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RESEARCH ARTICLE

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# Modified gravitational equations on braneworld with Lorentz invariant violation

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**Abstract** The modified gravitational equations to describe a four-dimensional braneworld in the case with the Lorentz invariant violation in a bulk spacetime is presented. It contains a trace part of the brane energy-momentum tensor and the coefficients of all terms describe the Lorentz violation effects from the bulk spacetime. As an application, we apply this formalism to study cosmology. In respect to standard effective Friedmann equations on the brane, Lorentz invariance violation in the bulk causes a modification of this equations that can lead to significant physical consequences. In particular, the effective Friedmann equation on the brane explicitly depends on the equation of state of the brane matter and the Lorentz violating parameters. We show that the components of five-dimensional Weyl curvature are related to the matter on brane even at low energies. We also find that the constraints on the theory parameters are depend on the equation of state of the energy components of the brane matter. Finally, the stability of the model depend on the specific choices of initial conditions and the parameters $\beta_i$.

**Keywords** Modified gravity, Lorentz invariant violation

## 1 Introduction

There has been a growing appreciation of the importance of the violations of Lorentz invariance recently. The intriguing possibility of the Lorentz violation is that an unknown physics at high-energy scales could lead to a spontaneous breaking of Lorentz invariance by giving an expectation value to certain non Standard

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Model fields that carry Lorentz indices, such as vectors, tensors, and gradients of scalar fields [1]. A relativistic theory of gravity where gravity is mediated by a tensor, a vector, and a scalar field, thus called TeVeS gravitational theory [2], provides modified Newtonian dynamics (MOND) and Newtonian limits in the weak field nonrelativistic limit. TeVeS could also explain the large-scale structure formation of the Universe without recurring to cold dark matter [3; 4], which is composed of very massive slowly moving and weakly interacting particles. On the other hand, the Einstein–Aether theory [5] is a vector-tensor theory, and TeVeS can be written as a vector-tensor theory which is the extension of the Einstein–Aether theory [6]. In the case of generalized Einstein–Aether theory [7], the effect of a general class of such theories on the solar system has been considered in Ref. [8]. On small scales the Einstein-Aether vector field will in general lead to a renormalization of the local Newton Constant [9]. Moreover, as has been shown by authors in Ref. [10], the Einstein–Aether theory may lead to significant modifications of the power spectrum of tensor perturbation. The existence of vector fields in a scalar–vector–tensor theory of gravity also leads to its applications in modern cosmology and it might explain inflationary scenarios [11; 12; 13; 14] and accelerated expansion of the universe [7; 15]. Based on a dynamical vector field coupled to the gravitation and scalar fields, we have studied to some extent the cosmological implications of a scalar–vector–tensor theory of gravity [16; 17; 18]. The models also allow crossing of phantom divide line [19].

Motivated by string theory and its extension M-theory, the standard model particles may be confined on a hypersurface, called brane, embedded in a higher dimensional space, called bulk. Only gravity and other exotic matter such as the dilaton can propagate in the bulk [20]. The braneworld models have been shown to be extremely rich in phenomena leading to modifications of General Relativity (GR) at both low and high energies (for a review see, e.g. [21]). In the context of gravity and cosmology, models proposed by Randall and Sundrum (RS) [22; 23] have attracted much attention, where four-dimensional gravity can be recovered at low energy despite the infinite size of the extra dimension. In RS II model [23], a positive tension brane is embedded in five-dimensional anti-de Sitter (AdS) spacetime. To study gravity on the brane, it is useful to derive the effective four-dimensional Einstein equation on the brane firstly developed by Shiromizu, Maeda, and Sasaki (SMS) [24]. There are two very important results that arise from the effective four-dimensional Einstein equations on the brane. The first one is quadratic energy-momentum tensor, $\pi_{\mu\nu}$, which is relevant in high energy and the second one is the projected Weyl tensor, $E_{\mu\nu}$, on the brane which is responsible for carrying on the brane the contribution of the bulk gravitational field. In the RS II models, this term supplies an additional matter-like effect to the brane. Thus, its contribution to the four-dimensional effective theory is of crucial importance as it is non-negligible already even in low energy limit. Then, the Friedmann equations on the brane, governing the cosmological evolution of the brane, are non conventional in that the Hubble parameter depends quadratically on the energy density instead of linearly as in standard cosmology, and one radiation like term, usually referred to as a dark radiation term in the homogeneous and isotropic background spacetime. This dark radiation modifies the expansion of the background universe in the same way as an usual radiation [25; 26; 27; 28].

Recently, a braneworld scenario with bulk broken Lorentz invariance has been developed, where a family of static self-tuning braneworld solutions was found [29; 30]. In a different approach braneworld model, a bulk vector field with a non-vanishing vacuum expectation value, allowing for the spontaneous breaking of the Lorentz symmetry. The breaking of Lorentz invariance the loss of this symmetry is transmitted to the gravitational sector of the model. By assuming that the vacuum expectation value of the component of the vector field normal to the brane vanishes, it found that Lorentz invariance on the brane can be made exact via the dynamics of the graviton, vector field, and the geometry of the extrinsic curvature of the surface of the brane. As a consequence of the exact reproduction of Lorentz symmetry on the brane, a condition for the matching of the observed cosmological constant in four dimensions is found [31]. The notion of Lorentz violation in four dimensions is extended to a five-dimensional braneworld scenario resulting the time variation in the gravitational coupling and cosmological constant. There exist also a relation between the maximal velocity in the bulk and the speed of light on the brane [32; 33]. Various Lorentz violating effects within the context of the braneworld scenario have also been studied in Refs. [34; 35; 36; 37; 38; 39].

In this paper we address the issue of cosmological evolution on a brane in a theory of gravity whose action includes, in addition to the familiar Einstein term, a Lorentz violating vector field contribution. We generalize the gravitational effects of the vector fields in four dimensions [5; 40] to include five dimensional braneworld gravity. In particular, we put a vector $n^a$ in the direction of the extra dimension such that the existence of the brane defines a preferred direction in the bulk.

This paper is organized as follows. In Sect. 2, we derive the four-dimensional effective Einstein equations on the brane in the case with the Lorentz invariant violation in a bulk spacetime. With non-ignoring of the Lorentz violation effects, this equation is modified by the trace of the brane energy–momentum tensor. Thus the relation between the projected Weyl tensor and the brane matter may be understood. In Sect. 3, we study the cosmological implications of the modified four-dimensional effective Einstein equations on the brane. In general, the effective four-dimensional Einstein equations on the brane cannot be solved without knowing $E_{\mu\nu}$, because it could have a non-trivial component of an anisotropic stress [41]. However, it is possible to know some features of this tensor by using constraint equations on the brane obtained by the four-dimensional Bianchi identity. In the background spacetime, the four-dimensional equations are sufficient to show that $E_{\mu\nu}$ induces the radiation fluid on the brane. We will take this strategy to determine the Friedmann equation on the brane. Interestingly, the Friedmann equation is found to depend on the equation of state of the matter explicitly, and the Lorentz violation parameters. In Sect. 4, we discuss a low energy limit of the theory. Remarkable, the parameters of the theory can be determined by equation of state of the brane matter. We discuss the stability of the model in Sect. 5. Section 6 is devoted to the conclusions.

## 2 Modified SMS effective equation on the brane

In this section, we derive the 4-dimensional effective gravitational equations in a $Z_2$-symmetric braneworld using the geometrical projection approach. For this purpose, we first write the 5-dimensional field equations in the form of the evolution equations along the extra dimension and the constraint equations.

The action we consider consists of the vector field $n^a$ minimally coupled to gravity:

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-\tilde{g}}\,(\mathscr{R} - 2\Lambda) + \int d^5x\sqrt{-\tilde{g}}\mathscr{L}_n + \int d^4x\sqrt{-g}(-\sigma + \mathscr{L}_m). \tag{1}$$

Here, $\mathscr{R}, \kappa, \Lambda$, and $\tilde{g}$ are the scalar curvature, the gravitational constant in 5-dimensions, the bulk cosmological constant, and the determinant of 5-dimensional metric, respectively. $\mathscr{L}_m$ and $\mathscr{L}_n$ are the Lagrangian density for the matter fields on the brane and the vector field Lagrangian, respectively. A metric $g$ is the induced metric on the brane while $\sigma$ denotes the brane tension. Note that we have assumed no coupling between the matter fields and the vector field in the action (1). Therefore, the brane observer does not feel the present of the preferred frame.

We write the coordinate system for the bulk spacetime in the form

$$ds^2 = g_{ab}dx^a dx^b = dy^2 + g_{\mu\nu}(y,x)dx^\mu dx^\nu, \tag{2}$$

and we may assume that the position of the brane is $y = 0$ in this coordinate system so that the induced metric on the brane is $g_{\mu\nu}(x) = \tilde{g}_{\mu\nu}(y = 0, x)$. We also assume a $Z_2$-symmetry across the brane and the extrinsic curvature is defined as $K_{\mu\nu} = -g_{\mu\nu,y}/2$.

The vector field Lagrangian, $\mathscr{L}_n$, is given by

$$\mathscr{L}_n = -\beta_1 \nabla^a n^b \nabla_a n_b - \beta_2 (\nabla_a n^a)^2 - \beta_3 \nabla^a n^b \nabla_b n_a + \lambda(n^a n_a - 1), \tag{3}$$

where $\beta_i$ are constant parameters and $\lambda$ is a Lagrangian multiplier. In this setup, we assume that $n^a$ is a vector field along the extra dimension and the preferred frame is selected by the constrained vector field $n^a$ which violates Lorentz symmetry. We take $n^a$ as the dimensionless vector. Hence, each $\beta_i$ has dimension of $(mass)^3$. In other words, $\beta_i^{1/3}$ gives the mass scale of symmetry breakdown in the bulk. Following the usual braneworld scenarios our spacetime is orthogonal to the extra dimension. Then one can introduce the normal unit vector $n^a$ which is orthogonal to the hypersurfaces at $y = const.$ as $n^a = \delta^a_y$. In particular, there is a background solution that 5-vector takes on a vacuum expectation value with components $(0,0,0,0,1)$, thus allowing for the spontaneous breaking of the Lorentz symmetry.

Varying the action (1) with respect to the metric, $\lambda$, and $n^a$, respectively, we have the field equations

$$^{(5)}G_{ab} = -\Lambda g_{ab} + \kappa^2(T_{ab} + \mathscr{T}_{ab}) + \kappa^2\delta^\mu_a\delta^\nu_b S_{\mu\nu}\delta(y), \tag{4}$$

$$g_{ab}n^a n^b = 1, \tag{5}$$

$$\nabla_a J^{ab} = \lambda n^b, \tag{6}$$

where current tensor $J^a{}_c$ is given by

$$J^a{}_b = -\beta_1 \nabla^a n_b - \beta_2 \delta^a_b \nabla_c n^c - \beta_3 \nabla_b n^a, \tag{7}$$

and $S_{\mu\nu} = -\sigma g_{\mu\nu} + \tau_{\mu\nu}$ is the energy momentum tensor on the brane, where $\tau_{\mu\nu}$ is the energy momentum tensor of the brane matter other than the tension. $T_{ab}$ is the energy-momentum tensor of the vector field. To be as general as possible, we also have included a bulk energy-momentum tensor in (4), denoted by $\mathscr{T}_{ab}$.

Using the extrinsic curvature, the components of the left hand side of Einstein equations (4) are

$$\begin{aligned} {}^{(5)}G^y{}_y &= -\frac{1}{2}R + \frac{1}{2}K^2 - \frac{1}{2}K^{\alpha\beta}K_{\alpha\beta} \\ &= -\Lambda + \kappa^2 T^y{}_y + \kappa^2 \mathscr{T}^y{}_y, \end{aligned} \tag{8}$$

$${}^{(5)}G^y{}_\mu = -D_\alpha K_\mu{}^\alpha + D_\mu K = \kappa^2 (T^y{}_\mu + \mathscr{T}^y{}_\mu), \tag{9}$$

$$\begin{aligned} {}^{(5)}G^\mu{}_\nu &= G^\mu{}_\nu + (K^\mu{}_\nu - \delta^\mu_\nu K)_{,y} + \frac{1}{2}\delta^\mu_\nu (K^2 + K^{\alpha\beta}K_{\alpha\beta}) \\ &= -\Lambda \delta^\mu_\nu + \kappa^2 (T^\mu{}_\nu + \mathscr{T}^\mu{}_\nu) + \kappa^2 S^\mu{}_\nu \delta(y), \end{aligned} \tag{10}$$

where $G^\mu{}_\nu$ is the 4-dimensional Einstein tensor and the covariant derivatives $D_\mu$ is calculated with respect to the four-dimensional metric $g_{\mu\nu}$. The components of the energy momentum tensor of the vector field are given by

$$T^y{}_y = \beta_2 K^2 + (\beta_1 + \beta_3) K^{\alpha\beta} K_{\alpha\beta}, \tag{11}$$

$$T^y{}_\mu = 0, \tag{12}$$

$$\begin{aligned} T^\mu{}_\nu &= 2(\beta_1 + \beta_3) K^\mu{}_\nu K + \beta_2 \delta^\mu{}_\nu K^2 - \delta^\mu_\nu (\beta_1 + \beta_3) K^{\alpha\beta} K_{\alpha\beta} \\ &\quad - 2(\beta_1 + \beta_3) K^\mu{}_{\nu,y} - 2\beta_2 \delta^\mu_\nu K_{,y}. \end{aligned} \tag{13}$$

Combining Eqs. (8) with (10) and using (11) and (13), we have

$$\begin{aligned} -\frac{1}{3}\left(R^\mu{}_\nu - \frac{1}{4}\delta^\mu_\nu R\right) &= \frac{1}{6}\delta^\mu_\nu \Lambda + \frac{(1-\alpha_0)}{12}\delta^\mu_\nu K^2 \\ &\quad - \frac{(1+\alpha_1)}{3}\left(K K^\mu{}_\nu - \frac{3}{4}\delta^\mu_\nu K_{\alpha\beta}K^{\alpha\beta}\right) + \frac{(1+\alpha_1)}{3} K^\mu{}_{\nu,y} \\ &\quad - \frac{(1-\alpha_0)}{3}\delta^\mu_\nu K_{,y} - \frac{\kappa^2}{3}\left(\mathscr{T}^\mu{}_\nu - \frac{1}{2}\delta^\mu_\nu \mathscr{T}^y{}_y\right), \end{aligned} \tag{14}$$

where we have defined

$$\alpha_0 = 2\kappa^2 \beta_2, \quad \alpha_1 = 2\kappa^2(\beta_1 + \beta_3). \tag{15}$$

The trace of equation (14) yields

$$\begin{aligned} (3 - 4\alpha_0 - \alpha_1) K_{,y} &= 2\Lambda - (\alpha_0 + \alpha_1) K^2 + 3(1+\alpha_1) K_{\alpha\beta}K^{\alpha\beta} \\ &\quad - \frac{\kappa^2}{3}\left(\mathscr{T}^\mu{}_\mu - 2\mathscr{T}^y{}_y\right). \end{aligned} \tag{16}$$

Substituting Eqs. (14) and (16) into the following components of the Weyl tensor

$$C_{y\mu y\nu} = -\frac{1}{3}\left(R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R\right) + \frac{1}{3}\left(KK_{\mu\nu} - \frac{1}{4}g_{\mu\nu}K^2\right) + \frac{1}{3}\left(K_\mu{}^\alpha K_{\alpha\nu} + \frac{3}{4}g_{\mu\nu}K_{\alpha\beta}K^{\alpha\beta}\right) + \frac{2}{3}\left(K_{\mu\nu,y} - \frac{1}{4}g_{\mu\nu}K_{,y}\right), \quad (17)$$

we have

$$\begin{aligned}\frac{3(1+\alpha_1)}{(3+\alpha_1)}C_{y\mu y\nu} = {} & \frac{1}{2}\Lambda g_{\mu\nu} - \frac{3\alpha_0 + (2+\alpha_0)\alpha_1}{4(3+\alpha_1)}g_{\mu\nu}K^2 \\ & - \frac{(1+\alpha_1)\alpha_1}{(3+\alpha_1)}KK_{\mu\nu} + \frac{(1+\alpha_1)(3+2\alpha_1)}{(3+\alpha_1)}K_\mu{}^\lambda K_{\lambda\nu} \\ & + \frac{3(1+\alpha_1)(4+\alpha_1)}{4(3+\alpha_1)}g_{\mu\nu}K_{\alpha\beta}K^{\alpha\beta} + (1+\alpha_1)K_{\mu\nu,y} \\ & - (1-\alpha_0)g_{\mu\nu}K_{,y} + \frac{\kappa^2}{2}g_{\mu\nu}\mathscr{T}^y{}_y - \frac{\kappa^2}{3}\left(\mathscr{T}_{\mu\nu} + \frac{1}{2}g_{\mu\nu}\mathscr{T}^\alpha{}_\alpha\right). \end{aligned} \quad (18)$$

Here, we have defined that the term $\mathscr{T}^\alpha{}_\alpha$ is the trace defined with respect to the four-dimensional metric $g$, and not the full trace defined with respect to $\tilde{g}$. Equation (10) can be expressed as

$$\begin{aligned} G_{\mu\nu} = {} & -\Lambda g_{\mu\nu} + (1+\alpha_1)KK_{\mu\nu} - \frac{(1-\alpha_0)}{2}g_{\mu\nu}K^2 \\ & - 2(1+\alpha_1)K_\mu{}^\alpha K_{\alpha\nu} - \frac{(1+\alpha_1)}{2}g_{\mu\nu}K_{\alpha\beta}K^{\alpha\beta} \\ & - (1+\alpha_1)K_{\mu\nu,y} + (1-\alpha_0)g_{\mu\nu}K_{,y} + \kappa^2\mathscr{T}_{\mu\nu}. \end{aligned} \quad (19)$$

Using Eq. (18), Eq. (19) is expressed as

$$\begin{aligned} G_{\mu\nu} = {} & -\frac{1}{2}\Lambda g_{\mu\nu} - \frac{3(1+\alpha_1)}{(3+\alpha_1)}E_{\mu\nu} - \frac{3(1+\alpha_1)}{(3+\alpha_1)}(K_\mu{}^\alpha K_{\alpha\nu} - KK_{\mu\nu}) \\ & - \frac{6+4\alpha_1 - (3+\alpha_1)\alpha_0}{4(3+\alpha_1)}g_{\mu\nu}K^2 + \frac{(1+\alpha_1)(6+\alpha_1)}{4(3+\alpha_1)}g_{\mu\nu}K_{\alpha\beta}K^{\alpha\beta} \\ & + \frac{\kappa^2}{2}g_{\mu\nu}\mathscr{T}^y{}_y + \frac{2\kappa^2}{3}\left(\mathscr{T}_{\mu\nu} - \frac{1}{4}g_{\mu\nu}\mathscr{T}^\alpha{}_\alpha\right), \end{aligned} \quad (20)$$

where the projected Weyl tensor is $E_{\mu\nu} = C_{y\mu y\nu}|_{y=0}$. Note that the coefficient of the four-dimensional Einstein tensor (20) is modified by factor $(3+\alpha_1)$. Here, we take $\alpha_1 \neq -3$. The case $\alpha_1 = -3$ provides a relation between the extrinsic curvature and the projected Weyl tensor. To eliminate the extrinsic curvature, we use the junction conditions. It can be obtained by collecting together the terms in field equations which contain a $\delta$-function. From Eqs. (10) and (13), we then obtain

$$\left[K^{\mu}{}_{\nu} - \delta^{\mu}_{\nu} K\right]|_{y=0} = \frac{\kappa^2}{2(1+\alpha_1)} \left(S^{\mu}{}_{\nu} + \alpha_2 \delta^{\mu}_{\nu} S\right), \tag{21}$$

where

$$\alpha_2 = \frac{\alpha_0 + \alpha_1}{3 - 4\alpha_0 - \alpha_1}. \tag{22}$$

For convenient we will take $\alpha_1 \neq -3$ and $\alpha_1 \neq -1$ in order to avoid unreal singularities in Eqs. (20) and (21). Substituting (21) into (20), we finally obtain the modified effective SMS equation on the brane as

$$G_{\mu\nu} = -\Lambda_b g_{\mu\nu} + 8\pi G \left(\tau_{\mu\nu} + \frac{\alpha_1}{12} g_{\mu\nu} \tau\right) + \kappa^4 \pi_{\mu\nu} - \widetilde{E}_{\mu\nu} + F_{\mu\nu}, \tag{23}$$

where we have defined the quantities

$$\Lambda_b = \frac{1}{2}\Lambda + \frac{\kappa^4}{4(3-4\alpha_0-\alpha_1)}\sigma^2, \tag{24}$$

$$8\pi G = \frac{3\kappa^4}{2(3+\alpha_1)(3-4\alpha_0-\alpha_1)}\sigma, \tag{25}$$

$$\pi_{\mu\nu} = \frac{3}{4(3+\alpha_1)(1+\alpha_1)}\left[\frac{(1-2\alpha_0-\alpha_1)}{(3-4\alpha_0-\alpha_1)}\tau\tau_{\mu\nu} - \tau_\mu{}^\alpha\tau_{\alpha\nu} \right.$$
$$\left. + \frac{(6+\alpha_1)}{12}g_{\mu\nu}\tau_{\alpha\beta}\tau^{\alpha\beta} - \frac{2(3-\alpha_1)-(9+\alpha_1)\alpha_0}{12(3-4\alpha_0-\alpha_1)}g_{\mu\nu}\tau^2\right], \qquad (26)$$

$$\widetilde{E}_{\mu\nu} = \frac{3(1+\alpha_1)}{(3+\alpha_1)}E_{\mu\nu}, \qquad (27)$$

and the bulk energy-momentum tensor projected on the brane is given by

$$F_{\mu\nu} = \left[\frac{\kappa^2}{2}g_{\mu\nu}\mathscr{T}^y{}_y + \frac{2\kappa^2}{3}\left(\mathscr{T}_{\mu\nu} - \frac{1}{4}g_{\mu\nu}\mathscr{T}^\alpha{}_\alpha\right)\right]_{y=0}. \qquad (28)$$

There are four features in the effective Einstein equations (23). The first one is the presence of the bulk energy-momentum tensor. This term allows exotic matter such as the dilaton can propagate in the bulk. The second departure from the standard four-dimensional Einstein equation arises from the presence of the Weyl tensor which is undetermined on the brane. The third is a quadratic in the brane energy-momentum tensor. The last one is a linear in addition to the brane energy-momentum tensor. It is our main result. This trace part of the brane energy-momentum tensor is measured by local observers at the brane and vanishes when $\alpha_1 = 2\kappa^2(\beta_1+\beta_3) = 0$.

Equation (9) and the junction conditions (21) imply

$$D_\mu\tau^\mu{}_\nu + \alpha_2 D_\nu\tau - (1+4\alpha_2)D_\nu\sigma = -2(1+\alpha_1)\mathscr{T}^y{}_\nu. \qquad (29)$$

This equation tell us that the energy momentum tensor $\tau_{\mu\nu}$ is not conserved on the brane. Taking the divergence of the four-dimensional effective equations and using four-dimensional Bianchi identity, we obtain the constraint equations for $E_{\mu\nu}$ as

$$D_\mu\widetilde{E}^\mu{}_\nu = -D_\nu\Lambda_b + 8\pi G\left(D_\mu\tau^\mu{}_\nu + \frac{\alpha_1}{12}D_\nu\tau\right) + \kappa^4 D_\mu\pi^\mu{}_\nu$$
$$+\frac{\kappa^2}{2}D_\nu\mathscr{T}^y{}_y + \frac{2\kappa^2}{3}\left(D_\mu\mathscr{T}_{\mu\nu} - \frac{1}{4}D_\nu\mathscr{T}^\alpha{}_\alpha\right). \qquad (30)$$

Equations (29) and (30) indicate a time variation of the brane tension, the cosmological constant, and the gravitational constant in general.

In the following section, we study analytically the cosmological consequences of Eqs. (23), (29) and (30). Here, for simplicity, we consider constant $\sigma$, because there are no theoretical observational arguments for the evolution of $\sigma$ in time. For cosmology on the brane, we suppose here that we can ignore the bulk matter, $F_{\mu\nu} = 0$. The bulk matter is important to get a well-behaved geometry in the bulk. We also assume that the bulk cosmological constant is truly constant. Then, Eqs. (29) and (30) become

$$D_\mu\tau^\mu{}_\nu = -\alpha_2 D_\nu\tau, \qquad (31)$$

$$D_\mu\widetilde{E}^\mu{}_\nu = -8\pi G\left(\alpha_2 - \frac{\alpha_1}{12}\right)D_\nu\tau + \kappa^4 D_\mu\pi^\mu{}_\nu. \qquad (32)$$

Note that the projected Weyl tensor is affected by the energy-momentum tensor on the brane even at low energies. Thus, the model is quite different from the conventional braneworld even at low energies.

## 3 Braneworld cosmology

The projected Weyl tensor in the modified Einstein equation (23) is a priori undetermined. This comes from the five-dimensional nature of the theory and the fact that the system of equations is not closed on the brane. This tensor mediates some information from the bulk to the brane. In this section, we will try to solve Einstein equation to study the cosmology braneworld from Eq. (23), by assuming that there is no cosmological constant on the brane and the constant vacuum energy. Although these assumptions are usual in braneworld scenario, we will show, which is the main result of present paper, the effective Friedmann equations is modified by the effect of Lorentz violation, and the components of the projected Weyl tensor are related to the matter on the brane. We then discuss the method to obtain the components of the projected Weyl tensor from the brane data. For cosmological applications, we consider a metric of the form

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \tag{33}$$

where $x^i$ are the three ordinary spatial coordinates and $a$ is the scale factor. The Hubble parameter $H$ on the brane, describing the cosmological dynamics of the Universe, is defined as $H = \dot{a}/a$. For simplicity, we ignore the bulk matter for the cosmology on the brane. Hereafter, we will consider only the matter on the brane. For further discussions on the gravitational field equations in the braneworld model with Lorentz violation and their cosmological applications see [32; 33]. We restrict the energy-momentum tensor on the brane of the form

$$\tau_{\mu\nu} = (\rho, Pa^2\delta_{ij}), \tag{34}$$

where $\rho$ is the energy density and $P$ the pressure. We will assume that the equation of state relating $\rho$ and $P$ has the form $P = \omega\rho$, where $\omega$ is constant. Similarly, the projected Weyl tensor is of the form

$$E_{\mu\nu} = (\rho_d, P_d a^2\delta_{ij}). \tag{35}$$

The traceless property of $E_{\mu\nu}$ implies: $-\rho_d + 3P_d = 0$. We will be interested in the relation between the components of the projected Weyl tensor and the brane energy-momentum tensor. The components of the quadratic in the energy-momentum tensor (26) are given by

$$\pi_{00} = \frac{1+3\alpha_3}{4(3+\alpha_1)(1+\alpha_1)^2}\rho^2, \tag{36}$$

$$\pi_{ij} = \frac{1+2\omega-3\alpha_4}{4(3+\alpha_1)(1+\alpha_1)^2}\rho^2 a^2\delta_{ij}, \tag{37}$$

where

$$\begin{aligned}\alpha_3 &= \frac{1}{12(3-4\alpha_0-\alpha_1)}\{[7-9(2+\omega)\omega-3(1+\omega)^2(2-\alpha_1)\alpha_1]\alpha_0 \\ &\quad -[17-3\omega(8+3\omega)+2(1-12\omega-3\omega^2+(1+3\omega^2)\alpha_1^2)]\alpha_1\}, \\ \alpha_4 &= \frac{1}{12(3-4\alpha_0-\alpha_1)}\{[-1-2\omega+15\omega^2+3(1+\omega)^2(6+\alpha_1)\alpha_1]\alpha_0 \\ &\quad -[15+32\omega-15\omega^2-(1+3\omega^2)\alpha_1^2-2(1+9\omega^2)\alpha_1]\alpha_1\}.\end{aligned} \tag{38}$$

Substituting metric (33) and tensors (34), (35) and (36), (37) in the effective Einstein equations (23), one finds

$$3H^2 = 8\pi G\left[1+\frac{(1-3\omega)\alpha_1}{12}\right]\rho+\frac{\kappa^4(1+3\alpha_3)}{4(1+\alpha_1)^2(3+\alpha_1)}\rho^2-\frac{3(1+\alpha_1)}{(3+\alpha_1)}\rho_d, \tag{39}$$

$$-2\dot{H}-3H^2 = 8\pi G\left[\omega-\frac{(1-3\omega)\alpha_1)}{12}\right]\rho+\frac{\kappa^4(1+2\omega-3\alpha_4)}{4(1+\alpha_1)^2(3+\alpha_1)}\rho^2-\frac{(1+\alpha_1)}{(3+\alpha_1)}\rho_d. \tag{40}$$

Obviously, these equations are quite different from the usual braneworld equations due to the effect of bulk Lorentz violation. From Eq. (31) and the constraint equation for the projected Weyl tensor (32), we have

$$[1+(1-\omega)\alpha_2]\dot{\rho}+3H\rho(1+\omega)=0, \tag{41}$$

and

$$\begin{aligned}\dot{\rho}_d+4H\rho_d &= \frac{8\pi G(1+\omega)(1-3\omega)(3+\alpha_1)(3-4\alpha_0-\alpha_1)}{3(1+\alpha_1)^3} \\ &\quad \times\left[\alpha_2-\frac{(1+\alpha_1)^2\alpha_1}{12(1-\omega\alpha_1-(1+\omega)\alpha_0)}\right]H\rho \\ &\quad -\frac{\kappa^4(1+\omega)\alpha_5}{4(1+\alpha_1)^3(1+(1-3\omega)\alpha_2)}H\rho^2,\end{aligned} \tag{42}$$

where

$$\begin{aligned}\alpha_5 &= \frac{(1+\alpha_1)}{2(3-4\alpha_0-\alpha_1)}\{12(1+\omega)^2\alpha_0^2\alpha_1+[9(1+3\omega^2) \\ &\quad +(1+3\omega^2)\alpha_1^2-2(1+12\omega-9\omega^2)\alpha_1]\alpha_1 \\ &\quad +[3(1-3\omega^2)-2(7+30\omega-9\omega^2)\alpha_1+(7+6\omega+15\omega^2)\alpha_1^2]\alpha_0\}.\end{aligned} \tag{43}$$

For $\omega \neq -1$, Eq. (41) is solved to yield

$$\rho = a^{-\frac{3(1+\omega)}{1+(1-\omega)\alpha_2}}. \tag{44}$$

Here, we have absorbed a constant factor into the scale factor by rescaling it. Equation (42) can be integrated. We find

$$\rho_d = -\frac{3C}{a^4} + \frac{8\pi G(1+\omega)(3+\alpha_1)(3-4\alpha_0-\alpha_1)^2}{9(1+\alpha_1)^4} \times \left\{[1+(1-3\omega)\alpha_2]\alpha_2 - \frac{\alpha_1(1+\alpha_1)^2}{4(3-4\alpha_0-\alpha_1)}\right\} a^{-\frac{3(1+\omega)}{1+(1-\omega)\alpha_2}} - \frac{\kappa^4(1+\omega)\alpha_5}{8(1+\alpha_1)^3[2(1-3\omega)\alpha_2-(1+3\omega)]} a^{-\frac{6(1+\omega)}{1+(1-\omega)\alpha_2}}, \quad (45)$$

where $C$ is a constant of integration. This effect of the bulk acts as radiation fluid, hence it is called as dark radiation. Substituting Eq. (45) into Eq. (39), we obtain the effective Friedmann equation

$$H^2 = \frac{8\pi G_{eff}}{3}\rho + A\rho^2 + \frac{\bar{C}}{a^4}, \quad (46)$$

where

$$G_{eff} = \left\{1 - \frac{[1-\omega\alpha_1-(1+\omega)\alpha_0][(2+3\omega-(2+\alpha_1)\alpha_1)\alpha_1+3(1+\omega)\alpha_0]}{3(1+\alpha_1)^3}\right\} G, \quad (47)$$

$$A = \frac{\kappa^4}{12(3+\alpha_1)(1+\alpha_1)^2}\left[1+3\alpha_3 - \frac{3(1+\omega)\alpha_5}{2[(1+3\omega)-2(1-3\omega)\alpha_2]}\right], \quad (48)$$

$$\bar{C} = \frac{3(1+\alpha_1)}{(3+\alpha_1)} C. \quad (49)$$

Note that the effective Newton constant depends on the Lorentz violating parameters and the equations of state. It is different from the conventional braneworld cosmology in five-dimensional case even at low energy. If the effects of Lorentz violations are ignored, $\beta_i = 0$, we have $G_{eff} = G, A = \kappa^4/36$ and $\bar{C} = C$. In the alternative theory of gravity including Brans–Dicke theory, the effective Newton constant need not be constant in time. Observational bounds on $\dot{G}/G$ then constrain the theory. In our case, we have the relation (47), hence the Newton constant is always constant.

If the effective cosmological constant is included, the Friedmann equation (46) becomes

$$H^2 = \frac{1}{3}\Lambda_b + \frac{8\pi G_{eff}}{3}\rho + A\rho^2 + \frac{\bar{C}}{a^4}, \quad (50)$$

where the relation between the vacuum energy and the effective cosmological constant on a brane is given by Eq. (24). It is different from the usual four-dimensional theory. In the RS braneworld, the vacuum energy in the brane is not directly related to the cosmological constant on the brane in the effective Einstein equation as in Eq. (24). In the RS braneworld, there should be a cancellation between the four-dimensional and five-dimensional contribution of the vacuum energy in order to have a vanishing cosmological constant on the brane. This requires a fine-tuning for the parameters in the action. In the present model the RS type relation is given by

$$\sigma = \frac{6}{\kappa^2 l}\left(1 - \frac{4}{3}\alpha_0 - \frac{1}{3}\alpha_1\right)^{1/2}, \quad (51)$$

and

$$8\pi G = \frac{3\kappa^2}{l(3+\alpha_1)(1+\alpha_1)^{1/2}}. \quad (52)$$

Here, the bulk cosmological constant is defined as $\Lambda = -6/\kappa^2 l^2$, where $l$ is the scale of the bulk curvature radius.

## 4 Low energy constraint on $\beta_i$

For a well-defined theory, the constraints on the theory parameters $\beta_i$ are given by [14] (see also [9]):

1. Subluminal propagation of spin-0 field: $(\beta_1 + \beta_2 + \beta_3)/\beta_1 \leq 1$,
2. Positivity of Hamiltonian: $\beta_1 > 0$,
3. Non-tachyonic propagation of spin-0 field: $(\beta_1 + \beta_2 + \beta_3)/\beta_1 \geq 0$,
4. Subluminal propagation of spin-2 field: $\beta_1 + \beta_3 \leq 0$.

All these conditions together imply $(\beta_1 + \beta_2 + \beta_3) \geq 0$ and $\beta_2 \geq 0$.

At low energies, we can neglect the quadratic term of the Friedmann equation (46). Then we have

$$H^2 = \frac{8\pi G_{eff}}{3}\rho + \frac{\bar{C}}{a^4}. \quad (53)$$

Here, we have assumed $3A/8\pi G_{eff} \ll 1$. Therefore, one can set $A \approx 0$ without loss of generality. Solving Eq. (48) one finds

$$\alpha_1 = \frac{1 - \alpha_0(1+\omega)}{\omega}, \quad \text{or} \quad \alpha_1 = \frac{2(1+3\omega) - 3\alpha_0(1+\omega)^2}{1+3\omega^2}, \quad (54)$$

where $\alpha_0$ and $\alpha_1$ is given by Eq. (15). In other word, the effect of Lorentz violation in the bulk is dependent on the equation of state of the energy components of the Universe. Remarkable, the first solution (54) yields $G_{eff} = G$. In this case, using the above constraints we find

1. For $\omega < -1$,

$$\alpha_0 > \frac{1+3\omega}{1+\omega}, \quad \alpha_1 < -3, \quad (55)$$

and

$$1 < \alpha_0 < \frac{1+3\omega}{1+\omega}, \quad -3 < \alpha_1 < -1. \quad (56)$$

2. For $-1 < \omega < 0$,

$$1 < \alpha_0 \leq \frac{1}{1+\omega}, \quad -1 < \alpha_1 \leq 0. \tag{57}$$

3. For $\omega > 0$,

$$\frac{1}{1+\omega} \leq \alpha_0 < 1, \quad -1 < \alpha_1 \leq 0. \tag{58}$$

The above constraints give the correction in the coefficient of the dark radiation. The second solution (54) gives the constraints:

1. For $\omega < -1$,

$$\alpha_0 > \frac{5+6\omega+9\omega^2}{3(1+\omega)^2}, \quad \alpha_1 < -3, \tag{59}$$

and

$$1 < \alpha_0 < \frac{5+6\omega+9\omega^2}{3(1+\omega)^2}, \quad -3 < \alpha_1 < -1. \tag{60}$$

2. For $-1 < \omega \leq -1/3$,

$$\alpha_0 > \frac{5+6\omega+9\omega^2}{3(1+\omega)^2}, \quad \alpha_1 < -3, \tag{61}$$

and

$$1 < \alpha_0 < \frac{5+6\omega+9\omega^2}{3(1+\omega)^2}, \quad -3 < \alpha_1 < -1. \tag{62}$$

3. For $\omega \geq -1/3$,

$$\frac{2(1+3\omega)}{3(1+\omega)^2} \leq \alpha_0 < 1, \quad -1 < \alpha_1 \leq 0. \tag{63}$$

These constraints give the corrections both in the effective Newton constant and the dark radiation.

## 5 Flat spacetime stability analysis

In this section we discuss a flat spacetime stability analysis. The classical stability of fixed-norm spacelike Lorentz violating vector field model has been studied in Ref. [42]. In this case, the Lagrangian of the vector field is as follows

$$\mathscr{L}_n = -\beta_1 \partial^a n^b \partial_a n_b - \beta_2 (\partial_a n^a)^2 - \beta_3 \partial^a n^b \partial_b n_a + \lambda (n^a n_a - 1), \tag{64}$$

where the equation of motion for the vector field is given by

$$\lambda n^a + \beta_1 \partial_b \partial^b n^a + (\beta_2 + \beta_3)\, \partial^a \left(\partial_b n^b\right) = 0. \tag{65}$$

We consider small perturbations about the background, $n^a = A^a + \varepsilon^a$. To the first order in perturbations we have the following equations for the non-trivial vector field perturbations:

$$\beta_1 \partial_b \partial^b \varepsilon^0 + (\beta_2 + \beta_3)\, \partial^0 \left(\partial_b n^b\right) = 0, \tag{66}$$

$$\beta_1 \partial_b \partial^b \varepsilon^i + (\beta_2 + \beta_3)\, \partial^i \left(\partial_b n^b\right) = 0, \tag{67}$$

where $i \in \{1,2,3\}$.

Using the Fourier transform the components of $\varepsilon^a$, and in terms of Fourier modes,

$$\varepsilon^0(\omega,\mathbf{k}) = k_5 \theta_1(\omega,\mathbf{k}), \quad \varepsilon^i(\omega,\mathbf{k}) = k_i \theta_2(\omega,\mathbf{k}) + \varepsilon_{ij} k_j \theta_3(\omega,\mathbf{k}), \tag{68}$$

we have

$$\left[\beta_1 \left(k_\perp^2 + k_5^2\right) - (\beta_1 + \beta_2 + \beta_3)\omega^2\right] k_5 \theta_1 + (\beta_2 + \beta_3)\, \omega k_\perp^2 \theta_2 = 0, \tag{69}$$

$$(\beta_2 + \beta_3)\, \omega k_5 \theta_1 - \left[\beta_1 \left(k_5^2 - \omega^2\right) + (\beta_1 + \beta_2 + \beta_3) k_\perp^2\right] \theta_2 = 0, \tag{70}$$

$$\beta_1 k_\perp^2 \left(k_\perp^2 + k_5^2 - \omega^2\right) \theta_3 = 0, \tag{71}$$

where $k_\perp^2 = \sum_{i=1}^3 k_i k_i$.

For the case $\beta_1 + \beta_2 + \beta_3 \neq 0$, the scalar mode $\theta_1 = (k_\perp^2/\omega k_5)\theta_2$ propagates with the dispersion relation

$$\omega^2 = k_\perp^2 + k_5^2. \tag{72}$$

while the scalar mode $\theta_1 = (\omega/k_5)\theta_2$ propagates with the dispersion relation

$$\omega^2 = k_\perp^2 + k_5^2 \left(\frac{\beta_1}{\beta_1 + \beta_2 + \beta_3}\right). \tag{73}$$

We see that the stability of the modes give the following conditions of $\beta_i$:

$$\beta_1 > 0, \quad \beta_1 + \beta_2 + \beta_3 \geq 0. \tag{74}$$

Himmetoglu et al. [43] have studied perturbation in the model for arbitrary $\beta_i$ coefficients. The authors have found that the ghost is absent for $\beta_1 + \beta_2 + \beta_3 \neq 0$.

For the case $\beta_1 + \beta_2 + \beta_3 = 0$, Eq. (69) becomes a constraint equation and the kinetic terms for the vector field take the form of a field strength tensor squared. The standard kinetic term corresponds to $\beta_1 = -\beta_3 = 1/2, \beta_2 = 0$.

The stability of the model also depends on whether the Hamiltonian density is positive over the full phase space. We use the Hamiltonian constraint analysis as same as the fixed-norm timelike Lorentz violating vector field model [44]. From the Lagrangian (64), we can calculate the conjugate momenta:

$$\begin{aligned}
\Pi^0 &= -2(\beta_1+\beta_2+\beta_3)(\partial_0 n_0)+2\beta_2(\partial_\mu n_\mu), &(75)\\
\Pi^\mu &= 2\beta_1(\partial_0 n_\mu)+2\beta_3(\partial_\mu n_0), &(76)\\
\Pi^{(\lambda)} &= 0, &(77)
\end{aligned}$$

where $\mu \in (1,2,3,5)$. Then we use the conjugate momenta to construct the Hamiltonian density

$$\begin{aligned}
\mathscr{H} =& -\left(\frac{\beta_1^2-\beta_3^2}{\beta_1}\right)(\partial_\mu n_0)^2+\frac{1}{4\beta_1}(\Pi^\mu)^2-\frac{\beta_3}{\beta_1}\Pi^\mu\partial_\mu n_0+\beta_1(\partial_\mu n_\nu)^2\\
&+\beta_2(\partial_\mu n_\mu)(\partial_\nu n_\nu)+\beta_3(\partial_\mu n_\nu)(\partial_\nu n_\mu)-\frac{1}{\beta_1+\beta_2+\beta_3}(\Pi^0)^2\\
&+\frac{\beta_2}{\beta_1+\beta_2+\beta_3}\Pi^0(\partial_\mu n_\mu)-\frac{\beta_2^2}{\beta_1+\beta_2+\beta_3}(\partial_\mu n_\mu)^2\\
&-\lambda\left(-n_0^2+n_\mu^2-1\right), &(78)
\end{aligned}$$

Four constraints are identified as

$$\begin{aligned}
\phi_1 =& \Pi^{(\lambda)}, &(79)\\
\phi_2 =& -\left(-n_0^2+n_\mu^2-1\right), &(80)\\
\phi_3 =& \frac{1}{\beta_1+\beta_2+\beta_3}\left[-\frac{1}{2}\Pi^0+\beta_2(\partial_\mu n_\mu)\right]n_0+\frac{1}{\beta_1}\left[\frac{1}{2}\Pi^\mu-\beta_3(\partial_\mu n_0)\right]n_\mu, &(81)\\
\phi_4 =& \lambda(n_0)^2-\lambda\left(\frac{\beta_1+\beta_2+\beta_3}{\beta_1}\right)(n_\mu)^2-\left[\frac{\beta_2\beta_3}{\beta_1}+\frac{(\beta_1+\beta_2+\beta_3)\beta_3^2}{\beta_1^2}\right](\partial_\mu n_0)^2\\
&-\left(\frac{\beta_2^2}{\beta_1+\beta_2+\beta_3}-\frac{\beta_2\beta_3}{\beta_1}\right)(\partial_\mu n_\mu)^2+(\beta_1+\beta_2+\beta_3)n_\mu\partial_\nu\partial_\nu n_\mu\\
&-\left[\frac{\beta_2^2}{\beta_1}-\frac{(\beta_1+\beta_2+\beta_3)(\beta_2+\beta_3)}{\beta_1}\right]n_\mu\partial_\mu\partial_\nu n_\nu-\left(\frac{\beta_1^2-\beta_3^2}{\beta_1}\right)n_0\partial_\mu\partial_\mu n_0\\
&+\left(\frac{\beta_2}{2\beta_1}\right)n_\mu\partial_\mu\Pi^0-\left(\frac{\beta_3}{2\beta_1}\right)n_0\partial_\mu\Pi^\mu-\left(\frac{\beta_2}{2\beta_1}-\frac{(\beta_1+\beta_2+\beta_3)\beta_3}{\beta_1^2}\right)\Pi^\mu\partial_\mu n_0
\end{aligned}$$

$$+\left(\frac{\beta_2}{\beta_1+\beta_2+\beta_3}-\frac{\beta_3}{2\beta_1}\right)\Pi^0\partial_\mu n_\mu+\frac{1}{4(\beta_1+\beta_2+\beta_3)}(\Pi^0)^2$$
$$+\frac{\beta_1+\beta_2+\beta_3}{4\beta_1^2}(\Pi^\mu)^2. \tag{82}$$

The constraint $\phi_1$ is primary, while $\phi_2, \phi_3$, and $\phi_4$ are secondary. All of our constraints are second class. According to Dirac's counting argument there are $6-0-4/2=4$ independent degrees of freedom in this model. In the spacelike case, the constraint $\phi_2$ allows for a solution $n^\mu$, effectively constraining away the spatial field component. A similar investigation, $\phi_3$ allows a solution of Eq. (81) with respect to $\Pi^\mu$. Finally, $\phi_3$ can be used to determine the Lagrange multiplier, $\lambda$.

Applying the boundary condition and $\phi_2 \approx 0$, the Hamiltonian density (78) can be written as

$$\mathscr{H}=\mathscr{H}_\Pi+\mathscr{H}_n, \tag{83}$$

where

$$\mathscr{H}_\Pi=-\frac{1}{\beta_1+\beta_2+\beta_3}\left[-\frac{1}{2}\Pi^0+\beta_2(\partial_\mu n_\mu)\right]^2+\frac{1}{\beta_1}\left[\frac{1}{2}\Pi^\mu-\beta_3(\partial_\mu n_0)\right]^2, \tag{84}$$

and

$$\mathscr{H}_n=\beta_1\left[(\partial_\mu n_\nu)^2-(\partial_\mu n_0)^2\right]+(\beta_2+\beta_3)(\partial_\mu n_\mu)^2. \tag{85}$$

Let us first consider the Hamiltonian depends only on the fields $n_\mu$ (85) which can used to examine the positivity of the Hamiltonian density. Applying the condition (74), the first term is nonnegative, while the second term is nonnegative if and only if

$$0<\frac{\beta_1}{\beta_1+\beta_2+\beta_3}\le 1, \tag{86}$$

then $\mathscr{H}_n$ becomes positive. Next let us consider the Hamiltonian includes dependence on the momenta (84). Using the condition (74) the first term is nonpositive, while the second term is nonnegative. Note that Eq. (84) is related to the constraint (81), then the two terms in Eq. (84) are not independent. If we choose an initial condition such that $-\frac{1}{2}\Pi^0+\beta_2(\partial_\mu n_\mu)=\frac{1}{2}\Pi^\mu-\beta_3(\partial_\mu n_0)=0$, we have the constraint $\phi_3\approx 0$, then $\mathscr{H}\ge 0$. However, for $\beta_1/(\beta_1+\beta_2+\beta_3)>1, \mathscr{H}_n$ can be made arbitrarily negative, and solutions with $\mathscr{H}<0$ can therefore exist. Thus the stability of the model depend heavily on specific choices of initial conditions and the parameters $\beta_i$.

## 6 Conclusions

In the present paper, we have considered a five-dimensional braneworld model with bulk Lorentz invariance violation, and derived the effective four-dimensional

Einstein equations on the brane. The main result of this paper is the existence of the trace part of the brane energy-momentum tensor in the modified Einstein equations on the brane, which is a modification of the SMS effective equation [24]. Thus, the divergence of the projected Weyl tensor is modified. Therefore, due to Lorentz violating effect, we have obtained an expression for the projected Weyl tensor as a function of the source on the brane. It becomes clear that the bulk effect can be determined by matter localized on the brane even at low energies. As an application, we have used the modified SMS effective equation to determine the Friedmann equation on the brane. We have showed the effective Newton constant that relates geometry to the matter density in Friedmann equation is dependent on the equation of state of the energy component of the Universe, and the Lorentz violating parameters. Note that if the brane was isotropic and homogeneous, the matter part would have the additional property, $D^{\mu}\pi_{\mu\nu} = 0$. However, due to effect of Lorentz violation in the bulk, the effect of matter still appears in Eq. (32). Thus, the brane matter will deform the bulk geometry. In other word, the back-reaction of this to the brane will modify the effective Friedmann equation even at low energies. It is interesting to understand the low energy description of this braneworld model. The low energy perturbation scheme proposed in [11] is a major achievement as it allows for the derivation of the effective theory on the brane and for the full comprehension of the Weyl tensor contribution to the effective theory. We leave this issue for future studies.

Finally, we also find that the effect of Lorentz violation in the bulk is dependent on the equation of state of the energy components of the brane matter. This model also provides a convenient framework within which one may study dark energy. Using the Hamiltonian constraint analysis, the stability of the model depend heavily on specific choices of initial conditions and the parameters $\beta_i$. Different aspects of Lorentz invariance violation in models with large extra dimensions were considered in Ref. [45]. In these model Lorentz invariance was broken spontaneously on the brane due to the presence of a symmetry-breaking potential and it has been shown that the model implies the absence of ghosts and/or tachyons to higher orders in classical perturbation theory.

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