Energy Loss of Heavy Quarks—A Signal of Plasma Properties

J. Aichelin

Abstract The possible observables for studying the properties of a plasma of quarks and gluons (QGP), which is presumably created in ultrarelativistic heavy-ion collisions, are discussed. While the light mesons do not contain the desired information about the QGP phase due to the strong final hadronic interaction, the 'heavy' mesons, i.e. those containing a c- or b- quark, are more useful. We demonstrate that our recent pQCD based approach for the energy loss of heavy quarks in a QGP combined with hydrodynamical model of Kolb and Heinz for the expansion of the plasma can successfully describe the variety of experimental data—as the transverse momentum spectra, R_{AA}, and the elliptic flow v_2 of heavy quarks—from RHIC to LHC energies.

1 Introduction

Hadrons have a finite radius. Therefore, if the density becomes too high one expects naively that the hadrons start to overlap and the constituents of the hadrons, the quarks and gluons, can move freely from one hadron to another. Thus hadrons loose their identity and form a new stage of matter—a plasma of unbound gluons and quarks which is in local thermal equilibrium. Because the density increases with temperature we expect such a transition at some given temperature in analogy with the boiling temperature of water when the phase transition from the liquid state to vapor occurs.

The phase transition from the hadronic to the partonic phase is also predicted by the fundamental theory for strongly interaction systems, the Quantum Chromo Dynamics (QCD). This follows from so called lattice gauge calculations [1], which are presently the only known way to predict expectation values of observables in strongly interacting systems. In this approach one creates by a Monte Carlo procedure

J. Aichelin (✉)
SUBATECH, University of Nantes, EMN, IN2P3/CNRS, 4 rue Alfred Kastler,
Nantes cedex 344307, France
e-mail: aichelin@subatech.in2p3.fr

W. Greiner (ed.), *Exciting Interdisciplinary Physics*,
FIAS Interdisciplinary Science Series, DOI: 10.1007/978-3-319-00047-3_21,

field configurations on a finite size lattice according the the QCD Lagrangian which are subsequently used to calculate expectation values of operators. They have then to be extrapolated to the continuum limit. This is a very subtle method and therefore only recently the different groups found agreement on the transition temperature from the QGP to hadronic phase ($\approx$ 170 MeV or $1.5 \cdot 10^6$ K at zero baryon chemical potential) . These calculations are done assuming an infinite system in global thermal equilibrium.

A QGP has been most probably created in the early universe—few microseconds after the big bang. Direct evidence for this is, however, not available. It may also exist in the interior of neutron stars [2] at a very low temperature and a high density, so one talks about quark matter and not about a QGP, but this is presently only speculation because the interior may also consist of neutron matter. Radius and mass measurements of neutron stars can provide information on the their interior. Both are related by the equation of state [3], which is different for quark and nuclear matter. The present available data do not allow for a conclusion about the phase of the interior of these stars.

If one wants to do systematic studies of the QGP and its properties there is only one possibility, to create a QPG in heavy ion reactions. There the situation differs substantially from the conditions used for the lattice QCD calculations, i.e. the infinite partonic matter in equilibrium: In heavy ion reactions such a state can be only created for a very short time (of the order of 10^{-23} s) and has an extension of a couple of 10^{-15} m. Then the system, which expands with almost the speed of light, forms hadrons which are finally observed in the detector. Thus, it is quite difficult to prove—from the theoretical as well as from the experimental side—that in a such small system in a very short time a partonic plasma has be formed.

The problem is therefore to reconstruct from the observed hadrons the existence and the properties of such a QGP. In this situation a physicists feel like a fire expert who has to determine the cause of the fire from the left overs long after the fire is extinguished. The vast majority of the observed hadrons are formed from light (u,d,s) quarks which, unfortunately, tell us little about the formation of a plasma. It turned out that the multiplicity of light hadrons is very well described by statistical models [4]. This means that the system of light quarks or hadrons (made of light quarks) has reached a state of equilibrium (and the temperature is close to that predicted by lattice gauge calculations for the transition temperature). From statistical mechanics we know that then all information about the properties of the system prior to the equilibrium is lost. As we cannot conclude from the presence of water at 0° that at a lower temperature ice is formed, we cannot conclude from the existence of an equilibrated hadron gas close to the transition temperature that at higher temperatures it goes over to a QGP. One may argue that the spectra of light hadrons may give additional information. Unfortunately hadronic final state interactions are too strong and contain too many experimentally unknown cross sections in order to infer from the measured spectra the spectra at the moment when hadrons move out of equilibrium.

There are some correlations like the ridge which could only be formed very early in the reaction and the mass, momentum and centrality dependence of collective variables, like the elliptic flow v_2, gives a glimpse on what happens early during the reaction but their interpretation is not unique.

The light particle spectra are well described by so called event generators [5] which model the whole reaction on a computer. They assume that after a violent initial phase hadrons are formed which, if the energy density exceeds a given value, form a QGP. The expansion of the QGP is modelled either in viscous or in non viscous hydrodynamics. At a given energy density which is around 1 GeV/fm^3 [6] a sudden transition to the hadronic world takes place followed by a final state interactions among the hadrons. The problem is that the physics of the initial state is little known and leaves a lot of room for different assumptions. As a consequence, some model assume that the measured centrality dependence of the elliptic flow presents evidence for the need of viscous hydrodynamics [7] whereas other models describe the data equally well in an ideal hydrodynamical approach [8] assuming that not all particles take part in the hydrodynamical expansion.

Also the fast equilibration is not yet understood. Therefore other models [9] do not assume that a thermal equilibrium is established and describe the strongly interacting quark-gluon plasma by relativistic transport equations, derived from many-body Kadanoff–Baym equations, with a dynamical hadronization of partons to hadrons. This concept leads as well to a very satisfying description of a variety of data.

In a situation like this it is evident that one looks for possible observables which do not suffer from this memory erasing equilibrium phase. There are essentially two: High p_t hadrons which originate from jets as well as the p_t and v_2 distribution of heavy mesons which contain either a c or a b quark because neither jets nor heavy quarks come to an equilibrium with the plasma. Jets have the problem that the leading particle, i.e. that with the highest momentum, may change by interactions with the plasma. This makes the understanding of jets difficult.

Heavy quarks are produced in hard binary initial collisions between the incoming protons. Their production cross sections are known from pp collisions and can as well be calculated in pQCD calculations. Therefore the initial transverse momentum distribution of the heavy quarks is known. Comparing this distribution with that measured in heavy ion collisions allows defining $R_{AA} = (d\sigma_{AA}/dp_t^2)/(N_c\, d\sigma_{pp}/dp_t^2)$, where N_c is the number of the initial binary collisions between projectile and target. The deviation of R_{AA} from one measures the interaction of the heavy quark with the plasma because the hadron cross sections of heavy mesons are small. The heavy quark does not come to thermal equilibrium with the QGP therefore R_{AA} contains the information on the interaction of the heavy quark while it traverses the plasma. In addition, the distribution of heavy quarks at the moment of their creation is isotropic in azimuthal direction, therefore the elliptic flow $v_2 = \langle \cos 2(\phi - \phi_R) \rangle$, where ϕ (ϕ_R) is the azimuthal angle of the emitted particle (reaction plane) is 0. The observed finite v_2 value of the observed heavy meson can only originate from interactions between light QGP constituents and the heavy quarks. The simultaneous description of R_{AA} and v_2 and their centrality dependence, presently the only observables for which data exist, give then the possibilities to understand the interactions inside the QGP.

Unfortunately the experimental results depend not only on the elementary interaction but also on the description of the expansion of the QGP [10]. Therefore the ultimate aim is to control the expansion by results on the light meson sector. This has not been achieved yet for the LHC and therefore it is difficult to asses the influence of the expansion on the observables. We use here the approach from Kolb and Heinz which has reasonably well described the midrapidity light mesons at RHIC [11]. We adjust only the charged particle multiplicity to the value measured at LHC.

The R_{AA} of 0.2 values observed for large p_t heavy mesons are much smaller than originally expected. Early theoretical approaches based on perturbative QCD (pQCD) calculation gave much larger values and it has been doubted, whether pQCD is the right tool to describe this interaction. This early calculation, however, used ad hoc assumptions on the coupling constant α_s and the infrared regulator μ. With a standard choice μ and α_s an artificial K factor, an overall multiplication factor of the elastic cross section of around 10 [12, 13] had to be introduced to match the experimental data.

A while ago we advanced an approach for the collisional energy loss of heavy quarks in the QGP [14–16] in which (a) μ has been fixed by the demand that more realistic calculations using the hard thermal loop approach give the same energy loss as our Born type pQCD calculation and (b) the coupling constant is running and fixed by the sum rule advanced by Dokshitzer and later used by Peshier. Both these improvements increased the cross section especially for small momentum transfers and reduced therefore the necessary K factor to 2. Here we include in addition the radiative energy loss [17, 18].

2 Model

Our approach extended by including radiative energy loss has been well described the heavy quark data at RHIC. Therefore it is worthwhile to calculate what we expect for LHC energies if we modify the model only in a minimal way by adjusting the initial condition do $dN/dy = 1600$, as observed at RHIC. To include radiation we have to consider the following 5 matrix elements, displayed in Fig. 1, which contributes to radiation. The commutation relation

$$T^b T^a = T^a T^b - i f_{abc} T^c \tag{1}$$

allows us to regroup the 5 matrix elements into 3 combinations, each of them being independently gauge invariant:

$$\begin{aligned} iM^{QED}_{h.q.} &= C_a i(M_1 + M_2) \\ iM^{QED}_{l.q.} &= C'_a i(M_3 + M_4) \\ iM^{QCD} &= C_c i(M_1 + M_3 + M_5). \end{aligned} \tag{2}$$

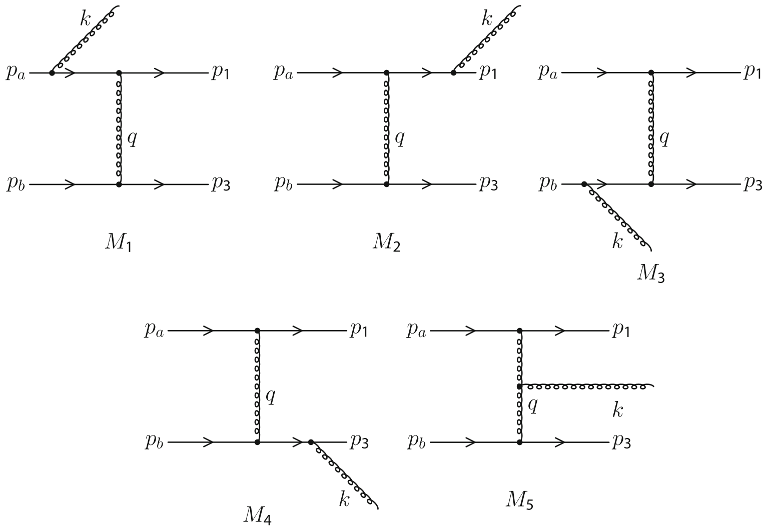

Fig. 1 The five matrix elements which contribute to the gluon bremsstrahlung

h.q. (l.q.) mark the emission of the gluon from the heavy (light quark) line. C_a, C'_a and C_c are the color algebra matrix elements. The matrix elements labeled as QED are the bremsstrahlung diagrams already observed in Quantum Electrodynamics (QED), whereas that labeled QCD is the genuine diagram of Quantum Chromodynamics (QCD). The QCD diagram is the main objet of interest here because it dominates the energy loss of heavy quarks.

We evaluate the matrix elements in scalar QCD (see Ref. [19]). They are given by

$$\begin{aligned} iM_1^{SQCD} &= C_A(ig)^3 \frac{(p_b+p_3)^\mu}{(p_3-p_b)^2} D_{\mu\nu}[p_3-p_b]\left(\frac{(p_a+p_1-k)^\nu(2p_a-k)\epsilon}{(p_a-k)^2-m^2} - \varepsilon^\nu\right) \\ iM_5^{SQCD} &= C_c(ig)^3 D^{\mu\mu'}[p_3-p_b]D^{\nu\nu'}[p_1-p_a]\left[g_{\mu'\nu'}(p_a-p_1+p_3-p_b)_\sigma\right. \\ &\quad \left.+ g_{\nu'\sigma}(p_1-p_a-k)_{\mu'} + g_{\sigma\mu'}(p_b-p_3+k)_{\nu'}\right]\varepsilon^\sigma \\ &\quad \cdot \frac{(p_3+p_b)^\mu(p_a+p_1)^\nu}{(p_3-p_b)^2(p_1-p_a)^2} \end{aligned} \tag{3}$$

M_3 is obtained by replacing $p_a \to p_b$ and $p_1 \to p_3$ in M_1. Using light cone gauge and keeping only the leading term in $\sqrt{s}$ we find that the square of the matrix element factorizes

$$|M|^2 = |M_{elast}(s,t)|^2 P_g(m,t,\mathbf{k_t},x) \tag{4}$$

with $|M_{elast}(s,t)|^2 = g^4 \frac{4s^2}{t^2}$ being the matrix element squared for the elastic cross section in a coulomb-like interaction between the heavy quark and a light quark (gluon). $P_g(m,t,s,\mathbf{k_t})$ describes the distribution function of the produced gluons.

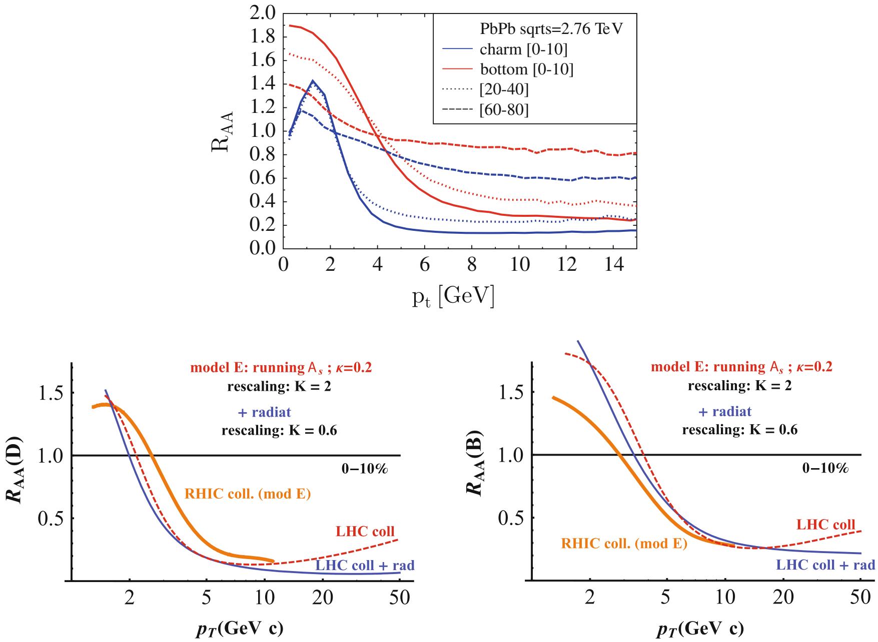

Fig. 2 The transverse momentum distribution of R_{AA} at midrapidity for different centralities and for bottom (*blue*) and charm (*red*) quarks. *Top* The details at small p_t, *bottom left* (*right*) R_{AA} at large p_t for D-mesons (B-mesons)

To discuss the physics we adopt the following light cone vectors

$$
\begin{aligned}
p_a &= \left\{\sqrt{s-m^2}, \frac{m^2}{\sqrt{s-m^2}}, 0, 0\right\} \\
p_b &= \{0, \sqrt{s-m^2}, 0, 0\} \\
k &= \{x\sqrt{s-m^2}, 0, \mathbf{k_t}\} \\
p_1 &= p_a + q - k = \left\{p_a^+(1-x) - \frac{q_t^2}{p_b^-}, \frac{(\mathbf{k_t}-\mathbf{q_t})^2+m^2}{(1-x)p_a^+}, \mathbf{q_t}-\mathbf{k_t}\right\} \\
p_3 &= p_b - q = \left\{\frac{q_t^2}{p_b^-}, p_b^- - \frac{(1-x)k_t^2 - x(\mathbf{k_t}-\mathbf{q_t})^2 + m^2x^2}{p_a^+(1-x)x}, -\mathbf{q_t}\right\}
\end{aligned}
\tag{5}
$$

The scalar product is defined as $p_a p_b = \frac{p_a^+ p_b^- + p_a^- p_b^+}{2} - p_{at} p_{bt}$ and $q^2 = t \approx q_t^2$. In this coordinate system x is given by $k^+ = x p_a^+$ and represents the relative longitudinal momentum fraction of the gluon with respect to the incoming heavy quark. In this coordinate system $|M_{SQCD}|^2$ has a very simple form:

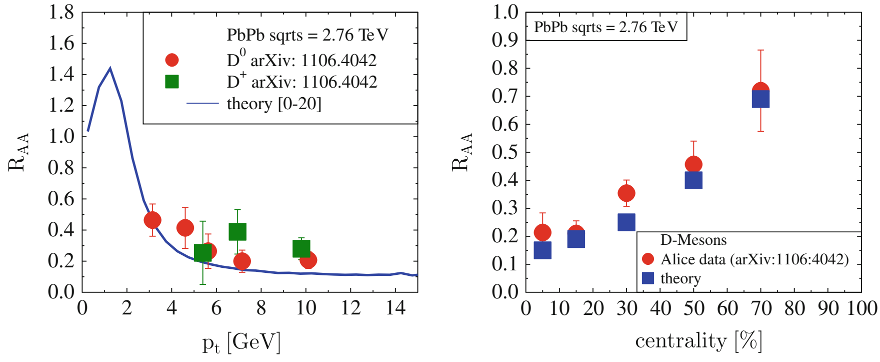

Fig. 3 *Left* R_{AA} as a function of p_t for 0–20 % centrality, *right* centrality dependence of R_{AA}. We compare data from the ALICE collaboration [21] with our prediction

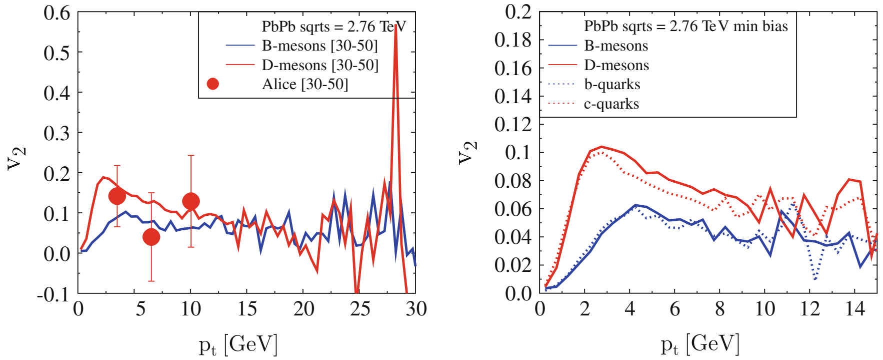

Fig. 4 p_t dependence of v_2. On the left hand side we compare our calculations for D and B meson for [30–50 %] centrality with the experimental data shown as this conference, on the right hand side we display v_2 for minimum bias separately for c-quarks and D-mesons and b-quarks and B-mesons, respectively

$$|M_{SQCD}|^2 = g^2 D^{QCD} 4(1-x)^2 |M_{elast}|^2 \left(\frac{\mathbf{k}_t}{k_t^2 + x^2 m^2} - \frac{\mathbf{k}_t - \mathbf{q}_t}{(\mathbf{q}_t - \mathbf{k}_t)^2 + x^2 m^2} \right)^2 \tag{6}$$

with the color factor $D^{QCD} = C_A * C_{el}^{qq} = \frac{2}{3}$. The first term in the bracket describes the emission from the incoming heavy quark line, the second term the emission from the gluon. This shows that in light cone gauge and in this coordinate system in leading order of $\sqrt{s}$ the matrix element for the emission from the light quark do not contribute. In the case of massless quarks we recover the squared matrix element of Gunion and Bertsch (GB) of Ref. [20].

3 Results

Having the matrix elements we can calculate the cross section of the elastic and radiative collisions of the heavy quarks with the plasma particles. At RHIC we found that the agreement is best when we multiply all cross section with a constant K factor of 0.6. A K factor of one is also compatible with the data but at the limits of the error bars. These cross sections are embedded in the plasma expansion as described in Refs. [14–16]. Figure 2 displays the p_t dependence of R_{AA} at midrapidity for different centrality bins and for c and b quarks separately. Charm quarks, being lighter, suffer a larger energy loss than bottom quarks and are therefore pushed more toward low p_t. Below a centrality of 40 % R_{AA} does not change substantially. At small momenta we see an enhancement. There the energy loss accumulates the heavy quarks. For large p_t, shown in the bottom part of Fig. 2, radiative collisions act differently than elastic collisions. If we employ only elastic collisions (model E, with a K factor of 2) we see an increase of R_{AA} with p_T whereas for elastic and radiative collisions (with a K-factor of 0.6) R_{AA} remains almost flat. If we include the Landau Pomeranchuck Migdal effect which suppresses radiation we would expect a moderate increase of R_{AA} with increasing pt. For comparison we display as well the calculation for the RHIC data which matched the experimental results.

Figure 3 shows the comparison of our calculations with R_{AA} ALICE data [21]. On the left hand side we display R_{AA} as a function of p_t of [0–20 %] centrality. The calculations follow closely the experimental data. On the right hand side we see R_{AA} for mesons with $p_t > 6$ GeV as a function for the centrality. Also here we see a good agreement between theory and experiment

Figure 4 show the comparison of our calculations with recent v_2 ALICE data [22]. We see that at low p_t v_2 for B-mesons is substantially smaller than for D-mesons. This is again the consequence of the smaller mass of the c-quarks which can more easily absorb the v_2 of the light plasma particles with whom they collide during the expansion. We see that the prediction of our model (the data have been presented for the first time at this conference when the calculations have been already performed) agrees with the experimental value in between the error bars. The right hand side highlights the difference of v_2 between b and c quarks at intermediate p_t. This difference is inherent in the model and may therefore serve as a verification if perturbative QCD is the right theory to describe the data. Whereas the v_2 of D-mesons is slightly higher than that of the c-quarks due to the hadronisation, the heavy B-meson has practically the same v_2 as the b-quark before hadronisation.

In conclusions we have shown that pQCD like models which include a running coupling constant as well as a infrared regulator of the gluon propagator in the elastic cross section which is based on hard thermal loop calculations reproduce the LHC data as they reproduced the RHIC data. The model predicts different v_2 values for D- and B-mesons as well as an increase of R_{AA} in central collisions with p_t for p_t larger than 15 GeV. The model can therefore be verified by future experimental data. The results show that collisional as well as radiative energy loss is necessary to describe the data. Both contribute to R_{AA} in a comparable way. In this analysis we used the hydrodynamical model of Heinz and Kolb. It remains to be seen how other models for the expansion of the plasma change the numerical values of R_{AA} and v_2. Studies of different expansion scenarios as well as of the influence of the Landau Pomernschuk Migdal effect are under way.

References

1. S. Borsanyi et al., J. Phys. G **38**, 124101 (2011)
2. S. Weissenborn, I. Sagert, G. Pagliara, M. Hempel, J. Schaffner-Bielich, Astrophys. J. **740**, L14 (2011) [arXiv:1102.2869 [astro-ph.HE]]
3. J. Aichelin, J. Schaffner-Bielich, arXiv:0812.1341 [nucl-th]
4. A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, J. Phys. G **38**, 124081 (2011). [arXiv:1106.6321 [nucl-h]]
5. K. Werner, I. Karpenko, T. Pierog, M. Bleicher, K. Mikhailov, Phys. Rev. C **82**, 044904 (2010). [arXiv:1004.0805 nucl-th]]
6. J. Steinheimer, J. Aichelin, M. Bleicher, arXiv:1203.5302 [nucl-th]
7. M. Luzum, P. Romatschke, Phys. Rev. C **78**, 034915 (2008) [Erratum-ibid. C 79 (2009) 039903] [arXiv:0804.4015 [nucl-th]]
8. J. Aichelin, K. Werner, J. Phys. G **37**, 094006 (2010). [arXiv:1008.5351 [nucl-th]]
9. W. Cassing, E.L. Bratkovskaya, Nucl. Phys. **A831**, 215 (2009) arXiv:0907.5331 [nucl-th]
10. P.B. Gossiaux, S. Vogel, H. van Hees, J. Aichelin, R. Rapp, M. He, M. Bluhm, arXiv:1102.1114 [hep-ph]
11. P.F. Kolb, U.W. Heinz, arXiv:nucl-th/0305084. Published in Qurak Gluon Plasma, World Scientific
12. G.D. Moore, D. Teaney, Phys. Rev. C **71**, 064904 (2005)
13. B. Svetitsky, Phys. Rev. D **37**, 2484 (1988)
14. P.B. Gossiaux, J. Aichelin, Phys. Rev. **78**, 014904 (2008) arXiv:0802.2525 [hep-ph]
15. P.B. Gossiaux, R. Bierkandt, J. Aichelin, arXiv:0901.0946 [hep-ph]
16. P. B. Gossiaux, J. Aichelin, arXiv:0901.2462 [nucl-th]
17. P.B. Gossiaux, J. Aichelin, T. Gousset, V. Guiho, J. Phys. G **37**, 094019 (2010) [arXiv:1001.4166 [hep-ph]]
18. J. Aichelin, P.B. Gossiaux, T. Gousset, to be published
19. E. Meggiolaro, Phys. Rev. D **53**, 3835 (1996). [arXiv:hep-th/9506043]
20. J.F. Gunion, G. Bertsch, Phys. Rev. D **25**, 746 (1982)
21. A. Dainese [ALICE Collaboration], arXiv:1203.2160 [nucl-ex]
22. F. Prino, C. Bianchin, *Contributions to the Conference "Strangeness in Quark Matter"*, Cracow, Sept 2011