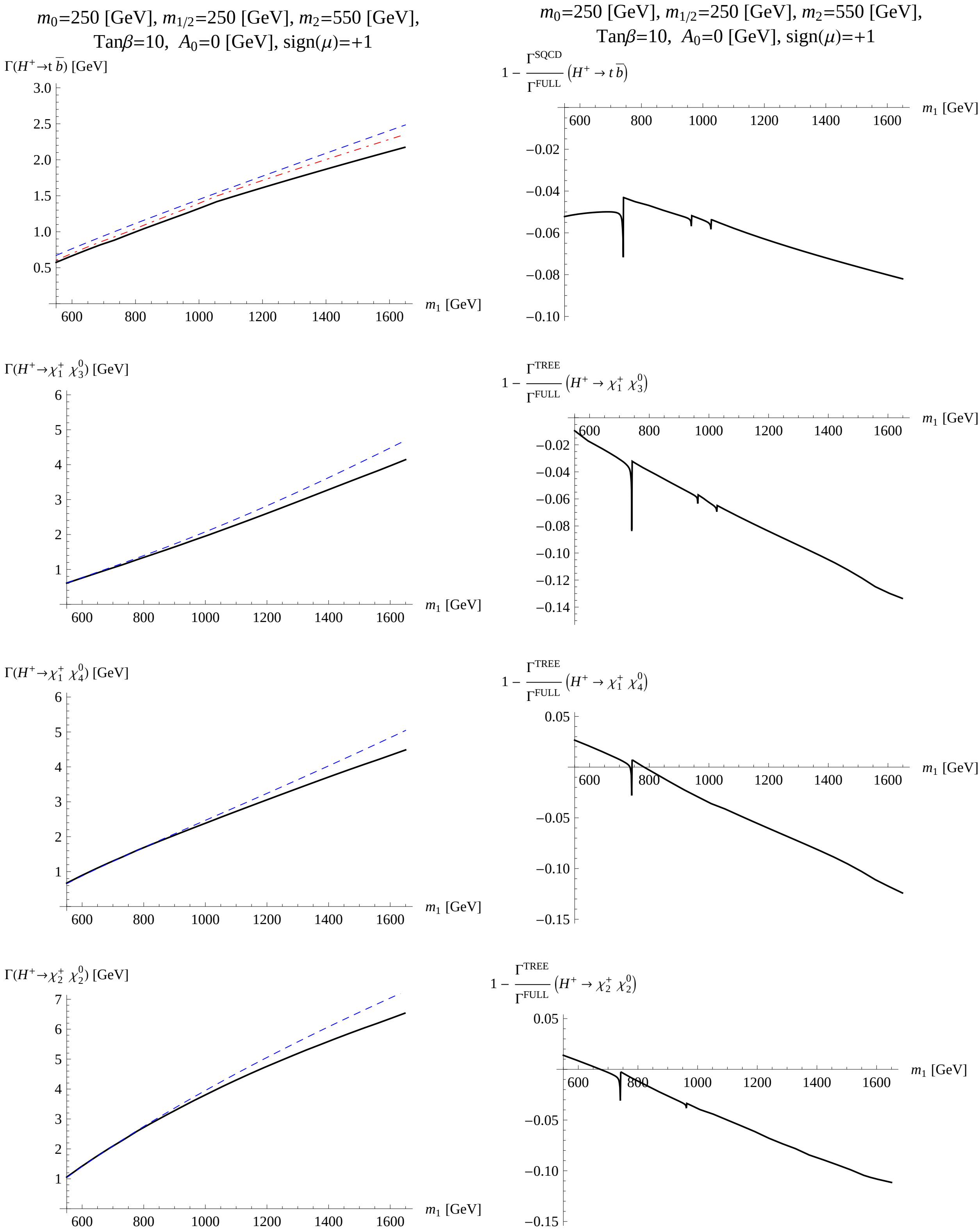


Figure 5.17: H^+ partial decay widths in GeV (blue=tree, solid = full one–loop)

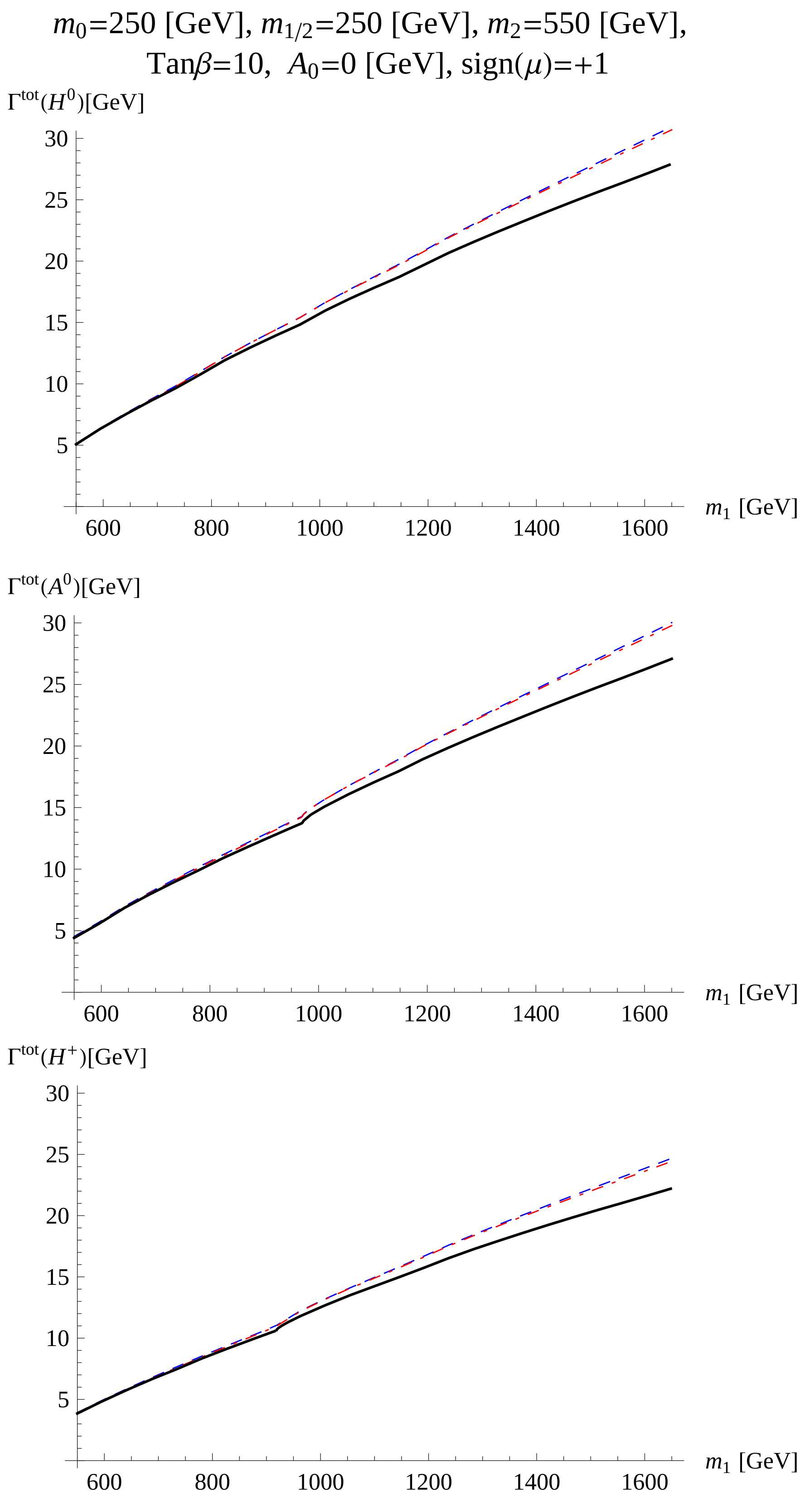


Figure 5.18: Total widths in GeV in scenario NUHM

Chapter 6

Program description

6.0.4 Requirements

- Fortran 77 (g77, ifort77)
- C compiler (e.g. gcc)
- LoopTools [94]

6.0.5 About version 1.0

- The CKM matrix is set diagonal
- Real SUSY input parameters

6.0.6 Installation

1. Download the file **hfold.tar** at

   http://www.hephy.at/tools

2. expand the file, go to the folder hfold/SLHALib-2.2 and type

   `./configure`

   `make`

3. to create the Fortran code for `hfold`, go back to the folder hfold and type

   `./configure`

   `make`

4. To run *HFOLD* type

   `hfold`

6.0.7 The input file `hfold.in`

1. **name of the spectrum (SLHA format)**

2. **Higgs boson = 1,2,3,4,5**
   $1 = h^0,\ 2 = H^0,\ 3 = A^0,\ 4 = H^+,\ 5 = All$

3. **contribution = 0,1,2**
   0 = tree–level calculation
   1 = full one–loop calculation
   2 = SUSY–QCD (only diagrams with a gluon/gluino are taken into account)

4. **bremsstrahlung = 0,1,2**
   0 = off, 1 = hard bremsstrahlung, 2 = soft bremsstrahlung

5. **resummation of bottom yukawa coupling = 0,1**
   0 = off, 1 = on

6. **esoftmax**
   cut on the soft photon(gluon) energy, if soft strahlung is used

7. **name of output-file**

6.0.8 Description of all subroutines

subroutine	**description**
hfold/1to2.F	calculates the soft bremsstrahlung for a $1 \to 2$ process
hfold/bremsstrahlung.F	contains all generic formulae for the hard Bremsstrahlung processes
hfold/CalcContributions.F	calculates the SUSY–QCD contributions
hfold/Contributions.F	sets right flags for different contributions
hfold/definitions.F	some initial definitions, necessary for the renormalization constants
hfold/getslhapara.F	loads the input spectrum (SLHA)
hfold/LHoutput.F	writes the total decay decay widths and the corresponding BRs to a file in the SLHA format
hfold/model_mssm.F	all parameters for the MSSM are initialized
hfold/mssmhdecay.F	calls all subroutines to calculate the partial decay widths and the corresponding branching ratios for the selected MSSM Higgs particle
hfold/resumbottomyuk.F	contains the formula for the resummation of the bottom Yukawa coupling
hfold/SetKinematics.F	sets up the kinematics
hfold/softstrahlung.F	adds up the contributions from soft-photon/gluon emission
print routines	
hfold/PrintBR.F	prints all relevant branching ratios and partial decay widths
hfold/PrintDrbarPara.F	prints Yukawa couplings, soft SUSY–breaking parameters
hfold/PrintMSSMPara.F	prints the MSSM mass spectrum
external programs	
hfold/util	utilities for e.g. kinematics (part of FormCalc)
hfold/SLHALib-2.2	Supersymmetry Les Houches Accord program library

automaticaly generated code	**description**
hfold/mssmhiggs/sqmes/sqmesh0 hfold/mssmhiggs/sqmes/sqmesHH hfold/mssmhiggs/sqmes/sqmesA0 hfold/mssmhiggs/sqmes/sqmesHp	all necessary squared amplitudes generated with FeynArts/FormCalc
hfold/renconst	all renormalization constants
hfold/mssmhiggs/processes/h0 hfold/mssmhiggs/processes/HH hfold/mssmhiggs/processes/A0 hfold/mssmhiggs/processes/Hp	Sets the right flags (masses, helicities, kinematics, bremsstrahlung) for each decay channel (for each decay channel there is a separate subroutine).

subroutine(s)	**description**
hfold/mssmhiggs/calcwidth0.F hfold/mssmhiggs/calcwidthHH.F hfold/mssmhiggs/calcwidthA0.F hfold/mssmhiggs/calcwidthHp.F	Calculates all partial decay widths
hfold/mssmhiggs/sumwidthh0.F hfold/mssmhiggs/sumwidthHH.F hfold/mssmhiggs/sumwidthA0.F hfold/mssmhiggs/sumwidthHp.F	Calculates branching ratios, sums up the total decay widths and sets the right SLHA code for each decay channel
hfold/osmasses/OSmasses.F hfold/osmasses/CalcOSMasses1.F hfold/osmasses/CalcOSMasses2.F hfold/osmasses/CalcOSMasses3.F hfold/osmasses/CalcOSMasses4.F hfold/osmasses/CalcOSMasses5.F hfold/osmasses/CalcOSMasses6.F hfold/osmasses/CalcOSMasses7.F hfold/osmasses/CalcOSMasses8.F hfold/osmasses/CalcOSMasses9.F hfold/osmasses/CalcOSMasses10.F	Calculates the on-shell masses of the SUSY particles; subroutine CalcOSMasses1.F - CalcOSMasses10.F contain all the necessary selfenergies

The screen output when running HFOLD is as following:

```
                        _    _ ______ ____  _      _____
                       | |  | |  ____/ __ \| |    |  __ \
                       | |__| | |__ | |  | | |    | |  | |
                       |  __  |  __|| |  | | |    | |  | |
                       | |  | | |   | |__| | |____| |__| |
                       |_|  |_|_|    \____/|______|_____/   1.0

                    Higgs Full One Loop Decays by W. Frisch,
                    H. Eberl, H. Hlucha


error 0
abort 173248520
nslhadata 5504
====================================================
  FF 2.0, a package to evaluate one-loop integrals
written by G. J. van Oldenborgh, NIKHEF-H, Amsterdam
====================================================
for the algorithms used see preprint NIKHEF-H 89/17,
’New Algorithms for One-loop Integrals’, by G.J. van
Oldenborgh and J.A.M. Vermaseren, published in
Zeitschrift fuer Physik C46(1990)425.
====================================================
ffxdb0: IR divergent B0’, using cutoff   1.
ffxc0i: infra-red divergent threepoint function,
working with a cutoff   1.
Flags:
----------------------------
Susyqcd calculation
----------------------------
resummation of bottom yukawa coupling off
----------------------------
Using onshell Higgs masses from : SPS1aprime.spc
----------------------------
hard bremsstrahlung on
----------------------------
the output SLHA will be written to : output.slha
----------------------------
============================
     Decay Table :
     Total width :  0.905677243
     H+ -> mu+ nu_mu        :   0.320441E-003 / BR :     0.00
     H+ -> tau+ nu_tau      :   0.906196E-001 / BR :     0.10
```

```
H+ -> c sb             :   0.349004E-003 / BR :     0.00
H+ -> t bb             :   0.617035E+000 / BR :     0.68
H+ -> ~chi_10 ~chi_1+  :   0.128997E+000 / BR :     0.14
H+ -> ~chi_30 ~chi_1+  :   0.699102E-003 / BR :     0.00
H+ -> h W+             :   0.277014E-002 / BR :     0.00
H+ -> ~nu_eL ~el_2+    :   0.126903E-002 / BR :     0.00
H+ -> ~nu_muL ~mu_1+   :   0.237453E-003 / BR :     0.00
H+ -> ~nu_muL ~mu_2+   :   0.124702E-002 / BR :     0.00
H+ -> ~nu_tauL ~tau_1+ :   0.609862E-001 / BR :     0.07
H+ -> ~nu_tauL ~tau_2+ :   0.114467E-002 / BR :     0.00
```

Appendix A

Loop–integrals

The integrals at one–loop level can be simply generalized by

$$T^N_{\mu_1...\mu_P}(k_1, ..., k_{N-1}, m_0^2, ..., m_{N-1}^2) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int dq^D \frac{q_{\mu_1}....q_{\mu_P}}{D_0...D_{N-1}} \ , \tag{A.1}$$

where $T^N_{\mu_1...\mu_P}$ is a tensor of rank P. We use the following abbreviations in the denominators $D_i = (q + k_i)^2 - m_i^2$. The inner loop–momenta k_i are defined using the following relations

$$p_{i+1} = k_{i+1} - k_i \quad \text{with} \quad j, j \in \{0, ..., N-1\} \ , \tag{A.2}$$

$$k_j = \sum_{i=1}^{j} p_i \quad \text{with} \quad k_0 = 0 \ , \tag{A.3}$$

$$\tag{A.4}$$

μ is an arbitrary mass (renormalization scale). The momenta in the nominator originate from inner fermion propagators or from momentum dependent couplings. A basis of scalar

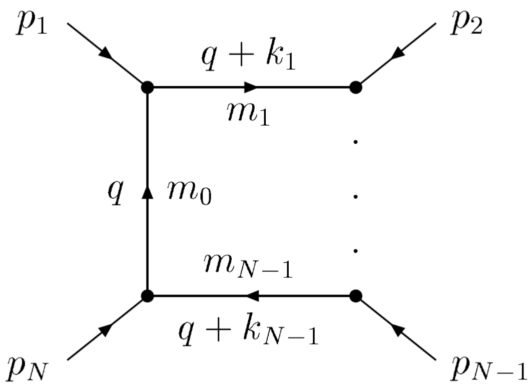


Figure A.1: Kinematics of the external momenta in the loops. The arrows denote the direction of the corresponding 4-momenta.

integrals can be defined by:

$$\begin{aligned} A_0(m_0^2) &= T_0^1(m_0^2) \ , \\ B_0(k_{10}^2, m_0^2, m_1^2) &= T_0^2(k_1, m_0^2, m_1^2) \ , \\ C_0(k_{10}^2, k_{12}^2, k_{20}^2, m_0^2, m_1^2, m_2^2) &= T_0^3(k_1, k_2, k_3, m_0^2, m_1^2, m_2^2) \ , \\ &\vdots \end{aligned} \tag{A.5}$$

Due dimensional regularization maintains Lorentz covariance it is possible to construct tensorial expressions with the use of the scalar basis

$$\begin{aligned} B_\mu &= k_{1\mu} B_1 \ , \\ B_{\mu\nu} &= g^{\mu\nu} \ , \\ C_\mu &= \sum_{i=1}^{2} k_{i\mu} C_i \ , \\ C_{\mu\nu} &= C_{00} + \sum_{i,j=1}^{2} k_{i\mu} k_{j\nu} C_{ij} \ , \\ &\vdots \end{aligned} \tag{A.6}$$

The coefficients $B_1, \ C_i, \ C_{i,j}, \ \ldots$ can be determined by multiplying the expressions in A.6 with their corresponding external momenta. This leads to a system of equation, which can then be solved.

A.0.9 Analytic expressions for scalar Passarino–Veltman integrals

Analytic epression for $A_0(m^2)$

The scalar integral $A_0(m_0^2)$ is defined as

$$A_0(m_0^2) = \frac{\mu^{4-D}}{i\pi^2} \int \frac{dq^D}{(2\pi)^{D-4}} \frac{1}{q^2 - m^2} = \alpha \int dq^D \frac{1}{q_0^2 - \vec{q}^2 - m^2} \ . \tag{A.7}$$

To process further we perform a Wick rotation from M^D to E^D by substituting $q_0 = i\, q_{0,E}$

$$A_0(m_0^2) = -i\alpha \int \frac{1}{q_{0,E}^2 + \vec{q}^2 + m^2} \ . \tag{A.8}$$

Using the Γ–function we can perform the spherical integration as follows

$$A_0(m_0^2) = -i\alpha \int d\Omega \int dr \frac{r^{D-1}}{r^2 + m^2} = -\alpha m^{-2} \Omega_D \int dr \frac{r^{D-1}}{\frac{r^2}{m^2} + 1} \ , \tag{A.9}$$

substituting $r = r^{'} m$ $(r^{'} \to r)$ leads to

$$\begin{aligned} A_0(m_0^2) &= -i\alpha m^{-2} \Omega_D \int drm \frac{(rm)^{D-1}}{r^2 + 1} = -i\alpha m^{-1} \Omega_D \int dr \frac{(rm)^{D-1}}{r^2 + 1} && \text{(A.10)} \\ &= -i\alpha m^{D-2} \Omega_D \int dr \frac{r^{D-1}}{r^2 + 1} \ , && \text{(A.11)} \end{aligned}$$

substituting again $r^2 = t$, we obtain the expression

$$
\begin{aligned}
A_0(m_0^2) &= -i\alpha m^{D-2}\Omega_D \frac{1}{2}\int dt \frac{t^{D/2-1}}{t+1} \,, && \text{(A.12)}\\
&= -i\alpha m^{D-2}\Omega_D \frac{1}{2}\frac{\Gamma(D/2)\Gamma(1-D/2)}{\Gamma(1)} \,, && \text{(A.13)}\\
&= -\frac{\mu^{4-D}}{\pi^2}\frac{1}{(2\pi)^{D-4}} m^{D-2}\pi^{\frac{D}{2}}\Gamma(1-\frac{D}{2}) \,. && \text{(A.14)}\\
& && \text{(A.15)}
\end{aligned}
$$

Setting $D = 4 - x\varepsilon$ leads to

$$
A_0(m_0^2) = -m^2\left(\frac{m^x}{2^x\pi^{\frac{x}{2}}\mu^x}\right)^{-\varepsilon}\Gamma(\frac{x\varepsilon}{2}-1) \,, \quad \text{(A.16)}
$$

using $y^{-x} = 1 - (\log y)x$ and $\Gamma(\epsilon - 1) = -\frac{1}{\epsilon} - 1 + \gamma_e + \mathcal{O}(\varepsilon)$, leads to

$$
\begin{aligned}
A_0(m_0^2) &= -m^2\left(1-\log\left(\frac{m^x}{2^x\pi^{\frac{x}{2}}\mu^x}\right)\varepsilon\right)\left(-\frac{2}{x\varepsilon}-1+\gamma_e+\mathcal{O}(\varepsilon)\right) \,, \quad \text{(A.17)}\\
&= -m^2\left(-\frac{2}{x\varepsilon}-1+\gamma_e+\frac{2}{x}\log\left(\frac{m^x}{2^x\pi^{\frac{x}{2}}\mu^x}\right)+\mathcal{O}(\varepsilon)\right) \,.
\end{aligned}
$$

Setting $x = 2$ we obtain the following result

$$
A_0(m_0^2) = m^2\left(\frac{1}{\varepsilon}+1+\log(4\pi)-\gamma_e-\log(\frac{m^2}{\mu^2})+\mathcal{O}(\varepsilon)\right) \,, \quad \text{(A.18)}
$$

the result can be expressed more compactly by introducing the UV–parameter $\Delta = \frac{1}{\varepsilon} + \log(4\pi) - \gamma_e$

$$
A_0(m_0^2) = m^2\left(\Delta-\log(\frac{m^2}{\mu^2})+1+\mathcal{O}(\varepsilon)\right) \,. \quad \text{(A.19)}
$$

In the Minimal Subtraction scheme MS, the counterterms contain only the UV-divergent parts proportional to $\frac{1}{\varepsilon}$. However the terms $+\log(4\pi) - \gamma_e$ lead to large finite shifts. To get rid of these large finite terms it is convenient to introduce the Modified Minimal Subtraction $\overline{\text{MS}}$, where the scale μ is transformed by

$$
\mu^2_{\text{MS}} \to \mu^2_{\overline{\text{MS}}}\, e^{-\ln 4\pi+\gamma_e} \,. \quad \text{(A.20)}
$$

Analytic epression for B_0

Using the technique of the Feynman–parameters the integral B_0 can be expressed by

$$
B_0(k^2, m_0^2, m_1^2) = \Delta - \int_0^1 \mathrm{d}x \log\frac{xm_0^2+(1-x)m_1^2-x(1-x)k^2}{\mu^2} + \mathcal{O}(\epsilon) \,. \quad \text{(A.21)}
$$

To evaluate the integral further, it is necessary to reexpress the argument of the log–function of eq. (A.21) in terms of the corresponding roots x_1 and x_2 with

$$x_{1/2} = \frac{k^2 + m_1^2 - m_0^2 \pm \sqrt{(k^2 + m_1^2 - m_0^2)^2 - 4m_1^2 k^2}}{2k^2} \,. \tag{A.22}$$

Now the integral can be written as

$$B_0(k^2, m_0^2, m_1^2) = -\log\frac{k^2}{\mu^2} - \int_0^1 \mathrm{d}x \log(x - x_1) - \int_0^1 \mathrm{d}x \log(x - x_2)\,. \tag{A.23}$$

Using $\int \mathrm{d}x \log x = x\log x - x$, we obtain the analytic solution for the scalar integral B_0 with general arguments

$$\begin{aligned} B_0(k^2, m_0^2, m_1^2) &= \Delta + 2 - \log\frac{k^2}{\mu^2} - \sum_{i=1}^{2}\left[(1 - x_i)\log(1 - x_i) + x_i\log(-x_i)\right] \\ &= \Delta + 2 + \log\frac{\mu^2}{m_0^2} + x_1\log\left(1 - \frac{1}{x_1}\right) + x_2\log\left(1 - \frac{1}{x_2}\right)\,. \end{aligned} \tag{A.24}$$

Now we want to investigate in some special cases of the integrals B_0. We will start with $B_0(k^2, 0, m_1^2)$, which can be expressed by

$$B_0(k^2, 0, m_1^2) = \Delta - \log\frac{k^2}{\mu^2} - \int_0^1 \mathrm{d}x \log(x - 1) - \int_0^1 \mathrm{d}x \log\left(\frac{m_1^2}{k^2} - 1\right)\,. \tag{A.25}$$

Using $\int_0^1 \mathrm{d}x \log(x-1) = -1$ and

$$\begin{aligned} \int_0^1 \mathrm{d}x \log(\frac{m_1^2}{k^2} - 1) &= -1 + \left(\frac{m_1^2}{k^2} - 1\right)\log\left(\frac{m_1^2}{k^2} - 1\right) - \frac{m_1^2}{k^2}\log\frac{m_1^2}{k^2} \\ &= -1 - \frac{m_1^2 - k^2}{k^2}\log\frac{m_1^2 - k^2}{m_1^2} + \log\frac{m_1^2}{k^2}\,, \end{aligned} \tag{A.26}$$

the result can be written as

$$B_0(k^2, 0, m_1^2) = \Delta + 2 + \log\frac{\mu^2}{m_1^2} + \frac{m_1^2 - k^2}{k^2}\log\frac{m_1^2 - k^2}{m_1^2}\,. \tag{A.27}$$

We can take the limit $m_1^2 \to k^2$, which leads to

$$B_0(m^2, 0, m^2) = \Delta + 2 + \log\frac{\mu^2}{m^2}\,. \tag{A.28}$$

Some important relations for the scalar integral B_0 with special arguments

$$\begin{aligned}
B_0(p^2,m_0^2,m_1^2) &= B_0(p^2,m_1^2,m_0^2)\,, && \text{(A.29)}\\
B_0(m^2,0,m^2) &= \frac{A_0(m^2)}{m^2}+1\,, && \text{(A.30)}\\
B_0(0,0,0) &= \Delta\,, && \text{(A.31)}\\
B_0(m^2,0,0) &= B_0(m^2,0,m^2)+i\pi\,, && \text{(A.32)}\\
B_0(0,0,m^2) &= B_0(m^2,0,m^2)-1\,, && \text{(A.33)}\\
B_0(0,m^2,m^2) &= B_0(m^2,0,m^2)-2\,, && \text{(A.34)}\\
m^2B_0(0,m^2,m^2) &= A_0(m^2)-m^2\,, && \text{(A.35)}\\
(m_0^2-m_1^2)B_0(0,m_0^2,m_1^2) &= A_0(m_0^2)-A_0(m_1^2)\,, && \text{(A.36)}\\
B_0(0,m_0^2,m_1^2) &= B_0(k^2,m_0^2,m_1^2)+\frac{k^2}{m_0^2-m_1^2}(2B_1+B_0)(k^2,m_0^2,m_1^2)\,. && \text{(A.37)}
\end{aligned}$$

Some expressions for the derivative of B_0 with respect to the infrared mass λ,

$$\begin{aligned}
\dot{B}_0(m^2,\lambda^2,m^2) &= -\frac{1}{2m^2}\left(2-\log\frac{m^2}{\lambda^2}\right)\,, && \text{(A.38)}\\
\dot{B}_0(m^2,m^2,\lambda^2) &= \dot{B}_0(m^2,\lambda^2,m^2)\,, && \text{(A.39)}\\
\dot{B}_0(0,m^2,m^2) &= \frac{1}{6m^2}\,, && \text{(A.40)}\\
\lim_{\lambda^2\to 0}\left(\lambda^2\dot{B}_0(\lambda^2,0,0)\right) &= -1\,. && \text{(A.41)}
\end{aligned}$$

Some important relations between B_1, B_0 and A_0

$$\begin{aligned}
B_1(k^2,m_1^2,m_0^2) &= -B_0(k^2,m_0^2,m_1^2)-B_1(k^2,m_0^2,m_1^2)\,, && \text{(A.42)}\\
2k^2B_1(k^2,m_0^2,m_1^2) &= A_0(m_0^2)-A_0(m_1^2)+(m_1^2-m_0^2-k^2)B_0(k^2,m_0^2,m_1^2)\,, && \text{(A.43)}\\
B_1(m^2,0,m^2) &= -\frac{1}{2}\left(\Delta+1+\log\frac{\mu^2}{m^2}\right)=-\frac{1}{2}\left(B_0(m^2,0,m^2)-1\right)\,, && \text{(A.44)}\\
B_1(m^2,m^2,0) &= B_1(m^2,0,m^2)-1\,. && \text{(A.45)}
\end{aligned}$$

Expressions for the derivative of the B_1

$$\begin{aligned}
\dot{B}_1(m^2,m^2,\lambda^2) &= \frac{1}{2m^2}\left(3-\log\frac{m^2}{\lambda^2}\right)=-\dot{B}_0(m^2,\lambda^2,m^2)+\frac{1}{2m^2}\,, && \text{(A.46)}\\
\dot{B}_1(m^2,\lambda^2,m^2) &= -\frac{1}{2m^2}\,, && \text{(A.47)}\\
\lim_{\lambda^2\to 0}\left(\lambda^2\dot{B}_1(\lambda^2,0,0)\right) &= \frac{1}{2}\,. && \text{(A.48)}
\end{aligned}$$

The scalar integral $C_0(m_1^2,m_0^2,m_2^2,\lambda^2,m_1^2,m_2^2)$ can be expressed in a simple analytic form given by

$$\begin{aligned}
\mathrm{Re}\Big\{C_0(m_1^2,m_0^2,m_2^2,\lambda^2,m_1^2,m_2^2)\Big\} &= \frac{1}{\kappa}\Bigg[\log\beta_0\log\frac{\kappa}{\lambda^2}-\frac{2\pi^2}{3}-\mathrm{Li}_2(\beta_1^2)-\mathrm{Li}_2(\beta_2^2)\\
&\quad -\log^2\beta_1-\log^2\beta_2+\log\frac{\kappa}{m_0^2}\log\beta_0\Bigg]\,, && \text{(A.49)}
\end{aligned}$$

with the factors β_0, β_1 and β_2 given by

$$\beta_0 = \frac{m_0^2 - m_1^2 - m_2^2 + \kappa}{2m_1 m_2}\,,\ \beta_1 = \frac{m_0^2 - m_1^2 + m_2^2 - \kappa}{2m_0 m_2}\,,\ \beta_2 = \frac{m_0^2 + m_1^2 - m_2^2 - \kappa}{2m_0 m_1}\,. \qquad \text{(A.50)}$$

integral	UV–divergent part
$A_0(m^2)$	$m^2\,\Delta$
$A_1(m^2)$	$-m^2\,\Delta$
$A_{00}(m^2)$	$\frac{m^4}{4}\,\Delta$
B_0	Δ
B_1	$-\frac{1}{2}\,\Delta$
$B_{00}(k^2, m_0^2, m_1^2)$	$-\frac{1}{4}(\frac{k^2}{3} - m_0^2 - m_1^2)\Delta$
B_{11}	$\frac{1}{3}\Delta$
C_{00}	$\frac{1}{4}\Delta$
C_{00i}	$-\frac{1}{12}\Delta$

Table A.1: UV–divergent parts of Passarino–Veltman integrals ($\Delta = \frac{1}{\varepsilon} + \log(4\pi) - \gamma_e$)

integral	IR–divergent part
$\dot{B}_0(m^2, \lambda^2, m^2) = \dot{B}_0(m^2, m^2, \lambda^2)$	$-\frac{\ln\lambda^2}{2m^2}$
$\dot{B}_1(m^2, m^2, \lambda^2)$	$\frac{\ln\lambda^2}{2m^2}$
$\dot{B}_1(m^2, \lambda^2, m^2)$	0
$\mathrm{Re}\,[C_0(m_1^2, m_0^2, m_2^2, \lambda^2, m_1^2, m_2^2)]$	$-\frac{\ln\beta_0}{\kappa}\,\ln\lambda^2$

Table A.2: IR–divergent parts of Passarino–Veltman integrals

Appendix B

Weyl spinors

Left handed spinors $\Psi_L = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ transform under the $(\frac{1}{2}, 0)$–representation of the Lorentz group. In contrast to the right handed spinors $\Psi_R = \begin{pmatrix} \bar{\psi}^{\dot{1}} \\ \bar{\psi}^{\dot{2}} \end{pmatrix}$, which transform under the $(0, \frac{1}{2})$–representation. Every dottet index belongs to the $(\frac{1}{2}, 0)$–representation and every undottet index belongs to the $(0, \frac{1}{2})$–representation. Lorentz invariant combinations can be written as

$$\Psi_R^\dagger \Psi_L = \psi^a \psi_a \ , \tag{B.1}$$
$$\Psi_L^\dagger \Psi_R = \psi_{\dot{a}} \psi^{\dot{a}} \ , \tag{B.2}$$

complex conjugation transforms an undottet into a dotted index and vice versa

$$(\psi_a)^* = \bar{\psi}_{\dot{a}} \qquad (\psi^a)^* = \bar{\psi}^{\dot{a}} \ . \tag{B.3}$$

Raising and lowering of spinor indices can be performed using spinor metrics ε and $\tilde{\varepsilon}$

$$\psi^a = \tilde{\varepsilon}^{ab} \psi_b \ , \qquad \psi_a = \tilde{\varepsilon}_{ab} \psi^b \ , \tag{B.4}$$
$$\bar{\psi}^{\dot{a}} = \varepsilon^{\dot{a}\dot{b}} \psi_{\dot{b}} \ , \qquad \bar{\psi}_{\dot{a}} = \varepsilon_{\dot{a}\dot{b}} \psi^{\dot{b}} \ , \tag{B.5}$$

with

$$\tilde{\varepsilon}_{ab} = \tilde{\varepsilon}^{ab} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \ , \tag{B.6}$$

$$\varepsilon^{\dot{a}\dot{b}} = \varepsilon_{\dot{a}\dot{b}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \ . \tag{B.7}$$

B.0.10 Dirac & Majorana spinors

Dirac and Majorana spinors are the composition of a pointed and an unpointed Weyl spinor, therefore they have four indices. The Dirac spinors Ψ and Φ can be expressed by

$$\Psi_a = \begin{pmatrix} \psi_a \\ \bar{\chi}^{\dot{a}} \end{pmatrix} \ , \quad \Phi_a = \begin{pmatrix} \psi_a \\ \bar{\lambda}^{\dot{a}} \end{pmatrix} \ , \tag{B.8}$$

$$\tag{B.9}$$

with

$$\bar{\Psi}_A = \Psi_A^\dagger \gamma_0 = (\bar{\psi}_{\dot{a}}, \chi^a) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (\chi^a, \bar{\psi}_{\dot{a}}) \ , \tag{B.10}$$

$$\bar{\Phi}_A = \Psi_A^\dagger \gamma_0 = (\bar{\phi}_{\dot{a}}, \lambda^a) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (\lambda^a, \bar{\psi}_{\dot{a}}) \ . \tag{B.11}$$

The Lorentz scalar $\bar{\Psi}\Phi = \bar{\Psi}_A \Phi_A$ takes the form

$$\bar{\Psi}\Phi = \chi\phi + \bar{\psi}\bar{\lambda} \ . \tag{B.12}$$

In the chiral representation the γ-matrices have the following index structure

$$\gamma^\mu_{AB} = \begin{pmatrix} 0 & \sigma^\mu_{a\dot{b}} \\ -\tilde{\sigma}^{\mu\dot{a}b} & 0 \end{pmatrix} \ , \tag{B.13}$$

$$\gamma^5_{AB} = \begin{pmatrix} -\delta_a^b & 0 \\ 0 & \delta^{\dot{a}}_{\dot{b}} \end{pmatrix} \ . \tag{B.14}$$

The charge conjugation operator C can be expressed in the chiral representation by

$$C_{AB} = \begin{pmatrix} -\tilde{\varepsilon}_{ab} & 0 \\ 0 & -\tilde{\varepsilon}^{\dot{a}\dot{b}} \end{pmatrix} \ , \tag{B.15}$$

$$\tag{B.16}$$

fulfilling the condition for Majorana spinors $\Psi_M = C\bar{\Psi}^T$.

B.0.11 Calculations with Weyl spinors

$$(\theta\theta) = 2\theta^1\theta^2 \tag{B.17}$$

$$(\bar{\theta}\bar{\theta}) = 2\bar{\theta}^{\dot{2}}\bar{\theta}^{\dot{1}} \tag{B.18}$$

the most important billinear forms

$$i\Phi_L^T\sigma^2\Psi_L = \phi^a\bar{\psi}^a = (\phi\psi) \tag{B.19}$$

$$-i\Phi_R^T\sigma^2\Psi_R = \bar{\phi}_{\dot{a}}\bar{\psi}^{\dot{a}} = (\phi\psi) \tag{B.20}$$

$$\Phi_R^\dagger\sigma^\mu\Psi_R = \phi^a\sigma^\mu_{a\dot{b}}\bar{\psi}^{\dot{b}} = (\phi\sigma^\mu\bar{\phi}) \tag{B.21}$$

$$\Phi_L^\dagger\tilde{\sigma}^\mu\Psi_R = \phi^a\tilde{\sigma}^\mu_{a\dot{b}}\bar{\psi}^{\dot{b}} = (\phi\tilde{\sigma}^\mu\bar{\phi}) \tag{B.22}$$

B.0.12 Differentation with Weyl spinors

$$\partial_a = \frac{\partial}{\partial\theta^a} \qquad \partial^a = \frac{\partial}{\partial\theta_a} \tag{B.23}$$

$$\partial_{\dot{a}} = \frac{\partial}{\partial\theta^{\dot{a}}} \qquad \partial^{\dot{a}} = \frac{\partial}{\partial\theta_{\dot{a}}} \tag{B.24}$$

Following the differentation rule for Grassmann numbers leads to the relations

$$\partial_a \theta^b = \delta_a^b \; , \qquad \partial^a \theta_b = \delta_b^a \; , \tag{B.25}$$

$$\bar\partial_{\dot a} \bar\theta^{\dot b} = \delta_{\dot a}^{\dot b} \; , \qquad \bar\partial^{\dot a} \bar\theta_{\dot b} = \delta_{\dot b}^{\dot a} \; . \tag{B.26}$$

$$\tag{B.27}$$

This rules can also be expressed using anti–commutators

$$\{\partial_a, \theta^b\} = \{\partial^b, \theta_a\} = \delta_a^b \; , \tag{B.28}$$

$$\{\partial_a, \theta_b\} = \tilde\varepsilon_{ab} \; , \tag{B.29}$$

$$\{\partial^a, \theta^b\} = -\tilde\varepsilon^{ab} \; , \tag{B.30}$$

$$\tag{B.31}$$

this relations are similar for dotted indices. Since θ and $\bar\theta$ belong to different representations we have

$$\{\bar\partial_{\dot a}, \theta_b\} = \{\partial_a, \bar\theta_{\dot b}\} = 0 \; . \tag{B.32}$$

B.0.13 Integration over Grassmann numbers

The Lorentz invariant volumes can be expressed by

$$d^2\theta = \frac{1}{4} d\theta^a d\theta_a \; , \tag{B.33}$$

$$d^2\bar\theta = \frac{1}{4} d\bar\theta_{\dot a} d\theta^{\dot a} \; , \tag{B.34}$$

$$d^4\theta = d^2\theta d^2\bar\theta \; , \tag{B.35}$$

$$d^2\theta = \frac{1}{2} d\theta_1 d\theta_2 \; , \tag{B.36}$$

$$d^2\bar\theta = \frac{1}{2} \bar\theta_{\dot 2} \bar\theta_{\dot 1} \; , \tag{B.37}$$

$$(d^2\theta)^\dagger = d^2\bar\theta \; . \tag{B.38}$$

Equivalence between integration and differentation leads to

$$\int d\theta_a = \frac{\partial}{\partial\theta_a} = \partial^a \; , \tag{B.39}$$

$$\int d\theta^a = \frac{\partial}{\partial\theta^a} = \partial_a \; . \tag{B.40}$$

Integrals can therefore be expressed by differentation operators

$$\int d^2\theta = -\frac{1}{4}\int d\theta^a \int d\theta_a = -\frac{1}{4}\partial_a\partial^a = \frac{1}{4}\partial^a\partial_a = \frac{1}{4}\partial^2 \; , \tag{B.41}$$

$$\int d^2\bar\theta = -\frac{1}{4}\int d\bar\theta_{\dot a} \int d\bar\theta^{\dot a} = -\frac{1}{4}\bar\partial^{\dot a}\bar\partial_{\dot a} = \frac{1}{4}\bar\partial_{\dot a}\bar\partial^{\dot a} = \frac{1}{4}\bar\partial^2 \; , \tag{B.42}$$

$$\int d^4\theta = \frac{1}{16}\partial^2\bar\partial^2 = \frac{1}{16}\bar\partial^2\partial^2 \; , \tag{B.43}$$

this leads to the following relations

$$\int d^2\theta(\theta\theta) = \frac{1}{4}\partial^a\partial_a(\theta\theta) = 1 \ , \tag{B.44}$$

$$\int d^2\bar{\theta}(\bar{\theta}\bar{\theta}) = \frac{1}{4}\bar{\partial}^a\bar{\partial}_a(\bar{\theta}\bar{\theta}) = 1 \ , \tag{B.45}$$

$$\int d^4\theta(\theta\theta)(\bar{\theta}\bar{\theta}) = 1 \ . \tag{B.46}$$

Let now assume a superfunction f given by

$$f(\theta) = f(0) + f^a\theta_a + f^{(2)}(\theta\theta) \ , \tag{B.47}$$

$$\int d\theta^2 f(\theta) = f^{(2)} \ . \tag{B.48}$$

As can be seen, the integration works as a projector, because

$$\int d\theta^2 1 \ = \ \int d\bar{\theta}^2 1 = 0 \ , \tag{B.49}$$

$$\int d\theta^2\theta \ = \ \int d\bar{\theta}^2\bar{\theta} = 0 \ . \tag{B.50}$$

Appendix C

Special functions

C.1 The Γ–function

The Γ–function is defined by

$$\Gamma(z) = \int_0^\infty dt\, t^{z-1} e^{-t} \ . \tag{C.1}$$

C.1.1 Properties

$$\begin{aligned}
\Gamma(1) &= 1 \ , && \text{(C.2)}\\
\Gamma(n) &= (n-1)! \ , && \text{(C.3)}\\
\Gamma(\epsilon - 1) &= -\frac{1}{\epsilon} - 1 + \gamma_e + \mathcal{O}(\varepsilon) \ , && \text{(C.4)}\\
\gamma_e &= \lim_{m\to\infty} (\sum_{k=1}^{m} \frac{1}{k} - \log m) \sim 0.577216 \ , && \text{(C.5)}\\
\int_0^\infty dt\, \frac{t^{\alpha-1}}{(t+1)^\beta} &= \frac{\Gamma(\alpha)\Gamma(\beta-\alpha)}{\Gamma(\beta)} \ . && \text{(C.6)}\\
& && \text{(C.7)}
\end{aligned}$$

D-dimensional spherical integration $\int d\Omega_D = \Omega_D$ can be expressed with the following relations and the Γ–function

$$\begin{aligned}
\int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} dx_1 ... dx_n &= \int d\Omega \int dr\ r^{n-1} \ , && \text{(C.8)}\\
\int_{-\infty}^{\infty} e^{-^2} dx &= \sqrt{\frac{1}{\pi}} \ , && \text{(C.9)}\\
\int d\Omega_D &= \frac{2\pi^{D/2}}{\Gamma(D/2)} \ . && \text{(C.10)}\\
& && \text{(C.11)}
\end{aligned}$$

C.1.2 The Spence–function Li_2

The Spence–function $\mathrm{Li}_2(x)$ is defined by

$$\mathrm{Li}_2(x) = -\int_0^x \frac{dt}{t} \log(1-t)\,. \tag{C.12}$$

C.1.3 The Källen function κ

the totally symmetric Källen function κ is defined by

$$\kappa(x,y,z) = \sqrt{(x-y-z)^2 - 4yz}\,. \tag{C.13}$$

Appendix D

SPheno flags

We used the following flags in *SPheno* for the generation of all spectra.

```
# SM values are taken from: C. Amsler et al. (Particle Data Group)
# Physics Letters B667, 1 (2008) (URL: http://pdg.lbl.gov)
# where MW = 80.398+/-0.025 GeV
#
Block MODSEL             # Select model
 1   1                   # mSugra
Block SMINPUTS           # Standard Model inputs
  2   1.16637000E-05     # G_F, Fermi constant
  3   1.17600000E-01     # alpha_s(MZ) SM MSbar
  4   9.11876000E+01     # Z-boson pole mass
  5   4.20000000E+00     # m_b(mb) SM MSbar
  6   1.71200000E+02     # m_top(pole)
  7   1.77684000E+00     # m_tau(pole)
Block MINPAR             # Input parameters
  1   2.50000000E+02     # m0
  2   7.00000000E+01     # m12
  3   1.00000000E+01     # tanb
  4   1.00000000E+00     # sign(mu)
  5   0.00000000E+00     # A0
Block SPhenoInput        # SPheno specific input
  1  -1.00000000E+00     # error level
  2   1.00000000E+00     # if =1, then SPA conventions are used ~ scale = 1000 GeV
 11   0.00000000E+00     # calculate branching ratios
 21   0.00000000E+00     # calculate cross section
 25   0.00000000E+00     # if 0 no ISR is calculated, if 1 ISR is caculated
 31  -1.00000000E+00     # m_GUT, if < 0 than it determined via g_1=g_2
 32   0.00000000E+00     # require strict unification g_1=g_2=g_3 if '1' is set
 33  -1.00000000E+00     # Q_EWSB, if < 0 than  Q_EWSB=sqrt(m_~t1 m_~t2)
 51   5.10998910E-04     # electron mass
 52   1.05658367E-01     # muon mass
 61   2.00000000E+00     # scale where quark masses of first 2 gen. are defined
```

```
62   3.00000000E-03   # m_u(Q)  1.5-3.3 MeV
63   1.20000000E+00   # m_c(Q)  1.27 +0.07 -0.11 MeV
64   6.00000000E-03   # m_d(Q)  3.5-6.0 MeV
65   1.20000000E-01   # m_s(Q)  104 +26 -34 MeV
```

Appendix E

Bremsstrahlung

E.1 Infrared divergences

Infrared (IR) divergences originate from the fact that photons and gluons are massless particles, this causes a divergent behavior in loop integrals for vanishing photon/gluon momenta. To regulate the divergent expressions, one introduces a small photon/gluon mass λ e.g.

$$\frac{\partial}{\partial p^2}\frac{1}{i\pi^2}\int d^dq \frac{1}{(q^2-\lambda^2)[(q+p)^2-m^2]}\bigg|_{p^2=m^2} = -\frac{1}{2m^2}\Big(2-\log\frac{m^2}{\lambda^2}\Big)\,, \tag{E.1}$$

as can be seen easily this integral diverges for $\lambda \to 0$.
Following a theorem by Bloch and Nordsieck [53], the IR–divergences can be canceled by the inclusion of hard Bremsstrahlung processes which contain one additional emission of a single photon/gluon in the final state.

E.2 Hard Bremsstrahlung

For an 1 to 3 process with a massless particle the three-body phase space can still be integrated out analytically. We have implemented this radiation by using self-derived generic formulae for all four generic decay structures fig. 4.1 where every charged line can radiate off a photon (or a gluon for colored particles). The width containing the real Bremsstrahlung processes can be written as

$$\Gamma^{\text{hard}} = \frac{N_C}{2m_0}\int\frac{d^3k_1}{(2\pi)^3 2E_1}\int\frac{d^3k_2}{(2\pi)^3 2E_2}\int\frac{d^3k_3}{(2\pi)^3 2E_3}(2\pi)^4\delta^4(p-k_1+k_2-k_3)|\mathcal{M}^{\text{hard}}|^2\,,$$

where N_C denotes the color factor. The phase–space integrals I_n and I_{mn} in the convention of [32] are given by

$$I_{i_1\ldots i_n} = \frac{1}{\pi^2}\int\frac{d^3k_1}{2E_1}\frac{d^3k_2}{2E_2}\frac{d^3k_3}{2E_3}\,\delta^4(p-k_1+k_2-k_3)\frac{1}{(\pm 2k_{i_1}\cdot k_3)\ldots(\pm 2k_{i_n}\cdot k_3)}\,, \tag{E.2}$$

where the plus signs belong to the momenta of the outgoing particles k_1 and k_3 and the minus signs to the momenta p and k_2. The IR–convergent decay width Γ^{corr} for the physical value $\lambda = 0$ is then given by

$$\Gamma^{\rm corr}(\Phi \to f_A f_B) = \Gamma(\Phi_k^0 \to f_A f_B) + \Gamma^{\rm hard}(\Phi \to f_A f_B \gamma/g) \tag{E.3}$$
$$\Gamma^{\rm hard}(\Phi \to f_A f_B \gamma/g) = \frac{1}{2^6 m_\Phi \pi^3} \left(|\mathcal{M}_{SFF}|^2, |\mathcal{M}_{SSS}|^2, |\mathcal{M}_{SSV}|^2, |\mathcal{M}_{SVV}|^2\right) \tag{E.4}$$
$$\Phi = h^0, H^0, A^0, H^\pm \,. \tag{E.5}$$

The generic couplings $g_0,\ g_1$ and g_2 for the hard photon radiation are setup by

$$\begin{aligned} g_0 &= -e\ charge(0)\ , \\ g_1 &= -e\ charge(1)\ , \\ g_2 &= -e\ charge(2)\ , \end{aligned} \tag{E.6}$$

where e.g. $charge(0) = -1$ for a H^+ decay. For the hard gluon radiation $g_0,\ g_1$ and g_2 are defined by

$$\begin{aligned} g_0 &= -g_S\ strongC(0)\ , \\ g_1 &= -g_S\ strongC(1)\ , \\ g_2 &= -g_S\ strongC(2)\ , \end{aligned} \tag{E.7}$$

where g_S denotes the strong coupling constant, $strongC(i)$ is equal to one for colored particles and zero for non–colored particles.

E.2.1 Analytic expressions of the relevant phase–space integrals

In this section we will give the Analytic expressions for all phase–space integrals, necessary for all the two–body decays in the program package *HFOLD*.
For the discussion of the phase–space integrals it is necessary to introduce the functions

$$\beta_0(m_0^2, m_1^2, m_2^2) = \frac{(m_0^2 - m_1^2 - m_2^2 + \kappa(m_0^2, m_1^2, m_2^2))}{(2m_1 m_2)}\,,$$
$$\beta_1(m_0^2, m_1^2, m_2^2) = \frac{(m_0^2 - m_1^2 + m_2^2 - \kappa(m_0^2, m_1^2, m_2^2))}{(2m_0 m_2)}\,,$$
$$\beta_2(m_0^2, m_1^2, m_2^2) = \frac{(m_0^2 + m_1^2 - m_2^2 - \kappa(m_0^2, m_1^2, m_2^2))}{(2m_0 m_1)}\,,$$

where κ denotes the Källen function. In the further discussion we will skip the arguments of the Källen function $\kappa(m_0^2, m_1^2, m_2^2)$ and just write κ.

Type I_i

$$\begin{aligned}
I_0 &= \frac{1}{4m_0^2}\big(-2m_1^2\log(\beta_2)-2m_2^2\log(\beta_1)-\kappa\big)\\
I_1 &= \frac{1}{4m_0^2}\big(-2m_0^2\log(\beta_2)-2m_2^2\log(\beta_0)-\kappa\big)\\
I_2 &= \frac{1}{4m_0^2}\big(-2m_0^2\log(\beta_1)-2m_1^2\log(\beta_0)-\kappa\big)
\end{aligned}$$

Type I_i^j

$$\begin{aligned}
Ii &= \frac{1}{4m_0^2}\big(\frac{\kappa}{2}(m_0^2+m_1^2+m_2^2)+2m_0^2m_1^2\log(\beta_2)+2m_0^2m_2^2\log(\beta_1)+2m_1^2m_2^2\log(\beta_0)\big)\\
I_0^1 &= \frac{1}{4m_0^2}\big(m_1^4\log(\beta_2)-m_2^2(2m_0^2-2m_1^2+m_2^2)\log(\beta_1)-\frac{\kappa}{4}(m_0^2-3m_1^2+5m_2^2)\big)\\
I_0^2 &= \frac{1}{4m_0^2}\big(m_2^4\log(\beta_1)-m_1^2(2m_0^2-2m_2^2+m_1^2)\log(\beta_2)-\frac{\kappa}{4}(m_0^2-3m_2^2+5m_1^2)\big)\\
I_1^0 &= \frac{1}{4m_0^2}\big(m_0^4\log(\beta_2)-m_2^2(2m_1^2-2m_0^2+m_2^2)\log(\beta_0)-\frac{\kappa}{4}(m_1^2-3m_0^2+5m_2^2)\big)\\
I_1^2 &= \frac{1}{4m_0^2}\big(m_2^4\log(\beta_0)-m_0^2(2m_1^2-2m_2^2+m_0^2)\log(\beta_2)-\frac{\kappa}{4}(m_1^2-3m_2^2+5m_0^2)\big)\\
I_2^0 &= \frac{1}{4m_0^2}\big(m_0^4\log(\beta_1)-m_1^2(2m_2^2-2m_0^2+m_1^2)\log(\beta_0)-\frac{\kappa}{4}(m_2^2-3m_0^2+5m_1^2)\big)\\
I_2^1 &= \frac{1}{4m_0^2}\big(m_1^4\log(\beta_0)-m_0^2(2m_2^2-2m_1^2+m_0^2)\log(\beta_1)-\frac{\kappa}{4}(m_2^2-3m_1^2+5m_0^2)\big)
\end{aligned}$$

Type $I_{i_1 i_2}$

$$\begin{aligned}
I_{00} &= \frac{1}{4m_0^4}\Big(\kappa\log(\frac{\kappa^2}{\lambda m_0 m_1 m_2}) - \kappa - (m_1^2 - m_2^2)\log(\frac{\beta_1}{\beta_2}) - m_0^2\log(\beta_0)\Big)\\
I_{11} &= \frac{1}{4m_1^2 m_0^2}\Big(\kappa\log(\frac{\kappa^2}{\lambda m_0 m_1 m_2}) - \kappa - (m_0^2 - m_2^2)\log(\frac{\beta_0}{\beta_2}) - m_1^2\log(\beta_1)\Big)\\
I_{22} &= \frac{1}{4m_2^2 m_0^2}\Big(\kappa\log(\frac{\kappa^2}{\lambda m_0 m_1 m_2}) - \kappa - (m_0^2 - m_1^2)\log(\frac{\beta_0}{\beta_1}) - m_2^2\log(\beta_2)\Big)\\
I_{21} &= \frac{1}{4m_0^2}\Big(- 2\log(\frac{\lambda m_0 m_1 m_2}{\kappa^2})\log(\beta_0) + 2\log(\beta_0)^2 - \log(\beta_1)^2 - \log(\beta_2)^2\\
&+ 2\mathrm{Li}_2(1-\beta_0^2) - \mathrm{Li}_2(1-\beta_1^2) - \mathrm{Li}_2(1-\beta_2^2)\Big)\\
I_{10} &= \frac{1}{4m_0^2}\Big(- 2\log(\frac{\lambda m_0 m_1 m_2}{\kappa^2})\log(\beta_2) + 2\log(\beta_2)^2 - \log(\beta_0)^2 - \log(\beta_1)^2\\
&+ 2\mathrm{Li}_2(1-\beta_2^2) - \mathrm{Li}_2(1-\beta_0^2) - \mathrm{Li}_2(1-\beta_1^2)\Big)\\
I_{20} &= \frac{1}{4m_0^2}\Big(- 2\log(\frac{\lambda m_0 m_1 m_2}{\kappa^2})\log(\beta_1) + 2\log(\beta_1)^2 - \log(\beta_0)^2 - \log(\beta_2)^2\\
&+ 2\mathrm{Li}_2(1-\beta_1^2) - \mathrm{Li}_2(1-\beta_0^2) - \mathrm{Li}_2(1-\beta_2^2)\Big)
\end{aligned}$$

Type $I_{i_1 i_2}^{j_1 j_2}$

$$\begin{aligned}
I_{00}^{11} &= \frac{1}{4m_0^2}\Big((2m_2^2(m_0^2 + m_2^2 - m_1^2)\log(\beta_1) + \frac{\kappa^3}{6m_0^2} + 2\kappa m_2^2\Big)\\
I_{00}^{21} &= -\frac{1}{4m_0^2}\Big((m_1^4\log(\beta_2) + m_2^4\log(\beta_1) + \frac{\kappa^3}{6m_0^2} + \frac{\kappa}{4}(3m_1^2 + 3m_2^2 - m_0^2)\Big)\\
I_{00}^{22} &= \frac{1}{4m_0^2}\Big((2m_1^2(m_0^2 + m_1^2 - m_2^2)\log(\beta_2) + \frac{\kappa^3}{6m_0^2} + 2\kappa m_1^2\Big)\\
I_{11}^{00} &= \frac{1}{4m_0^2}\Big((2m_2^2(m_1^2 + m_2^2 - m_0^2)\log(\beta_0) + \frac{\kappa^3}{6m_1^2} + 2\kappa m_2^2\Big)\\
I_{11}^{20} &= -\frac{1}{4m_0^2}\Big((m_0^4\log(\beta_2) + m_2^4\log(\beta_0) + \frac{\kappa^3}{6m_1^2} + \frac{\kappa}{4}(3m_0^2 + 3m_2^2 - m_1^2)\Big)\\
I_{11}^{22} &= \frac{1}{4m_0^2}\Big((2m_0^2(m_0^2 + m_1^2 - m_2^2)\log(\beta_2) + \frac{\kappa^3}{6m_1^2} + 2\kappa m_0^2\Big)\\
I_{22}^{00} &= \frac{1}{4m_0^2}\Big((2m_1^2(m_2^2 + m_1^2 - m_0^2)\log(\beta_0) + \frac{\kappa^3}{6m_2^2} + 2\kappa m_1^2\Big)\\
I_{22}^{10} &= -\frac{1}{4m_0^2}\Big((m_0^4\log(\beta_1) + m_1^4\log(\beta_0) + \frac{\kappa^3}{6m_2^2} + \frac{\kappa}{4}(3m_0^2 + 3m_1^2 - m_2^2)\Big)\\
I_{22}^{11} &= \frac{1}{4m_0^2}\Big((2m_0^2(m_0^2 + m_2^2 - m_1^2)\log(\beta_1) + \frac{\kappa^3}{6m_2^2} + 2\kappa m_0^2\Big)
\end{aligned}$$

E.3 Scalar→ Fermion1 + Fermion2 (SFF)

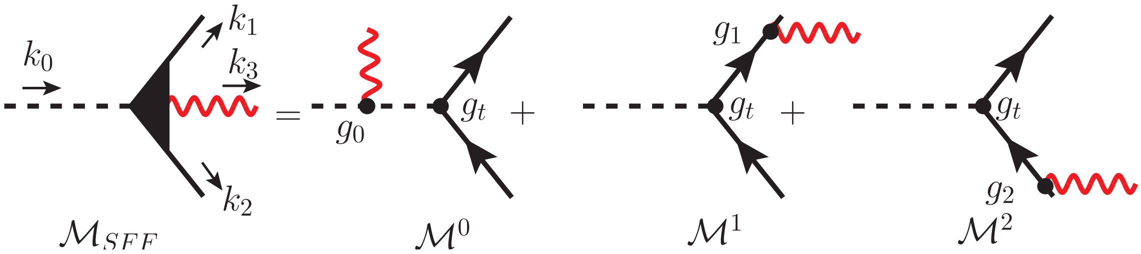


Figure E.1: Hard Bremsstrahlung process with one scalar and two fermions

The generic matrix elements are given by

$$\mathcal{M}_0 = \bar{u}(k_1)\, i(g_t^L P_L + g_t^R P_R) \frac{i}{(k_0 - k_3)^2 - m_0^2}\, i g_0\, (2k_0 - k_3)_\mu\, \varepsilon^{*\mu}(k_3, \lambda)\, v(k_2)\ , \tag{E.8}$$

$$\mathcal{M}_1 = \bar{u}(k_1)\, i g_1 \gamma_\mu \frac{i(\not{k}_3 + \not{k}_1 + m_1)}{(k_1 + k_3)^2 - m_1^2}\, i(g_t^L P_L + g_t^R P_R)\, \varepsilon^{*\mu}(k_3, \lambda)\, v(k_2)\ , \tag{E.9}$$

$$\mathcal{M}_2 = \bar{u}(k_1)\, i(g_t^L P_L + g_t^R P_R) \frac{i(-\not{k}_2 - \not{k}_3 + m_2)}{(k_2 + k_3)^2 - m_2^2}\, i g_2 \gamma_\mu\, \varepsilon^{*\mu}(k_3, \lambda)\, v(k_2)\ . \tag{E.10}$$

The squared matrix element $|\mathcal{M}_{SFF}|^2 = |\mathcal{M}_0 + \mathcal{M}_1 + \mathcal{M}_2|^2$ reads as

$$
\begin{aligned}
|\mathcal{M}_{SFF}|^2 &= 2g_1^2 g_t^L g_t^{L*} I_1^2 + 2g_1^2 g_t^R g_t^{R*} I_1^2 + 2g_2^2 g_t^L g_t^{L*} I_2^1 + 2g_2^2 g_t^R g_t^{R*} I_2^1 - 2g_0^2 g_t^L g_t^{L*} Ii \qquad \text{(E.11)} \\
&+ 2g_0g_1 g_t^L g_t^{L*} Ii + 2g_0g_2 g_t^L g_t^{L*} Ii - 4g_1g_2 g_t^L g_t^{L*} Ii - 2g_0^2 g_t^R g_t^{R*} Ii + 2g_0g_1 g_t^R g_t^{R*} Ii \\
&- 4g_1g_2 g_t^R g_t^{R*} Ii - 6g_0^2 g_t^L g_t^{L*} I_0 m_0^2 + 2g_0g_1 g_t^L g_t^{L*} I_0 m_0^2 + 2g_0g_2 g_t^L g_t^{L*} I_0 m_0^2 - 6g_0^2 g_t^R g_t^{R*} I_0 m_0^2 \\
&+ 2g_0g_1 g_t^R g_t^{R*} I_0 m_0^2 + 2g_0g_2 g_t^R g_t^{R*} I_0 m_0^2 - 4g_0g_1 g_t^L g_t^{L*} I_1 m_0^2 + 4g_1g_2 g_t^L g_t^{L*} I_1 m_0^2 \\
&+ 4g_1g_2 g_t^R g_t^{R*} I_1 m_0^2 - 4g_0g_2 g_t^L g_t^{L*} I_2 m_0^2 + 4g_1g_2 g_t^L g_t^{L*} I_2 m_0^2 - 4g_0g_2 g_t^R g_t^{R*} I_2 m_0^2 \\
&- 4g_0^2 g_t^L g_t^{L*} I_{00} m_0^4 - 4g_0^2 g_t^R g_t^{R*} I_{00} m_0^4 - 4g_0g_1 g_t^L g_t^{L*} I_{10} m_0^4 - 4g_0g_1 g_t^R g_t^{R*} I_{10} m_0^4 \\
&- 4g_0g_2 g_t^L g_t^{L*} I_{20} m_0^4 - 4g_0g_2 g_t^R g_t^{R*} I_{20} m_0^4 - 4g_1g_2 g_t^L g_t^{L*} I_{21} m_0^4 - 4g_1g_2 g_t^R g_t^{R*} I_{21} m_0^4 \\
&+ 2g_0^2 g_t^L g_t^{L*} I_0 m_1^2 + 2g_0g_1 g_t^L g_t^{L*} I_0 m_1^2 + 2g_0g_2 g_t^L g_t^{L*} I_0 m_1^2 + 2g_0^2 g_t^R g_t^{R*} I_0 m_1^2 \\
&+ 2g_0g_2 g_t^R g_t^{R*} I_0 m_1^2 + 4g_1^2 g_t^L g_t^{L*} I_1 m_1^2 + 4g_1^2 g_t^R g_t^{R*} I_1 m_1^2 + 4g_0g_2 g_t^L g_t^{L*} I_2 m_1^2 \\
&+ 4g_0g_2 g_t^R g_t^{R*} I_2 m_1^2 - 4g_1g_2 g_t^R g_t^{R*} I_2 m_1^2 + 4g_0^2 g_t^L g_t^{L*} I_{00} m_0^2 m_1^2 + 4g_0^2 g_t^R g_t^{R*} I_{00} m_0^2 m_1^2 \\
&- 4g_1^2 g_t^L g_t^{L*} I_{11} m_0^2 m_1^2 - 4g_1^2 g_t^R g_t^{R*} I_{11} m_0^2 m_1^2 + 8g_0g_2 g_t^L g_t^{L*} I_{20} m_0^2 m_1^2 + 8g_0g_2 g_t^R g_t^{R*} I_{20} m_0^2 m_1^2 \\
&+ 8g_1g_2 g_t^L g_t^{L*} I_{21} m_0^2 m_1^2 + 8g_1g_2 g_t^R g_t^{R*} I_{21} m_0^2 m_1^2 + 4g_0g_1 g_t^L g_t^{L*} I_{10} m_1^4 + 4g_0g_1 g_t^R g_t^{R*} I_{10} m_1^4 \\
&+ 4g_1^2 g_t^L g_t^{L*} I_{11} m_1^4 + 4g_1^2 g_t^R g_t^{R*} I_{11} m_1^4 - 4g_0g_2 g_t^L g_t^{L*} I_{20} m_1^4 - 4g_0g_2 g_t^R g_t^{R*} I_{20} m_1^4 \\
&- 4g_1g_2 g_t^L g_t^{L*} I_{21} m_1^4 - 4g_1g_2 g_t^R g_t^{R*} I_{21} m_1^4 + 4g_0^2 g_t^{L*} g_t^R I_0 m_1 m_2 + 4g_0g_1 g_t^{L*} g_t^R I_0 m_1 m_2 \\
&+ 4g_0g_2 g_t^{L*} g_t^R I_0 m_1 m_2 + 4g_0^2 g_t^L g_t^{R*} I_0 m_1 m_2 + 4g_0g_1 g_t^L g_t^{R*} I_0 m_1 m_2 + 4g_0g_2 g_t^L g_t^{R*} I_0 m_1 m_2 \\
&+ 4g_0g_1 g_t^{L*} g_t^R I_1 m_1 m_2 + 4g_1^2 g_t^{L*} g_t^R I_1 m_1 m_2 - 4g_1g_2 g_t^{L*} g_t^R I_1 m_1 m_2 + 4g_0g_1 g_t^L g_t^{R*} I_1 m_1 m_2 \\
&+ 4g_1^2 g_t^L g_t^{R*} I_1 m_1 m_2 - 4g_1g_2 g_t^L g_t^{R*} I_1 m_1 m_2 + 4g_0g_2 g_t^{L*} g_t^R I_2 m_1 m_2 - 4g_1g_2 g_t^{L*} g_t^R I_2 m_1 m_2 \\
&+ 4g_2^2 g_t^{L*} g_t^R I_2 m_1 m_2 + 4g_0g_2 g_t^L g_t^{R*} I_2 m_1 m_2 - 4g_1g_2 g_t^L g_t^{R*} I_2 m_1 m_2 + 4g_2^2 g_t^L g_t^{R*} I_2 m_1 m_2 \\
&+ 8g_0^2 g_t^{L*} g_t^R I_{00} m_0^2 m_1 m_2 + 8g_0^2 g_t^L g_t^{R*} I_{00} m_0^2 m_1 m_2 + 8g_0g_1 g_t^{L*} g_t^R I_{10} m_0^2 m_1 m_2 \\
&+ 8g_0g_2 g_t^{L*} g_t^R I_{20} m_0^2 m_1 m_2 + 8g_0g_2 g_t^L g_t^{R*} I_{20} m_0^2 m_1 m_2 + 8g_1g_2 g_t^{L*} g_t^R I_{21} m_0^2 m_1 m_2 \\
&+ 8g_0g_1 g_t^{L*} g_t^R I_{10} m_1^3 m_2 + 8g_0g_1 g_t^L g_t^{R*} I_{10} m_1^3 m_2 + 8g_1^2 g_t^{L*} g_t^R I_{11} m_1^3 m_2 \\
&- 8g_0g_2 g_t^{L*} g_t^R I_{20} m_1^3 m_2 - 8g_0g_2 g_t^L g_t^{R*} I_{20} m_1^3 m_2 - 8g_1g_2 g_t^{L*} g_t^R I_{21} m_1^3 m_2 \\
&+ 8g_0g_1 g_t^{L*} g_t^R I_{10} m_1 m_2^3 - 8g_0g_1 g_t^L g_t^{R*} I_{10} m_1 m_2^3 + 8g_0g_2 g_t^{L*} g_t^R I_{20} m_1 m_2^3 \\
&- 8g_1g_2 g_t^{L*} g_t^R I_{21} m_1 m_2^3 - 8g_1g_2 g_t^L g_t^{R*} I_{21} m_1 m_2^3 + 8g_2^2 g_t^{L*} g_t^R I_{22} m_1 m_2^3 + 8g_2^2 g_t^L g_t^{R*} I_{22} m_1 m_2^3 \\
&+ 2g_0^2 g_t^L g_t^{L*} I_0 m_2^2 + 2g_0g_1 g_t^L g_t^{L*} I_0 m_2^2 + 2g_0g_2 g_t^L g_t^{L*} I_0 m_2^2 + 2g_0^2 g_t^R g_t^{R*} I_0 m_2^2 \\
&+ 2g_0g_2 g_t^R g_t^{R*} I_0 m_2^2 + 4g_0g_1 g_t^L g_t^{L*} I_1 m_2^2 - 4g_1g_2 g_t^L g_t^{L*} I_1 m_2^2 + 4g_0g_1 g_t^R g_t^{R*} I_1 m_2^2 \\
&+ 4g_2^2 g_t^L g_t^{L*} I_2 m_2^2 + 4g_2^2 g_t^R g_t^{R*} I_2 m_2^2 + 4g_0^2 g_t^L g_t^{L*} I_{00} m_0^2 m_2^2 + 4g_0^2 g_t^R g_t^{R*} I_{00} m_0^2 m_2^2 \\
&+ 8g_0g_1 g_t^L g_t^{L*} I_{10} m_0^2 m_2^2 + 8g_0g_1 g_t^R g_t^{R*} I_{10} m_0^2 m_2^2 + 8g_1g_2 g_t^L g_t^{L*} I_{21} m_0^2 m_2^2 \\
&- 4g_2^2 g_t^L g_t^{L*} I_{22} m_0^2 m_2^2 - 4g_2^2 g_t^R g_t^{R*} I_{22} m_0^2 m_2^2 + 4g_1^2 g_t^L g_t^{L*} I_{11} m_1^2 m_2^2 + 4g_1^2 g_t^R g_t^{R*} I_{11} m_1^2 m_2^2 \\
&- 8g_1g_2 g_t^L g_t^{L*} I_{21} m_1^2 m_2^2 - 8g_1g_2 g_t^R g_t^{R*} I_{21} m_1^2 m_2^2 + 4g_2^2 g_t^L g_t^{L*} I_{22} m_1^2 m_2^2 + 4g_2^2 g_t^R g_t^{R*} I_{22} m_1^2 m_2^2 \\
&- 4g_0g_1 g_t^L g_t^{L*} I_{10} m_2^4 - 4g_0g_1 g_t^R g_t^{R*} I_{10} m_2^4 + 4g_0g_2 g_t^L g_t^{L*} I_{20} m_2^4 + 4g_0g_2 g_t^R g_t^{R*} I_{20} m_2^4 \\
&- 4g_1g_2 g_t^L g_t^{L*} I_{21} m_2^4 - 4g_1g_2 g_t^R g_t^{R*} I_{21} m_2^4 + 4g_2^2 g_t^L g_t^{L*} I_{22} m_2^4 + 4g_2^2 g_t^R g_t^{R*} I_{22} m_2^4 \\
&+ 8g_0g_1 g_t^L g_t^{R*} I_{10} m_0^2 m_1 m_2 + 8g_1g_2 g_t^L g_t^{R*} I_{21} m_0^2 m_1 m_2 + 2g_0g_2 g_t^R g_t^{R*} Ii - 4g_0g_1 g_t^R g_t^{R*} I_1 m_0^2 \\
&+ 4g_1g_2 g_t^R g_t^{R*} I_2 m_0^2 + 8g_1^2 g_t^L g_t^{R*} I_{11} m_1^3 m_2 - 8g_1g_2 g_t^L g_t^{R*} I_{21} m_1^3 m_2 + 8g_0g_2 g_t^L g_t^{R*} I_{20} m_1 m_2^3 \\
&+ 2g_0g_1 g_t^R g_t^{R*} I_0 m_2^2 - 4g_1g_2 g_t^R g_t^{R*} I_1 m_2^2 + 8g_1g_2 g_t^R g_t^{R*} I_{21} m_0^2 m_2^2 + 2g_0g_1 g_t^R g_t^{R*} I_0 m_1^2 \\
&- 4g_1g_2 g_t^L g_t^{L*} I_2 m_1^2 \,.
\end{aligned}
$$

E.3.1 One massless external fermion

If one external fermion has zero mass, one has to take the limit $m_2 \to 0$ and reexpress the phase–space integrals.

$$\begin{aligned}
\beta_2 &= \frac{m_0^2 + m_1^2 - m_2^2 - \kappa}{2m_0 m_1} \\
I_0 &= \frac{1}{4m_0^2}\big(- 2m_1^2 \log(\beta_2) - \kappa\big) \\
I_1 &= \frac{1}{4m_0^2}\big(- 2m_0^2 \log(\beta_2) - \kappa\big) \\
I_{00} &= -\frac{1}{4m_0^4}\big(m_0^2 \log(\frac{m_0}{m_1} - \frac{m_1}{m_0}) - (m_0 - m_1)(m_0 + m_1)(-1 + \log(\frac{(m_0^2 - m_1^2)^2}{\lambda m_0^2 m_1})) \\
&+ m_1^2 \log(\frac{m_0^3}{m_0^2 m_1 - m_1^3})\big) \\
I_{11} &= -\frac{1}{4m_0^2 m_1^2}\big(m_0^2 \log(-1 + \frac{m_0^2}{m_1^2}) + m_1^2 \log(\frac{m_0^2}{m_0^2 - m_1^2}) \\
&- (m_0 - m_1)(m_0 + m_1)(-1 + \log(\frac{(m_0^2 - m_1^2)^2}{\lambda m_0^2 m_1}))\big) \\
I_{10} &= \frac{1}{4m_0^2}\big(6 \log(\frac{m_1}{m_0})^2 + \log(\frac{m_0}{m_1} - \frac{m_1}{m_0})^2 - 2 \log(\frac{m_1}{m_0}) \log(\frac{\lambda m_0^2 m_1}{(m_0^2 - m_1^2)^2}) \\
&- \log(\frac{m_0^2}{m_0^2 - m_1^2})^2 + 2\mathrm{Li}_2(1 - \frac{m_0^2}{m_1^2})\big) \\
I_0^2 &= \frac{1}{4m_0^2}\big(- m_1^2(2m_0^2 - 2m_2^2 + m_1^2) \log(\beta_2) + \frac{\kappa}{4}(m_0^2 - 3m_2^2 + 5m_1^2)\big) \\
Ii &= \frac{1}{4m_0^2}\big(\frac{\kappa}{2}(m_0^2 + m_1^2 + m_2^2) + 2m_0^2 m_1^2 \log(\beta_2)\big) \\
I_1^2 &= \frac{1}{4m_0^2}\big(- m_0^2(2m_1^2 - 2m_2^2 + m_0^2) \log(\beta_2) - \frac{\kappa}{4}(m_1^2 + 5m_0^2)\big)
\end{aligned}$$

The squared matrix element $|\mathcal{M}_{SFF}|^2$ then takes a much simpler form compared to $m_2 \neq 0$, given by

$$\begin{aligned}
|\mathcal{M}_{SFF}|^2 &= 2g_1^2 g_t^R g_t^{R*} I_1^2 - 2g_0^2 g_t^R g_t^{R*} Ii + 2g_0 g_1 g_t^R g_t^{R*} Ii - 6g_0^2 g_t^R g_t^{R*} I_0 m_0^2 \\
&- 2g_0 g_1 g_t^R g_t^{R*} I_0 m_0^2 - 4g_0 g_1 g_t^R g_t^{R*} I_1 m_0^2 - 4g_0^2 g_t^R g_t^{R*} I_{00} m_0^4 \\
&- 4g_0 g_1 g_t^R g_t^{R*} I_{10} m_0^4 + 2g_0^2 g_t^R g_t^{R*} I_0 m_1^2 + 2g_0 g_1 g_t^R g_t^{R*} I_0 m_1^2 \\
&+ 4g_1^2 g_t^R g_t^{R*} I_1 m_1^2 + 4g_0^2 g_t^R g_t^{R*} I_{00} m_0^2 m_1^2 - 4g_1^2 g_t^R g_t^{R*} I_{11} m_0^2 m_1^2 \\
&+ 4g_0 g_1 g_t^R g_t^{R*} I_{10} m_1^4 + 4g_1^2 g_t^R g_t^{R*} I_{11} m_1^4 \,.
\end{aligned} \tag{E.12}$$

E.4 Scalar → Scalar1 + Scalar2 (SSS)

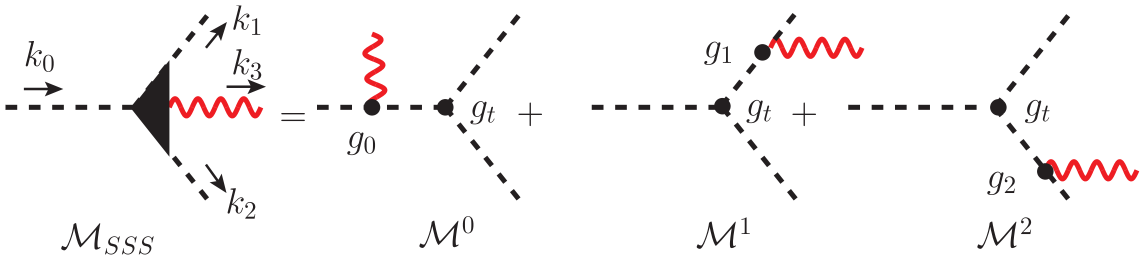


Figure E.2: Hard Bremsstrahlung process with 3 scalars

The generic matrix elements are given by

$$\mathcal{M}_0 = \frac{i g_t g_0 (2k_0 - k_3)_\mu \epsilon^\mu(k_3,\lambda)}{(k_0-k_3)^2 - m_0^2}, \tag{E.13}$$

$$\mathcal{M}_1 = \frac{i g_t g_1 (2k_1 + k_3)_\mu \epsilon^\mu(k_3,\lambda)}{(k_1+k_3)^2 - m_1^2}, \tag{E.14}$$

$$\mathcal{M}_2 = \frac{i g_t g_2 (2k_2 + k_3)_\mu \epsilon^\mu(k_3,\lambda)}{(k_2+k_3)^2 - m_2^2}. \tag{E.15}$$

$$\tag{E.16}$$

The squared matrix element $|\mathcal{M}_{SSS}|^2 = |\mathcal{M}_0 + \mathcal{M}_1 + \mathcal{M}_2|^2$ then reads as

$$\begin{aligned} |\mathcal{M}_{SSS}|^2 &= -4g_0g_1g_tg_t^*I_0 - 4g_0g_2g_tg_t^*I_0 - 4g_0g_1g_tg_t^*I_1 + 4g_1g_2g_tg_t^*I_1 \\ &+ 4g_1g_2g_tg_t^*I_2 - 4g_0^2g_tg_t^*I_{00}m_0^2 - 4g_0g_1g_tg_t^*I_{10}m_0^2 - 4g_0g_2g_tg_t^*I_{20}m_0^2 \\ &- 4g_1g_2g_tg_t^*I_{21}m_0^2 - 4g_0g_1g_tg_t^*I_{10}m_1^2 - 4g_1^2g_tg_t^*I_{11}m_1^2 + 4g_0g_2g_tg_t^*I_{20}m_1^2 \\ &+ 4g_1g_2g_tg_t^*I_{21}m_1^2 + 4g_0g_1g_tg_t^*I_{10}m_2^2 - 4g_0g_2g_tg_t^*I_{20}m_2^2 + 4g_1g_2g_tg_t^*I_{21}m_2^2 \\ &- 4g_2^2g_tg_t^*I_{22}m_2^2 - 4g_0g_2g_tg_t^*I_2 . \end{aligned} \tag{E.17}$$

E.5 Scalar → Vector1 + Vector2 (SVV)

Since there is only one Higgs decay into two charged on–shell vector bosons allowed in the MSSM, there is no necessity to calculate the full generic formula. Therefore we just give the result for $H^+ \to W^+W^-$.

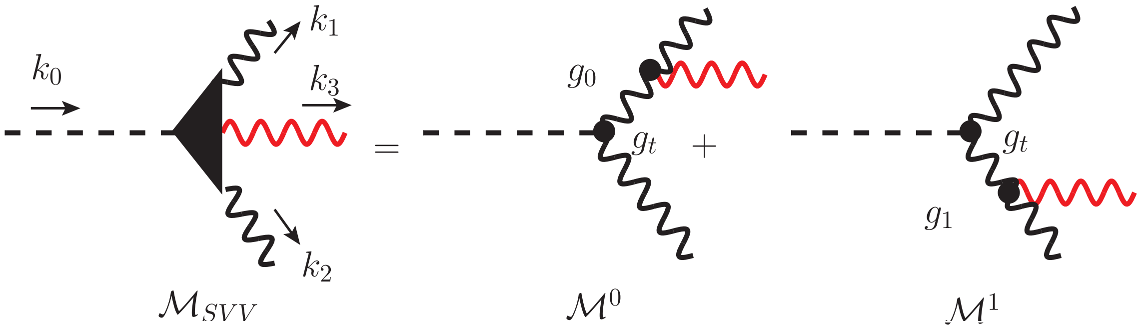


Figure E.3: Hard Bremsstrahlung process with 2 vector bosons in the final state

The squared matrix element $|\mathcal{M}_{SVV}|^2 = |\mathcal{M}_0 + \mathcal{M}_1|^2$ then reads as

$$
\begin{aligned}
|\mathcal{M}_{SVV}|^2 \; = \; & \frac{\alpha^2 c_{\beta\alpha}^2 \pi^2}{s_w^2}\big(- I_{11}^{00} - 46 I_{11}^{20} - 13 I_{11}^{22} - 100 I_1^0 - 80 I_1^2 - I_{22}^{00} - 46 I_{22}^{10} \\
- \; & 100 I_2^0 - 80 I_2^1 - 134 Ii + 64 I_1 m_0^2 + 64 I_2 m_0^2 - 16 I_{11} m_0^4 - 96 I_{21} m_0^4 \\
- \; & 16 \frac{I_2 m_0^4}{m_1^2} + 16 \frac{I_{21} m_0^6}{m_1^2} - 192 I_1 m_1^2 - 192 I_2 m_1^2 - 13 I_{22}^{11} - 16 I_{22} m_0^4 \\
+ \; & 64 I_{11} m_0^2 m_1^2 + 320 I_{21} m_0^2 m_1^2 + 64 I_{22} m_0^2 m_1^2 - 192 I_{11} m_1^4 - 384 I_{21} m_1^4 \\
- \; & 192 I_{22} m_1^4 + 42 \frac{I_1^0 m_0^2}{m_1^2} + 42 \frac{I_1^2 m_0^2}{m_1^2} + 42 \frac{I_2^0 m_0^2}{m_1^2} + 42 \frac{I_2^1 m_0^2}{m_1^2} + 84 \frac{Ii m_0^2}{m_1^2} - 16 \frac{I_1 m_0^4}{m_1^2}\big)
\end{aligned}
\tag{E.18}
$$

E.6 Scalar → Scalar1 + Vector2 (SSV)

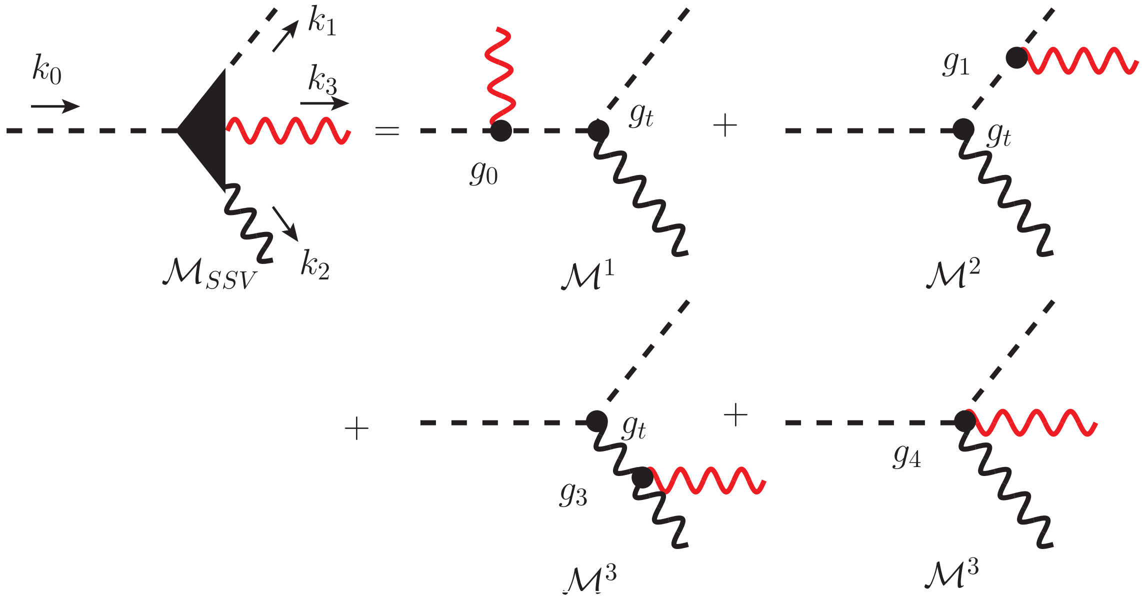


Figure E.4: Hard Bremsstrahlung process with one scalar and a vector boson in the final state

The generic matrix elements are given by

$$\mathcal{M}_0 = \frac{i}{(k_0-k_3)^2-m_0^2} g_t g_0 (k_0+k_1-k_3)_\nu (2k_0-k_3)_\mu \epsilon_\nu^{W*}(k_2,\lambda_2)\epsilon_\mu^*(k_3,\lambda) \ , \tag{E.19}$$

$$\mathcal{M}_1 = \frac{i}{(k_1+k_3)^2-m_1^2} g_t g_1 (k_0+k_1+k_3)_\nu (2k_1+k_3)_\mu \epsilon_\nu^{W*}(k_2,\lambda_2)\epsilon_\mu^*(k_3,\lambda) \ , \tag{E.20}$$

$$\mathcal{M}_2 = \frac{i\left(g^{\rho\sigma}-\frac{(k_2+k_3)_\rho(k_2+k_3)_\sigma}{m_2^2}\right)}{(k_2+k_3)^2-m_2^2}(k_0+k_1)_\sigma g_t g_2 \ , \tag{E.21}$$

$$\left((-2k_2-k_3)_\mu g^{\rho\nu}+(k_2-k_3)_\rho g^{\mu\nu}+(k_2+2k_3)_\nu g^{\mu\rho}\right)\epsilon_\nu^{W*}(k_2,\lambda_2)\epsilon_\mu^*(k_3,\lambda)$$

$$\mathcal{M}_3 = i g_t (g_0+g_1) g^{\mu\nu}\epsilon_\nu^{W*}(k_2,\lambda_2)\epsilon_\mu(k_3,\lambda) \ . \tag{E.22}$$

The squared matrix element $|\mathcal{M}_{SVV}|^2 = |\mathcal{M}_0 + \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3|^2$ then reads as

$$\begin{aligned}
|\mathcal{M}_{SSV}|^2 &= g_0^2 g_t g_t^* I_0^2 - g_0 g_1 g_t g_t^* I_0^2 - g_0 g_2 g_t g_t^* I_0^2 - g_0 g_1 g_t g_t^* I_1^2 + g_1^2 g_t g_t^* I_1^2 + g_1 g_2 g_t g_t^* I_1^2 \\
&+ 2g_2^2 g_t g_t^* I_{22}^{00} - 4g_2^2 g_t g_t^* I_{22}^{10} + 2g_2^2 g_t g_t^* I_{22}^{11} - 8g_0 g_2 g_t g_t^* I_2^1 - 8g_1 g_2 g_t g_t^* I_2^1 \\
&+ 5g_0^2 g_t g_t^* Ii - 2g_0 g_1 g_t g_t^* Ii + 5g_1^2 g_t g_t^* Ii - 6g_0 g_2 g_t g_t^* Ii - 2g_1 g_2 g_t g_t^* Ii - g_2^2 g_t g_t^* Ii \\
&+ 8g_0^2 g_t g_t^* I_0 m_0^2 + 8g_0 g_1 g_t g_t^* I_1 m_0^2 - 8g_1 g_2 g_t g_t^* I_1 m_0^2 + 4g_0 g_2 g_t g_t^* I_2 m_0^2 - 4g_1 g_2 g_t g_t^* I_2 m_0^2 \\
&+ 4g_2^2 g_t g_t^* I_2 m_0^2 + 8g_0^2 g_t g_t^* I_{00} m_0^4 + 12g_0 g_1 g_t g_t^* I_{10} m_0^4 + 4g_0 g_2 g_t g_t^* I_{20} m_0^4 \\
&+ 12g_1 g_2 g_t g_t^* I_{21} m_0^4 - 4g_2^2 g_t g_t^* I_{22} m_0^4 + 8g_0 g_1 g_t g_t^* I_0 m_1^2 + 8g_0 g_2 g_t g_t^* I_0 m_1^2 + 8g_1^2 g_t g_t^* I_1 m_1^2 \\
&+ 4g_0 g_2 g_t g_t^* I_2 m_1^2 - 4g_1 g_2 g_t g_t^* I_2 m_1^2 + 4g_2^2 g_t g_t^* I_2 m_1^2 + 8g_0^2 g_t g_t^* I_{00} m_0^2 m_1^2 + 8g_0 g_1 g_t g_t^* I_{10} m_0^2 m_1^2 \\
&+ 8g_1^2 g_t g_t^* I_{11} m_0^2 m_1^2 + 8g_0 g_2 g_t g_t^* I_{20} m_0^2 m_1^2 - 8g_1 g_2 g_t g_t^* I_{21} m_0^2 m_1^2 + 8g_2^2 g_t g_t^* I_{22} m_0^2 m_1^2 \\
&+ 12g_0 g_1 g_t g_t^* I_{10} m_1^4 + 8g_1^2 g_t g_t^* I_{11} m_1^4 - 12g_0 g_2 g_t g_t^* I_{20} m_1^4 - 4g_1 g_2 g_t g_t^* I_{21} m_1^4 \\
&- 4g_2^2 g_t g_t^* I_{22} m_1^4 - (g_0 g_2 g_t g_t^* I^1 m_0^2)/m_2^4 + (g_1 g_2 g_t g_t^* I^1 m_0^2)/m_2^4 + (g_2^2 g_t g_t^* I^1 m_0^2)/m_2^4 \\
&- (g_0 g_2 g_t g_t^* I^2 m_0^2)/(2m_2^4) + (g_1 g_2 g_t g_t^* I^2 m_0^2)/(2m_2^4) + (g_2^2 g_t g_t^* I^2 m_0^2)/(2m_2^4) \\
&+ (g_0 g_2 g_t g_t^* Iim_0^4)/m_2^4 - (g_1 g_2 g_t g_t^* Iim_0^4)/m_2^4 - (g_2^2 g_t g_t^* Iim_0^4)/m_2^4 + (g_0 g_2 g_t g_t^* I^1 m_1^2)/m_2^4 \\
&- (g_1 g_2 g_t g_t^* I^1 m_1^2)/m_2^4 - (g_2^2 g_t g_t^* I^1 m_1^2)/m_2^4 + (g_0 g_2 g_t g_t^* I^2 m_1^2)/(2m_2^4) \\
&- (g_1 g_2 g_t g_t^* I^2 m_1^2)/(2m_2^4) - (g_2^2 g_t g_t^* I^2 m_1^2)/(2m_2^4) - (2g_0 g_2 g_t g_t^* Iim_0^2 m_1^2)/m_2^4 \\
&+ (2g_1 g_2 g_t g_t^* Iim_0^2 m_1^2)/m_2^4 + (2g_2^2 g_t g_t^* Iim_0^2 m_1^2)/m_2^4 + (g_0 g_2 g_t g_t^* Iim_1^4)/m_2^4 \\
&- (g_1 g_2 g_t g_t^* Iim_1^4)/m_2^4 - (g_2^2 g_t g_t^* Iim_1^4)/m_2^4 + (2g_0 g_2 g_t g_t^* I_2^1 1)/m_2^2 - (2g_1 g_2 g_t g_t^* I_2^1 1)/m_2^2 \\
&- (2g_2^2 g_t g_t^* I_2^1 1)/m_2^2 + (2g_0 g_2 g_t g_t^* I^1)/m_2^2 - (2g_1 g_2 g_t g_t^* I^1)/m_2^2 - (2g_2^2 g_t g_t^* I^1)/m_2^2 \\
&+ (g_0^2 g_t g_t^* I^2)/m_2^2 - (2g_0 g_1 g_t g_t^* I^2)/m_2^2 + (g_1^2 g_t g_t^* I^2)/m_2^2 - (3g_0 g_2 g_t g_t^* I^2)/(2m_2^2) \\
&+ (3g_1 g_2 g_t g_t^* I^2)/(2m_2^2) + (g_2^2 g_t g_t^* I^2)/(2m_2^2) - (4g_0^2 g_t g_t^* I_{00} m_0^6)/m_2^2 \\
&- (4g_0 g_1 g_t g_t^* I_{10} m_0^6)/m_2^2 - (4g_0 g_2 g_t g_t^* I_{20} m_0^6)/m_2^2 - (4g_1 g_2 g_t g_t^* I_{21} m_0^6)/m_2^2 \\
&- (g_0^2 g_t g_t^* I_{00}^{11} m_0^2)/m_2^2 - (2g_0^2 g_t g_t^* I_{00}^{21} m_0^2)/m_2^2 - (g_0^2 g_t g_t^* I_{00}^{22} m_0^2)/m_2^2 \\
&+ (g_0^2 g_t g_t^* I_0^2 m_0^2)/m_2^2 - (g_0 g_1 g_t g_t^* I_0^2 m_0^2)/m_2^2 - (g_0 g_2 g_t g_t^* I_0^2 m_0^2)/m_2^2 + (g_0 g_1 g_t g_t^* I_1^2 m_0^2)/m_2^2 \\
&- (g_1^2 g_t g_t^* I_1^2 m_0^2)/m_2^2 - (g_1 g_2 g_t g_t^* I_1^2 m_0^2)/m_2^2 - (2g_0 g_2 g_t g_t^* I_2^1 m_0^2)/m_2^2 \\
&+ (2g_1 g_2 g_t g_t^* I_2^1 m_0^2)/m_2^2 + (2g_2^2 g_t g_t^* I_2^1 m_0^2)/m_2^2 - (3g_0^2 g_t g_t^* Iim_0^2)/m_2^2 + (4g_0 g_1 g_t g_t^* Iim_0^2)/m_2^2 \\
&+ (5g_0 g_2 g_t g_t^* Iim_0^2)/m_2^2 - (g_1 g_2 g_t g_t^* Iim_0^2)/m_2^2 - (g_2^2 g_t g_t^* Iim_0^2)/m_2^2 - (8g_0^2 g_t g_t^* I_0 m_0^4)/m_2^2 \\
&+ (4g_0 g_1 g_t g_t^* I_0 m_0^4)/m_2^2 + (4g_0 g_2 g_t g_t^* I_0 m_0^4)/m_2^2 - (4g_0 g_1 g_t g_t^* I_1 m_0^4)/m_2^2 \\
&+ (4g_1 g_2 g_t g_t^* I_1 m_0^4)/m_2^2 - (6g_0 g_2 g_t g_t^* I_2 m_0^4)/m_2^2 + (6g_1 g_2 g_t g_t^* I_2 m_0^4)/m_2^2 \\
&+ (2g_2^2 g_t g_t^* I_2 m_0^4)/m_2^2 - (4g_0 g_1 g_t g_t^* I_{10} m_1^6)/m_2^2 - (4g_1^2 g_t g_t^* I_{11} m_1^6)/m_2^2 \\
&+ (4g_0 g_2 g_t g_t^* I_{20} m_1^6)/m_2^2 + (4g_1 g_2 g_t g_t^* I_{21} m_1^6)/m_2^2 - (g_0^2 g_t g_t^* I_0^2 m_1^2)/m_2^2 \\
&+ (g_0 g_1 g_t g_t^* I_0^2 m_1^2)/m_2^2 + (g_0 g_2 g_t g_t^* I_0^2 m_1^2)/m_2^2 - (g_1^2 g_t g_t^* I_{11}^{00} m_1^2)/m_2^2 \\
&- (2g_1^2 g_t g_t^* I_{11}^{20} m_1^2)/m_2^2 - (g_1^2 g_t g_t^* I_{11}^{22} m_1^2)/m_2^2 - (g_0 g_1 g_t g_t^* I_1^2 m_1^2)/m_2^2
\end{aligned} \tag{E.23}$$

$$
\begin{aligned}
&+ \quad (g_1^2 g_t g_t^* I_1^2 m_1^2)/m_2^2 + (g_1 g_2 g_t g_t^* I_1^2 m_1^2)/m_2^2 + (2g_0 g_2 g_t g_t^* I_2^1 m_1^2)/m_2^2 \\
&- \quad (2g_1 g_2 g_t g_t^* I_2^1 m_1^2)/m_2^2 - (2g_2^2 g_t g_t^* I_2^1 m_1^2)/m_2^2 + (4g_0 g_1 g_t g_t^* Iim_1^2)/m_2^2 - (3g_1^2 g_t g_t^* Iim_1^2)/m_2^2 \\
&+ \quad (3g_0 g_2 g_t g_t^* Iim_1^2)/m_2^2 - (7g_1 g_2 g_t g_t^* Iim_1^2)/m_2^2 - (3g_2^2 g_t g_t^* Iim_1^2)/m_2^2 + (8g_0^2 g_t g_t^* I_0 m_0^2 m_1^2)/m_2^2 \\
&+ \quad (8g_1^2 g_t g_t^* I_1 m_0^2 m_1^2)/m_2^2 + (12g_0 g_2 g_t g_t^* I_2 m_0^2 m_1^2)/m_2^2 - (12g_1 g_2 g_t g_t^* I_2 m_0^2 m_1^2)/m_2^2 \\
&- \quad (4g_2^2 g_t g_t^* I_2 m_0^2 m_1^2)/m_2^2 + (8g_0^2 g_t g_t^* I_{00} m_0^4 m_1^2)/m_2^2 + (4g_0 g_1 g_t g_t^* I_{10} m_0^4 m_1^2)/m_2^2 \\
&- \quad (4g_1^2 g_t g_t^* I_{11} m_0^4 m_1^2)/m_2^2 + (12g_0 g_2 g_t g_t^* I_{20} m_0^4 m_1^2)/m_2^2 + (12g_1 g_2 g_t g_t^* I_{21} m_0^4 m_1^2)/m_2^2 \\
&- \quad (4g_0 g_1 g_t g_t^* I_0 m_1^4)/m_2^2 - (4g_0 g_2 g_t g_t^* I_0 m_1^4)/m_2^2 + (4g_0 g_1 g_t g_t^* I_1 m_1^4)/m_2^2 \\
&- \quad (8g_1^2 g_t g_t^* I_1 m_1^4)/m_2^2 - (4g_1 g_2 g_t g_t^* I_1 m_1^4)/m_2^2 - (6g_0 g_2 g_t g_t^* I_2 m_1^4)/m_2^2 \\
&+ \quad (6g_1 g_2 g_t g_t^* I_2 m_1^4)/m_2^2 + (2g_2^2 g_t g_t^* I_2 m_1^4)/m_2^2 - (4g_0^2 g_t g_t^* I_{00} m_0^2 m_1^4)/m_2^2 \\
&+ \quad (4g_0 g_1 g_t g_t^* I_{10} m_0^2 m_1^4)/m_2^2 + (8g_1^2 g_t g_t^* I_{11} m_0^2 m_1^4)/m_2^2 - (12g_0 g_2 g_t g_t^* I_{20} m_0^2 m_1^4)/m_2^2 \\
&- \quad (12g_1 g_2 g_t g_t^* I_{21} m_0^2 m_1^4)/m_2^2 - 4g_0 g_1 g_t g_t^* I_0 m_2^2 - 4g_0 g_2 g_t g_t^* I_0 m_2^2 - 4g_0 g_1 g_t g_t^* I_1 m_2^2 \\
&+ \quad 4g_1 g_2 g_t g_t^* I_1 m_2^2 + 2g_0 g_2 g_t g_t^* I_2 m_2^2 - 2g_1 g_2 g_t g_t^* I_2 m_2^2 - 6g_2^2 g_t g_t^* I_2 m_2^2 - 4g_0^2 g_t g_t^* I_{00} m_0^2 m_2^2 \\
&- \quad 12g_0 g_1 g_t g_t^* I_{10} m_0^2 m_2^2 + 4g_0 g_2 g_t g_t^* I_{20} m_0^2 m_2^2 - 12g_1 g_2 g_t g_t^* I_{21} m_0^2 m_2^2 + 8g_2^2 g_t g_t^* I_{22} m_0^2 m_2^2 \\
&- \quad 12g_0 g_1 g_t g_t^* I_{10} m_1^2 m_2^2 - 4g_1^2 g_t g_t^* I_{11} m_1^2 m_2^2 + 12g_0 g_2 g_t g_t^* I_{20} m_1^2 m_2^2 - 4g_1 g_2 g_t g_t^* I_{21} m_1^2 m_2^2 \\
&+ \quad 8g_2^2 g_t g_t^* I_{22} m_1^2 m_2^2 + 4g_0 g_1 g_t g_t^* I_{10} m_2^4 - 4g_0 g_2 g_t g_t^* I_{20} m_2^4 + 4g_1 g_2 g_t g_t^* I_{21} m_2^4 \\
&- \quad 4g_2^2 g_t g_t^* I_{22} m_2^4 \; .
\end{aligned}
$$

Appendix F

Electroweak Interactions in the MSSM

This chapter has been obtained from [107]. In the following we give all couplings which are necessary for the calculation of electroweak corrections to Higgs decay processes. For the whole set of Feynman rules and a complete list of all terms of the MSSM Lagrangian we refer to [60].

F.1 Higgs–Sfermion–Sfermion couplings

For the neutral and charged Higgs fields we use the notation $H_k^0 = \{h^0, H^0, A^0, G^0\}$, $H_k^+ = \{H^+, G^+, H^-, G^-\}$ and $H_k^- \equiv (H_k^+)^* = \{H^-, G^-, H^+, G^+\}$. $t/\tilde{t}$ stands for an up–type (s)fermion and $b/\tilde{b}$ for a down–type one. Following [44, 45] the Higgs–Sfermion–Sfermion couplings for neutral Higgs bosons, $G_{ijk}^{\tilde{f}}$, can be written as

$$G_{ijk}^{\tilde{f}} \equiv G\big(H_k^0 \tilde{f}_i^* \tilde{f}_j\big) = \Big[R^{\tilde{f}} G_{LR,k}^{\tilde{f}} (R^{\tilde{f}})^T\Big]_{ij}. \tag{F.1}$$

The 3rd generation left–right couplings $G^{\tilde f}_{LR,k}$ for up– and down–type sfermions are given by

$$G^{\tilde t}_{LR,1} = \begin{pmatrix} -\sqrt{2}h_t m_t c_\alpha + g_Z m_Z (I^{3L}_t - e_t s^2_W) s_{\alpha+\beta} & -\frac{h_t}{\sqrt{2}}(A_t\, c_\alpha + \mu s_\alpha) \\ -\frac{h_t}{\sqrt{2}}(A_t\, c_\alpha + \mu s_\alpha) & -\sqrt{2}h_t m_t c_\alpha + g_Z m_Z e_t s^2_W s_{\alpha+\beta} \end{pmatrix}, \quad (F.2)$$

(F.3)

$$G^{\tilde b}_{LR,1} = \begin{pmatrix} \sqrt{2}h_b m_b s_\alpha + g_Z m_Z (I^{3L}_b - e_b s^2_W) s_{\alpha+\beta} & \frac{h_b}{\sqrt{2}}(A_b\, s_\alpha + \mu c_\alpha) \\ \frac{h_b}{\sqrt{2}}(A_b\, s_\alpha + \mu c_\alpha) & \sqrt{2}h_b m_b s_\alpha + g_Z m_Z e_b s^2_W s_{\alpha+\beta} \end{pmatrix}, \quad (F.4)$$

$$G^{\tilde f}_{LR,2} = G^{\tilde f}_{LR,1} \qquad \text{with } \alpha \to \alpha - \pi/2\,, \quad (F.5)$$

$$G^{\tilde t}_{LR,3} = -\sqrt{2}h_t \begin{pmatrix} 0 & -\frac{i}{2}\Big(A_t\, c_\beta + \mu\, s_\beta\Big) \\ \frac{i}{2}\Big(A_t\, c_\beta + \mu\, s_\beta\Big) & 0 \end{pmatrix}, \quad (F.6)$$

$$G^{\tilde b}_{LR,3} = -\sqrt{2}h_b \begin{pmatrix} 0 & -\frac{i}{2}\Big(A_b\, s_\beta + \mu\, c_\beta\Big) \\ \frac{i}{2}\Big(A_b\, s_\beta + \mu\, c_\beta\Big) & 0 \end{pmatrix}, \quad (F.7)$$

$$G^{\tilde f}_{LR,4} = G^{\tilde f}_{LR,3} \qquad \text{with } \beta \to \beta - \pi/2\,, \quad (F.8)$$

where we have used the abbreviations $s_x \equiv \sin x$, $c_x \equiv \cos x$ and $s_W \equiv \sin\theta_W$. α denotes the mixing angle of the $\{h^0, H^0\}$–system, and h_t and h_b are the Yukawa couplings

$$h_t = \frac{g\, m_t}{\sqrt{2}m_W \sin\beta}\,, \qquad h_b = \frac{g\, m_b}{\sqrt{2}m_W \cos\beta}\,. \quad (F.9)$$

The couplings of charged Higgs bosons to two sfermions are given by ($l = 1, 2$)

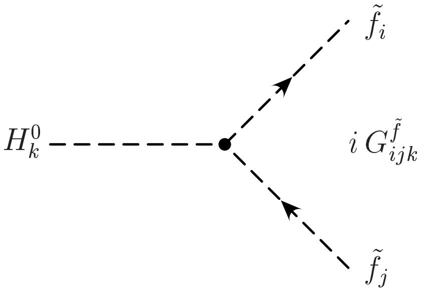
$$G^{\tilde f\tilde f'}_{ijl} \equiv G\Big(H^\pm_l \tilde f^*_i \tilde f'_j\Big) = G^{\tilde f'\tilde f}_{jil} = \left(R^{\tilde f}\, G^{\tilde f\tilde f'}_{LR,l} \left(R^{\tilde f'}\right)^T\right)_{ij}, \quad (F.10)$$

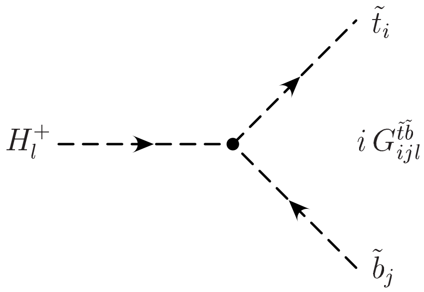
$$G^{\tilde t\tilde b}_{LR,1} = \begin{pmatrix} h_b m_b \sin\beta + h_t m_t \cos\beta - \frac{g m_W}{\sqrt{2}}\sin 2\beta & h_b(A_b \sin\beta + \mu\cos\beta) \\ h_t(A_t\cos\beta + \mu\sin\beta) & h_t m_b \cos\beta + h_b m_t \sin\beta \end{pmatrix}, \quad (F.11)$$

$$G^{\tilde b\tilde t}_{LR,1} = \begin{pmatrix} h_b m_b \sin\beta + h_t m_t \cos\beta - \frac{g m_W}{\sqrt{2}}\sin 2\beta & h_t(A_t \cos\beta + \mu\sin\beta) \\ h_b(A_b\sin\beta + \mu\cos\beta) & h_t m_b \cos\beta + h_b m_t \sin\beta \end{pmatrix}, \quad (F.12)$$

$$G^{\tilde f\tilde f'}_{LR,2} = G^{\tilde f\tilde f'}_{LR,1} \qquad \text{with} \quad \beta \;\to\; \beta - \frac{\pi}{2}\,. \quad (F.13)$$

f' denotes the isospin partner of the fermion f, i. e. $t' = b$, $\tilde b'_i = \tilde t_i$ etc. Note that only the angle β explicitly given in the matrices above has to be substituted; the dependence of β in the Yukawa couplings has to remain the same.
The Feynman rules for the Higgs–sfermion–sfermion couplings are (for $k = 1, \ldots, 4$; $l = 1, 2$)





F.2 Higgs–Fermion–Fermion couplings

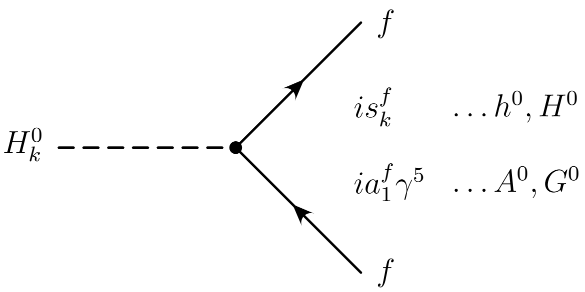
For the Higgs–fermion–fermion couplings the interaction Lagrangian reads

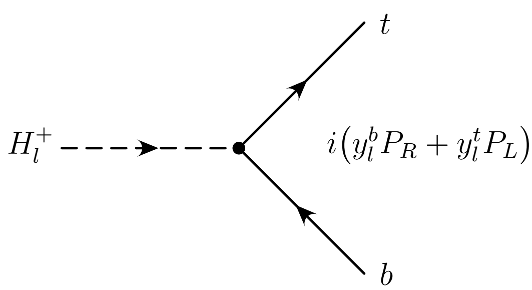
$$\mathcal{L} = \sum_{k=1}^{2} s_k^f\, H_k^0 \bar{f} f + \sum_{k=3}^{4} s_k^f\, H_k^0 \bar{f}\gamma^5 f + \sum_{l=1}^{2} \Big[H_l^+ \bar{t}\left(y_l^b P_R + y_l^t P_L\right) b + \text{h.c.}\Big] \tag{F.14}$$

with the couplings

$$\begin{aligned}
s_1^t &= -\,g\tfrac{m_t\cos\alpha}{2m_W\sin\beta} = -\,\tfrac{h_t}{\sqrt{2}}\cos\alpha, & s_1^b &= g\tfrac{m_b\sin\alpha}{2m_W\cos\beta} = \tfrac{h_b}{\sqrt{2}}\sin\alpha\,,\\
s_2^t &= -\,g\tfrac{m_t\sin\alpha}{2m_W\sin\beta} = -\,\tfrac{h_t}{\sqrt{2}}\sin\alpha, & s_2^b &= -\,g\tfrac{m_b\cos\alpha}{2m_W\cos\beta} = -\,\tfrac{h_b}{\sqrt{2}}\cos\alpha\,,\\
s_3^t &= ig\tfrac{m_t\cot\beta}{2m_W} = i\tfrac{h_t}{\sqrt{2}}\cos\beta, & s_3^b &= ig\tfrac{m_b\tan\beta}{2m_W} = i\tfrac{h_b}{\sqrt{2}}\sin\beta\,,\\
s_4^t &= ig\tfrac{m_t}{2m_W} = i\tfrac{h_t}{\sqrt{2}}\sin\beta, & s_4^b &= -\,ig\tfrac{m_b}{2m_W} = -\,i\tfrac{h_b}{\sqrt{2}}\cos\beta\,,\\
y_1^t &= g\tfrac{m_t\cot\beta}{\sqrt{2}m_W} = h_t\cos\beta, & y_1^b &= g\tfrac{m_b\tan\beta}{\sqrt{2}m_W} = h_b\sin\beta\,,\\
y_2^t &= g\tfrac{m_t}{\sqrt{2}m_W} = h_t\sin\beta, & y_2^b &= -\,g\tfrac{m_b}{\sqrt{2}m_W} = -\,h_b\cos\beta\,.
\end{aligned} \tag{F.15}$$

The Feynman rules for the couplings to the Higgs bosons are





F.3 Higgs–Gaugino–Gaugino couplings

The interaction Lagrangian for Higgs bosons and gauginos is given by

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{g}{2}\sum_{k=1}^{2} H_k^0\, \bar{\tilde{\chi}}_l^0 F_{lmk}^0\, \tilde{\chi}_m^0 - i\frac{g}{2}\sum_{k=3}^{4} H_k^0\, \bar{\tilde{\chi}}_l^0\, F_{lmk}^0 \gamma_5\, \tilde{\chi}_m^0 \\
& -g\sum_{k=1}^{2} H_k^0\, \bar{\tilde{\chi}}_i^+ \left(F_{ijk}^+ P_R + F_{jik}^+ P_L\right) \tilde{\chi}_j^+ + ig\sum_{k=3}^{4} H_k^0\, \bar{\tilde{\chi}}_i^+ \left(F_{ijk}^+ P_R + F_{jik}^+ P_L\right) \tilde{\chi}_j^+ \\
& -g\sum_{k=1}^{2} \Big[H_k^+\, \bar{\tilde{\chi}}_i^+ \left(F_{ilk}^R P_R + F_{ilk}^L P_L\right) \tilde{\chi}_l^0 + \text{h.c.}\Big]\,. \qquad \text{(F.16)}
\end{aligned}
$$

with

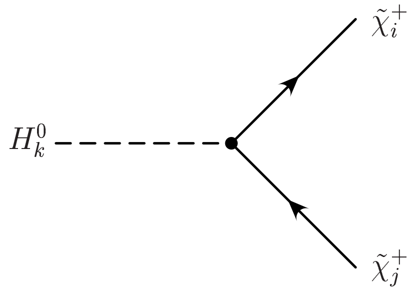
$$
\begin{aligned}
F_{lmk}^0 \;=\; & \frac{e_k}{2}\Big[Z_{l3}Z_{m2} + Z_{m3}Z_{l2} - \tan\theta_W \left(Z_{l3}Z_{m1} + Z_{m3}Z_{l1}\right)\Big] \\
& +\frac{d_k}{2}\Big[Z_{l4}Z_{m2} + Z_{m4}Z_{l2} - \tan\theta_W \left(Z_{l4}Z_{m1} + Z_{m4}Z_{l1}\right)\Big] \;=\; F_{mlk}^0\,, \qquad \text{(F.17)} \\
F_{ijk}^+ \;=\; & \frac{1}{\sqrt{2}}\left(e_k V_{i1} U_{j2} - d_k V_{i2} U_{j1}\right)\,, \qquad \text{(F.18)}
\end{aligned}
$$

and

$$
\begin{aligned}
F_{ilk}^R \;=\; & d_{k+2}\Big[V_{i1} Z_{l4} + \frac{1}{\sqrt{2}}(Z_{l2} + Z_{l1}\tan\theta_W) V_{i2}\Big]\,, \\
F_{ilk}^L \;=\; & -e_{k+2}\Big[U_{i1} Z_{l3} - \frac{1}{\sqrt{2}}(Z_{l2} + Z_{l1}\tan\theta_W) U_{i2}\Big]\,. \qquad \text{(F.19)}
\end{aligned}
$$

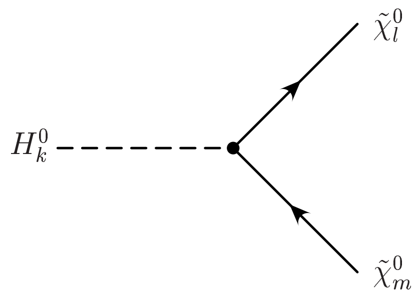
U, V and Z are rotation matrices which diagonalize the chargino and neutralino mass matrices (see chapter 2.3.4), and d_k and e_k take the values

$$
d_k = \{-\cos\alpha, -\sin\alpha, \cos\beta, \sin\beta\}\,, \qquad e_k = \{-\sin\alpha, \cos\alpha, -\sin\beta, \cos\beta\}\,.
$$



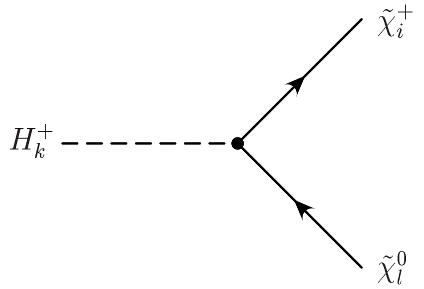
$-ig\left(F_{ijk}^+ P_R + F_{jik}^+ P_L\right) \qquad \ldots h^0, H^0$

$-g\left(F_{ijk}^+ P_R + F_{jik}^+ P_L\right) \qquad \ldots A^0, G^0$



$-igF^0_{lmk}$ $\quad\ldots h^0, H^0$

$-gF^0_{lmk}(P_R - P_L)$ $\quad\ldots A^0, G^0$



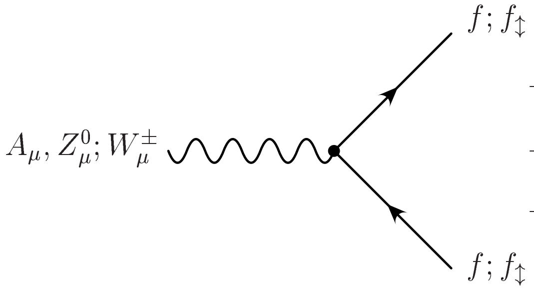
$-ig\left(F^R_{ilk}P_R + F^L_{ilk}P_L\right)$

F.4 Vector boson–Fermion–Fermion couplings

The Lagrangian describing interactions of vector bosons to fermions in the MSSM is given by

$$\begin{aligned}\mathcal{L} &= -e\,e_f\,A_\mu \bar{f}\gamma^\mu f - g_Z Z^0_\mu\,\bar{f}\gamma^\mu(C^f_L P_L + C^f_R P_R)f \\ &\quad -\frac{g}{\sqrt{2}}\Big(W^+_\mu \bar{f}_\uparrow \gamma^\mu P_L f_\downarrow + W^-_\mu \bar{f}_\downarrow \gamma^\mu P_L f_\uparrow\Big)\,, \end{aligned} \tag{F.20}$$

where C^f_L and C^f_R are defined as $C^f_L = I^{3L}_f - e_f s^2_W$ and $C^f_R = -e_f s^2_W$. Here and in the following the arrows $\uparrow$ and $\downarrow$ attached at (s)fermions denote up– and down–type (s)fermions, respectively.



$-ie\,e_f\gamma_\mu$ $\quad\ldots A_\mu$

$-ig_Z\gamma_\mu(C^f_L P_L + C^f_R P_R)$ $\quad\ldots Z^0_\mu$

$-i\dfrac{g}{\sqrt{2}}\gamma_\mu P_L$ $\quad\ldots W^\pm_\mu$

F.5 Vector boson–Gaugino–Gaugino couplings

The interaction of a vector boson with two gauginos is described by the Lagrangian

$$
\begin{aligned}
\mathcal{L} \;=\;& g\,\bar{\tilde{\chi}}_j^+\gamma^\mu\Big(O_{ij}^L P_L + O_{ij}^R P_R\Big)\tilde{\chi}_i^0\, W_\mu^+ + g\,\bar{\tilde{\chi}}_i^0\Big(O_{ij}^L P_L + O_{ij}^R P_R\Big)\tilde{\chi}_j^+\, W_\mu^- \\
& -e\,A_\mu\,\bar{\tilde{\chi}}_i^+\gamma^\mu\tilde{\chi}_i^+ + g_Z Z_\mu^0\,\bar{\tilde{\chi}}_i^+\gamma^\mu\Big(O_{ij}^{\prime L} P_L + O_{ij}^{\prime R} P_R\Big)\tilde{\chi}_j^+ \\
& +\frac{g_Z}{2} Z_\mu^0\,\bar{\tilde{\chi}}_i^0\gamma^\mu\Big(O_{ij}^{\prime\prime L} P_L + O_{ij}^{\prime\prime R} P_R\Big)\tilde{\chi}_j^0\,,
\end{aligned}
\tag{F.21}
$$

with the 4×2 coupling matrices for the $W^\pm$–chargino–neutralino vertex

$$
O_{ij}^L \;=\; Z_{i2}V_{j1} - \tfrac{1}{\sqrt{2}} Z_{i4}V_{j2}\,, \qquad O_{ij}^R \;=\; Z_{i2}U_{j1} + \tfrac{1}{\sqrt{2}} Z_{i3}U_{j2}\,, \tag{F.22}
$$

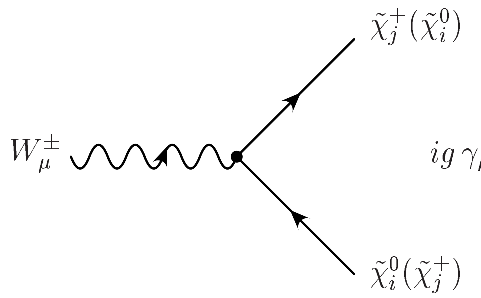
and the symmetric 2×2 and 4×4 coupling matrices

$$
O_{ij}^{\prime L} \;=\; -V_{i1}V_{j1} - \tfrac{1}{2}V_{i2}V_{j2} + \delta_{ij}s_W^2\,, \tag{F.23}
$$

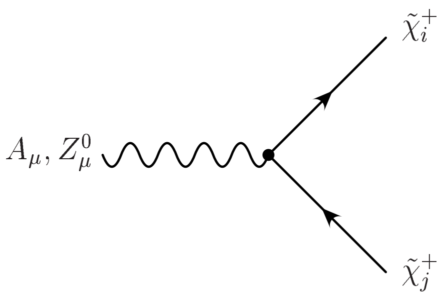
$$
O_{ij}^{\prime R} \;=\; -U_{i1}U_{j1} - \tfrac{1}{2}U_{i2}U_{j2} + \delta_{ij}s_W^2\,, \tag{F.24}
$$

$$
O_{ij}^{\prime\prime L} \;=\; -\tfrac{1}{2}Z_{i3}Z_{j3} + \tfrac{1}{2}Z_{i4}Z_{j4} \;=\; -\,O_{ij}^{\prime\prime R}\,. \tag{F.25}
$$

Z_{ij}, U_{ij}, V_{ij} are the neutralino and chargino mixing matrices, respectively.

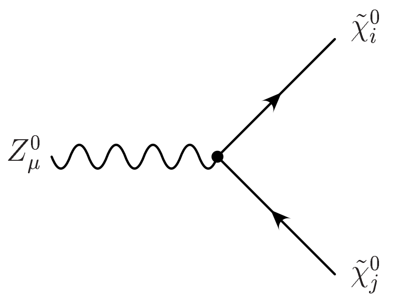


$$ig\,\gamma_\mu\Big(O_{ij}^L P_L + O_{ij}^R P_R\Big)$$



$$-ie\,\delta_{ij}\gamma_\mu \qquad \ldots A_\mu$$

$$ig_Z\gamma_\mu\Big(O_{ij}^{\prime L} P_L + O_{ij}^{\prime R} P_R\Big) \qquad \ldots Z_\mu^0$$



$$-ig_Z\,\gamma_\mu\,O_{ij}^{\prime\prime L}(P_R - P_L)$$

F.6 Four–Scalar couplings ($F-$ and $D-$terms)

In this section we will derive all couplings involving four scalar particles in the MSSM. These interactions have two different sources originating from 'F–term' and 'D–term' contributions, which build up the complete scalar potential $V = V_F + V_D$ discussed in the following.

F.6.1 F–term potential

We start with the superpotential

$$W = h_t\,\tilde{t}_R^*\tilde{t}_L H_2^0 + h_b\,\tilde{b}_R^*\tilde{b}_L H_1^0 + h_\tau\,\tilde{\tau}_R^*\tilde{\tau}_L H_1^0 - h_t\,\tilde{t}_R^*\tilde{b}_L H_2^1 - h_b\,\tilde{b}_R^*\tilde{t}_L H_1^2 - h_\tau\,\tilde{\tau}_R^*\tilde{\nu}_\tau H_1^2 \tag{F.26}$$

from which we can derive the F–term interaction potential $V_F = -\mathcal{L}_{int} = F_i^\dagger F_i$ with $F_i = \frac{\partial W}{\partial A_i}$, where A_i denotes all scalar (super)fields numbered by the index i, $A_i = \{\tilde{t}_L, \tilde{b}_L, \tilde{\nu}_\tau, \tilde{\tau}_L, \tilde{t}_R^*, \tilde{b}_R^*, \tilde{\tau}_R^*, H_1^0, H_2^0, H_1^2, H_2^1\}$.
The potential V_F then reads

$$\begin{aligned}
V_F \;=\;& \big(h_t\,\tilde{t}_R H_2^{0*} - h_b\,\tilde{b}_R H_1^{2*}\big)\big(h_t\,\tilde{t}_R^* H_2^0 - h_b\,\tilde{b}_R^* H_1^2\big) + \big(h_b\,\tilde{b}_R H_1^{0*} - h_t\,\tilde{t}_R H_2^{1*}\big)\big(h_b\,\tilde{b}_R^* H_1^0 - h_t\,\tilde{t}_R^* H_2^1\big)\\
&+ h_t^2\big(\tilde{t}_L^* H_2^{0*} - \tilde{b}_L^* H_2^{1*}\big)\big(\tilde{t}_L H_2^0 - \tilde{b}_L H_2^1\big) + h_b^2\big(\tilde{b}_L^* H_1^{0*} - \tilde{t}_L^* H_1^{2*}\big)\big(\tilde{b}_L H_1^0 - \tilde{t}_L H_1^2\big)\\
&+ h_\tau^2\,\tilde{\tau}_R H_1^{2*}\tilde{\tau}_R^* H_1^2 + h_\tau^2\,\tilde{\tau}_R H_1^{0*}\tilde{\tau}_R^* H_1^0 + h_\tau^2\big(\tilde{\tau}_L^* H_1^{0*} - \tilde{\nu}_\tau^* H_1^{2*}\big)\big(\tilde{\tau}_L H_1^0 - \tilde{\nu}_\tau H_1^2\big)\\
&+ \big(h_b\tilde{b}_R\tilde{b}_L^* + h_\tau\tilde{\tau}_R\tilde{\tau}_L^*\big)\big(h_b\tilde{b}_R^*\tilde{b}_L + h_\tau\tilde{\tau}_R^*\tilde{\tau}_L\big) + h_t^2\,\big(\tilde{t}_R\tilde{t}_L^*\big)\big(\tilde{t}_R^*\tilde{t}_L\big)\\
&+ \big(h_b\tilde{b}_R\tilde{t}_L^* + h_\tau\tilde{\tau}_R\tilde{\nu}_\tau^*\big)\big(h_b\tilde{b}_R^*\tilde{t}_L + h_\tau\tilde{\tau}_R^*\tilde{\nu}_\tau\big) + h_t^2\,\big(\tilde{t}_R\tilde{b}_L^*\big)\big(\tilde{t}_R^*\tilde{b}_L\big)\\
=\;& h_t^2\,\big|H_2^0\big|^2\,\big(\tilde{t}_R^*\tilde{t}_R + \tilde{t}_L^*\tilde{t}_L\big) + h_b^2\,\big|H_1^0\big|^2\,\big(\tilde{b}_R^*\tilde{b}_R + \tilde{b}_L^*\tilde{b}_L\big) + h_\tau^2\,\big|H_1^0\big|^2\,\big(\tilde{\tau}_R^*\tilde{\tau}_R + \tilde{\tau}_L^*\tilde{\tau}_L\big)\\
&+ h_t^2\,\big|H_2^1\big|^2\,\big(\tilde{t}_R^*\tilde{t}_R + \tilde{b}_L^*\tilde{b}_L\big) + h_b^2\,\big|H_1^2\big|^2\,\big(\tilde{b}_R^*\tilde{b}_R + \tilde{t}_L^*\tilde{t}_L\big) + h_\tau^2\,\big|H_1^2\big|^2\,\big(\tilde{\tau}_R^*\tilde{\tau}_R + \tilde{\nu}_\tau^*\tilde{\nu}_\tau\big)\\
&- h_t^2\big(\tilde{t}_L^*\tilde{b}_L H_2^{0*} H_2^1 + \tilde{b}_L^*\tilde{t}_L H_2^{1*} H_2^0\big) - h_b^2\big(\tilde{b}_L^*\tilde{t}_L H_1^{0*} H_1^2 + \tilde{t}_L^*\tilde{b}_L H_1^{2*} H_1^0\big)\\
&- h_\tau^2\big(\tilde{\tau}_L^*\tilde{\nu}_\tau H_1^{0*} H_1^2 + \tilde{\nu}_\tau^*\tilde{\tau}_L H_1^{2*} H_1^0\big)\\
&- h_t h_b\,\Big(\tilde{t}_R^*\tilde{b}_R H_1^{2*} H_2^0 + \tilde{b}_R^*\tilde{t}_R H_2^{0*} H_1^2 + \tilde{b}_R^*\tilde{t}_R H_2^{1*} H_1^0 + \tilde{t}_R^*\tilde{b}_R H_1^{0*} H_2^1\Big)\\
&+ h_t^2\Big(\big(\tilde{t}_R^*\tilde{t}_L\big)\big(\tilde{t}_L^*\tilde{t}_R\big) + \big(\tilde{t}_R^*\tilde{b}_L\big)\big(\tilde{b}_L^*\tilde{t}_R\big)\Big) + h_b^2\Big(\big(\tilde{b}_R^*\tilde{b}_L\big)\big(\tilde{b}_L^*\tilde{b}_R\big) + \big(\tilde{b}_R^*\tilde{t}_L\big)\big(\tilde{t}_L^*\tilde{b}_R\big)\Big)\\
&+ h_\tau^2\Big(\tilde{\tau}_R^*\tilde{\tau}_L\tilde{\tau}_L^*\tilde{\tau}_R + \tilde{\tau}_R^*\tilde{\nu}_\tau\tilde{\nu}_\tau^*\tilde{\tau}_R\Big)\\
&+ h_b h_\tau\Big(\tilde{b}_R^*\tilde{b}_L\tilde{\tau}_L^*\tilde{\tau}_R + \tilde{b}_L^*\tilde{b}_R\tilde{\tau}_R^*\tilde{\tau}_L + \tilde{t}_L^*\tilde{b}_R\tilde{\tau}_R^*\tilde{\nu}_\tau + \tilde{b}_R^*\tilde{t}_L\tilde{\nu}_\tau^*\tilde{\tau}_R\Big)
\end{aligned} \tag{F.27}$$

with the couplings of the neutral Higgs bosons and sfermions in the first line, those of the charged Higgs bosons in the second, the couplings of a neutral Higgs boson with a charged one and two sfermions and the four–sfermion couplings. Note that in the detailed calculation of the Feynman rules for the four–sfermion couplings we have to take care about the colour flow, see section F.6.5.

Transforming the interaction fields into the mass eigenstate fields,

$$\begin{aligned}
H_1^0 &= v_1 + \tfrac{1}{\sqrt{2}}\left[\cos\alpha\, H^0 - \sin\alpha\, h^0 + i\left(-\cos\beta\, G^0 + \sin\beta\, A^0\right)\right] , \\
H_1^2 &= -\cos\beta\, G^- + \sin\beta\, H^- , \\
H_2^1 &= \sin\beta\, G^+ + \cos\beta\, H^+ , \\
H_2^0 &= v_2 + \tfrac{1}{\sqrt{2}}\left[\sin\alpha\, H^0 + \cos\alpha\, h^0 + i\left(\sin\beta\, G^0 + \cos\beta\, A^0\right)\right] ,
\end{aligned} \tag{F.28}$$

$$\begin{aligned}
\tilde{f}_L &= R^{\tilde{f}}_{i1}\tilde{f}_i = \cos\theta_{\tilde{f}}\, \tilde{f}_1 - \sin\theta_{\tilde{f}}\, \tilde{f}_2 , \\
\tilde{f}_R &= R^{\tilde{f}}_{i2}\tilde{f}_i = \sin\theta_{\tilde{f}}\, \tilde{f}_1 + \cos\theta_{\tilde{f}}\, \tilde{f}_2 ,
\end{aligned} \tag{F.29}$$

we simplify our notations as follows:

$$\begin{aligned}
\left|H_1^0\right|^2 &= \frac{1}{2}\Big[\sin^2\!\alpha\,(h^0)^2 - \sin 2\alpha\, h^0 H^0 + \cos^2\!\alpha\,(H^0)^2 + \sin^2\!\beta\,(A^0)^2 - \sin 2\beta\, A^0 G^0 \\
&\quad + \cos^2\!\beta\,(G^0)^2\Big] = \frac{1}{2}\, H_k^0\, c^{\tilde{b}}_{kl}\, H_l^0 ,
\end{aligned} \tag{F.30}$$

$$\begin{aligned}
\left|H_2^0\right|^2 &= \frac{1}{2}\Big[\cos^2\!\alpha\,(h^0)^2 + \sin 2\alpha\, h^0 H^0 + \sin^2\!\alpha\,(H^0)^2 + \cos^2\!\beta\,(A^0)^2 + \sin 2\beta\, A^0 G^0 \\
&\quad + \sin^2\!\beta\,(G^0)^2\Big] = \frac{1}{2}\, H_k^0\, c^{\tilde{t}}_{kl}\, H_l^0 ,
\end{aligned} \tag{F.31}$$

$$\left|H_1^2\right|^2 = \sin^2\!\beta\, H^+H^- - \frac{1}{2}\sin 2\beta\, H^+G^- - \frac{1}{2}\sin 2\beta\, H^-G^+ + \cos^2\!\beta G^+G^- = \frac{1}{2}\, H_k^+\, d^{\tilde{b}}_{kl}\, H_l^- , \tag{F.32}$$

$$\left|H_2^1\right|^2 = \cos^2\!\beta\, H^+H^- + \frac{1}{2}\sin 2\beta\, H^+G^- + \frac{1}{2}\sin 2\beta\, H^-G^+ + \sin^2\!\beta G^+G^- = \frac{1}{2}\, H_k^+\, d^{\tilde{t}}_{kl}\, H_l^- , \tag{F.33}$$

with $H_k^0 = \{h^0, H^0, A^0, G^0\}$, $H_k^+ = \{H^+, G^+, H^-, G^-\}$, $H_k^- \equiv (H_k^+)^\dagger = \{H^-, G^-, H^+, G^+\}$ and

$$c^{\tilde{b}}_{kl} = \begin{pmatrix} \sin^2\!\alpha & -\frac{1}{2}\sin 2\alpha & 0 & 0 \\ -\frac{1}{2}\sin 2\alpha & \cos^2\!\alpha & 0 & 0 \\ 0 & 0 & \sin^2\!\beta & -\frac{1}{2}\sin 2\beta \\ 0 & 0 & -\frac{1}{2}\sin 2\beta & \cos^2\!\beta \end{pmatrix} , \tag{F.34}$$

$$c^{\tilde{t}}_{kl} = \begin{pmatrix} \cos^2\alpha & \frac{1}{2}\sin 2\alpha & 0 & 0 \\ \frac{1}{2}\sin 2\alpha & \sin^2\alpha & 0 & 0 \\ 0 & 0 & \cos^2\beta & \frac{1}{2}\sin 2\beta \\ 0 & 0 & \frac{1}{2}\sin 2\beta & \sin^2\beta \end{pmatrix}, \tag{F.35}$$

$$d^{\tilde{b}}_{kl} = \begin{pmatrix} \sin^2\beta & -\frac{1}{2}\sin 2\beta & 0 & 0 \\ -\frac{1}{2}\sin 2\beta & \cos^2\beta & 0 & 0 \\ 0 & 0 & \sin^2\beta & -\frac{1}{2}\sin 2\beta \\ 0 & 0 & -\frac{1}{2}\sin 2\beta & \cos^2\beta \end{pmatrix}, \tag{F.36}$$

$$d^{\tilde{t}}_{kl} = \begin{pmatrix} \cos^2\beta & \frac{1}{2}\sin 2\beta & 0 & 0 \\ \frac{1}{2}\sin 2\beta & \sin^2\beta & 0 & 0 \\ 0 & 0 & \cos^2\beta & \frac{1}{2}\sin 2\beta \\ 0 & 0 & \frac{1}{2}\sin 2\beta & \sin^2\beta \end{pmatrix}. \tag{F.37}$$

F.6.2 D–term potential

The D–term potential reads

$$V_D = \frac{1}{2}\left(D'D' + \sum_{i=1}^{3} D^i D^i + \sum_{a=1}^{8} D^a D^a\right) \tag{F.38}$$

with $D' = g' A_i^* \frac{Y_i}{2}\,\delta_{ij} A_j$, $D^i = g A_k^* \frac{\sigma^i_{kl}}{2}\, A_l$ and $D^a = g_s A_i^* \frac{\lambda^a_{ij}}{2}\, A_j$ being the terms according to U(1)–hypercharge, SU(2)–weak isospin and SU(3)–strong interaction. The matrices σ^i_{kl} and λ^a_{ij} are the well known Pauli and Gell–Mann matrices. A_i stands for the scalar (super)fields, $A_i = \{\tilde{Q}, \tilde{L}, \tilde{U}, \tilde{D}, \tilde{E},\ H_1, H_2\}$,

$$\tilde{Q} = \begin{pmatrix} \tilde{t}_L \\ \tilde{b}_L \end{pmatrix}, \qquad \tilde{L} = \begin{pmatrix} \tilde{\nu}_\tau \\ \tilde{\tau}_L \end{pmatrix}, \qquad \tilde{U} = \tilde{t}^*_R\,, \qquad \tilde{D} = \tilde{b}^*_R\,, \qquad \tilde{E} = \tilde{\tau}^*_R\,,$$

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} H_1^0 \\ H_1^2 \end{pmatrix}, \qquad H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} H_2^1 \\ H_2^0 \end{pmatrix}.$$

The U(1)–hypercharge term reads with

$$\frac{Y}{2} = Q - I_3 \qquad \to \qquad Y_{H_1} = -1,\ Y_{H_2} = 1 \tag{F.39}$$

$$D' = \frac{g'}{2}\left[\sum_f \left(Y_{\tilde{f}_L} \tilde{f}^*_L \tilde{f}_L - Y_{\tilde{f}_R} \tilde{f}^*_R \tilde{f}_R\right) - \left|H_1^0\right|^2 + \left|H_2^0\right|^2 - \left|H_1^2\right|^2 + \left|H_2^1\right|^2\right]. \tag{F.40}$$

Inserting the Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{F.41}$$

we get for the electroweak parts

$$
\begin{aligned}
D^1 &= \frac{g}{2}\left(\tilde{t}_L^*\tilde{b}_L + \tilde{b}_L^*\tilde{t}_L + \tilde{\nu}_\tau^*\tilde{\tau}_L + \tilde{\tau}_L^*\tilde{\nu}_\tau + H_1^{0*}H_1^2 + H_1^{2*}H_1^0 + H_2^{1*}H_2^0 + H_2^{0*}H_2^1\right) , \\
D^2 &= -i\frac{g}{2}\left(\tilde{t}_L^*\tilde{b}_L - \tilde{b}_L^*\tilde{t}_L + \tilde{\nu}_\tau^*\tilde{\tau}_L - \tilde{\tau}_L^*\tilde{\nu}_\tau + H_1^{0*}H_1^2 - H_1^{2*}H_1^0 + H_2^{1*}H_2^0 - H_2^{0*}H_2^1\right) , \\
D^3 &= \frac{g}{2}\left(\tilde{t}_L^*\tilde{t}_L - \tilde{b}_L^*\tilde{b}_L + \tilde{\nu}_\tau^*\tilde{\nu}_\tau - \tilde{\tau}_L^*\tilde{\tau}_L + \left|H_1^0\right|^2 - \left|H_1^2\right|^2 + \left|H_2^1\right|^2 - \left|H_2^0\right|^2\right) \\
&= g\sum_f I_f^{3L}\tilde{f}_L^*\tilde{f}_L + \sum_{i=1,2}\left(I^3_{H_1^i}\left|H_1^i\right|^2 + I^3_{H_2^i}\left|H_2^i\right|^2\right) .
\end{aligned}
$$

(The meaning of $I^3_{H_j^i}$ should be clear.)
When we take the square of the single terms we have to take care about the colours of the sfermion fields. The result is

$$
\begin{aligned}
D'D' = \frac{g'^2}{4}\Bigg\{ & \left[\sum_f\left(Y_{\tilde{f}_L}(\tilde{f}_L^*\tilde{f}_L) - Y_{\tilde{f}_R}(\tilde{f}_R^*\tilde{f}_R)\right)\right]^2 + \left[-\left|H_1^0\right|^2 + \left|H_2^0\right|^2 - \left|H_1^2\right|^2 + \left|H_2^1\right|^2\right]^2 \\
& + 2\sum_f\left(Y_{\tilde{f}_L}(\tilde{f}_L^*\tilde{f}_L) - Y_{\tilde{f}_R}(\tilde{f}_R^*\tilde{f}_R)\right)\left(-\left|H_1^0\right|^2 + \left|H_2^0\right|^2 - \left|H_1^2\right|^2 + \left|H_2^1\right|^2\right)\Bigg\},
\end{aligned}
\tag{F.42}
$$

$$
\begin{aligned}
D^1D^1 + D^2D^2 = g^2\Bigg[& \left((\tilde{t}_L^*\tilde{b}_L) + \tilde{\nu}_\tau^*\tilde{\tau}_L\right)\left((\tilde{b}_L^*\tilde{t}_L) + \tilde{\tau}_L^*\tilde{\nu}_\tau\right) + \left|H_1^0\right|^2\left|H_1^2\right|^2 + \left|H_2^0\right|^2\left|H_2^1\right|^2 \\
& + H_1^{0*}H_1^2H_2^{0*}H_2^1 + H_2^{1*}H_2^0H_1^{2*}H_1^0 + \left((\tilde{t}_L^*\tilde{b}_L) + \tilde{\nu}_\tau^*\tilde{\tau}_L\right)\left(H_1^{2*}H_1^0 + H_2^{0*}H_2^1\right) \\
& + \left((\tilde{b}_L^*\tilde{t}_L) + \tilde{\tau}_L^*\tilde{\nu}_\tau\right)\left(H_1^{0*}H_1^2 + H_2^{1*}H_2^0\right)\Bigg],
\end{aligned}
\tag{F.43}
$$

$$
\begin{aligned}
D^3D^3 = \frac{g^2}{4}\Bigg\{ & \sum_f(\tilde{f}_L^*\tilde{f}_L)\left[(\tilde{f}_L^*\tilde{f}_L) - (\tilde{f}_L'^*\tilde{f}_L') + (\hat{\tilde{f}}_L^*\hat{\tilde{f}}_L) - (\hat{\tilde{f}}_L'^*\hat{\tilde{f}}_L')\right] \\
& + \left[\left|H_1^0\right|^2 - \left|H_1^2\right|^2 + \left|H_2^1\right|^2 - \left|H_2^0\right|^2\right]^2 \\
& + 4\sum_f I_f^{3L}(\tilde{f}_L^*\tilde{f}_L)\left(\left|H_1^0\right|^2 - \left|H_1^2\right|^2 + \left|H_2^1\right|^2 - \left|H_2^0\right|^2\right)\Bigg\},
\end{aligned}
\tag{F.44}
$$

where we have used $\left(I_f^{3L}\right)^2 = \frac{1}{4}$ and $I_f^{3L}I_{f'}^{3L} = -\frac{1}{4}$. Now we are able to calculate the Feynman rules, beginning with the easiest ones, those of two sfermions and two neutral Higgs bosons.

F.6.3 Higgs–Higgs–Sfermion–Sfermion

$H_k^0 H_l^0 \tilde{f}_i \tilde{f}_j$ couplings

The interesting part in the F–term potential for this coupling is (see eq. (F.27))

$$
\begin{aligned}
V_F \;\supset\;& h_t^2 \left|H_2^0\right|^2 \left(\tilde{t}_R^* \tilde{t}_R + \tilde{t}_L^* \tilde{t}_L\right) + h_b^2 \left|H_1^0\right|^2 \left(\tilde{b}_R^* \tilde{b}_R + \tilde{b}_L^* \tilde{b}_L\right) + h_\tau^2 \left|H_1^0\right|^2 \left(\tilde{\tau}_R^* \tilde{\tau}_R + \tilde{\tau}_L^* \tilde{\tau}_L\right) \\
=\;& \frac{h_t^2}{2} H_k^0\, c_{kl}^{\tilde{t}}\, H_l^0 \big(\tilde{t}_R^* \tilde{t}_R + \tilde{t}_L^* \tilde{t}_L\big) + \frac{h_b^2}{2} H_k^0\, c_{kl}^{\tilde{b}}\, H_l^0 \big(\tilde{b}_R^* \tilde{b}_R + \tilde{b}_L^* \tilde{b}_L\big) \\
&+ \frac{h_\tau^2}{2} H_k^0\, c_{kl}^{\tilde{b}}\, H_l^0 \big(\tilde{\tau}_R^* \tilde{\tau}_R + \tilde{\tau}_L^* \tilde{\tau}_L\big) \\
=\;& \sum_f \frac{h_f^2}{2} H_k^0\, c_{kl}^{\tilde{f}}\, H_l^0 \left(\tilde{f}_R^* \tilde{f}_R + \tilde{f}_L^* \tilde{f}_L\right) \;=\; \sum_f \frac{h_f^2}{2} H_k^0\, c_{kl}^{\tilde{f}}\, H_l^0\, \delta_{ij}\, \tilde{f}_i^* \tilde{f}_j\,, \qquad \text{(F.45)}
\end{aligned}
$$

where in the last step we have transformed the sfermion interaction fields to the mass eigenstate fields (see eq. (F.29)) and made use of the relation $R_{i1}^{\tilde{f}} R_{j1}^{\tilde{f}} + R_{i2}^{\tilde{f}} R_{j2}^{\tilde{f}} = \delta_{ij}$.

From the D–term potential we need the terms $\propto H_k^0 H_l^0 \tilde{f}_i \tilde{f}_j$ of

$$
\begin{aligned}
V_D \;\supset\; \frac{1}{2}\left(D' D' + D^3 D^3\right) \;\sim\;& -\frac{g'^2}{4} \sum_f \left(Y_{\tilde{f}_L} \tilde{f}_L^* \tilde{f}_L - Y_{\tilde{f}_R} \tilde{f}_R^* \tilde{f}_R\right) \left(\left|H_1^0\right|^2 - \left|H_2^0\right|^2\right) \\
&+ \frac{g^2}{2} \sum_f I_f^{3L} \big(\tilde{f}_L^* \tilde{f}_L\big) \left(\left|H_1^0\right|^2 - \left|H_2^0\right|^2\right) .
\end{aligned}
$$

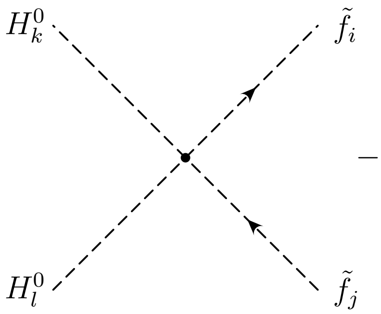
Using the abbreviations defined in eqs. (F.30) – (F.37), we get

$$
\begin{aligned}
V_D \;\supset\;& \frac{g^2}{4} \sum_f \left[\left(I_f^{3L} + (I_f^{3L} - e_f)\, t_W^2\right) \tilde{f}_L^* \tilde{f}_L + e_f\, t_W^2\, \tilde{f}_R^* \tilde{f}_R\right] H_k^0 \left(c_{kl}^{\tilde{b}} - c_{kl}^{\tilde{t}}\right) H_l^0 \\
=\;& \frac{g^2}{4 c_W^2} \sum_f \left[(I_f^{3L} - e_f s_W^2) R_{i1}^{\tilde{f}} R_{j1}^{\tilde{f}} + e_f s_W^2 R_{i2}^{\tilde{f}} R_{j2}^{\tilde{f}}\right] H_k^0 \left(c_{kl}^{\tilde{b}} - c_{kl}^{\tilde{t}}\right) H_l^0\, \tilde{f}_i^* \tilde{f}_j \\
=\;& \sum_f \frac{g^2}{2} H_k^0 \left(c_{kl}^{\tilde{b}} - c_{kl}^{\tilde{t}}\right) H_l^0\, e_{ij}^{\tilde{f}}\, \tilde{f}_i^* \tilde{f}_j\,. \qquad \text{(F.46)}
\end{aligned}
$$

with

$$
e_{ij}^{\tilde{f}} \;=\; \frac{1}{2 c_W^2}\left[(I_f^{3L} - e_f s_W^2) R_{i1}^{\tilde{f}} R_{j1}^{\tilde{f}} + e_f s_W^2 R_{i2}^{\tilde{f}} R_{j2}^{\tilde{f}}\right] . \qquad \text{(F.47)}
$$

Therefore the Feynman rule for this coupling becomes



$$-\,i\Big[h_f^2\, c_{kl}^{\tilde{f}}\,\delta_{ij} + g^2\left(c_{kl}^{\tilde{b}} - c_{kl}^{\tilde{t}}\right) e_{ij}^{\tilde{f}}\Big]\,.$$

$H_k^+ H_l^+ \tilde{f}_i \tilde{f}_j$ couplings

For the couplings of the charged Higgs bosons and two sfermions we start with the interaction potential

$$\begin{aligned} V_F \;&\supset\; h_t^2\left|H_2^1\right|^2\left(\tilde{t}_R^*\tilde{t}_R + \tilde{b}_L^*\tilde{b}_L\right) + h_b^2\left|H_1^2\right|^2\left(\tilde{b}_R^*\tilde{b}_R + \tilde{t}_L^*\tilde{t}_L\right) + h_\tau^2\left|H_1^2\right|^2\left(\tilde{\tau}_R^*\tilde{\tau}_R + \tilde{\nu}_\tau^*\tilde{\nu}_\tau\right) \\ &= \frac{h_t^2}{2}\,H_k^+\,d_{kl}^{\tilde{t}}\,H_l^-\left(\tilde{t}_R^*\tilde{t}_R + \tilde{b}_L^*\tilde{b}_L\right) + \frac{h_b^2}{2}\,H_k^+\,d_{kl}^{\tilde{b}}\,H_l^-\left(\tilde{b}_R^*\tilde{b}_R + \tilde{t}_L^*\tilde{t}_L\right) \\ &\quad + \frac{h_\tau^2}{2}\,H_k^+\,d_{kl}^{\tilde{b}}\,H_l^-\left(\tilde{\tau}_R^*\tilde{\tau}_R + \tilde{\nu}_\tau^*\tilde{\nu}_\tau\right) = \sum_f \frac{h_f^2}{2}\,H_k^+\,d_{kl}^{\tilde{f}}\,H_l^-\left(\tilde{f}_R^*\tilde{f}_R + \tilde{f}_L^{\prime *}\tilde{f}_L^{\prime}\right)\,. \end{aligned}$$

To get the Feynman rule for this coupling we have to calculate the first (nontrivial) term of the S–matrix;

$$S_{fi}^{(1)} \;=\; -i\sum_f \frac{h_f^2}{2}\int d^4x\,\langle f|\,d_{mn}^{\tilde{f}}\,:H_m^+H_n^-\left(\tilde{f}_R^*\tilde{f}_R + \tilde{f}_L^{\prime *}\tilde{f}_L^{\prime}\right):|i\rangle \tag{F.48}$$

with

$$|\,i\,\rangle = a_{H_k^+}^\dagger\, a_{H_l^-}^\dagger\,|\,0\,\rangle \qquad \text{and} \qquad \langle\, f\,| = \langle\,0\,|\,a_i\,b_j$$

for two incoming Higgs bosons (H_k^+ and H_l^-) and two outgoing sfermions (sfermion $\tilde{f}_i$ and anti–sfermion $\tilde{f}_j$).

Contracting the sfermions gives (here we use a short notation neglecting all space coordinates and momenta, p_i belongs to the particle with index i)

$$\begin{aligned} S_{fi}^{(1)} \;&=\; -i\sum_f \frac{h_f^2}{2}\,d_{mn}^{\tilde{f}}\int d^4x\,\langle 0|\,a_i\,b_j\,:H_m^+H_n^-\left(R_{i'2}^{\tilde{f}}R_{j'2}^{\tilde{f}}\,\tilde{f}_{i'}^*\tilde{f}_{j'} + R_{i'1}^{\tilde{f}'}R_{j'1}^{\tilde{f}'}\,\tilde{f}_{i'}^{\prime *}\tilde{f}_{j'}^{\prime}\right):\,a_{H_k^+}^\dagger\,a_{H_l^-}^\dagger|\,0\,\rangle \\ &=\; -i\sum_f \frac{h_f^2}{2}\,d_{mn}^{\tilde{f}}\left(R_{i2}^{\tilde{f}}R_{j2}^{\tilde{f}} + R_{i1}^{\tilde{f}'}R_{j1}^{\tilde{f}'}\right)\int d^4x\,e^{i(p_i-p_j)}\,\langle\,0|\,:H_m^+H_n^-:\,a_{H_k^+}^\dagger\,a_{H_l^-}^\dagger|\,0\,\rangle \end{aligned}$$

where we have used

$$\langle\,0\,|a_i(p_i)\tilde{f}_{i'}^*(x)|\,0\rangle \;=\; \underbracket{a_i(p_i)\,\tilde{f}_{i'}^*(x)} \;=\; \delta_{ii'}\,e^{ip_ix} \tag{F.49}$$

$$\langle 0\,|b_j(p_j)\,\tilde{f}_{j'}(x)|\,0\rangle \;=\; b_j(p_j)\,\tilde{f}_{j'}(x) \;=\; \delta_{jj'}\,e^{-ip_jx} \tag{F.50}$$

In the contractions of the Higgs fields we have to take care of the Higgs creation and annihilation operators e.g. $a^\dagger_{H^+}$ which can create a Higgs boson H^+ and therefore gives **two** contributions from H_1^+ **and** H_3^-:

$$H_k^+(x)\,a_{H^+_{k'}}(p_{k'}) \;=\; \delta_{kk'}\,e^{ip_kx} \tag{F.51}$$

$$H_k^-(x)\,a_{H^+_{k'}}(p_{k'}) \;=\; \begin{pmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 \end{pmatrix}_{kk'} e^{ip_kx} \tag{F.52}$$

$$\begin{aligned}
\langle 0|:H_m^+H_n^-:\,a^\dagger_{H_k^+}\,a^\dagger_{H_l^-}|\,0\,\rangle \;&=\; H_m^+H_n^-a^\dagger_{H_k^+}\,a^\dagger_{H_l^-} + H_m^+H_n^-a^\dagger_{H_k^+}\,a^\dagger_{H_l^-}\\
&\propto\; \delta_{mk}\,\delta_{nl} \;+\; (\delta_{n1}\,\delta_{k3}+\delta_{n2}\,\delta_{k4}+\delta_{n3}\,\delta_{k1}+\delta_{n4}\,\delta_{k2})\\
&\qquad \times\;(\delta_{m1}\,\delta_{l3}+\delta_{m2}\,\delta_{l4}+\delta_{m3}\,\delta_{l1}+\delta_{m4}\,\delta_{l2})\\
&=\; \delta_{mk}\,\delta_{nl} \;+\; \begin{pmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 \end{pmatrix}_{nk} \begin{pmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 \end{pmatrix}_{ml}
\end{aligned} \tag{F.53}$$

For the Feynman amplitude $\mathcal{M}$ we finally get

$$\begin{aligned}
\mathcal{M} \;&=\; -i\sum_f \frac{h_f^2}{2}\left(R^{\tilde{f}}_{i2}R^{\tilde{f}}_{j2}+R^{\tilde{f}'}_{i1}R^{\tilde{f}'}_{j1}\right)\left[d^{\tilde{f}}_{kl} + \begin{pmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 \end{pmatrix}_{nk} d^{\tilde{f}}_{mn} \begin{pmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 \end{pmatrix}_{ml}\right]\\
&=\; -i\sum_f h_f^2\left(R^{\tilde{f}}_{i2}R^{\tilde{f}}_{j2}+R^{\tilde{f}'}_{i1}R^{\tilde{f}'}_{j1}\right)d^{\tilde{f}}_{kl}\,.
\end{aligned}$$

The D–terms read with eqs. (F.42), (F.44) and (F.30)–(F.37)

$$\begin{aligned}
V_D \;&\supset\; -\frac{g'^2}{4}\sum_f\left(Y_{\tilde{f}_L}\tilde{f}^*_{L\alpha}\tilde{f}_{L\alpha} - Y_{\tilde{f}_R}\tilde{f}^*_{R\alpha}\tilde{f}_{R\alpha}\right)\left(\left|H_1^2\right|^2-\left|H_2^1\right|^2\right)\\
&\quad -\frac{g^2}{2}\sum_f I_f^{3L}\big(\tilde{f}^*_L\tilde{f}_L\big)\left(\left|H_1^2\right|^2-\left|H_2^1\right|^2\right)\\
&=\; \frac{g^2}{4c_W^2}\sum_f\left[\big(-I_f^{3L}\cos 2\theta_W - e_f s_W^2\big)\,R^{\tilde{f}}_{i1}R^{\tilde{f}}_{j1} + e_f s_W^2 R^{\tilde{f}}_{i2}R^{\tilde{f}}_{j2}\right]H_k^+\big(d^{\tilde{b}}_{kl}-d^{\tilde{t}}_{kl}\big)H_l^-\;\tilde{f}^*_i\tilde{f}_j
\end{aligned}$$

$$\equiv \frac{g^2}{2} \sum_f H_k^+ \big(d_{kl}^{\tilde{b}} - d_{kl}^{\tilde{t}}\big) H_l^- f_{ij}^{\tilde{f}}\, \tilde{f}_i^* \tilde{f}_j\,. \tag{F.54}$$

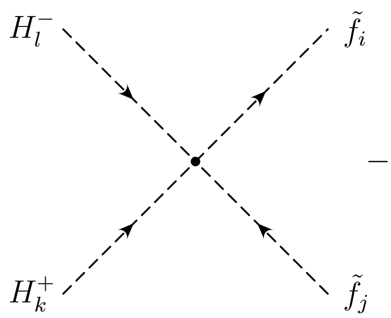
with

$$f_{ij}^{\tilde{f}} \;=\; \frac{1}{2c_W^2}\Big[\big(-I_f^{3L}\cos 2\theta_W - e_f s_W^2\big)\, R_{i1}^{\tilde{f}} R_{j1}^{\tilde{f}} + e_f s_W^2 R_{i2}^{\tilde{f}} R_{j2}^{\tilde{f}}\Big] \tag{F.55}$$

Analogously to the previous calculation we get for the Feynman amplitude

$$\mathcal{M} \;=\; -i \sum_f g^2 f_{ij}^{\tilde{f}} \left(d_{kl}^{\tilde{b}} - d_{kl}^{\tilde{t}}\right). \tag{F.56}$$

In the Feynman rule we have to take into consideration that only the terms $\propto R_{ij}^{\tilde{f}}$ are valid for a coupling with sfermions and not the terms $\propto R_{ij}^{\tilde{f}'}$!



$$-i\left[h_{f'}^2\, d_{kl}^{\tilde{f}'}\, R_{i1}^{\tilde{f}} R_{j1}^{\tilde{f}} + h_f^2\, d_{kl}^{\tilde{f}}\, R_{i2}^{\tilde{f}} R_{j2}^{\tilde{f}} + g^2 f_{ij}^{\tilde{f}}\left(d_{kl}^{\tilde{b}} - d_{kl}^{\tilde{t}}\right)\right]$$

$H_k^0 H_l^+ \tilde{f}_i \tilde{f}_j'$ couplings

The terms of the superpotential V_F which are necessary for calculating these couplings can be picked out of eq. (F.27):

$$\begin{aligned} V_F \;\supset\; & - h_t^2\big(\tilde{t}_L^* \tilde{b}_L H_2^{0*} H_2^1 + \tilde{b}_L^* \tilde{t}_L H_2^{1*} H_2^0\big) - h_b^2\big(\tilde{b}_L^* \tilde{t}_L H_1^{0*} H_1^2 + \tilde{t}_L^* \tilde{b}_L H_1^{2*} H_1^0\big) \\ & - h_\tau^2\big(\tilde{\tau}_L^* \tilde{\nu}_\tau H_1^{0*} H_1^2 + \tilde{\nu}_\tau^* \tilde{\tau}_L H_1^{2*} H_1^0\big) \\ & - h_t h_b \Big(\tilde{t}_R^* \tilde{b}_R H_1^{2*} H_2^0 + \tilde{b}_R^* \tilde{t}_R H_2^{0*} H_1^2 + \tilde{b}_R^* \tilde{t}_R H_2^{1*} H_1^0 + \tilde{t}_R^* \tilde{b}_R H_1^{0*} H_2^1\Big) \end{aligned} \tag{F.57}$$

Accordingly to the abbreviations before we introduce a few more coupling matrices for better reading:

$$\begin{aligned} H_2^{0*} H_2^1 \;&=\; \frac{1}{\sqrt{2}}\left[\cos\alpha\, h^0 + \sin\alpha\, H^0 - i\left(\cos\beta\, A^0 + \sin\beta\, G^0\right)\right]\left[\cos\beta\, H^+ + \sin\beta\, G^+\right] \\ &=\; \frac{1}{\sqrt{2}}\left(h^0, H^0, A^0, G^0\right)\begin{pmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & 0 & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & 0 & 0 \\ -i\cos^2\beta & -\frac{i}{2}\sin 2\beta & 0 & 0 \\ -\frac{i}{2}\sin 2\beta & -i\sin^2\beta & 0 & 0 \end{pmatrix}\begin{pmatrix} H^+ \\ G^+ \\ H^- \\ G^- \end{pmatrix} \end{aligned}$$

$$= \frac{1}{\sqrt{2}} H_k^0 \, c_{kl}^{\tilde{t},0+} H_l^+ \tag{F.58}$$

$$H_2^{1*} H_2^0 = \left(H_2^{0*} H_2^1\right)^* = \frac{1}{\sqrt{2}} \left(H_k^0 \, c_{kl}^{\tilde{t},0+} H_l^+\right)^* \tag{F.59}$$

$$\begin{aligned}
H_1^{0*} H_1^2 &= \frac{1}{\sqrt{2}} \left[-\sin\alpha \, h^0 + \cos\alpha \, H^0 - i\left(\sin\beta \, A^0 - \cos\beta \, G^0\right)\right] \left[\sin\beta \, H^- - \cos\beta \, G^-\right] \\
&= \frac{1}{\sqrt{2}} \left(h^0, H^0, A^0, G^0\right) \begin{pmatrix} 0 & 0 & -\sin\alpha\sin\beta & \sin\alpha\cos\beta \\ 0 & 0 & \cos\alpha\sin\beta & -\cos\alpha\cos\beta \\ 0 & 0 & -i\sin^2\beta & \frac{i}{2}\sin 2\beta \\ 0 & 0 & \frac{i}{2}\sin 2\beta & -i\cos^2\beta \end{pmatrix} \begin{pmatrix} H^+ \\ G^+ \\ H^- \\ G^- \end{pmatrix} \\
&= \frac{1}{\sqrt{2}} H_k^0 \, c_{kl}^{\tilde{b},0+} H_l^+
\end{aligned} \tag{F.60}$$

$$H_1^{2*} H_1^0 = \left(H_1^{0*} H_1^2\right)^* = \frac{1}{\sqrt{2}} \left(H_k^0 \, c_{kl}^{\tilde{b},0+} H_l^+\right)^* \tag{F.61}$$

$$\begin{aligned}
H_1^{0*} H_2^1 &= \frac{1}{\sqrt{2}} \left[-\sin\alpha \, h^0 + \cos\alpha \, H^0 - i\left(\sin\beta \, A^0 - \cos\beta \, G^0\right)\right] \left[\cos\beta \, H^+ + \sin\beta \, G^+\right] \\
&= \frac{1}{\sqrt{2}} \left(h^0, H^0, A^0, G^0\right) \begin{pmatrix} -\sin\alpha\cos\beta & -\sin\alpha\sin\beta & 0 & 0 \\ \cos\alpha\cos\beta & \cos\alpha\sin\beta & 0 & 0 \\ -\frac{i}{2}\sin 2\beta & -i\sin^2\beta & 0 & 0 \\ i\cos^2\beta & \frac{i}{2}\sin 2\beta & 0 & 0 \end{pmatrix} \begin{pmatrix} H^+ \\ G^+ \\ H^- \\ G^- \end{pmatrix} \\
&= \frac{1}{\sqrt{2}} H_k^0 \, c_{kl}^{\tilde{t}\tilde{b},0+} H_l^+
\end{aligned} \tag{F.62}$$

$$H_2^{1*} H_1^0 = \left(H_1^{0*} H_2^1\right)^* = \frac{1}{\sqrt{2}} \left(H_k^0 \, c_{kl}^{\tilde{t}\tilde{b},0+} H_l^+\right)^* \tag{F.63}$$

$$\begin{aligned}
H_2^{0*} H_1^2 &= \frac{1}{\sqrt{2}} \left[\cos\alpha \, h^0 + \sin\alpha \, H^0 - i\left(\cos\beta \, A^0 + \sin\beta \, G^0\right)\right] \left[\sin\beta \, H^- - \cos\beta \, G^-\right] \\
&= \frac{1}{\sqrt{2}} \left(h^0, H^0, A^0, G^0\right) \begin{pmatrix} 0 & 0 & \cos\alpha\sin\beta & -\cos\alpha\cos\beta \\ 0 & 0 & \sin\alpha\sin\beta & -\sin\alpha\cos\beta \\ 0 & 0 & -\frac{i}{2}\sin 2\beta & i\cos^2\beta \\ 0 & 0 & -i\sin^2\beta & \frac{i}{2}\sin 2\beta \end{pmatrix} \begin{pmatrix} H^+ \\ G^+ \\ H^- \\ G^- \end{pmatrix} \\
&= \frac{1}{\sqrt{2}} H_k^0 \, c_{kl}^{\tilde{b}\tilde{t},0+} H_l^+
\end{aligned} \tag{F.64}$$

$$H_1^{2*} H_2^0 = \left(H_2^{0*} H_1^2\right)^* = \frac{1}{\sqrt{2}} \left(H_k^0 \, c_{kl}^{\tilde{b}\tilde{t},0+} H_l^+\right)^* \tag{F.65}$$

After inserting this into eq. (F.57) we get

$$V_F \supset -\frac{1}{\sqrt{2}} \sum_f \left(h_f^2 \, \tilde{f}_L^* \tilde{f}_L' \, H_k^0 \, c_{kl}^{\tilde{f},0+} H_l^+ + h_t h_b \, \tilde{f}_R^* \tilde{f}_R' \, H_k^0 \, c_{kl}^{\tilde{f}\tilde{f}',0+} H_l^+\right) \quad + \text{ c.c.} \tag{F.66}$$

Rotating the sfermion fields into the mass eigenstate fields (see eq. (F.29)) gives

$$V_F \;\supset\; -\frac{1}{\sqrt{2}}\sum_f \left(h_f^2\, R^{\tilde{f}}_{i1}R^{\tilde{f}'}_{j1}\, c^{\tilde{f},0+}_{kl} + h_f h_{f'}\, R^{\tilde{f}}_{i2}R^{\tilde{f}'}_{j2}\, c^{\tilde{f}\tilde{f}',0+}_{kl}\right) H^0_k H^+_l \tilde{f}^*_i \tilde{f}'_j$$
$$-\frac{1}{\sqrt{2}}\sum_f \left(h_f^2\, R^{\tilde{f}}_{i1}R^{\tilde{f}'}_{j1}\, (H^0_k c^{\tilde{f}',0+}_{kl} H^+_l)^* + h_f h_{f'}\, R^{\tilde{f}}_{i2}R^{\tilde{f}'}_{j2}\, (H^0_k c^{\tilde{f}'\tilde{f},0+}_{kl} H^+_l)^*\right) \tilde{f}_i \tilde{f}'^*_j$$

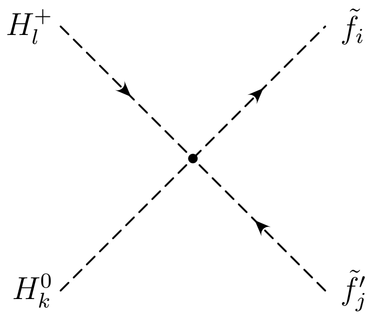
and with

$$\left(c^{\tilde{f}',0+}_{kl} H^+_l\right)^* = \left(c^{\tilde{f}',0+}_{kl}\right)^* H^-_l = \left(c^{\tilde{f}',0+}_{kl}\right)^* \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{4\text{x}4} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{4\text{x}4}}_{1_{4\text{x}4}} H^-_l = \left(c^{\tilde{f}',0+}_{kl'}\right)^* H^+_l$$

(look at the index l' which is $l' = 3, 4$ for $l = 1, 2$ and vice versa !) we arrive at

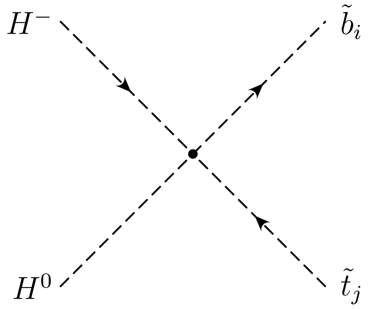
$$V_F \;\supset\; -\frac{1}{\sqrt{2}}\sum_f \left[R^{\tilde{f}}_{i1}R^{\tilde{f}'}_{j1}\left(h_f^2 c^{\tilde{f},0+}_{kl} + h_{f'}^2 \left(c^{\tilde{f}',0+}_{kl'}\right)^*\right) + R^{\tilde{f}}_{i2}R^{\tilde{f}'}_{j2}\, h_f h_{f'}\left(c^{\tilde{f}\tilde{f}',0+}_{kl} + \left(c^{\tilde{f}'\tilde{f},0+}_{kl'}\right)^*\right)\right]$$
$$\times\, H^0_k\, H^+_l\, \tilde{f}^*_i\, \tilde{f}'_j \qquad \text{(F.67)}$$

Now the Feynman rules take the form



$$\frac{i}{\sqrt{2}}\left[R^{\tilde{f}}_{i1}R^{\tilde{f}'}_{j1}\left(h_f^2\, c^{\tilde{f},0+}_{kl} + h_{f'}^2 \left(c^{\tilde{f}',0+}_{kl'}\right)^*\right)\right.$$
$$\left. +\; R^{\tilde{f}}_{i2}\, R^{\tilde{f}'}_{j2}\, h_f h_{f'}\left(c^{\tilde{f}\tilde{f}',0+}_{kl} + \left(c^{\tilde{f}'\tilde{f},0+}_{kl'}\right)^*\right)\right]$$

Be careful which Higgs boson you should take in a particular graph! As an illustrative example we give the result for a coupling which we will need in the main part of this work:



$$\frac{i}{\sqrt{2}}\left[R^{\tilde{b}}_{i1}R^{\tilde{t}}_{j1}\left(h_b^2 \cos\alpha\sin\beta + h_t^2 \sin\alpha\cos\beta\right)\right.$$
$$\left. +\; R^{\tilde{b}}_{i2}R^{\tilde{t}}_{j2}\, h_t h_b \left(\sin\alpha\sin\beta + \cos\alpha\cos\beta\right)\right]$$

Here we have to take the negative charged Higgs boson H^- (the H^+ would not be allowed due to charge conservation).

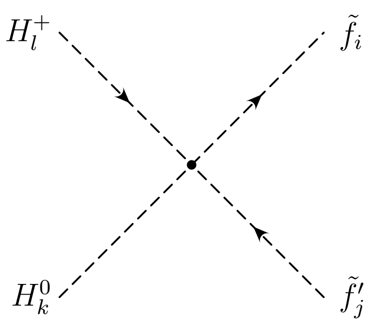
Setting $k = 2$ and $l = 3$ leaves

$$\begin{aligned}
&\frac{i}{\sqrt{2}}\left[R^{\tilde{b}}_{i1}R^{\tilde{t}}_{j1}\left(h_b^2 c^{\tilde{b},0+}_{23}+h_t^2 c^{\tilde{t},0+}_{23'}\right)+R^{\tilde{b}}_{i2}R^{\tilde{t}}_{j2}\,h_t h_b\left(c^{\tilde{b}\tilde{t},0+}_{23}+c^{\tilde{t}\tilde{b},0+}_{23'}\right)\right]\\
&=\frac{i}{\sqrt{2}}\left[R^{\tilde{b}}_{i1}R^{\tilde{t}}_{j1}\left(h_b^2\cos\alpha\sin\beta+h_t^2\sin\alpha\cos\beta\right)+R^{\tilde{b}}_{i2}R^{\tilde{t}}_{j2}\,h_t h_b(\sin\alpha\sin\beta+\cos\alpha\cos\beta)\right].
\end{aligned}$$

The D–term potential terms coming from the off–diagonal Pauli matrices σ^1_{kl} and σ^2_{kl} are given by

$$\begin{aligned}
V_D &\supset \frac{1}{2}\left(D^1D^1+D^2D^2\right)\\
&\supset \left(\left(\tilde{t}^*_L\tilde{b}_L\right)+\tilde{\nu}^*_\tau\tilde{\tau}_L\right)\left(H_1^{2*}H_1^0+H_2^{0*}H_2^1\right)+\left(\left(\tilde{b}^*_L\tilde{t}_L\right)+\tilde{\tau}^*_L\tilde{\nu}_\tau\right)\left(H_1^{0*}H_1^2+H_2^{1*}H_2^0\right)\\
&= \left(\left(\tilde{t}^*_L\tilde{b}_L\right)+\tilde{\nu}^*_\tau\tilde{\tau}_L\right)\frac{g^2}{2\sqrt{2}}\left[\left(H_k^0\,c^{\tilde{b},0+}_{kl}H_l^+\right)^*+H_k^0\,c^{\tilde{t},0+}_{kl}H_l^+\right]\\
&\quad+\left(\left(\tilde{b}^*_L\tilde{t}_L\right)+\tilde{\tau}^*_L\tilde{\nu}_\tau\right)\frac{g^2}{2\sqrt{2}}\left[H_k^0\,c^{\tilde{b},0+}_{kl}H_l^++\left(H_k^0\,c^{\tilde{t},0+}_{kl}H_l^+\right)^*\right]\\
&= \frac{g^2}{2\sqrt{2}}\left[\left(\left(\tilde{t}^*_L\tilde{b}_L\right)+\tilde{\nu}^*_\tau\tilde{\tau}_L\right)\left(H_k^0\left(c^{\tilde{b},0+}_{kl'}\right)^*H_l^++H_k^0\,c^{\tilde{t},0+}_{kl}H_l^+\right)\right.\\
&\qquad\left.+\left(\left(\tilde{b}^*_L\tilde{t}_L\right)+\tilde{\tau}^*_L\tilde{\nu}_\tau\right)\left(H_k^0\,c^{\tilde{b},0+}_{kl}H_l^++H_k^0\left(c^{\tilde{t},0+}_{kl'}\right)^*H_l^+\right)\right]\\
&= \frac{g^2}{2\sqrt{2}}\sum_f R^{\tilde{f}}_{i1}R^{\tilde{f}'}_{j1}\left(c^{\tilde{f},0+}_{kl}+\left(c^{\tilde{f}',0+}_{kl'}\right)^*\right)H_k^0H_l^+\tilde{f}^*_i\tilde{f}'_j\,, \qquad \text{(F.68)}
\end{aligned}$$

which results in the Feynman rules



$$-\frac{i}{\sqrt{2}}\frac{g^2}{2}R^{\tilde{f}}_{i1}R^{\tilde{f}'}_{j1}\left(c^{\tilde{f},0+}_{kl}+\left(c^{\tilde{f}',0+}_{kl'}\right)^*\right)$$

F.6.4 Higgs–Higgs–Higgs–Higgs

The 4–Higgs couplings are only coming from the D–terms, and therefore we have (see eqs. (F.42)–(F.44))

$$\begin{aligned} V_D &\supset \frac{g'^2}{8}\left[\left|H_1^0\right|^2-\left|H_2^0\right|^2+\left|H_1^2\right|^2-\left|H_2^1\right|^2\right]^2+\frac{g^2}{8}\left[\left|H_1^0\right|^2-\left|H_2^0\right|^2-\left|H_1^2\right|^2+\left|H_2^1\right|^2\right]^2 \\ &+\frac{g^2}{2}\left[\left|H_1^0\right|^2\left|H_1^2\right|^2+\left|H_2^0\right|^2\left|H_2^1\right|^2+H_1^{0*}H_1^2H_2^{0*}H_2^1+H_2^{1*}H_2^0H_1^{2*}H_1^0\right], \end{aligned} \tag{F.69}$$

or in a more compact way

$$V_D \supset \frac{1}{8}(g^2+g'^2)\left(H_1^{i*}H_1^i-H_2^{i*}H_2^i\right)^2+\frac{g^2}{2}\left|H_1^{i*}H_2^i\right|^2 . \tag{F.70}$$

$H_k^0 H_l^0 H_m^0 H_n^0$ couplings

The 4–neutral Higgs couplings are obtained from the following term of the D–term potential,

$$V_D \supset \frac{1}{8}(g^2+g'^2)\left(\left|H_1^0\right|^2-\left|H_2^0\right|^2\right)^2 , \tag{F.71}$$

which can be expressed in components,

$$\begin{aligned} V_D \supset \frac{g^2}{32c_W^2}\Big[&\Big((h^0)^2-(H^0)^2\Big)\cos 2\alpha+\Big((A^0)^2-(G^0)^2\Big)\cos 2\beta+2\Big(h^0H^0\sin 2\alpha \\ &+A^0G^0\sin 2\beta\Big)\Big]^2, \end{aligned} \tag{F.72}$$

as well as in index notation (see eqs.(F.30), (F.31), (F.34) and (F.35)),

$$V_D \supset \frac{g^2}{32c_W^2}\sum_{k,l,m,n} H_k^0H_l^0H_m^0H_n^0\left(c^{\tilde b}_{kl}c^{\tilde b}_{mn}+c^{\tilde t}_{kl}c^{\tilde t}_{mn}-c^{\tilde b}_{kl}c^{\tilde t}_{mn}-c^{\tilde t}_{kl}c^{\tilde b}_{mn}\right) . \tag{F.73}$$

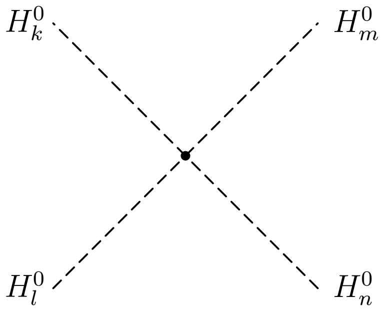
To get a special coupling out of the potential we either can sum over all indices an then pick out the single terms which belong to the required coupling or we symmetrize the coupling matrices $c^{\tilde b}$ and $c^{\tilde t}$ in eq. (F.73), fix the indices belonging to the Higgs fields H_k^0 and take a combinatorial factor for multiple counting into consideration. Here we choose the second possibility:

$$V_D \supset \frac{g^2}{32c_W^2}H_k^0H_l^0H_m^0H_n^0\left(c^{\tilde b}_{(kl}c^{\tilde b}_{mn)}+c^{\tilde t}_{(kl}c^{\tilde t}_{mn)}-c^{\tilde b}_{(kl}c^{\tilde t}_{mn)}-c^{\tilde t}_{(kl}c^{\tilde b}_{mn)}\right)\times \mathrm{CF} \tag{F.74}$$

(no sum over indices, this is respected in the combinatorial factor!) Here the brackets around the indices denote symmetrization and CF stands for a combinatorial factor, which is given for a general coupling $(h^0)^a(H^0)^b(A^0)^c(G^0)^d$ with $a+b+c+d=4$ by

$$\mathrm{CF} = \begin{pmatrix}4\\a\end{pmatrix}\cdot\begin{pmatrix}4-a\\b\end{pmatrix}\cdot\begin{pmatrix}4-a-b\\c\end{pmatrix}\cdot\begin{pmatrix}4-a-b-c\\d\end{pmatrix} = \frac{4!}{a!\ b!\ c!\ d!} \tag{F.75}$$

In the Feynman rule we have to take the symmetry factor (SF) of the diagram into account ($n!$ for n equal neutral particles):



$$-i\,\frac{g^2}{32c_W^2}\left(c^{\tilde b}_{(kl}c^{\tilde b}_{mn)}+c^{\tilde t}_{(kl}c^{\tilde t}_{mn)}-c^{\tilde b}_{(kl}c^{\tilde t}_{mn)}-c^{\tilde t}_{(kl}c^{\tilde b}_{mn)}\right)\\ \times\,\mathrm{CF}\times\mathrm{SF}$$

As an example we take the coupling $h^0H^0(A^0)^2$. For the indices we choose $k=1, l=2$ and $m=n=3$. The combinatorial factor then is given by

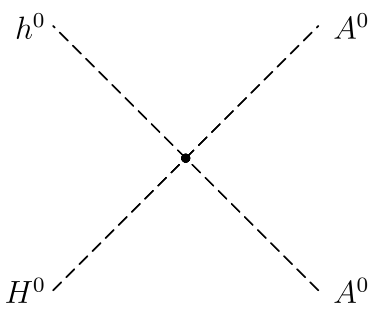
$$\mathrm{CF}\;=\;\begin{pmatrix}4\\1\end{pmatrix}\cdot\begin{pmatrix}3\\1\end{pmatrix}\cdot\begin{pmatrix}2\\2\end{pmatrix}\cdot\begin{pmatrix}0\\0\end{pmatrix}\;=\;4\cdot3\cdot1\cdot1\;=\;12\,,$$

so the interaction Lagrangian reads with (note that the matrices $c^{\tilde b}$ and $c^{\tilde t}$ are symmetric)

$$\begin{aligned}\left(c^{\tilde b}_{(12}c^{\tilde b}_{33)}+c^{\tilde t}_{(12}c^{\tilde t}_{33)}-c^{\tilde b}_{(12}c^{\tilde t}_{33)}-c^{\tilde t}_{(12}c^{\tilde b}_{33)}\right)\;&=\;\frac{1}{3}\Bigg[\left(c^{\tilde b}_{12}c^{\tilde b}_{33}+c^{\tilde b}_{13}c^{\tilde b}_{23}+c^{\tilde b}_{13}c^{\tilde b}_{23}\right)\\ -\;\left(c^{\tilde b}_{12}c^{\tilde t}_{33}+c^{\tilde b}_{13}c^{\tilde t}_{23}+c^{\tilde b}_{13}c^{\tilde t}_{23}\right)\;+\;\tilde b\leftrightarrow\tilde t\,\Bigg]\;&=\;\frac{1}{3}\sin2\alpha\cos2\beta\qquad,\end{aligned}$$

and therefore

$$\mathcal{L}\;=\;-\frac{g^2}{8c_W^2}\sin2\alpha\cos2\beta\,h^0H^0(A^0)^2\,. \tag{F.76}$$



$$-i\,\frac{g^2}{4c_W^2}\sin2\alpha\cos2\beta$$

$\boldsymbol{H_k^+H_l^-H_m^+H_n^-}$ couplings

Like in the case of 4–neutral Higgs boson couplings we get the 4–charged Higgs couplings from the D–term potential

$$
\begin{aligned}
V_D &\supset \frac{1}{8}(g^2+g'^2)\left(\left|H_1^2\right|^2-\left|H_2^1\right|^2\right)^2 \\
&= \frac{g^2}{8c_W^2}\left[H^-\Big(\sin 2\beta G^+ + \cos 2\beta H^+\Big) + G^-\Big(-\cos 2\beta G^+ + \sin 2\beta H^+\Big)\right]^2 . \quad \text{(F.77)}
\end{aligned}
$$

In index notation this reads with the abbreviations defined in eqs. (F.30) –(F.37)

$$
V_D \supset \frac{g^2}{32c_W^2}\sum_{k,l,m,n} H_k^+H_l^-H_m^+H_n^-\left(d_{kl}^{\tilde b}d_{mn}^{\tilde b}+d_{kl}^{\tilde t}d_{mn}^{\tilde t}-d_{kl}^{\tilde b}d_{mn}^{\tilde t}-d_{kl}^{\tilde t}d_{mn}^{\tilde b}\right) . \quad \text{(F.78)}
$$

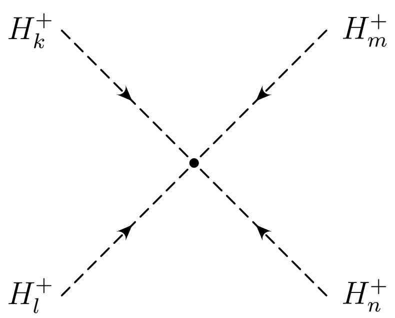
To be able to pick out the various couplings out of eq. (F.78) in the same way as we did in the case of the neutral Higgs bosons we first must express all fields in one single base, H_k^+ or H_k^-:

$$
V_D \supset \frac{g^2}{32c_W^2}\sum_{k,l,m,n} H_k^+H_l^+H_m^+H_n^+\left(d_{kl}^{\tilde b}d_{mn}^{\tilde b}+d_{kl}^{\tilde t}d_{mn}^{\tilde t}-d_{kl}^{\tilde b}d_{mn}^{\tilde t}-d_{kl}^{\tilde t}d_{mn}^{\tilde b}\right)_{\text{rot}} , \quad \text{(F.79)}
$$

with

$$
\left(d_{kl}^{\tilde f}\,d_{mn}^{\tilde f}\right)_{\text{rot}} = \left(d_{kl}^{\tilde f}\right)_{\text{rot}}\left(d_{mn}^{\tilde f}\right)_{\text{rot}} = \sum_{l',n'=1}^{4} d_{kl'}^{\tilde f}\begin{pmatrix}0 & 1_{2\text{x}2}\\ 1_{2\text{x}2} & 0\end{pmatrix}_{l'l} d_{mn'}^{\tilde f}\begin{pmatrix}0 & 1_{2\text{x}2}\\ 1_{2\text{x}2} & 0\end{pmatrix}_{n'n} .
$$

Now we can symmetrize the rotated coupling matrices as before if we want to fix the indices in the Higgs fields. The combinatorial factor CF stays the same but we don't have to take a symmetry factor in the Feynman rule because of charged particles:



$$
-i\,\frac{g^2}{32c_W^2}\left(d_{kl}^{\tilde b}d_{mn}^{\tilde b}+d_{kl}^{\tilde t}d_{mn}^{\tilde t}-d_{kl}^{\tilde b}d_{mn}^{\tilde t}-d_{kl}^{\tilde t}d_{mn}^{\tilde b}\right)_{\text{rot, symm}} \times \text{CF}
$$

F.6.5 Sfermion–Sfermion–Sfermion–Sfermion

As we already mentioned before, we have to take care about the colour indices in this term of the superpotential,

$$
W \supset h_t\,\tilde t_R^*\tilde t_L H_2^0 + h_b\,\tilde b_R^*\tilde b_L H_1^0 + h_\tau\,\tilde\tau_R^*\tilde\tau_L H_1^0 - h_t\,\tilde t_R^*\tilde b_L H_2^1 - h_b\,\tilde b_R^*\tilde t_L H_1^2 - h_\tau\,\tilde\tau_R^*\tilde\nu_\tau H_1^2 , \quad \text{(F.80)}
$$

which leads to the F–term potential

$$\begin{aligned} V_F \;\supset\; & h_t^2\big(\tilde{t}_R^*\tilde{t}_L\big)\big(\tilde{t}_L^*\tilde{t}_R\big) + h_b^2\big(\tilde{b}_R^*\tilde{b}_L\big)\big(\tilde{b}_L^*\tilde{b}_R\big) + h_\tau^2\big(\tilde{\tau}_R^*\tilde{\tau}_L\big)\big(\tilde{\tau}_L^*\tilde{\tau}_R\big) \\ & + h_t^2\big(\tilde{t}_R^*\tilde{b}_L\big)\big(\tilde{b}_L^*\tilde{t}_R\big) + h_b^2\big(\tilde{b}_R^*\tilde{t}_L\big)\big(\tilde{t}_L^*\tilde{b}_R\big) + h_\tau^2\big(\tilde{\tau}_R^*\tilde{\nu}_\tau\big)\big(\tilde{\nu}_\tau^*\tilde{\tau}_R\big) \\ & + h_b h_\tau\Big(\tilde{b}_R^*\tilde{b}_L\tilde{\tau}_L^*\tilde{\tau}_R + \tilde{b}_L^*\tilde{b}_R\tilde{\tau}_R^*\tilde{\tau}_L + \tilde{t}_L^*\tilde{b}_R\tilde{\tau}_R^*\tilde{\nu}_\tau + \tilde{b}_R^*\tilde{t}_L\tilde{\nu}_\tau^*\tilde{\tau}_R\Big) \\ = \; & \sum_f \bigg[h_f^2\Big(\big(\tilde{f}_R^*\tilde{f}_L\big)\big(\tilde{f}_L^*\tilde{f}_R\big) + \big(\tilde{f}_R^*\tilde{f}_L'\big)\big(\tilde{f}_L'^*\tilde{f}_R\big)\Big) + h_f h_{\hat{f}}\Big(\tilde{f}_R^*\tilde{f}_L\hat{\tilde{f}}_L^*\hat{\tilde{f}}_R + \tilde{f}_R^*\tilde{f}_L'\hat{\tilde{f}}_L'^*\hat{\tilde{f}}_R\Big)\bigg]. \end{aligned} \tag{F.81}$$

$\tilde{f}_i\tilde{f}_j\tilde{f}_k\tilde{f}_l$ couplings

With the initial and final states for the first term,

$$\begin{aligned} |i\rangle &= a_{l\delta}^\dagger(p_1)b_{k\gamma}^\dagger(p_2)|0\rangle\,, \\ \langle f| &= \langle 0|a_{i\alpha}(k_1)b_{j\beta}(k_2)\,, \end{aligned}$$

we get for the S–matrix element

$$\begin{aligned} S_{fi}^{(1)} &= i\int d^4x\,\langle f|:\mathcal{L}_{int}(x):|i\rangle = -ih_f^2\,R_{m1}^{\tilde{f}}R_{n1}^{\tilde{f}}R_{p2}^{\tilde{f}}R_{q2}^{\tilde{f}} \\ &\times \int d^4x\,\langle 0|a_{i\alpha}(k_1)b_{j\beta}(k_2):\Big(\tilde{f}_{m\gamma'}^*\tilde{f}_{n\beta'}\tilde{f}_{p\alpha'}^*\tilde{f}_{q\delta'}\Big)(x):a_{l\delta}^\dagger(p_1)b_{k\gamma}^\dagger(p_2)|0\rangle\,\delta_{\alpha'\beta'}\delta_{\gamma'\delta'} \end{aligned} \tag{F.82}$$

In order to evaluate the vacuum expectation value we make use of Wick's theorem in taking all possible contractions:

$$\begin{aligned} S_{fi}^{(1)} &= -ih_f^2\,R_{m1}^{\tilde{f}}R_{n1}^{\tilde{f}}R_{p2}^{\tilde{f}}R_{q2}^{\tilde{f}}\,(2\pi)^4\delta^4(p_1+p_2-k_1-k_2)\,\delta_{\alpha'\beta'}\delta_{\gamma'\delta'} \\ &\times\big(\delta_{im}\delta_{\alpha\gamma'}\,\delta_{jn}\delta_{\beta\beta'}\,\delta_{ql}\delta_{\delta'\delta}\,\delta_{pk}\delta_{\alpha'\gamma} + \delta_{im}\delta_{\alpha\gamma'}\,\delta_{jq}\delta_{\beta\delta'}\,\delta_{nl}\delta_{\beta'\delta}\,\delta_{pk}\delta_{\alpha'\gamma} \\ &\quad + \delta_{ip}\delta_{\alpha\alpha'}\,\delta_{jn}\delta_{\beta\beta'}\,\delta_{ql}\delta_{\delta'\delta}\,\delta_{mk}\delta_{\gamma'\gamma} + \delta_{ip}\delta_{\alpha\alpha'}\,\delta_{jq}\delta_{\beta\delta'}\,\delta_{mk}\delta_{\gamma'\gamma}\,\delta_{nl}\delta_{\beta'\delta}\big) \end{aligned} \tag{F.83}$$

In the last equation we have used the notation

$$\delta_{\alpha\beta\gamma\delta,\alpha'\beta'\gamma'\delta'} \equiv \delta_{\alpha\alpha'}\,\delta_{\beta\beta'}\,\delta_{\gamma\gamma'}\,\delta_{\delta\delta'}\,, \qquad \rightarrow \qquad \delta_{\alpha\beta\gamma\delta,\alpha'\beta'\alpha'\beta'} = \delta_{\alpha\gamma}\,\delta_{\beta\delta} \tag{F.84}$$

and

$$R_{ijkl}^{\tilde{f}} \equiv R_{i1}^{\tilde{f}}R_{j1}^{\tilde{f}}R_{k2}^{\tilde{f}}R_{l2}^{\tilde{f}}\,. \tag{F.85}$$

The corresponding D–term potential is given by (see eqs. (F.42) and (F.44))

$$V_D \;\supset\; \frac{g'^2}{8}\sum_f\Big(Y_{\tilde{f}_L}\big(\tilde{f}_L^*\tilde{f}_L\big) - Y_{\tilde{f}_R}\big(\tilde{f}_R^*\tilde{f}_R\big)\Big)\Big(Y_{\tilde{f}_L}\big(\tilde{f}_L^*\tilde{f}_L\big) - Y_{\tilde{f}_R}\big(\tilde{f}_R^*\tilde{f}_R\big)\Big)$$

$$+\frac{g^2}{8}\sum_f \left(\tilde{f}_L^*\tilde{f}_L\right)\left(\tilde{f}_L^*\tilde{f}_L\right). \tag{F.86}$$

Rotating the sfermion fields into their mass eigenstates, $\tilde{f}_L = R^{\tilde{f}}_{i1}\tilde{f}_i$, $\tilde{f}_R = R^{\tilde{f}}_{i2}\tilde{f}_i$, the D–term potential for 4–sfermions reads

$$\begin{aligned} V_D \;&\supset\; \frac{g^2}{8}\sum_{\tilde{f}}\left[R^{\tilde{f}_L}_{ijkl} + t_W^2 Y^2_{\tilde{f}_L} R^{\tilde{f}_L}_{ijkl} + t_W^2 Y^2_{\tilde{f}_R} R^{\tilde{f}_R}_{ijkl} - t_W^2 Y_{\tilde{f}_L} Y_{\tilde{f}_R}\left(R^{\tilde{f}}_{ijkl} + R^{\tilde{f}}_{klij}\right)\right]\left(\tilde{f}_i^*\tilde{f}_j\right)\left(\tilde{f}_k^*\tilde{f}_l\right) \\ &\equiv\; \sum_{\tilde{f}} C^{\tilde{f}\tilde{f}}_{ijkl}\left(\tilde{f}_i^*\tilde{f}_j\right)\left(\tilde{f}_k^*\tilde{f}_l\right), \end{aligned} \tag{F.87}$$

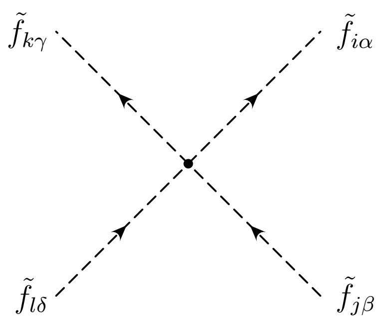
where we have used the relation $g' = g\tan\theta_W$ as well as the abbreviations

$$R^{\tilde{f}_L}_{ijkl} = R^{\tilde{f}}_{i1}R^{\tilde{f}}_{j1}R^{\tilde{f}}_{k1}R^{\tilde{f}}_{l1}, \qquad R^{\tilde{f}_R}_{ijkl} = R^{\tilde{f}}_{i2}R^{\tilde{f}}_{j2}R^{\tilde{f}}_{k2}R^{\tilde{f}}_{l2}. \tag{F.88}$$

Like in the calculation of the Yukawa coupling terms in the previous section we get for the Feynman amplitude

$$\mathcal{M} \;=\; -i\left[\left(C^{\tilde{f}\tilde{f}}_{ijkl} + C^{\tilde{f}\tilde{f}}_{klij}\right)\delta_{\alpha\beta}\delta_{\gamma\delta} + \left(C^{\tilde{f}\tilde{f}}_{ilkj} + C^{\tilde{f}\tilde{f}}_{kjil}\right)\delta_{\alpha\delta}\delta_{\beta\gamma}\right]. \tag{F.89}$$

Both, $F-$ and D–terms, result in the Feynman rule



$$\begin{aligned} &-i\,h_f^2\left[\left(R^{\tilde{f}}_{ijkl} + R^{\tilde{f}}_{klij}\right)\delta_{\alpha\delta}\delta_{\beta\gamma} \;+\; \left(R^{\tilde{f}}_{ilkj} + R^{\tilde{f}}_{kjil}\right)\delta_{\alpha\beta}\delta_{\gamma\delta}\right] \\ &-i\left[\left(C^{\tilde{f}\tilde{f}}_{ijkl} + C^{\tilde{f}\tilde{f}}_{klij}\right)\delta_{\alpha\beta}\delta_{\gamma\delta} + \left(C^{\tilde{f}\tilde{f}}_{ilkj} + C^{\tilde{f}\tilde{f}}_{kjil}\right)\delta_{\alpha\delta}\delta_{\beta\gamma}\right] \end{aligned}$$

$\tilde{f}_i\tilde{f}_j\tilde{f}'_k\tilde{f}'_l$ couplings

For the second term of eq. (F.81) we take the initial and final states

$$\begin{aligned} |i\rangle \;&=\; a'^{\dagger}_{l\delta}(p_1)b'^{\dagger}_{k\gamma}(p_2)|0\rangle, \\ \langle f| \;&=\; \langle 0|a_{i\alpha}(k_1)b_{j\beta}(k_2), \end{aligned}$$

which leads with

$$R^{\tilde{f}'\tilde{f}_D}_{ijkl} \equiv R^{\tilde{f}'}_{i1}R^{\tilde{f}'}_{j1}R^{\tilde{f}}_{k2}R^{\tilde{f}}_{l2}, \qquad R^{\tilde{f}\tilde{f}'_D}_{ijkl} \equiv R^{\tilde{f}}_{i1}R^{\tilde{f}}_{j1}R^{\tilde{f}'}_{k2}R^{\tilde{f}'}_{l2} \tag{F.90}$$

to the Feynman amplitude

$$\mathcal{M} \;=\; -i\left(h_f^2\,R^{\tilde{f}'\tilde{f}_D}_{klij} + h^2_{f'}R^{\tilde{f}\tilde{f}'_D}_{ijkl}\right)\delta_{\alpha\delta}\delta_{\beta\gamma}. \tag{F.91}$$

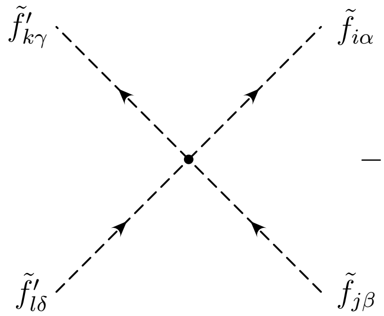
For the D–term potential we get with the eqs. (F.42) – (F.44)

$$
\begin{aligned}
V_D \;\supset\;& \frac{g^2}{4}\sum_f \Big((\tilde{f}_L^* \tilde{f}'_L)(\tilde{f}'^*_L \tilde{f}_L) - \frac{1}{2}(\tilde{f}_L^* \tilde{f}_L)(\tilde{f}'^*_L \tilde{f}'_L) \Big) \\
&+ \frac{g'^2}{8}\sum_f \Big(Y_{\tilde{f}_L}(\tilde{f}_L^* \tilde{f}_L) - Y_{\tilde{f}_R}(\tilde{f}_R^* \tilde{f}_R) \Big)\Big(Y_{\tilde{f}'_L}(\tilde{f}'^*_L \tilde{f}'_L) - Y_{\tilde{f}'_R}(\tilde{f}'^*_R \tilde{f}'_R) \Big) \qquad \text{(F.92)}
\end{aligned}
$$

and with a little bit of cosmetics we have

$$
\begin{aligned}
V_D \;\supset\;& \frac{g^2}{8}\sum_f \Bigg\{ \bigg[\Big(t_W^2 \big(Y_{\tilde{f}_L} Y_{\tilde{f}'_L}\big) - 1 \Big) R^{\tilde{f}\tilde{f}'_L}_{ijkl} + t_W^2 \big(Y_{\tilde{f}_R} Y_{\tilde{f}'_R}\big)\, R^{\tilde{f}\tilde{f}'_R}_{ijkl} - t_W^2 \big(Y_{\tilde{f}_L} Y_{\tilde{f}'_R}\big)\, R^{\tilde{f}\tilde{f}'_D}_{ijkl} \\
&\qquad - t_W^2 \big(Y_{\tilde{f}_R} Y_{\tilde{f}'_L}\big)\, R^{\tilde{f}'\tilde{f}_D}_{klij} \bigg] \delta_{\alpha\beta}\delta_{\gamma\delta} + 2R^{\tilde{f}\tilde{f}'_L}_{ijkl}\,\delta_{\alpha\delta}\delta_{\beta\gamma} \Bigg\} \tilde{f}^*_{i\alpha}\tilde{f}_{j\beta}\tilde{f}'^*_{k\gamma}\tilde{f}'_{l\delta} \\
\equiv\;& \sum_f C^{\tilde{f}\tilde{f}'}_{ijkl,\alpha\beta\gamma\delta}\; \tilde{f}^*_{i\alpha}\tilde{f}_{j\beta}\tilde{f}'^*_{k\gamma}\tilde{f}'_{l\delta}\,. \qquad \text{(F.93)}
\end{aligned}
$$

The Yukawa and electroweak contributions to the Feynman rule are then given by



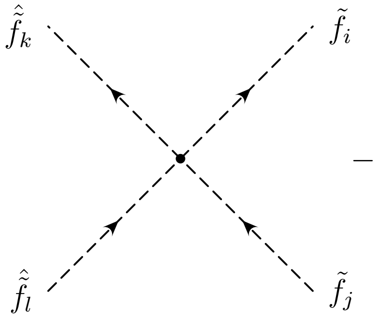
$$
-i\Big(h_f^2\, R^{\tilde{f}'\tilde{f}_D}_{klij} + h_{f'}^2 R^{\tilde{f}\tilde{f}'_D}_{ijkl}\Big)\delta_{\alpha\delta}\delta_{\beta\gamma} - i\Big(C^{\tilde{f}\tilde{f}'}_{ijkl,\alpha\beta\gamma\delta} + C^{\tilde{f}'\tilde{f}}_{klij,\gamma\delta\alpha\beta}\Big)
$$

$\tilde{f}_i\tilde{f}_j\hat{\tilde{f}}_k\hat{\tilde{f}}_l$ couplings

The Feynman rule for couplings with two sfermions and two 'family partner' sfermions can be obtained from eqs. (F.42), (F.44) and (F.81):

$$
\begin{aligned}
V_{F+D} \;\supset\;& \sum_f \bigg[h_f h_{\hat{f}} (\tilde{f}_R^* \tilde{f}_L)(\hat{\tilde{f}}_L^* \hat{\tilde{f}}_R) + \frac{g^2}{8}(\tilde{f}_L^* \tilde{f}_L)(\hat{\tilde{f}}_L^* \hat{\tilde{f}}_L) \\
&\qquad + \frac{g'^2}{8}\Big(Y_{\tilde{f}_L}(\tilde{f}_L^* \tilde{f}_L) - Y_{\tilde{f}_R}(\tilde{f}_R^* \tilde{f}_R) \Big)\Big(Y_{\hat{\tilde{f}}_L}(\hat{\tilde{f}}_L^* \hat{\tilde{f}}_L) - Y_{\hat{\tilde{f}}_R}(\hat{\tilde{f}}_R^* \hat{\tilde{f}}_R) \Big) \bigg] \\
=\;& \sum_f \bigg\{ h_f h_{\hat{f}}\, R^{\tilde{f}\hat{\tilde{f}}_F}_{jikl} + \frac{g^2}{8} R^{\tilde{f}\hat{\tilde{f}}_L}_{ijkl} + \frac{g'^2}{8}\Big[Y_{\tilde{f}_L} Y_{\hat{\tilde{f}}_L} R^{\tilde{f}\hat{\tilde{f}}_L}_{ijkl} + Y_{\tilde{f}_R} Y_{\hat{\tilde{f}}_R} R^{\tilde{f}\hat{\tilde{f}}_R}_{ijkl} - Y_{\tilde{f}_L} Y_{\hat{\tilde{f}}_R} R^{\tilde{f}\hat{\tilde{f}}_D}_{ijkl} \\
&\qquad - Y_{\tilde{f}_R} Y_{\hat{\tilde{f}}_L} R^{\hat{\tilde{f}}\tilde{f}_D}_{klij} \Big]\bigg\} (\tilde{f}_i^* \tilde{f}_j)(\hat{\tilde{f}}_k^* \hat{\tilde{f}}_l)
\end{aligned}
$$

$$\equiv \sum_f \Big[h_f h_{\hat f}\, R^{\tilde f \hat{\tilde f}_F}_{jikl} + C^{\tilde f \hat{\tilde f}}_{ijkl} \Big] \big(\tilde f_i^* \tilde f_j\big)\big(\hat{\tilde f}_k^* \hat{\tilde f}_l\big) \tag{F.94}$$

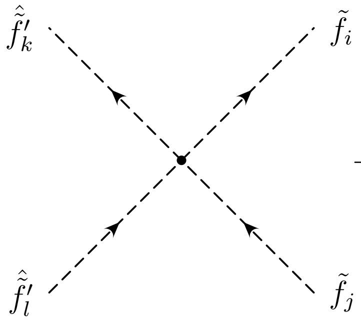


$$-ih_f h_{\hat f}\Big(R^{\tilde f \hat{\tilde f}_F}_{jikl} + R^{\hat{\tilde f} \tilde f_F}_{lkij}\Big) - i\Big(C^{\tilde f \hat{\tilde f}}_{ijkl} + C^{\hat{\tilde f} \tilde f}_{klij}\Big)$$

$\tilde f_i \tilde f_j \hat{\tilde f}'_k \hat{\tilde f}'_l$ couplings

For the coupling with two sfermions $\tilde f_i$ and two family partner sfermions with different isospin, $\hat{\tilde f}'_i$, we get with eqs. (F.42) and (F.44)

$$\begin{aligned} V_D \;\supset\; & \sum_f \Big[-\frac{g^2}{8}\big(\tilde f_L^* \tilde f_L\big)\big(\hat{\tilde f}_L^* \hat{\tilde f}_L\big) + \frac{g'^2}{8}\Big(Y_{\tilde f_L}\big(\tilde f_L^* \tilde f_L\big) - Y_{\tilde f_R}\big(\tilde f_R^* \tilde f_R\big)\Big) \\ & \qquad \Big(Y_{\hat{\tilde f}'_L}\big(\hat{\tilde f}'^*_L \hat{\tilde f}'_L\big) - Y_{\hat{\tilde f}'_R}\big(\hat{\tilde f}'^*_R \hat{\tilde f}'_R\big)\Big)\Big] \\ = \; & \sum_f \Big\{ -\frac{g^2}{8} R^{\tilde f \hat{\tilde f}'_L}_{ijkl} + \frac{g'^2}{8}\Big[Y_{\tilde f_L} Y_{\hat{\tilde f}'_L} R^{\tilde f \hat{\tilde f}'_L}_{ijkl} + Y_{\tilde f_R} Y_{\hat{\tilde f}'_R} R^{\tilde f \hat{\tilde f}'_R}_{ijkl} - Y_{\tilde f_L} Y_{\hat{\tilde f}'_R} R^{\tilde f \hat{\tilde f}'_D}_{ijkl} - Y_{\tilde f_R} Y_{\hat{\tilde f}'_L} R^{\hat{\tilde f}' \tilde f_D}_{klij} \Big]\Big\} \\ & \qquad \times \big(\tilde f_i^* \tilde f_j\big)\big(\hat{\tilde f}'^*_k \hat{\tilde f}'_l\big) \;\equiv\; \sum_f C^{\tilde f \hat{\tilde f}'}_{ijkl} \big(\tilde f_i^* \tilde f_j\big)\big(\hat{\tilde f}'^*_k \hat{\tilde f}'_l\big)\,. \end{aligned} \tag{F.95}$$



$$-i\Big(C^{\tilde f \hat{\tilde f}'}_{ijkl} + C^{\hat{\tilde f}' \tilde f}_{klij}\Big)$$

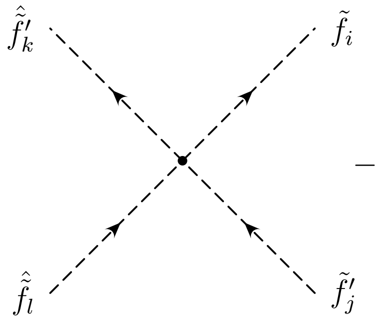
$\tilde{f}_i \tilde{f}'_j \hat{\tilde{f}}'_k \hat{\tilde{f}}_l$ couplings

Finally, eqs. (F.43) and (F.81) give the Feynman rule for the mixed four–sfermion coupling,

$$\begin{aligned} V_{F+D} &\supset \sum_f \Big(- h_f h_{\hat{f}} \tilde{f}_R^* \tilde{f}'_L \hat{\tilde{f}}_L'^* \hat{\tilde{f}}_R + \frac{g^2}{4} \tilde{f}_L^* \tilde{f}'_L \hat{\tilde{f}}_L'^* \hat{\tilde{f}}_L \Big) \\ &= \sum_f \Big(- h_f h_{\hat{f}} R^{\tilde{f}' \tilde{f} \hat{\tilde{f}}' \hat{\tilde{f}}_F}_{jikl} + \frac{g^2}{4} R^{\tilde{f} \tilde{f}' \hat{\tilde{f}}' \hat{\tilde{f}}_L}_{ijkl} \Big) \tilde{f}_i^* \tilde{f}'_j \hat{\tilde{f}}_k'^* \hat{\tilde{f}}_l \end{aligned} \tag{F.96}$$

with

$$R^{\tilde{f}' \tilde{f} \hat{\tilde{f}}' \hat{\tilde{f}}_F}_{ijkl} \equiv R^{\tilde{f}'}_{i1} R^{\tilde{f}}_{j2} R^{\hat{\tilde{f}}'}_{k1} R^{\hat{\tilde{f}}}_{l2}\,, \qquad R^{\tilde{f} \tilde{f}' \hat{\tilde{f}}' \hat{\tilde{f}}_L}_{ijkl} \equiv R^{\tilde{f}}_{i1} R^{\tilde{f}'}_{j1} R^{\hat{\tilde{f}}'}_{k1} R^{\hat{\tilde{f}}}_{l1}\,.$$



$$-i h_f h_{\hat{f}} R^{\tilde{f}' \tilde{f} \hat{\tilde{f}}' \hat{\tilde{f}}_F}_{jikl} - i h_{\hat{f}'} h_{f'} R^{\hat{\tilde{f}} \hat{\tilde{f}}' \tilde{f} \tilde{f}'_F}_{lkij} - i \frac{g^2}{2} R^{\tilde{f} \tilde{f}' \hat{\tilde{f}}' \hat{\tilde{f}}_L}_{ijkl}$$

F.7 Vector boson–Sfermion–Sfermion couplings

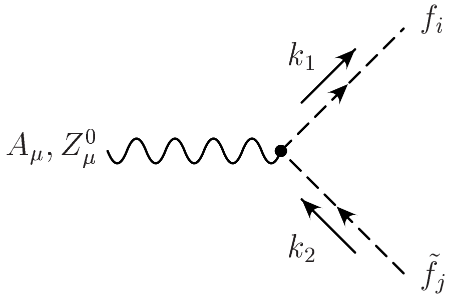
The interaction Lagrangian of a vector boson and two sfermions is given by

$$\mathcal{L} = -i\, e\, e_f\, A_\mu\, \tilde{f}_i^* \overleftrightarrow{\partial^\mu} \tilde{f}_j - i\, g_Z\, z^{\tilde{f}}_{ij}\, Z^0_\mu\, \tilde{f}_i^* \overleftrightarrow{\partial^\mu} \tilde{f}_j + \left(-i \frac{g}{\sqrt{2}} W^+_\mu\, R^{\tilde{f}_\uparrow}_{i1} R^{\tilde{f}_\downarrow}_{j1} \tilde{f}^*_{\uparrow i} \overleftrightarrow{\partial^\mu} \tilde{f}_{\downarrow j} + \text{h.c.} \right)\,, \tag{F.97}$$

with the abbreviations

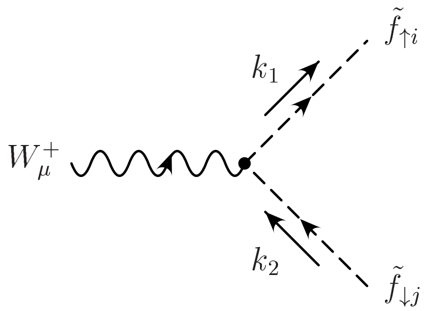
$$z^{\tilde{f}}_{ij} = C^f_L\, R^{\tilde{f}}_{i1} R^{\tilde{f}}_{j1} + C^f_R\, R^{\tilde{f}}_{i2} R^{\tilde{f}}_{j2} \tag{F.98}$$

and $g_Z = g/\cos\theta_W$, $C^f_L = I^{3L}_f - e_f s^2_W$ and $C^f_R = -e_f s^2_W$.



$$-ie\, e_f \delta_{ij} (k_1 + k_2)_\mu \qquad \ldots A_\mu$$

$$i g_Z z^{\tilde{f}}_{ij} (k_1 + k_2)_\mu \qquad \ldots Z^0_\mu$$



$$-i\,\frac{g}{\sqrt{2}}\,R^{\tilde{f}_\uparrow}_{i1}R^{\tilde{f}_\downarrow}_{j1}(k_1+k_2)_\mu$$

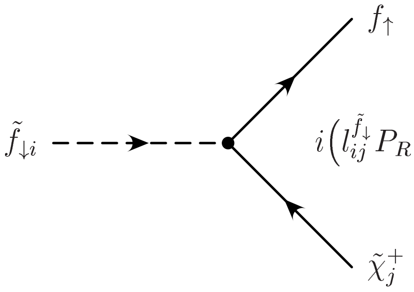
F.8 Gaugino–Fermion–Sfermion couplings

The interaction Lagrangian of the chargino–sfermion–fermion couplings is given by

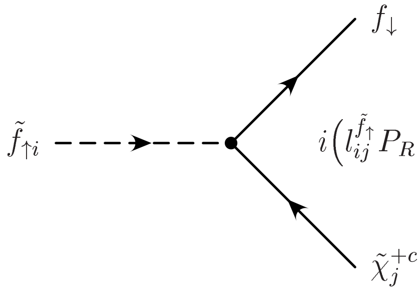
$$\begin{aligned}\mathcal{L} \;&=\; \bar{f}_\uparrow\Big(l^{\tilde{f}_\downarrow}_{ij}P_R+k^{\tilde{f}_\downarrow}_{ij}P_L\Big)\tilde{\chi}^+_j\,\tilde{f}_{\downarrow i}+\bar{f}_\downarrow\Big(l^{\tilde{f}_\uparrow}_{ij}P_R+k^{\tilde{f}_\uparrow}_{ij}P_L\Big)\tilde{\chi}^{+c}_j\,\tilde{f}_{\uparrow i}\\ &\quad+\bar{\tilde{\chi}}^+_j\Big(l^{\tilde{f}_\downarrow}_{ij}P_L+k^{\tilde{f}_\downarrow}_{ij}P_R\Big)f_\uparrow\,\tilde{f}^*_{\downarrow i}+\bar{\tilde{\chi}}^{+c}_j\Big(l^{\tilde{f}_\uparrow}_{ij}P_L+k^{\tilde{f}_\uparrow}_{ij}P_R\Big)f_\downarrow\,\tilde{f}^*_{\uparrow i} \qquad \text{(F.99)}\end{aligned}$$

with the coupling matrices

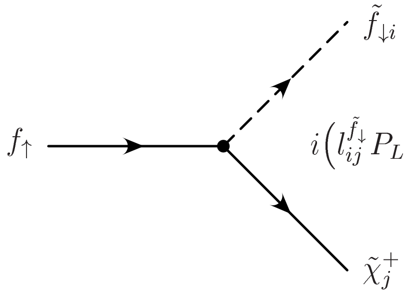
$$\begin{aligned} l^{\tilde{f}_\uparrow}_{ij} &= -gV_{j1}R^{\tilde{f}_\uparrow}_{i1}+h_{f_\uparrow}V_{j2}R^{\tilde{f}_\uparrow}_{i2}\,, &\qquad l^{\tilde{f}_\downarrow}_{ij} &= -gU_{j1}R^{\tilde{f}_\downarrow}_{i1}+h_{f_\downarrow}U_{j2}R^{\tilde{f}_\downarrow}_{i2}\,,\\ k^{\tilde{f}_\uparrow}_{ij} &= h_{f_\downarrow}U_{j2}R^{\tilde{f}_\uparrow}_{i1}\,, &\qquad k^{\tilde{f}_\downarrow}_{ij} &= h_{f_\uparrow}V_{j2}R^{\tilde{f}_\downarrow}_{i1}\,. \end{aligned} \qquad \text{(F.100)}$$



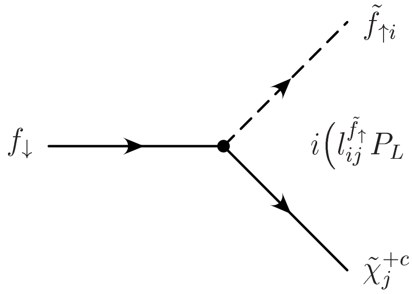
$$i\Big(l^{\tilde{f}_\downarrow}_{ij}P_R+k^{\tilde{f}_\downarrow}_{ij}P_L\Big)$$



$$i\Big(l^{\tilde{f}_\uparrow}_{ij}P_R+k^{\tilde{f}_\uparrow}_{ij}P_L\Big)$$



$$i\Big(l^{\tilde{f}_\downarrow}_{ij}P_L+k^{\tilde{f}_\downarrow}_{ij}P_R\Big)$$



$$i\Big(l^{\tilde{f}_\uparrow}_{ij}P_L+k^{\tilde{f}_\uparrow}_{ij}P_R\Big)$$

For the neutralino–sfermion–fermion couplings the Lagrangian reads

$$\mathcal{L} \;=\; \bar{f}\Big(a^{\tilde{f}}_{ik}P_R+b^{\tilde{f}}_{ik}P_L\Big)\tilde{\chi}^0_k\,\tilde{f}_i+\bar{\tilde{\chi}}^0_k\Big(a^{\tilde{f}}_{ik}P_L+b^{\tilde{f}}_{ik}P_R\Big)f\,\tilde{f}^*_i \qquad \text{(F.101)}$$

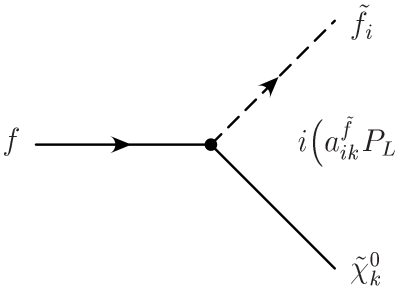
with the coupling matrices

$$a_{ik}^{\tilde{f}} \;=\; h_f Z_{kx} R_{i2}^{\tilde{f}} + g f_{Lk}^{f} R_{i1}^{\tilde{f}}\,, \qquad b_{ik}^{\tilde{f}} \;=\; h_f Z_{kx} R_{i1}^{\tilde{f}} + g f_{Rk}^{f} R_{i2}^{\tilde{f}} \tag{F.102}$$

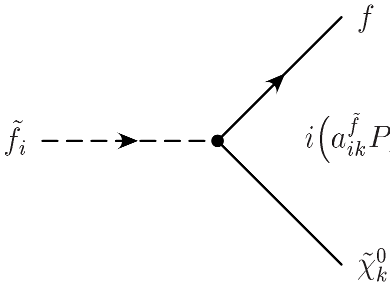
and

$$f_{Lk}^{f} = \sqrt{2}\left((e_f - I_f^{3L})\tan\theta_W Z_{k1} + I_f^{3L} Z_{k2}\right)\,, \quad f_{Rk}^{f} = -\sqrt{2} e_f \tan\theta_W Z_{k1}\,. \tag{F.103}$$

x takes the values $\{3,4\}$ for {down, up}–type case, respectively.



$$i\left(a_{ik}^{\tilde{f}} P_L + b_{ik}^{\tilde{f}} P_R\right)$$



$$i\left(a_{ik}^{\tilde{f}} P_R + b_{ik}^{\tilde{f}} P_L\right)$$

F.9 Higgs–Vector boson–Vector boson couplings

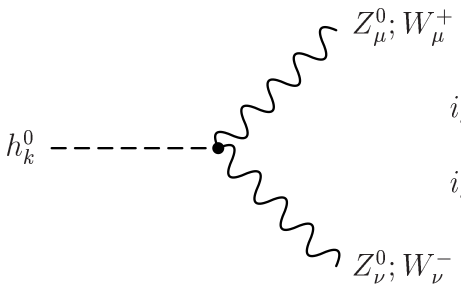
The interaction Lagrangian describing the couplings of one Higgs boson to two gauge bosons in the MSSM is given by

$$\begin{aligned}
\mathcal{L} \;=\; & \frac{g_Z\, m_Z}{2}\left[\cos(\alpha-\beta) H^0 Z_\mu^0 Z^{0\mu} - \sin(\alpha-\beta) h^0 Z_\mu^0 Z^{0\mu}\right] \\
& + g\, m_W \left[\cos(\alpha-\beta) H^0 W_\mu^+ W^{-\mu} - \sin(\alpha-\beta) h^0 W_\mu^+ W^{-\mu}\right] \\
& - g_Z\, m_W s_W^2\, G^- W_\mu^+ Z^{0\mu} + g s_W m_W\, G^- W_\mu^+ A^\mu + \text{h.c.}
\end{aligned} \tag{F.104}$$

With the usual form of rotation matrices, used throughout this paper,

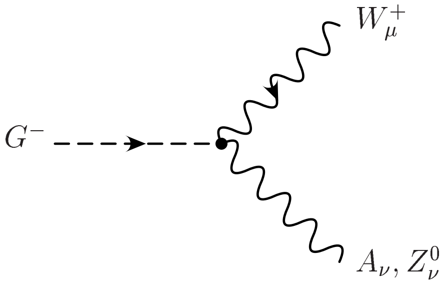
$$R_{kl}(\phi) \;\equiv\; \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}_{kl}\,, \tag{F.105}$$

the Feynman rules can then be written as



$$i g_Z\, m_Z R_{2k}(\alpha-\beta) g_{\mu\nu} \qquad \ldots Z_\mu^0$$

$$i g\, m_W R_{2k}(\alpha-\beta) g_{\mu\nu} \qquad \ldots W_\mu^\pm$$



$$ig s_W m_W g_{\mu\nu} \qquad \ldots A_\nu$$

$$-i g_Z\, m_W s_W^2 g_{\mu\nu} \qquad \ldots Z_\nu^0$$

Bibliography

[1] S. Glashow, Nucl. Phys. 22 (1961) 579; S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264; A. Salam, in "Elementary Particle Theory", ed. N. Svartholm, Almqvist and Wiksells, Stockholm (1969) p. 367.

[2] M. Gell-Mann, Phys. Lett. 8 (1964) 214; G. Zweig, CERN-Report 8182/TH401 (1964); H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. B47 (1973) 365; D. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343; H.D. Politzer, Phys. Rev. Lett. 30 (1973) 1346; G. 't Hooft, Marseille Conference on Yang–Mills fields (1972).

[3] P.W. Higgs, Phys. Rev. Lett. 13 (1964) 508; *ibid.* Phys. Rev. 145 (1966) 1156; F. Englert and R. Brout, Phys. Rev. Lett. 13 (1964) 321; G.S. Guralnik, C.R. Hagen and T. Kibble, Phys. Rev. Lett. 13 (1965) 585; T. Kibble, Phys. Rev. 155 (1967) 1554.

[4] W. Marciano, in Proceedings of the Eighth Workshop on Grand Unification, 16-18 April, 1987, Syracuse University, Syracuse, NY, ed. K. Wali (World Scientific, Singapore, 1988), pp. 185-189.

[5] A. D. Sakharov, JETP Lett. **91B**, (1967) 24.

[6] L. Girardello and M.T. Grisaru, Nucl. Phys. B194 (1982) 65.

[7] E. Witten, Nucl. Phys. B188 (1981) 513; *ibid* Nucl. Phys. B202 (1982) 253; N. Sakai, Z. Phys. C11 (1981) 153; S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150; R.K. Kaul and P. Majumdar, Nucl. Phys. B199 (1982) 36.

[8] L. E. Ibanez and G. G. Ross, Phys. Lett. B **110**, 215 (1982).

[9] L. Alvarez-Gaume, M. Claudson and M. Wise, Nucl. Phys. B **207**, 96 (1982).

[10] U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B **260**, 447 (1991).

[11] G. R. Farrar and P. Fayet, Phys. Lett. B **76** (1978) 575.

[12] H. E. Haber and G. L. Kane, Phys. Rept. **117** (1985) 75.

[13] S. Dimopoulos and D. Sutter, Nucl. Phys. B452 (1995) 496; see also the discussion given by H.E. Haber, hep-ph/9709450.

[14] A. Djouadi and S. Rosiers–Lees (conv.) *et al.*, Summary Report of the MSSM Working Group for the "GDR–Supersymétrie", hep-ph/9901246.

[15] A.H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 49 (1982) 970; R. Barbieri, S. Ferrara and C.A Savoy, Phys. Lett. B119 (1982) 343; L. Hall, J. Lykken and S. Weinberg, Phys. Rev. D27 (1983) 2359.

[16] H.P. Nilles, Phys. Lett. B115 (1982) 193; *ibid.* Nucl. Phys. B217 (1983) 366; S.K. Soni and H.A. Weldon, Phys. Lett. B126 (1983) 215; N. Ohta, Prog. Theor. Phys. 70 (1983) 542.

[17] J. Ellis, S. Kelley and D.V. Nanopoulos, Phys. Lett. B260 (1991) 131; U. Amaldi, W. de Boer and H. Fürstenau, Phys. Lett. B260 (1991) 447; P. Langacker and M. Luo, Phys. Rev. D44 (1991) 817; C. Giunti, C.W. Kim and U.W. Lee, Mod. Phys. Lett. A6 (1991) 1745.

[18] K. Inoue, A. Komatsu and S. Takeshita, Prog. Theor. Phys 68 (1982) 927; (E) *ibid.* 70 (1983) 330.

[19] M. Machacek and M.T. Vaughn, Nucl. Phys. B222 (1983) 83; *ibid.* Nucl. Phys. B236 (1984) 221; *ibid.* Nucl. Phys. B249 (1985) 70; I. Jack, Phys. Lett. B147 (1984) 405; W. de Boer, R. Ehret and D.I. Kazakov, Z. Phys. C67 (1994) 647; Y. Yamada, Phys. Rev. D50 (1994) 3537; I. Jack and D.R.T. Jones, Phys. Lett. B333 (1994) 372; D.J. Castaño, E.J. Piard and P. Ramond, Phys. Rev. D49 (1994) 4882; V. Barger, M.S. Berger and P. Ohmann, Phys. Rev. D47 (1993) 1093.

[20] I. Jack, D. Jones, S. Martin, M. Vaughn and Y. Yamada, Phys. Rev. D50 (1994) 5481; S. Martin and M. Vaughn, Phys. Lett. B318 (1993) 331; *ibid.* Phys. Rev. D50 (1994) 47.

[21] L. Randall and R. Sundrum, Nucl. Phys. B557 (1999) 79; G. Giudice, M. Luty, H. Murayama and R. Rattazzi, JHEP 9812 (1998) 027; J.A. Bagger, T. Moroi and E. Poppitz, JHEP 0004 (2000) 009.

[22] For a phenomenological review of AMSB models, see for instance: K. Huitu, J. Laamanen and P. N. Pandita, Phys. Rev. D65 (2002) 115003.

[23] V. Berezinsky, A. Bottino, J. R. Ellis, N. Fornengo, G. Mignola and S. Scopel, Astropart. Phys. **5** (1996) 1 [arXiv:hep-ph/9508249]; P. Nath and R. Arnowitt, Phys. Rev. D **56** (1997) 2820 [arXiv:hep-ph/9701301]; A. Bottino, F. Donato, N. Fornengo and S. Scopel, Phys. Rev. D **63**, 125003 (2001) [arXiv:hep-ph/0010203]; V. Bertin, E. Nezri and J. Orloff, arXiv:hep-ph/0210034.

[24] M. Drees, M. M. Nojiri, D. P. Roy and Y. Yamada, Phys. Rev. D **56** (1997) 276 [Erratum-ibid. D **64** (1997) 039901] [arXiv:hep-ph/9701219]; see also M. Drees, Y. G. Kim, M. M. Nojiri, D. Toya, K. Hasuko and T. Kobayashi, Phys. Rev. D **63** (2001) 035008 [arXiv:hep-ph/0007202].

[25] J. R. Ellis, T. Falk, G. Ganis, K. A. Olive and M. Schmitt, Phys. Rev. D **58** (1998) 095002 [arXiv:hep-ph/9801445]; J. R. Ellis, T. Falk, G. Ganis and K. A. Olive, Phys. Rev. D **62** (2000) 075010 [arXiv:hep-ph/0004169].

[26] Abdelhak Djouadi, Phys. Rept. 459 (2008) 1.

[27] M. Dine and A. E. Nelson, Phys. Rev. D48 (1993) 1277; M. Dine, A. E. Nelson and Y. Shirman, Phys. Rev. D51 (1995) 1362; M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D53 (1996) 2658.

[28] For a general review of GMSB models, see: G.F. Giudice and R. Rattazzi, Phys. Rept. 322 (1999) 419.

[29] For phenomenological reviews on GMSB models, see for instance: S. Ambrosanio, G.D. Kribs and S.P. Martin, Phys. Rev. D56 (1997) 1761; C.H. Chen and J.F. Gunion, Phys. Rev. D58 (1998) 075005.

[30] D. A. Ross and J. C. Taylor, Nucl. Phys. B **51** (1973) 125 [Erratum-ibid. B **58** (1973) 643];
A. Sirlin, Phys. Rev. D **22** (1980) 971.

[31] M. Böhm, H. Spiesberger and W. Hollik, Fortsch. Phys. **34** (1986) 687.

[32] A. Denner, Fortsch. Phys. **41** (1993) 307.

[33] H. Burkhardt, F. Jegerlehner, G. Penso and C. Verzegnassi, Z. Phys. C **43** (1989) 497.

[34] H. Eberl, M. Kincel, W. Majerotto and Y. Yamada, Nucl. Phys. B **625** (2002) 372 [arXiv:hep-ph/0111303].

[35] W. Öller, H. Eberl and W. Majerotto, Phys. Rev. D **71** (2005) 115002 [arXiv:hep-ph/0504109].

[36] F. Jegerlehner, Nucl. Phys. Proc. Suppl. **131** (2004) 213 [arXiv:hep-ph/0312372].

[37] P. H. Chankowski, S. Pokorski, J. Rosiek, Phys. Lett. B **274** (1992) 191; Nucl. Phys. B **423** (1994) 437; 497;
A. Dabelstein, Z. Phys. C **67** (1995) 495; Nucl. Phys. B **456** (1995) 25.

[38] J. Guasch, J. Sola and W. Hollik, Phys. Lett. B **437** (1998) 88 [arXiv:hep-ph/9802329].

[39] H. Eberl, S. Kraml and W. Majerotto, JHEP **9905** (1999) 016 [arXiv:hep-ph/9903413].
H. Eberl, S. Kraml and W. Majerotto, JHEP **9905** (1999) 016.

[40] H. Eberl, M. Kincel, W. Majerotto and Y. Yamada, Phys. Rev. D **64** (2001) 115013 [arXiv:hep-ph/0104109].

[41] W. Öller, H. Eberl, W. Majerotto and C. Weber, Eur. Phys. J. C **29** (2003) 563 [arXiv:hep-ph/0304006].

[42] A. Freitas and D. Stockinger, Phys. Rev. D **66** (2002) 095014 [arXiv:hep-ph/0205281].

[43] CERN–EP/2003-011, submitted to Phys. Lett. B.

[44] J. F. Gunion, H. E. Haber, Nucl. Phys B **272** (1986) 1; B **402** (1993) 567 (E).

[45] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, The Higgs Hunter's Guide, Addison-Wesley (1990).

[46] H. Baer, D. Dicus, M. Drees, and X. Tata, Phys. Rev. D **36** (1987) 1363;
J. F. Gunion and H. E. Haber, Nucl. Phys. B **307** (1988) 445;
J. F. Gunion and H. E. Haber, Nucl. Phys. B **402** (1993) 569(E);
K. Griest and H. E. Haber, Phys. Rev. D **37** (1988) 719.

[47] A. Djouadi, J. Kalinowski, and P. M. Zerwas, Z. Phys. C **57** (1993) 569;
A. Djouadi, P. Janot, J. Kalinowski, and P. M. Zerwas, Phys. Lett. B **376** (1996) 220;
A. Djouadi, J. Kalinowski, P. Ohmann, and P. M. Zerwas, Z. Phys. C **74** (1997) 93;
A. Djouadi, Mod. Phys. Lett. A **14** (1999) 359;
G. Bélanger, F. Boudjema, F. Donato, R. Godbole, and S. Rosier-Lees, Nucl. Phys. B **581** (2000) 3.

[48] A. Bartl, K. Hidaka, Y. Kizukuri, T. Kon and W. Majerotto, Phys. Lett **315** (1993) 360;
A. Djouadi, J. Kalinowski, P. Ohmann, P.M. Zerwas, Z. Phys. C **74** (1997) 93.

[49] A. Bartl, H.Eberl, K. Hidaka, T. Kon W. Majerotto and Y. Yamada, Phys. Lett **378** (1996) 167 and references therein.

[50] A. Bartl, H. Eberl, K. Hidaka, T. Kon, W. Majerotto and Y. Yamada, Phys. Lett. B **402** (1997) 303.

[51] A. Arhrib, A. Djouadi, W. Hollik, C. Jünger, Phys. Rev. D **57** (1998) 5860.

[52] A. Dabelstein, Z. Phys. C **67** (1995) 495; Nucl. Phys. B **456**, 25 (1995) [arXiv:hep-ph/9409375].

[53] F. Bloch and A. Nordsieck, Phys. Rev. **52** (1937) 54.

[54] R. Hempfling, Phys. Rev. D **49** (1994) 6168;
L.J. Hall, R. Rattazzi, and U. Sarid, Phys. Rev. D **50** (1994) 7048;
M. Carena, M. Olechowski, S. Pokorski, and C.E.M. Wagner, Nucl. Phys. B **426** (1994) 269;
D.M. Pierce, J.A. Bagger, K. Matchev, and R. Zhang, Nucl. Phys. B **491** (1997) 3.

[55] H. Eberl, K. Hidaka, S. Kraml, W. Majerotto and Y. Yamada, Phys. Rev. D **62** (2000) 055006.

[56] E. Braaten and J.P. Leveille, Phys. Rev. D **22** (1980) 715;
M. Drees and K. Hikasa, Phys. Lett. B **240** (1990) 455; **262** (1991) 497 (E);
A. Méndez and A. Pomarol, Phys. Lett. B **252** (1990) 461.

[57] S.G. Gorishny, A.L. Kataev, S.A. Larin, and L.R. Surguladze, Mod. Phys. Lett. **A5** (1990) 2703; Phys. Rev. D **43** (1991) 1633;
A. Djouadi, M. Spira, and P.M. Zerwas, Z. Phys. C **70** (1996) 427;
A. Djouadi, J. Kalinowski, and M. Spira, Comput. Phys. Commun. **108** (1998) 56;
M. Spira, Fortschr. Phys. **46** (1998) 203.

[58] M. Carena, S. Mrenna, and C.E.M. Wagner, Phys. Rev. D **60** (1999) 075010; Phys. Rev. D **62** (2000) 055008.

[59] P.H. Chankowski and S. Pokorski, hep-ph/9707497, in *Perspectives on Supersymmetry*, ed. by G. L. Kane (World Scientific, 1997);
K.S. Babu and C. Kolda, Phys. Lett. B **451** (1999) 77.

[60] M. Kuroda, arXiv:hep-ph/9902340.

[61] G. 't Hooft and M. Veltman, Nucl. Phys. B **153** (1979) 365;
G. Passarino and M. Veltman, Nucl. Phys. B **160** (1979) 151.

[62] C. Weber, H. Eberl and W. Majerotto, Phys. Lett. B **572** (2003) 56 [arXiv:hep-ph/0305250].

[63] ATLAS Technical Design Report, CERN/LHCC/99-15, ATLAS TDR 15 (1999); CMS Technical Proposal, CERN/LHCC/94-38 (1994).

[64] J.A. Aguilar-Saavedra *et al.*, TESLA Technical Design Report, DESY 01-011 and arXiv:hep-ph/0106315; T. Abe *et al.* [American LC WG], in *Proceedings of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)* , SLAC-R-570 and arXiv:hep-ex/0106055-58; T. Abe *et al.* [Asian LC WG], KEK-Report–2001-011 and arXiv: hep-ph/0109166.

[65] LHC/LC Study Group, G. Weiglein *et al.*, *LHC+LC Report*, Phys. Rept. 426 (2006) 47.

[66] A. Arhrib, A. Djouadi, W. Hollik, C. Jünger,
Phys. Rev. D57 (1998) 5860.

[67] A. Djouadi, M. Spira, P. M. Zerwas, Z. Phys. C70 (1996) 427.

[68] S. Heinemeyer, W. Hollik, G. Weiglein, Eur. Phys. J. C16 (2000) 139.

[69] A. Bartl, H. Eberl, K. Hidaka, T, Kon, W. Majerotto, Y. Yamada,
Phys. Lett. B402 (1997) 303.

[70] A. Bartl, H. Eberl, K. Hidaka, T, Kon, W. Majerotto, Y. Yamada,
Phys. Lett. B378 (1996) 167.

[71] J. A. Coarasa, R. A. Jiménez, J. Solà,
Phys. Lett. B389 (1996) 312.

[72] C. Weber, K. Kovarik, H. Eberl, W. Majerotto, Nucl. Phys. B776 (2007) 138.

[73] H. Eberl, W. Majerotto, Y. Yamada, Phys. Lett. B597 (2004) 275.

[74] C. Weber, H. Eberl, W. Majerotto, Phys. Rev. D68 (2003) 093011.

[75] C. Weber, H. Eberl, W. Majerotto, Phys. Lett. B572 (2003) 56.

[76] H. Eberl, M. Kincel, W. Majerotto, Y. Yamada, Nucl. Phys. B625 (2002) 372.

[77] H. Eberl, K. Hidaka, S. Kraml, W. Majerotto, Y. Yamada, Phys. Rev. D62 (2000) 055006.

[78] A. Bartl, H. Eberl, K. Hidaka, S. Kraml, W. Majerotto, W. Porod, Y. Yamada, Phys. Rev. D59 (1999) 115007.

[79] A. Dabelstein, Nucl. Phys. B456 (1995) 25.

[80] S. Heinemeyer, Int. J. Mod. Phys. A21 (2006) 2659.

[81] S. Heinemeyer, Nucl. Phys. B474 (1996) 32.

[82] N. Baro, F. Boudjema, A. Semenov, Phys. Rev. D78 (2008) 115003.

[83] N. Baro, F. Boudjema, Phys. Rev. D80 (2009) 076010.

[84] J. Fujimoto, T. Ishikawa, M. Jimbo, T. Kaneko, T. Kon, Y. Kurihara, M. Kuroda and Y. Shimizu,
Nucl. Phys. Proc. Suppl. (2006) 157 157.

[85] The tool HFOLD can be downloaded from `http:/www.hephy.at/tools`.

[86] P. Skands et al., JHEP 0407 (2004) 36.

[87] J. A. Aguilar-Saavedra *et al.*, EPJ C46 (2006) 43;
J. Kalinowski, Acta Phys. Polon. B37 (2006) 1215.

[88] A. Denner, Fortschr. Phys. 41 (1993) 307.

[89] W. Öller, H. Eberl, W. Majerotto,
Phys. Lett. B590 (2004) 273; Phys. Rev. D71 (2005) 115002.

[90] K. Kovařík, C. Weber, H. Eberl, W. Majerotto, Phys. Rev. D72 (2005) 053010.

[91] T. Hahn, Comp. Phys. Comm. 140 (2001) 418; T. Hahn, C. Schappacher, Comp. Phys. Comm. 143 (2002) 54.

[92] M. Carena, D. Garcia, U. Nierste, C. E. M. Wagner, Nucl. Phys. B577 (2000) 88.

[93] Lars Hofer, Ulrich Nierste, Dominik Scherer, JHEP (2009) 0910.

[94] T. Hahn, M. Perez-Victoria, Comp. Phys. Comm. 118 (1999) 153.

[95] W. Porod, Comp. Phys. Comm. (2003) 153.

[96] T. Hahn, S. Heinemeyer,
`FeynHiggs2.7.4` from `http://www.feynhiggs.de`.

[97] A. Djouadi, J. Kalinowski, M. Spira, Comp. Phys. Comm. (1) 153.

[98] A. Djouadi, M. M. Mühlleitner, M. Spira, Acta Pol. B38 (2007) 635.

[99] A. Dabelstein, Z. Phys. C67 (1995) 495.

[100] D.Pierce, A. Papadopoulos Phys. Rev. D47 (1993) 222.

[101] PhD thesis, Helmut Eberl, Hephy Vienna 1998.

[102] D. Stockinger, JHEP 0503 (2005) 076.

[103] L. Ibañez and G.G. Ross, Phys. Lett. B110 (1982) 227; L. Alvarez–Gaumé, J. Polchinski and M.B. Wise, Nucl. Phys. B221 (1983) 495; J. Ellis, J. Hagelin, D. Nanopoulos and K. Tamkavis, Phys. Lett. B125 (1983) 275; L.E. Ibanez and C. Lopez, Nucl. Phys. B233 (1984) 511; L.E. Ibanez, C. Lopez and C. Munoz, Nucl. Phys. B256 (1985) 218.

[104] A. Djouadi, J. Kalinowski, P. Ohmann and P.M. Zerwas, Z. Phys. C74 (1997) 93.

[105] G. t Hooft and M. Veltman, Nucl. Phys. B153 (1979) 365.

[106] CERN-PH-EP-2011-022, Submitted to Phys. Lett. B

[107] PhD thesis, Christian Weber, Hephy Vienna 2005.

[108] A. Djouadi, W. Kilian, M.M. Muhlleitner and P.M. Zerwas, Eur. Phys. J.C10 (1999) 27; *ibid* hep-ph/0001169.

Curriculum Vitae

Persönliche Daten:

Name:	Dipl.-Ing. Wolfgang Frisch
Geburtsdaten:	11.12.1982 in Wien
Nationalität:	Österreich
Sprachen:	Deutsch, Englisch(fließend)
Anschrift:	Senefeldergasse 15/14, 1100 Wien
eMail:	frisch@hephy.oeaw.ac.at

Schule & Studium:

11/2007–06/2011	Dissertation: Precise predictions of Higgs boson decays including the full one-loop corrections in Supersymmetry Institut für Hochenergiephysik der ÖAW / TU Wien
10/2001–06/2007	Studium der Technischen Physik, TU Wien Diplomarbeit: Simulation and topology of two-dimensional noncommutative U(1) gauge theory
09/1993–06/2001	Gymnasium Neulandschule Ludwig-von-Höhnel-Gasse 17, 1100 Wien

Berufserfahrung:

11/2007–06/2011	Anstellung als wissenschaftlicher Mitarbeiter am Institut für Hochenergiephysik

June 20, 2011

List of Publications

- H. Eberl, W. Frisch, H. Hlucha
  "Decay widths of all MSSM scalar particles at full one-loop level",
  Nuclear Physics B (Proc. Suppl.) 205-206 (2010) 277.

- W. Frisch, H. Eberl, H. Hlucha
  "HFOLD - a program package for calculating two-body MSSM Higgs decays at full one-loop level"
  accepted for publication in Comp. Phys. Com. [arXiv:hep-ph/1012.5025].

- H. Hlucha, H. Eberl, W. Frisch
  "SFOLD - a program package for calculating two-body sfermion decays at full one-loop level in the MSSM"
  submitted to Comp. Phys. Com. [arXiv:hep-ph/1104.2151].