UNDERSTANDING SEXTUPOLE

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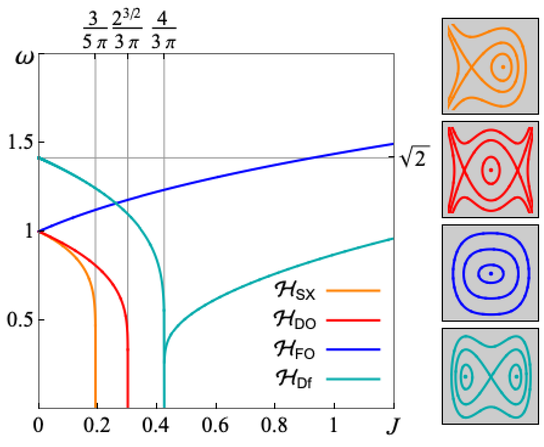
1D Henon-Héiles Hamiltonian



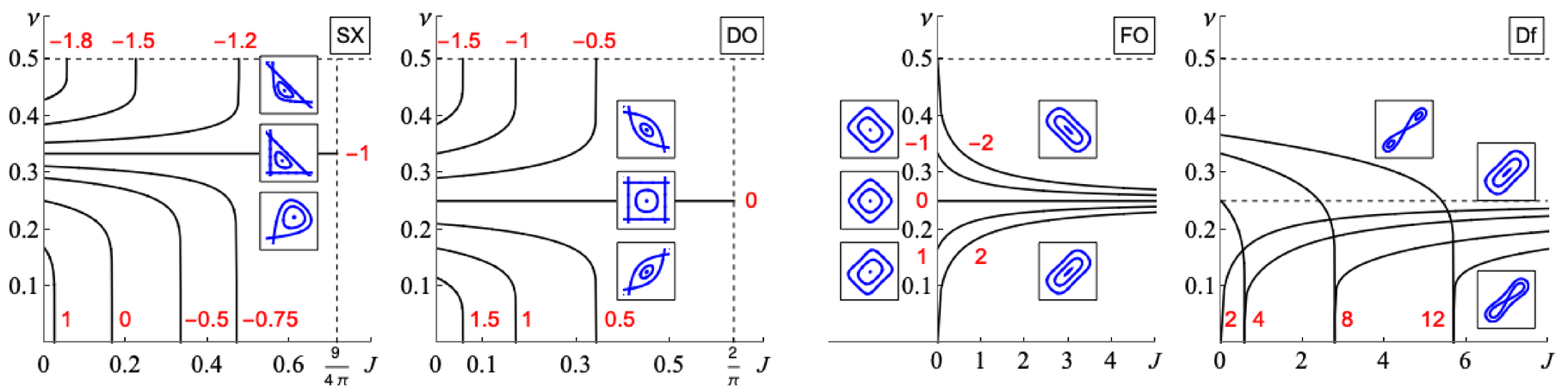
$$\mathcal{H}_{\rm sxt}[p,q;t] = \frac{p^2}{2}+\frac{q^2}{2}+\frac{q^3}{3}, \qquad \text{(SX)}$$

$$\mathcal{H}^{-}_{\rm oct}[p,q;t] = \frac{p^2}{2}+\frac{q^2}{2}-\frac{q^4}{4}, \qquad \text{(DO)}$$

$$\mathcal{H}^{+}_{\rm oct}[p,q;t] = \frac{p^2}{2}+\frac{q^2}{2}+\frac{q^4}{4}, \qquad \text{(FO)}$$

$$\mathcal{H}_{\rm Duf}[p,q;t] = \frac{p^2}{2}-\frac{q^2}{2}+\frac{q^4}{4}. \qquad \text{(Df)}$$

McMillan multipoles



$$\mathcal{K}_{\rm sxt}[p,q] = p^2 - a\,p\,q + q^2 + p^2 q + q\,p^2, \quad |a|\le 2, \qquad \text{(SX)}$$

$$\mathcal{K}^{-}_{\rm oct}[p,q] = p^2 - a\,p\,q + q^2 - p^2 q^2, \quad |a|\le 2, \qquad \text{(DO)}$$

$$\mathcal{K}^{+}_{\rm oct}[p,q] = p^2 - a\,p\,q + q^2 + p^2 q^2, \quad |a|\le 2, \qquad \text{(FO)}$$

$$\mathcal{K}^{+}_{\rm oct}[p,q] = p^2 - a\,p\,q + q^2 + p^2 q^2, \quad |a|> 2, \qquad \text{(Df)}$$

$$q' = p,$$

$$p' = -q + f(p),$$

$$f_{\rm sxt}(p) = p\,(a-p)/(1+p),$$

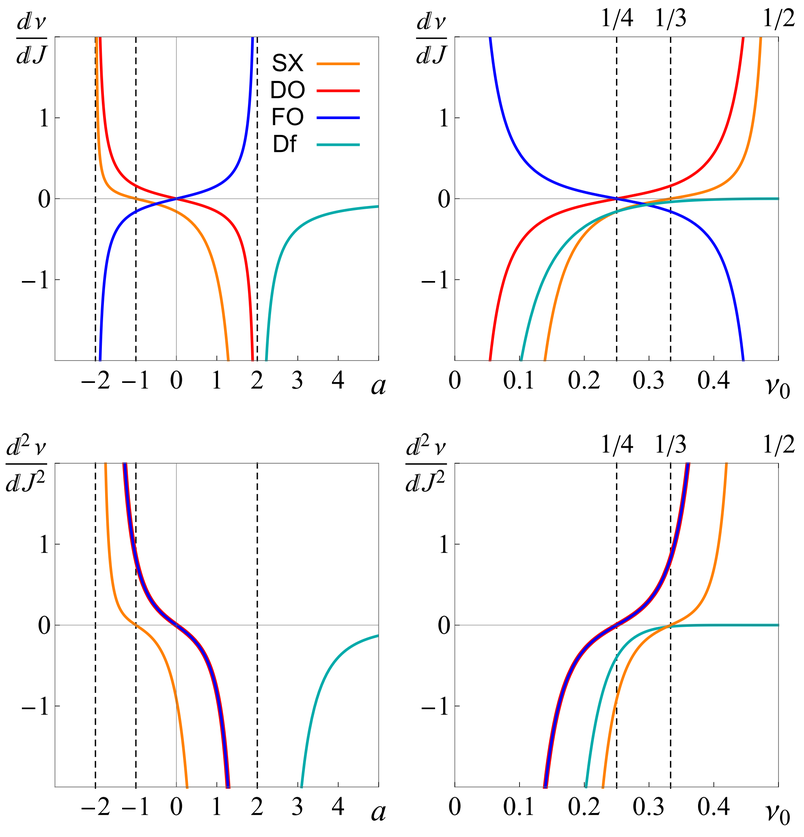
$$f_{\rm oct}(p) = a\,p\,/(1\pm p^2).$$

Detuning

Henon-Héiles Hamiltonians

$$\left.\frac{{\rm d}\omega_{\rm SX}}{{\rm d}J_{\rm SX}}\right|_{J=0} = -\frac{5}{6}, \qquad \left.\frac{{\rm d}\omega_{\rm FO}}{{\rm d}J_{\rm FO}}\right|_{J=0} = -\left.\frac{{\rm d}\omega_{\rm DO}}{{\rm d}J_{\rm DO}}\right|_{J=0} = \frac{3}{4}.$$

McMillan multipoles

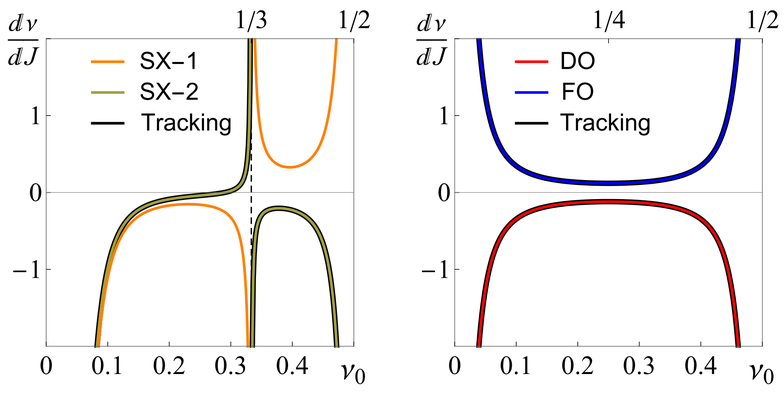


$$\left.\frac{{\rm d}\nu_{\rm SX}}{{\rm d}J_{\rm SX}}\right|_{J=0} = -\frac{1}{2\,\pi}\frac{(1+a)(8+a)}{(2+a)(2-a)^2},$$

$$\left.\frac{{\rm d}\nu_{\rm FO,DO}}{{\rm d}J_{\rm FO,DO}}\right|_{J=0} = \pm\frac{3}{2\,\pi}\frac{a}{4-a^2}, \qquad \left.\frac{{\rm d}\nu_{\rm Df}}{{\rm d}J_{\rm Df}}\right|_{J=0} = \frac{1}{2\,\pi}\frac{4+a}{a\,(2-a)}.$$

Hénon quadratic and cubic mappings

$$f_{\rm sxt}(x) = a\,q + q^2, \qquad f_{\rm oct}(x) = a\,q \pm q^3.$$

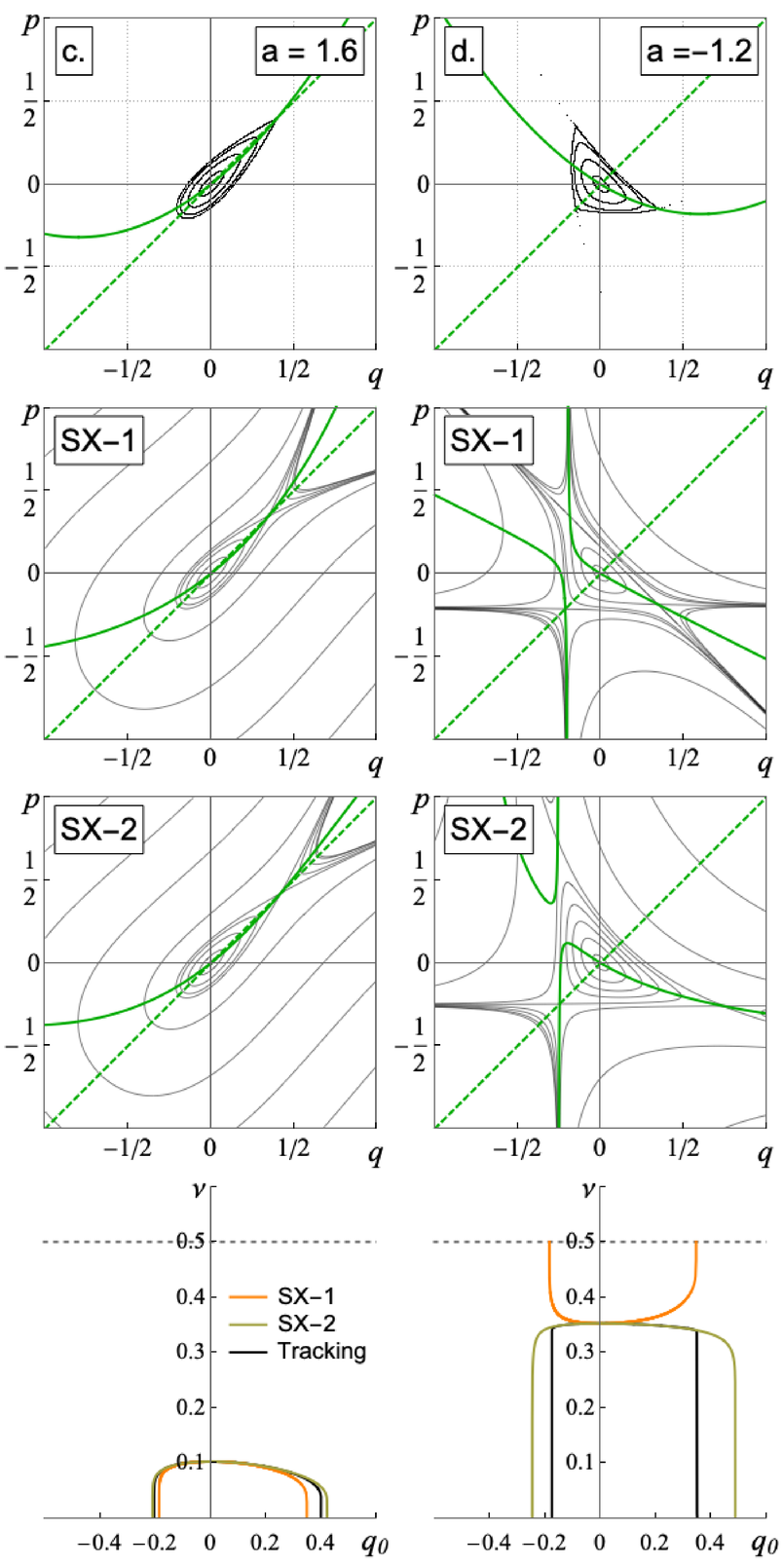


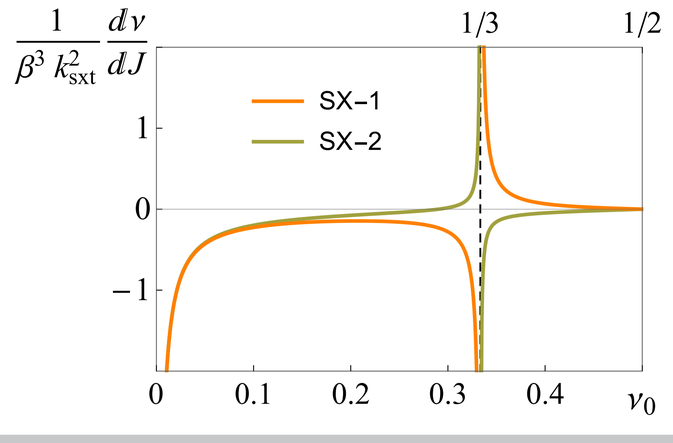
$$\left.\frac{{\rm d}\nu^{\rm (MH)}_{\rm SX\text{-}1}}{{\rm d}J^{\rm (MH)}_{\rm SX\text{-}1}}\right|_{J=0} = -\frac{1}{16\,\pi}\frac{9\,\cos(\pi\,\nu_0)+\cos(3\,\pi\,\nu_0)}{\sin^3(2\,\pi\,\nu_0)\,\sin(3\,\pi\,\nu_0)},$$

$$\left.\frac{{\rm d}\nu^{\rm (MH)}_{\rm SX\text{-}2}}{{\rm d}J^{\rm (MH)}_{\rm SX\text{-}2}}\right|_{J=0} = -\frac{1}{16\,\pi}\frac{3\,\cot(\pi\,\nu_0)+\cot(3\,\pi\,\nu_0)}{\sin^3(2\,\pi\,\nu_0)},$$

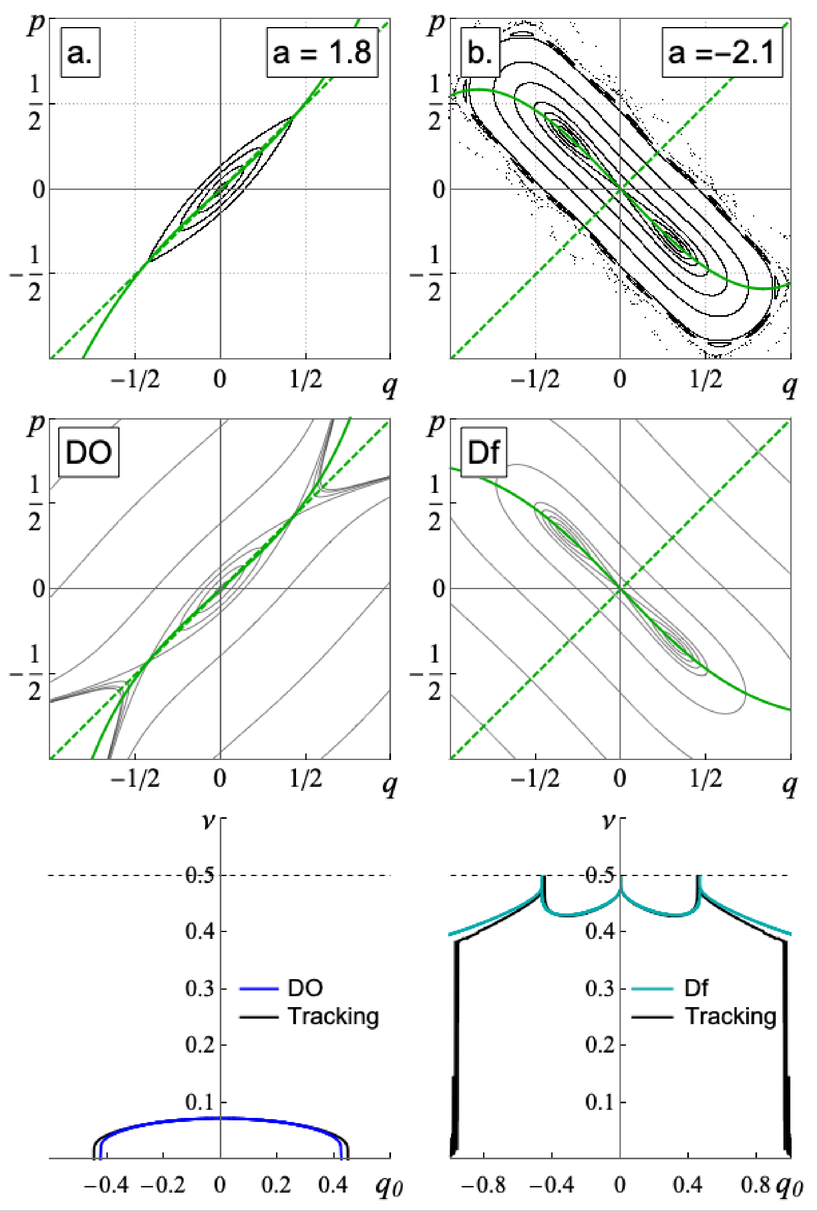
$$\left.\frac{{\rm d}\nu^{\rm (MH)}_{\rm DO,FO}}{{\rm d}J^{\rm (MH)}_{\rm DO,FO}}\right|_{J=0} = \mp\frac{3}{8\,\pi}\frac{1}{\sin^2(2\,\pi\,\nu_0)}.$$

Sextupole magnet

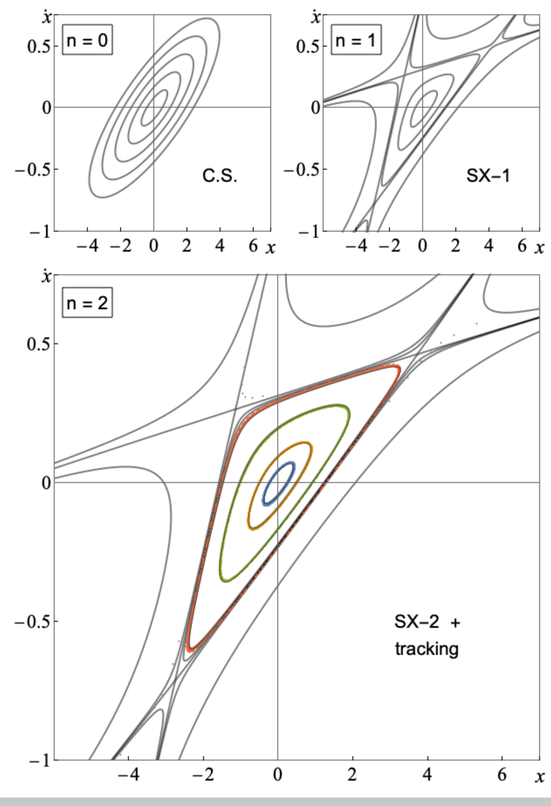




Octupole magnet



Third-integer resonant extraction for Mu2e



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