

Analyzing Powers for $\vec{p}\vec{p} \rightarrow p\Delta^+$

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Introduction

The theoretical study of neutral pion production in proton-proton collisions has excited considerable attention since the experimental measurements for the total cross-section were found to be more than five times larger than the then existing theoretical predictions. Advances in technologies not only led to the complete identification of the three-body final state kinematically, but also to measurements employing polarized beam on a polarized target [1]. Hanhart et.al., [2] observed: “As far as microscopic model calculations of the reaction $NN \rightarrow NN\pi$ are concerned, one has to concede that the theory is definitely lagging behind the development of experimental sector”. The Julich meson exchange model [2] which was comparatively more successful with the less complete data on $\vec{p}\vec{p} \rightarrow d\pi^+$ and $\vec{p}\vec{p} \rightarrow np\pi^+$, failed to provide an overall satisfactory reproduction of the complete set of polarization observables in the case of $\vec{p}\vec{p} \rightarrow np\pi^0$. A model independent approach [3] using the irreducible tensor techniques [4] was developed to analyse the data. A comparison [5] was made to incisively analyse the findings of the Julich model which not only identified some of the deficiencies in the model but also revealed the importance of Δ contributions. A phase ambiguity inherent in [5] was pointed out in [6].

The purpose of this contribution is to present a model independent theoretical study of the analyzing powers in $\vec{p}\vec{p} \rightarrow p\Delta^+$. We supplement the analysing powers to the unpolarized differential cross-section in Δ production amplitudes.

Theoretical formalism

In an experiment like [1], where both the beam and target are polarized, the initial polarization state is described by the spin density matrix

$$\rho^i = \frac{1}{4} (1 + \sigma \cdot P) \cdot (1 + \sigma \cdot Q) \quad (1)$$

We express ρ^i as

$$\begin{aligned} \rho^i &= \frac{1}{4} \cdot \sum_{k_1, k_2=0}^1 (\sigma^{k_1}(1) \cdot P^{k_1}) \cdot (\sigma^{k_2}(2) \cdot Q^{k_2}) \\ &= \frac{1}{4} \sum_{k_1, k_2=0}^1 \sum_{k=|k_1-k_2|}^{k_1+k_2} (-1)^{k_1+k_2-k} (\sigma^{k_1} \otimes \sigma^{k_2}) (P^{k_1} \otimes Q^{k_2}) \end{aligned} \quad (2)$$

introducing the notations

$$\begin{aligned} P_0^0 &= 1; P_0^1 = P_z; P_{\pm 1}^1 = \mp \frac{P_x \pm iP_y}{\sqrt{2}} \\ Q_0^0 &= 1; Q_0^1 = Q_z; Q_{\pm 1}^1 = \mp \frac{Q_x \pm iQ_y}{\sqrt{2}} \\ \sigma_0^0(j) &= 1; \sigma_0^1(j) = \sigma_z(j); \sigma_{\pm 1}^1(j) = \mp \frac{\sigma_x(j) \pm i\sigma_y}{\sqrt{2}} \\ j &= 1, 2 \end{aligned} \quad (3)$$

The differential cross-section for $pp \rightarrow p\Delta^+$ is then given by

$$\frac{d\sigma}{d\Omega_2} = \text{tr} M \rho^i M^\dagger \quad (4)$$

where $d\Omega_2$ refers to the solid angle associated with the outgoing proton with cm energy

$$E_2 = (E_{cm}^2 - M_\Delta^2 + M_p^2)/2E_{cm} \quad (5)$$

where M_p and M_Δ denote the masses of the proton and delta respectively. The reaction matrix M for $pp \rightarrow p\Delta^+$ may be written in the form

$$M = \sum_{s_i=0}^1 \sum_{s=1}^2 \sum_{\Lambda=|s-s_i|}^{(s+s_i)} (S^\Lambda(s, s_i) \cdot M^\Lambda(s, s_i)) \quad (6)$$

where the irreducible spin tensor operators are defined following [4] and the corresponding irreducible tensor amplitudes $M_\mu^\Lambda(s, s_i)$ are expressible in terms of the partial wave amplitudes $M_{l_2 s; l_1 s_i}^j$ which include also all factors dependent on cm energy E_{cm} .

$$M_\mu^A(s, s_i) = \sum_{l_i, l_2, j} (-1)^{s_i-j} \frac{[j]^2}{[s]} \cdot W(s_i l_i s l_2; j A) \times M_{l_2 s_i, l_i s_i}^j \left(Y_{l_2}(\hat{p}_2) \otimes Y_{l_i}(\hat{p}_i) \right)_\mu^A \quad (7)$$

Using the eq.(2) and eq.(6) in eq.(4), we have

$$\frac{d\sigma}{d\Omega_2} = \frac{d\sigma_0}{d\Omega_2} \sum_{k_1, k_2=0}^1 \sum_{k=|k_1-k_2|}^{(k_1+k_2)} (P^{k_1} \otimes Q^{k_2}) \cdot A^k(k_1, k_2) \quad (8)$$

where, the unpolarized differential cross-section is of the form

$$\frac{d\sigma_0}{d\Omega_2} = \frac{1}{4} \sum_{s, s_i, A, \mu} |M_\mu^A(s, s_i)|^2 = a + b \cos^2 \theta_2 \quad (9)$$

The analyzing powers $A_q^k(k_1, k_2)$ are obtained through

$$\frac{d\sigma_0}{d\Omega_2} \cdot A_q^k(k_1, k_2) = B_q^k(k_1, k_2) =$$

$$\frac{1}{2} \sum_{s_i, s_i', s, A, A'} (-1)^A (-1)^{k_1+k_2-k} [k_1][k_2][s_i][s_i'] [A][A'] \times \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & s_i \\ \frac{1}{2} & \frac{1}{2} & s_i' \\ k_1 & k_2 & k \end{Bmatrix} \cdot (M^A(s, s_i) \otimes (M^{A'}(s, s_i'))_q^k \quad (10)$$

The $B_q^k(k_1, k_2)$ can be expressed in the form

$$B_q^k(k_1, k_2) = a_q^k(k_1, k_2) + b_q^k(k_1, k_2) \sin \theta_2 + c_q^k(k_1, k_2) \cos \theta_2 + d_q^k(k_1, k_2) \sin \theta_2 \cos \theta_2 + g_q^k(k_1, k_2) \cos^2 \theta_2 \quad (11)$$

where the coefficients are functions of partial wave amplitudes $M_{l_2 s_2; l_i s_i}^j$. If we consider only the first three amplitudes

$$\begin{aligned} F_1 &= M_{02;20}^2; \\ F_2 &= M_{11;11}^0; \\ F_3 &= M_{11;11}^1 \end{aligned} \quad (12)$$

close to threshold, we can determine their modulus values and the relative phase between F_2 and F_3 using the following equations

$$|F_1|^2 = 64\pi^2 (a_0^0(0,0) - 2g_0^0(0,0) - a_2^2(1,1) + a_{-2}^2(1,1)) \quad (13)$$

$$|F_2|^2 = 192\pi^2 (g_0^0(0,0) + a_2^2(1,1) - a_{-2}^2(1,1)) \quad (14)$$

$$|F_3|^2 = \frac{256\pi^2}{3} g_0^0(0,0) \quad (15)$$

$$Re F_2 F_3^* = 32\pi^2 (6g_0^0(0,0) - 8a_2^2(1,1) - 2a_{-2}^2(1,1)) \quad (16)$$

$$Im F_2 F_3^* = \frac{64\pi^2 i}{9+3\sqrt{3}-\sqrt{15}} \{ (14 - 2\sqrt{3} - \sqrt{15}) a_2^2(1,1) + (91 - 13\sqrt{3} + \sqrt{15}) a_{-2}^2(1,1) - (99 - 18\sqrt{3}) g_0^0(0,0) - 36c_1^1(1,0) \} \quad (17)$$

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