Research Article

Black Component of Dark Matter

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A mechanism of primordial black hole formation with specific mass spectrum is discussed. It is shown that these black holes could contribute to the energy density of dark matter. Our approach is elaborated in the framework of universal extra dimensions.

1. Introduction

After discovery of the Higgs boson, dark matter remains the most challenging unsolved problem. A nature of this dominant component of matter density is so far unknown. Extensions of standard model propose a number of possible candidates for the dark matter particle, so-called WIMPs (Weakly Interacting Massive Particles). Most popular candidates—neutralino, sneutrino, or gravitino—come from the SUSY models. Axions represent another way to describe the dark matter phenomenon by means of particle physics [1, 2]. The third widely discussed method is various extra dimensional models producing different sorts of Kaluza-Klein particles that can be considered as the dark matter particles [3].

There are many experiments aimed to clarifying the essence of dark matter. One of the most known Fermi LAT experiments is searching for the dark matter footprints in gamma rays since 2008. According to the recent Fermi LAT data [4] there is some signature of a gamma-ray line at the energy about 130 GeV in the direction of the Galactic Center. Unfortunately, in spite of continual efforts it is not clear yet if this is a real line. Up to now, there are no signatures of dark matter in the existing spectra of cosmic rays. The successor of Fermi LAT, Russian-Italian satellite GAMMA-400, was announced to have an unprecedented energy and angular resolution and believed to be more effective.

Another explanation for dark matter content is the existence of massive compact halo objects (MACHOs) [5–7]. Brown dwarfs, interstellar gas, comets, cosmic dust, neutron stars, and black holes of stellar origin may be considered as MACHOs. All of these constituents are made from baryons which drastically reduces their possible contribution to the energy density of dark matter [8].

Primordial black holes (PBH) with a wide range of masses provide additional contribution to an invisible part of the average energy density of the universe. There are several models describing PBH formation soon after the end of inflation. Starting from the first articles [5–7], various models of such kind have appeared during last decades; see substantial review [9]. In contrast to the black holes originating from stars for PBHs there is no lower mass limit (actually [10] there is black hole minimum mass $\simeq 0.04\sqrt{g_*}M_{\rm pl}$).

Mechanism of PBH formation as a result of phase transitions during the inflationary stage was developed in [11–13]. The basic of this mechanism is quantum fluctuations of scalar field with a potential possessing at least two minima. Constraints on a PBH abundance associated with cosmological nucleosynthesis are not applicable because the formation of PBHs is a result of first order phase transitions of some scalar field.

The origin of the scalar field remains uncertain in this model. Meantime, extra dimensions provide a wide range of such potentials which can influence the evolution of the universe; see also [14, 15].

In this paper we elaborate the model of PBH formation based on universal extra dimensions. Mass distribution and number of PBH do not contradict observational limits and could contribute substantially to dark matter content.

The rest of the paper is organized as follows. In Section 2 we describe the mechanism of primordial black holes formation. In Section 3 we discuss initial conditions and modern limits on PBHs. In Section 4 we conclude.

2. Primordial Black Holes Formation and Extra Space

As was mentioned above our study is based on the mechanism of PBH formation revealed in [11]. Its necessary ingredient is a potential of scalar field with two minima which is postulated from the beginning. In the framework of multidimensional gravity, metric components of extra space are perceived as scalar fields acting in 4-dim space. In this connection we shortly remind some effective way to reduce a D-dim theory to 4-dim low energy effective theory [16].

Consider a $D = 4 + d$ dimensional manifold with the metric

$$ds^2 = g_{MN}dx^M dx^N = g_{\mu\nu}dx^\mu dx^\nu + e^{2\beta(x)} b_{ab} dx^a dx^b, \quad (1)$$

where $M, N = 1, 2, \ldots, d+4$ and the extra dimensional metric components b_{ab} are independent of x^μ, the observable space-time coordinates. In the framework of this metric let us consider the curvature-nonlinear theory of gravity with the action which has various nontrivial consequences:

$$S = \frac{1}{2} m_D^{D-2} \int \sqrt{{}^D g}\, d^D x \left(F(R) + c_1 R^{AB} R_{AB} + c_2 \mathcal{K} \right), \quad (2)$$

where m_D is a D-dimensional analogue of the Planck mass. For example, action (2) was used [17] to develop the model describing both primary and secondary inflations. Action (2) contains an arbitrary smooth function $F(R)$, c_1 and c_2 are arbitrary constants, R^{AB} and $\mathcal{K} = R^{ABCD} R_{ABCD}$ are Ricci tensor and Kretschmann scalar, and ${}^D g = |\det(g_{MN})|$.

Let us express all quantities in terms of 4-dimensional variables and $\beta(x)$; notice that now

$$R = R_4 + \phi + f_1,$$
$$f_1 = 2d\beta + d(d+1)(\partial\beta)^2; \quad (3)$$

here we introduce scalar field

$$\phi(x) = \pm m_D^2 d(d-1) e^{-2\beta(x)}. \quad (4)$$

We suppose that all quantities are slowly varying; that is, consider each derivative ∂_μ (including those in the definition of R_4 (Ricci scalar corresponding to $g_{\mu\nu}$)) as an expression containing a small parameter ε, and neglect all quantities of orders higher than $O(\varepsilon^2)$. Then we have the following decompositions:

$$F(R) = F(\phi + R_4 + f_1) \simeq F(\phi) + F'(\phi) \cdot (R_4 + f_1) + \ldots, \quad (5)$$

where $F'(\phi) \equiv dF/d\phi$. For detailed analysis we refer to [16, 18, 19].

We implement the conformal mapping leading to the Einstein frame:

$$g_{\mu\nu} \longmapsto \tilde{g}_{\mu\nu} = |f(\phi)|\, g_{\mu\nu}, \qquad f(\phi) = e^{d\beta} F'(\phi). \quad (6)$$

After reduction to 4 dimensions the dynamics of the model is defined by the following action:

$$S = \frac{\nu[d]\, m_D^2}{2} \int d^4 x \sqrt{{}^4 g} \left(\operatorname{sign} F'\right) L, \quad (7)$$

where $\nu[d] = \int d^d x \sqrt{{}^d g}$ is the volume of a compact hyperbolic d-dimensional extra space of a unit curvature. The volume of such a space could be arbitrary large without contradiction to observations [20–22].

The Lagrangian in the Einstein frame has the form

$$L = \tilde{R}_4 + \frac{1}{2} K_{\text{Ein}}(\phi)(\partial\phi)^2 - V_{\text{Ein}}(\phi), \quad (8)$$

$$K_{\text{Ein}}(\phi) = \frac{1}{2\phi^2}\left(6\phi^2 \left[\frac{F''}{F'}\right]^2 - 2d\phi \frac{F''}{F'} + \frac{1}{2} d(d+2) \right) + \frac{2(c_1 + c_2)}{F'\phi}, \quad (9)$$

$$V_{\text{Ein}}(\phi) = -\frac{\operatorname{sign}(F')}{F'^2} \left[\frac{|\phi|\, m_D^{-2}}{d(d-1)}\right]^{d/2} \left(F(\phi) + c_V \frac{\phi^2}{d} \right),$$
$$c_V = c_1 + \frac{2c_2}{d-1}, \quad (10)$$

$$F(\phi) = \phi + c\phi^2 + b_1\phi^3 + b_2\phi^4 - 2\Lambda. \quad (11)$$

$F(\phi)$ is the function introduced in (2), b_1, b_2 are initial parameters of the model, d is a dimension of extra space, and $\tilde{R}_4$ is the Ricci scalar constructed from the conformal metric $\tilde{g}_{\mu\nu}$ of the Einstein frame.

The action acquires the standard form after the renormalization of the kinetic and potential terms

$$K(\phi) = \frac{M_{\text{pl}}^2}{2} K_{\text{Ein}}, \qquad V(\phi) = \frac{M_{\text{pl}}^2}{2} V_{\text{Ein}}, \quad (12)$$

where the Planck mass M_{pl} is expressed in terms of the initial parameters as follows:

$$\nu[d]\, m_D^2 \simeq M_{\text{pl}}^2. \quad (13)$$

In what follows we will use Hubble units ($H = 1$), where H is the value of Hubble parameter at the inflationary stage (we assume $H = 10^{-6} M_{\text{pl}}$).

The potential (10) possesses one maximum; see Figure 2 with some parameter values represented in captions of Figure 3. The observable cosmological constant is small comparing to the energy density at early stages and corresponding conditions

$$V(\phi_{\min}) = 0, \qquad V'_\phi(\phi_{\min}) = 0 \quad (14)$$

give two constraints to parameters of our model.

To simplify the further analysis we will consider the kinetic term in the form (see Figure 1)

$$K(\phi) \approx \frac{c_5}{\phi^2} \quad (15)$$

valid at small ϕ (see Figure 2).

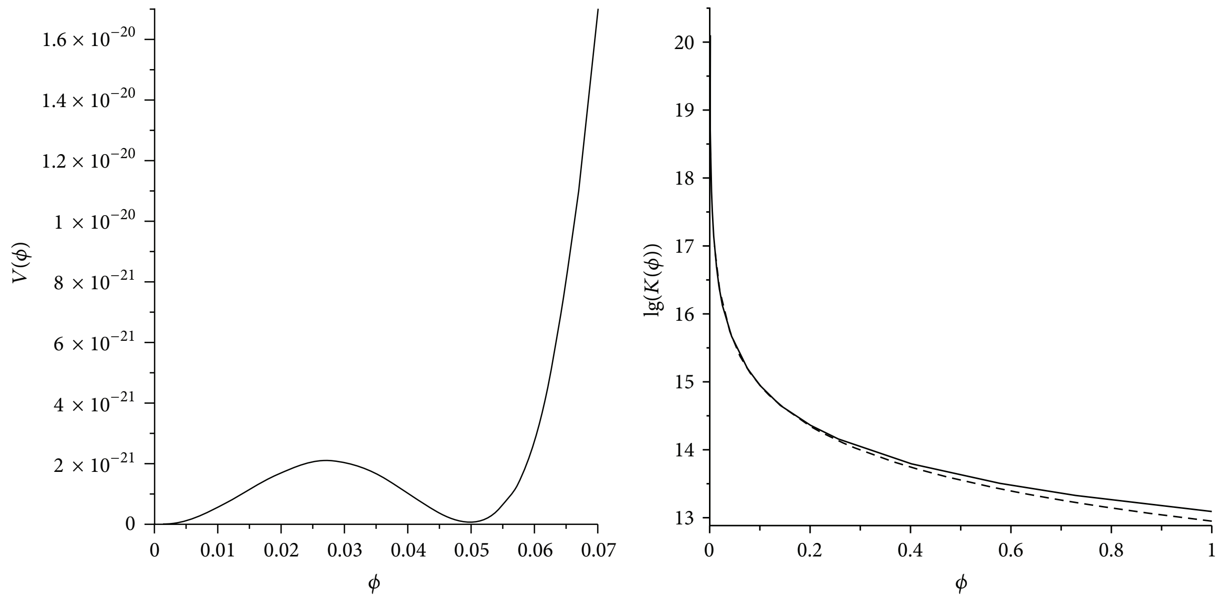

Figure 1: Right panel: Solid line: $\lg K(\phi)$ function; dotted line: approximation of $\lg K(\phi)$; see (15). One can see that this approximation is valid for the small values of ϕ where a dynamic is developed; see Left panel: Potential $V(\phi)$. Numerical values of the parameters are listed in the caption of Figure 3.

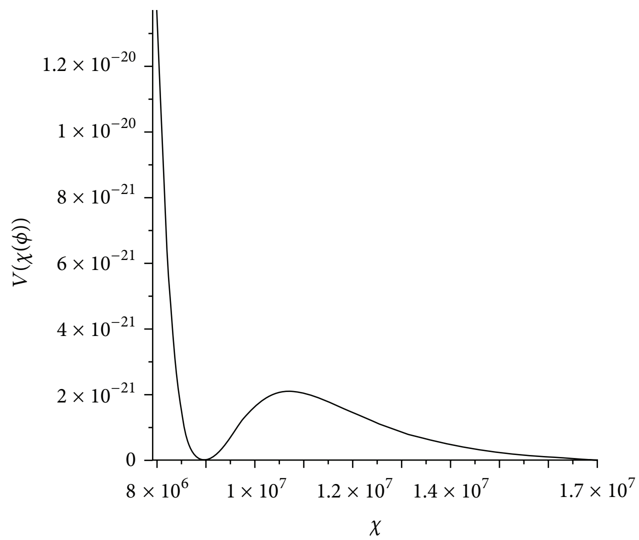

Figure 2: Potential possesses one maximum at $\chi = \chi_{\text{crit}}$ and one minimum at $\chi = \chi_{\min}$. When the field in some region reaches the value $\chi = \chi_{\text{crit}}$ some part of causally independent domains can flip to the other side of the potential with the field value $\chi > \chi_{\text{crit}}$.

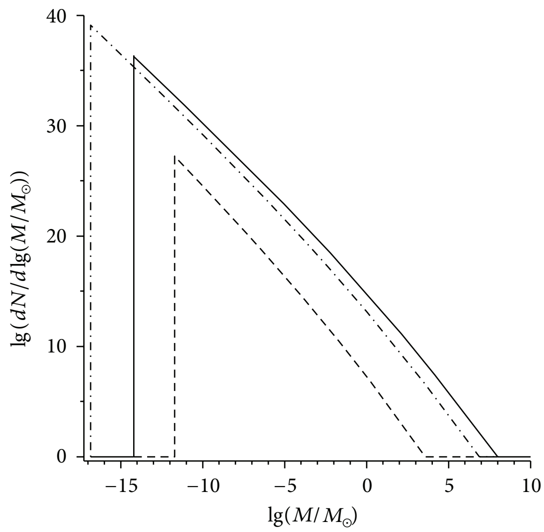

Figure 3: Solid line: mass spectrum of primordial BHs for the parameter values: $c_5 = 8.9 \cdot 10^{12}$, $c = 5.6 \cdot 10^{-6} H^{-2}$, $d = 5$, $b_1 = 12.89 \cdot 10^{-6} H^{-4}$, $b_2 = -8 \cdot 10^{-8} H^{6}$, $c_1 = 2H^{-2}$, $c_V = -50H^{-2}$, $\Lambda = 0.0125H^2$, $\chi_0 = 1.069 \cdot 10^7 H$, $\nu[d] = 10^3$, and $m_D = 4.4 \cdot 10^4 H$. The spectrum of the PBHs is very sensitive to the initial conditions, so that slight change of χ (in order of $10^{-5} H$) drastically changes the picture (dashed and dot-dashed lines).

In terms of a new field $\chi = -\sqrt{c_5} \ln \phi$ the Lagrangian takes the standard form:

$$L = \frac{1}{2} (\partial \chi)^2 - V(\phi(\chi)). \tag{16}$$

Quantum fluctuations during inflation lead to the appearance of lots of causally independent space regions. Due to quantum fluctuations, each of these regions is characterized by specific value of the field. Eventually field value reaches the maximum at $\chi = \chi_{\text{crit}}$ (see Figure 2) in some region and some part of causally independent domains can flip to the other side of the potential with the field value $\chi > \chi_{\text{crit}}$. In the following the field in such domains moves classically from the maximum to $\chi \to \infty$. After the end of inflation the universe is filled with the phase $\chi = \chi_{\min}$ except some set of regions filled with the phase $\chi \to \infty$.

As was established [11] domain walls are formed between the regions characterized by different phase. These walls are expanding along with the rest of the space. When the inflationary period is over and the universe stops expanding exponentially, domain walls begin to collapse. In the case when domain wall collapses down to its Schwarzschild radius, the black hole forms.

The detailed description of quantum fluctuations during the inflationary stage can be found in [23, 24]. The corresponding mathematical tool was developed in [25, 26]. An amplitude of quantum fluctuations obeys the Gauss distribution with the average value $\phi(N)$ equal to the classical component of the field at the e-fold number N:

$$dP\left(\chi_{\text{crit}}\right) = \frac{1}{\sqrt{2\pi\left(N_u - N\right)}} \exp\left(-\frac{\left(\chi_{\text{crit}} - \chi\left(N\right)\right)^2}{2\delta\chi^2\left(N_u - N\right)}\right). \quad (17)$$

Here $\delta\chi = H/(2\pi)$ is the average value of the amplitude of the field fluctuation and $N_u = 60$ is a duration of the inflationary period.

Number of regions filled with the phase $\chi > \chi_{\text{crit}}$ that appear during one e-fold and N e-folds before the end of inflation and that lead to the formation of BHs is defined by the probability (17). The volume filled with the false vacuum can be calculated using the iterative procedure. Let $v(t)$ be a volume with new phase. This volume is expanding in e^3 times during one e-fold. Besides, new volume $\Delta v(t)$ will be filled with the new phase due to quantum transitions. Finally we obtain the connection

$$\begin{aligned} v\left(t+1\right) &\simeq v\left(t\right)e^3 + \Delta v\left(t\right), \\ \Delta v\left(t\right) &\equiv \left[v_U\left(t+1\right) - e^3 v\left(t\right)\right] dP\left(t\right); \end{aligned} \quad (18)$$

here we substitute $t = 60 - N$, where N is a number of e-folds before the end of inflation and $v_U = e^{3(N_U - N)}H^{-3}$ is the volume of the universe. We also put $N_U = 60$ followed from observations. Number of closed walls is estimated as

$$N_{\text{walls}} = H^3 \Delta v\left(t\right). \quad (19)$$

After the end of the inflationary period these regions are surrounded by closed domain walls. Due to the collapse of such a domain wall its energy is concentrated in a small volume inside the Schwarzschild radius—fulfilling the necessary condition for the formation of PBH. To calculate the amount of energy concentrated during the collapse of a domain wall we should estimate the surface energy density of the wall σ that depends on the parameters of the potential (10).

To simplify the analysis let us approximate the potential term of our model in the vicinity of its maximum by Higgs-like potential; see Figure 2. In this case the soliton solution is well known [27] together with its width d_w and surface energy density σ.

The radius R of the region that is formed N e-folds before the end of inflation was found in [11]

$$R = \left(2^8\pi\sigma\right)^{-1/3}\left(\frac{M_{\text{pl}}}{H}\right)^{2/3} N^{-2/3} \exp\left(\frac{4N}{3}\right). \quad (20)$$

Thereby we get the total amount of energy of the black hole as $E \sim 4\pi R^2\sigma$. Masses of the forming BHs depend on the typical size of the closed wall. This size is the bigger; the earlier space region surrounded by the domain wall starts to form.

3. Primordial Black Holes as MACHO Objects

Discrete spectrum $N_{\text{PBH}}(M)$ of specific masses M of PBH can be easily obtained by iterative procedure (18). It will be used later (see (25)) for an estimation of PBH density. Much more informative is the differential spectrum of the PBHs presented in Figure 3. It is obtained approximately on the basis of numerical solution $N_{\text{PBH}}(M)$.

We presented three different spectra, to show the dependence of spectra on the initial conditions and parameters of the Lagrangian. Since PBHs are forming during the phase transitions when the domain wall collapses inside the Schwarzschild radius r_{Sch} there exists a natural requirement

$$r_{\text{Sch}} < d_w \quad (21)$$

that cuts off PBHs of smaller masses. If the width d_w of the domain wall is bigger than r_{Sch} then PBHs of that mass cannot form.

There exist strong constraints on the PBHs of the mass about 10^{15} g, so the fact that PBHs with masses $<10^{16}$ g do not appear under certain choice of the parameters allows us to avoid the constraints related to the evaporations of PBHs during Big Bang nucleosynthesis.

According to [9] the current PBHs density is limited from above by parameter

$$\begin{aligned} \Omega_{\text{PBH}} &= \frac{M n_{\text{PBH}}\left(t_0\right)}{\rho_c} \\ &\approx \left(\frac{\beta\left(M\right)}{1.15\times10^{-8}}\right)\left(\frac{h}{0.72}\right)^{-2}\gamma^{1/2}\left(\frac{g_{*i}}{106.75}\right)^{-1/4} \\ &\quad \times \left(\frac{M}{M_{\odot}}\right)^{-1/2} \end{aligned} \quad (22)$$

for PBHs which have not been evaporated yet. Here M is the PBHs mass, $n_{\mathrm{PBH}}(t_0)$ is their concentration, ρ_c is the critical density, $\beta(M) = \rho_{\mathrm{PBH}}(t_i)/\rho(t_i)$ and t_i indicates an epoch of PBH formation, $h = 0.72$ is the Hubble parameter, γ is a numerical factor of order unity related to the gravitational collapse details, and g_{*i} is the number of relativistic degrees of freedom.

It is convenient to define a new parameter $\beta'(M)$:

$$\beta'(M) = \gamma^{1/2}\left(\frac{g_{*i}}{106.75}\right)^{-1/4}\beta(M) \tag{23}$$

so that (22) gives

$$\beta'(M) = 1.15 \times 10^{-8}\Omega_{\mathrm{PBH}}\left(\frac{M}{M_\odot}\right)^{1/2}. \tag{24}$$

Contribution of PBH into the average energy density Ω_{PBH} may be estimated as follows:

$$\Omega_{\mathrm{PBH}}(t_0) = \frac{M \cdot N_{\mathrm{PBH}}(M)}{\rho_c v_U}, \tag{25}$$

where uniform space distribution of PBH is supposed. Here $N_{\mathrm{PBH}}(M)$ is the number of PBHs of mass M discussed in the beginning of this section, and $v_U \approx l_U^3$ is the volume of the universe with $l_U \approx 6000$ Mpc.

To satisfy the constraints (see [9]), we should have

$$\begin{aligned} &\beta'(M) < 10^{-21} \quad \text{for } M \approx 10^{16} g\left(\approx 10^{-17} M_\odot\right), \\ &\beta'(M) < 10^{-18} \quad \text{for } M \approx 10^{17} g\left(\approx 10^{-16} M_\odot\right). \end{aligned} \tag{26}$$

They are true for the solid and dashed lines while the dot-dashed line does not satisfy these conditions. According to the chaotic inflation, a lot of universes with different initial conditions are formed due to quantum fluctuations. In our context it means that there are a lot of universes filled be various number of PBH and we live in one of them. Comparison of solid and dashed distribution reveals strong dependence on initial conditions.

The differential spectrum represented by solid line in Figure 3 gives total mass of all PBHs $M_{\mathrm{total}} \approx 2 \times 10^{22} M_\odot$, what could explain most of the hidden mass of the universe.

The mass of the particles corresponding to the field χ is about 10 KeV that is small enough for them to remain unseen. Axions and gravitino in GMSB model are other examples of very light particles that have not been detected yet [28–30].

4. Conclusion

In this paper, we propose a way to explain the hidden mass of the universe. Our approach is based on the scalar fields that appear in frame of the multidimensional gravity. Components of extra space metric are perceived as scalar fields. Appropriate Lagrangian in low energy limit is obtained from the initial pure gravitational Lagrangian in $d + 4$ dimensions.

Closed domain walls formed during the inflationary phase of the universe begin to collapse immediately after the end of inflation. If due to the collapse of domain wall its energy is concentrated within the gravitational radius, one can expect the formation of a primordial black hole.

Primordial black holes with wide range of masses are uniformly dispersed in the early universe. Their mass spectrum satisfies the observational constraints on the density of primordial black holes. Their total mass is large enough to explain substantial of the dark matter in the universe. This approach is an alternative to the WIMP-explanation of dark matter. Nevertheless, there is a room for particles beyond the standard model.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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