



Enhancing variational quantum classifier performance with meta-heuristic feature selection for credit card fraud detection

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Abstract A transformative hybrid approach is proposed, combining Quantum Machine Learning (QML) with both traditional and meta-heuristic feature selection algorithms to overcome the complexities and limitations of conventional credit card fraud detection methods. In this study, advanced data balancing techniques such as SMOTE-ENN (Synthetic Minority Over-sampling Technique–Edited Nearest Neighbors) and Random Under Sampler (RUS) are employed on the imbalanced European Cardholder Dataset to address class imbalance and enhance model resilience. Core feature selection algorithms—both traditional methods like K-Best and meta-heuristic techniques including Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), and Atom Search Optimization (ASO)—are systematically evaluated, each paired with the Variational Quantum Classifier (VQC) for classification. Remarkably, the PSO+VQC combination achieves an accuracy rate of 94.54%, underscoring the efficacy of integrating meta-heuristic algorithms with VQC to manage complex, high-dimensional data in fraud detection. These findings highlight QML and meta-heuristic algorithms’ potential to surpass conventional methods, delivering superior accuracy and efficiency in critical, data-intensive applications such as financial fraud detection.

1 Introduction

Credit card fraud has become a significant problem with the growth of the digital economy. This problem poses serious risks to individual users, financial institutions, and the global economy. The proliferation of the Internet and the rapid adoption of online payment systems have led to increased fraudulent activity. This has made timely detection of fraud threats a vital requirement. The complex nature of fraud and the constant evolution of the methods used by fraudsters make detection a difficult process. Due to this dynamic process, there is an increasing need for more advanced technologies to ensure financial security and prevent fraud. Although traditional Machine Learning (ML) methods have been used for fraud detection for many years, these approaches face various challenges with large and complex data sets [1, 2]. The large size of data sets, imbalances, and the complexity of the data structure are the main factors limiting traditional algorithms’ accuracy. Especially with imbalanced data sets, it becomes difficult to detect rare fraud events, which jeopardizes financial security. Traditional methods often face high computational costs, take long periods, and in some cases lead to misclassification. In recent years, research on quantum computing technologies has accelerated to overcome these challenges and achieve more effective results [3, 4]. Quantum computing has emerged as a new approach that goes beyond classical computing and

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has emerged with the ability to process large data sets faster. Quantum computing has the potential to solve more complex problems faster by offering capabilities that exceed the processing capacity of classical computers. Quantum Machine Learning (QML) exploits these advantages of quantum technology to provide more efficient methods for data processing and classification. Especially when working with large datasets, the speed and accuracy advantages of QML make a significant difference compared to traditional ML methods. For financial institutions, accurate and fast classification in fraud detection is of great importance both in terms of preventing financial losses and ensuring customer safety. Improving the accuracy of fraud detection systems also directly affects operational efficiency in the financial sector. Therefore, research on advanced classification methods plays a key role in ensuring financial security. This study takes an in-depth look at the innovations offered by quantum technology as a solution to the challenges faced by traditional ML methods. In particular, the advantages of QML are evaluated in a general framework and its potential contributions to fraud detection are emphasized. Traditional ML methods often suffer from overfitting or underfitting, especially due to the imbalanced nature of datasets and the underrepresentation of minority classes. In imbalanced datasets, rare fraud cases are under-detected and the model tends to favor majority classes and neglect minority classes. Furthermore, their limited generalization capabilities cause traditional methods to achieve high accuracy on training data, but perform poorly on new and unexpected datasets. In addition, deep learning models often require large datasets and face high computational costs when working on these datasets. This makes it difficult to obtain accurate results on dynamic and fast-moving datasets such as credit card fraud. However, deep learning-based approaches offer significant advantages, especially when working with complex data structures and large datasets. Deep learning models, especially using architectures such as CNN and RNN, achieve better classification results and provide successful results in fraud detection. However, these methods can sometimes lose efficiency due to their dependence on large datasets. QML-based models [5–8] are becoming an important focus of research on their contribution to fraud detection. These methods overcome the limitations of traditional ML methods by reducing the dependence on large data sets and better adapting to dynamic processes. Recent studies have also shown that quantum-classical hybrid optimization methods can be successfully applied to financial transaction problems, including settlement optimization tasks under real-world constraints, further emphasizing the practical relevance of QML approaches in data-limited environments [9]. When combined with meta-heuristic algorithms, the processing time of QML methods is shortened and high performance is achieved with less data. In this way, quantum computing has the potential to provide more advanced solutions by overcoming challenges where traditional methods are limited. The literature review reveals limited application of QML approaches in the field of fraud detection. However, QML methods have successful applications in various disciplines such as health [10–12], chemistry [13–16], finance [17–19], agriculture [20, 21], cyber security [22, 23], and materials science [24, 25]. Studies in these fields point to the potential of quantum computing to provide a wide range of solutions. An overview of the literature is shown in Table 1. Although QML algorithms currently have some limitations due to their low fault tolerance, research on quantum hardware is paving the way for the development of devices with higher fault tolerance in the future. In this context, advances and discoveries in this area will play a critical role in enabling the effective use of quantum technologies in ML processes. Limitations of traditional ML and other techniques are shown in Table 2.

1.1 Motivation and contribution

The main motivation of this work is to present an innovative integration of QML and meta-heuristic algorithms to overcome the performance limitations of traditional ML methods on large and unstable datasets. Ever-changing and complex data structures, such as credit card fraud, are areas where traditional methods often fall short. In particular, these methods are limited by their high computational cost and low generalization capabilities, while struggling with problems such as overfitting and underfitting. This paper aims to show how meta-heuristics can be combined with QML to optimize classification performance and to demonstrate the contributions of this hybrid approach, which is rare in the literature. To the best of our knowledge, there is no work in the literature that integrates QML and meta-heuristics in this way.

The primary contributions of the study are as follows:

- We use quantum simulators to study how the model performs in simulation environments, thus providing the precursors for a model that could run on a real quantum computer in the future.
- Using a qubit-based mechanism, we project features into quantum space, thus performing well compared to conventional methods in metrics such as accuracy, speed, and data stability.
- By minimizing the limitations of traditional ML algorithms, such as overfitting, we offer a model that produces more balanced and accurate results.
- We develop an innovative hybrid approach to feature selection using QML integration with meta-heuristics, in this context, we show that we have made a new contribution to the literature.
- We present a comparative analysis of traditional feature extraction methods and various meta-heuristic algorithms, demonstrating their impact on feature extraction and classification performance with VQC.

Table 1 An overview of the literature

References	Application	Models	Technique	Dataset	Performance
Plakandaras et al. [26]	Credit card fraud detection	Just-Add-Data AutoML System	Automated ML algorithm	European Cardholder	Identified 32/39 fraud cases
Noviandy et al. [27]	Credit card fraud detection	XGBoost	Data augmentation and balancing techniques	European Cardholder	Accuracy: 99.96%
Mathew et al. [28]	Credit card fraud detection	Random Forest (RF)	Standard ML algorithms with fused feature selection	European Cardholder	Accuracy: 98.5%
Alarfaj et al. [29]	Credit card fraud detection	CNN	CNN architecture with additional layers	European Cardholder	Accuracy: 99.9%
Ileberi et al. [30]	Credit card fraud detection	XGBoost, Extra Tree (ET)	SMOTE, AdaBoost	European Cardholder	Accuracy: 99.98%
Ileberi et al. [31]	Credit card fraud detection	RF	Genetic algorithm for feature selection	European Cardholder	Accuracy: 99.98%
Mienye et al. [32]	Credit card fraud detection	LSTM, GRU, MLP	Ensemble	European Cardholder	Sensitivity: 100%
Esenogho et al. [33]	Credit card fraud detection	LSTM, AdaBoost	Ensemble Method	European and Taiwan Cardholder	Specificity: 99.8%
Khalid et al. [34]	Credit card fraud detection	SVM, KNN, RF, Bagging, Boosting	Ensemble method	European Cardholder	Accuracy: 94.73%
Islam et al. [35]	Credit card anomaly detection	XGBoost, Elliptical Envelope, Isolation Forest, LOF	Stacked Ensemble	CCF and CCDP	Sensitivity: 99.9%
Malik et al. [36]	Credit card fraud detection	AdaBoost + LGBM	Hybrid model	IEEE CIS	Specificity: 95.9%
Mienye et al. [37]	Credit card fraud detection	GA	Hybrid feature selection (IG + GA)	European Cardholder	Specificity: 99.4%
Berhane et al. [38]	Credit card fraud detection	CNN, SVM	Hybrid CNN–SVM	European Cardholder	Accuracy: 91.08%
Innan et al. [8]	Financial fraud detection	Quantum Support Vector Classifier (QSVC), QNN	Quantum and classical models	BankSim	F1 score: 98%
Our system	Credit card fraud detection	Meta-heuristic feature selection-VQC	Hybrid method	European Cardholder	Accuracy: 94.54%

Table 2 Limitations of traditional ML and other techniques

Technique	Limitations
Traditional ML	Overfitting and underfitting problems in imbalanced datasets High computational costs for large datasets Limited generalization capabilities, especially with unseen data
Traditional feature extraction	Ineffectiveness in handling complex and high-dimensional data Inability to capture dynamic patterns in evolving datasets
Ensemble methods	Requires large computational resources for training multiple models Efficiency may decrease as model complexity increases
Deep learning	Dependence on large datasets and high computational costs Susceptibility to overfitting without proper regularization

2 Methods

In our work, we aim to improve the effectiveness of traditional and meta-heuristic feature selection methods in credit card fraud detection by leveraging innovations in QML. By leveraging the power of quantum circuits in data projection and model training, we aim to achieve higher accuracy and efficiency in high-dimensional and imbalanced datasets. The flowchart diagram of the work is presented in detail in Fig. 1.

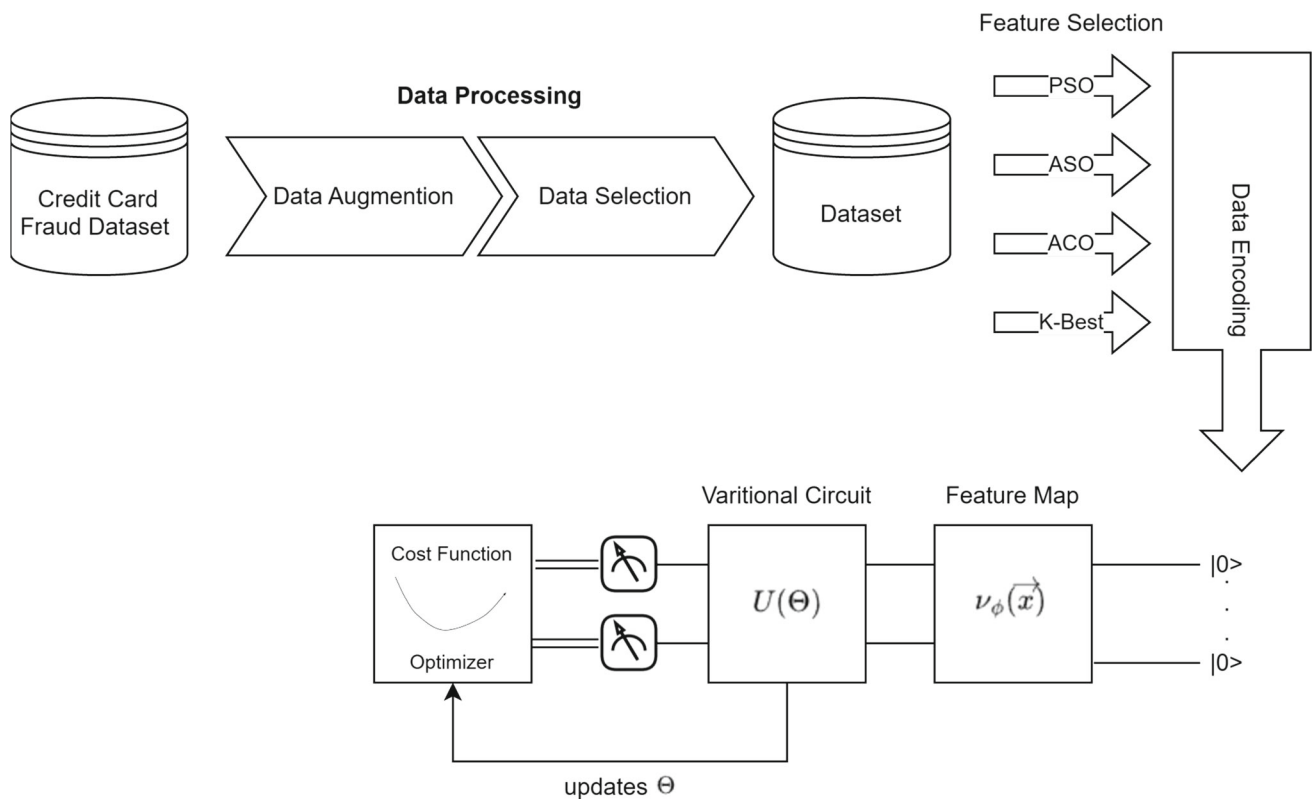


Fig. 1 The flowchart of the meta-heuristic+VQC classification approach

2.1 Dataset description

In this study, we use a dataset collected by the Machine Learning Group of ULB and Worldline from online transaction records of European Cardholders as part of their research on fraud detection [39]. The dataset consists of 284,807 transactions and has a highly unbalanced class distribution. Of these, 284,315 transactions are non-fraudulent, while only 492 are labeled as fraudulent. This indicates a low fraud rate of approximately 0.17 percent, indicating a severe class imbalance problem. Due to this imbalance, detecting rare fraud cases poses a significant challenge in training robust ML models. The dataset contains 29 features, including anonymised transaction details. In addition, ‘Time’ and ‘Quantity’ variables are also included in the dataset, and these variables were subjected to various scaling and normalization processes before being used in the classification models. In the preprocessing process, we removed duplicate records to ensure data integrity. As a result of the process, the dataset contains 283,726 transactions, of which 283,253 are labeled as non-fraud and 473 as fraud cases. This step was performed to eliminate the bias that repetitive transactions may bring to the training process. We also applied data balancing techniques to address the severe class imbalance problem in the dataset, which will be discussed in detail in the next section.

2.2 Feature selection: meta-heuristic and traditional approaches

Feature selection is a fundamental step to improve the accuracy and efficiency of ML models [40, 41]. Large and complex datasets can negatively impact model performance due to the presence of redundant and irrelevant features. Especially in scenarios with complex data structures, such as credit card fraud, the selection of appropriate features plays a decisive role in the success of the model. Traditional methods usually try to select the best features in the dataset based on certain statistical criteria. While these methods are computationally efficient, they may not be able to assess the relationships in the data set from a global perspective. Especially in complex and imbalanced datasets, traditional feature selection methods sometimes have limited performance [42]. On the other hand, meta-heuristic algorithms, inspired by optimization processes in nature, offer a highly effective approach to solving the feature selection problem [43]. These algorithms have the potential to find global optimal solutions in large and complex datasets. Meta-heuristic algorithms can improve the overall success of the model with the ability to discover complex relationships between features in the dataset [44]. In particular, they show the ability to optimize both model accuracy and processing time by making more efficient feature selection in imbalanced datasets [45]. In our study, both traditional and meta-heuristic methods are used and the effectiveness of these two approaches are compared in detail.

2.3 Meta-heuristic algorithms overview

Meta-heuristic algorithms are flexible and powerful methods for solving complex optimization problems [46]. These algorithms are inspired by biological and physical processes in nature and are capable of finding global optimum solutions without being trapped in local optimal [47]. Unlike traditional heuristics, meta-heuristic algorithms can scan a large search space and provide more efficient results for complex problems. Especially when working on multidimensional datasets, it is possible to explore more comprehensive solution spaces from different starting points depending on their stochastic nature [48].

The use of meta-heuristic algorithms in feature selection problems offers significant advantages, especially for large and unbalanced datasets. While traditional feature selection methods usually focus only on the surface relationships of the data, meta-heuristic algorithms can examine the complex relationships between features in more depth and select more appropriate feature subsets. This increases the accuracy of the model and allows for optimizing processing time by removing redundant features. Especially for unbalanced datasets such as credit card fraud, meta-heuristics have the potential to improve classification success by increasing the generalization capability of the model.

2.3.1 Particle swarm optimization

Particle Swarm Optimization (PSO) is a nature-inspired optimization algorithm developed by Kennedy and Eberhart in 1995 [49]. PSO is an approach based on the behavior of animal herds, such as adapting to new habitats, foraging for food, and avoiding traps. This population-based method keeps the knowledge of the herd constantly updated, making it possible to scan the search space widely and find near-perfect solutions [50]. In PSO, each individual is called a “particle” and each particle has information about its position and velocity in the swarm. As each particle moves towards the best known position, it also updates its position with information from other particles in the swarm. The velocity and position of the particles are updated by certain equations. The velocity and position of the particle in the next iteration are denoted by v_{t+1} and p_{t+1} and the following formulas are used,

taking into account the best positions updated in each iteration:

$$v_{t+1} = wv_t + c_1r_1(p_{\text{best}} - p_t) + c_2r_2(g_{\text{best}} - g_t) \quad (1)$$

$$p_{t+1} = p_t + v_{t+1} \quad (2)$$

In these equations, v_t represents the current velocity of the particle, and p_t represents the current position of the particle. The term p_{best} refers to the particle's own best known position, while g_{best} represents the best known global position in the swarm. Coefficients c_1 and c_2 determine how fast the particle moves towards its own and the swarm's best positions, respectively. The terms r_1 and r_2 are the randomness factors. The momentum coefficient w allows the algorithm to take into account the previous velocity of the particle, which helps to stabilize the search process. One of the biggest advantages of PSO is its ability to reach the global optimum solution in multidimensional and complex search spaces. Furthermore, due to the nature of PSO, the sharing of information between all particles in the swarm at each iteration helps the algorithm quickly converge to the best solution.

In terms of feature selection, PSO offers an effective strategy to exclude redundant or irrelevant features, thereby improving the accuracy of the model and optimizing the processing time [51]. The comprehensive search capability and speed of optimization make PSO particularly well-suited for feature selection in large datasets.

2.3.2 Ant colony optimization

Ant Colony Optimization (ACO) is a widely recognized meta-heuristic algorithm inspired by the natural foraging behavior of ant colonies [52]. The algorithm operates on the principle of pheromone trails, which ants deposit while searching for food sources. As ants discover shorter and more efficient paths, they reinforce these paths by increasing the concentration of pheromones. This, in turn, enhances the likelihood that other ants will follow the same path, leading to the intensification of the most optimal route over time. Ultimately, the path with the highest pheromone concentration converges to the best solution, enabling the colony to identify the shortest or most efficient route. One of the key strengths of ACO lies in its capacity to find global optimum solutions within multidimensional and complex problem spaces. Furthermore, its parallel processing nature enhances computational efficiency, as the cooperation among ants facilitates exploration across diverse regions of the solution space. This inherent characteristic makes ACO particularly well-suited for high-dimensional optimization problems such as feature selection. Specifically, ACO's ability to identify and eliminate redundant or irrelevant features contributes to improving both the accuracy and efficiency of ML models, especially in the context of large and complex datasets [53]. In the context of feature selection, ACO proves highly effective in optimizing computational cost and improving model performance when applied to large datasets [54]. The pheromone-driven mechanism of ACO allows for a systematic exploration of feature subsets, guiding the algorithm to identify the most relevant features while discarding unnecessary ones. This approach is particularly beneficial when dealing with complex data structures, as ACO's global search capabilities contribute to significant improvements in model accuracy and computational efficiency.

2.3.3 Atom search optimization

Atom Search Optimization (ASO) is a physics-based algorithm built on the analysis of molecular dynamics. ASO applies the law of motion of atoms and the principle of reaching the global optimum using the fundamental concepts of molecular dynamics such as properties of the interaction force, potential function, and geometric restraint force [55, 56]. In this algorithm, the potential solution of each atom is obtained with the best fitness value as a result of interactions between the atoms themselves. ASO describes the motion of atoms in terms of velocity, position, and acceleration components. In the ASO algorithm, the motion of each atom is represented by the following formulas:

$$v_i^d(t+1) = \text{rand}_i^d \cdot v_i^d(t) + a_i^d(t) \quad (3)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \quad (4)$$

Here v_i^d is the atom's velocity, x_i^d is its position, and a_i^d is the acceleration. According to Newton's law of motion, acceleration is calculated as follows:

$$a_i^d(t) = \frac{F_i^d(t)}{m_i(t)} + \frac{G_i^d(t)}{m_i(t)} \quad (5)$$

In this formula, $F_i^d(t)$, is the total force of atom i at the t th iteration and $m_i(t)$ is its mass. Here, the forces between the interaction and the geometric constraints are denoted by $F_i^d(t)$ and $G_i^d(t)$. Atoms exchange information with each other and are oriented to the best positions based on this information. One of the most important advantages of ASO is its ability to reach the global optimum solution. Especially in multidimensional and complex search spaces, ASO allows optimizing redundant components while achieving high accuracy rates [43]. Especially in feature selection problems, the use of ASO produces more effective solutions in complex datasets, reducing processing time and increasing model accuracy.

By optimizing the interactions between different atoms and using fitness functions interactively with each other, this algorithm provides efficient solutions even for large datasets.

2.4 Traditional feature selection method (K-Best)

Feature selection is an important preprocessing step used to remove redundant or irrelevant features from high-dimensional datasets and improve model performance. Filter-based methods such as K-Best are based on the principle of ranking features according to their dependencies on the target variable and selecting a certain number of best features. While these methods have the advantage of reducing computational cost, they do not directly optimize model performance [57, 58]. This is because filter methods such as K-Best evaluate features independently and make selections without considering global relationships. In the K-Best method, each feature is evaluated according to a certain scoring function. In this evaluation process, a ranking based on F-score is made for each feature and the features that carry the most information with the target variable are selected. The computational cost of K-Best is expressed as $O(m \cdot n + m \log m)$. Here, m and n denote the number of features and the number of data points, respectively. The first term involves evaluating each feature independently and the second term involves ranking these features. This approach allows for more efficient use of memory and processing power, especially when working with large datasets. However, although the K-Best method is computationally efficient, it runs the risk of ignoring complex and multidimensional relationships in datasets. Especially for imbalanced datasets or data structures with a large number of features, the performance of K-Best may be limited and more complex, meta-heuristic algorithms may be more successful.

2.5 Data balancing techniques

The performance of ML models on imbalanced datasets presents considerable challenges, particularly in accurately classifying rare classes [59]. Data balancing techniques are commonly applied to address this issue. These techniques help mitigate class imbalance, enabling the model to learn both classes more accurately. In this study, we employ the Synthetic Minority Over-sampling Technique–Edited Nearest Neighbors (SMOTE-ENN) and Random Under Sampler (RUS) methods to tackle the imbalance problem in the dataset, and we assess the effectiveness of these approaches.

2.5.1 Synthetic minority over-sampling technique–edited nearest neighbors (SMOTE-ENN)

Synthetic Minority Over-sampling Technique–Edited Nearest Neighbors (SMOTE-ENN) is a hybrid technique used to augment the minority class and balance the dataset in imbalanced data scenarios. While SMOTE generates synthetic samples to increase the minority class, ENN removes noisy or borderline data based on their nearest neighbors [60]. In this study, the 473 fraud data points in the European Cardholder Dataset were oversampled to 283,253 using the SMOTE method and then reduced to 282,801 after noisy data were cleaned with ENN. The total dataset size reached 566,054. The process augmented the data and removed misclassified samples, resulting in a balanced and refined dataset that improved classification model performance.

2.5.2 Random under sampler application

Random Under Sampler (RUS) is a technique that aims to reduce class imbalance in imbalanced datasets by randomly extracting samples from the majority class [61]. This technique allows the model to learn the minority class better by removing class imbalance [59]. In our study, we reduced 283,253 non-fraudulent transactions in the European Cardholder Dataset to 25,000 by random sampling and equated it with 25,000 fraud cases to obtain a balanced dataset. As a result, we balanced the dataset with a total of 50,000 transactions, allowing the model to learn both classes effectively.

2.6 Quantum machine learning

QML algorithms enable efficient processing of high-dimensional and complex datasets, making them particularly suitable for tasks requiring complex data processing and pattern recognition.

In QML, classical data is encoded into quantum states using quantum circuits, where gates such as Hadamard gates create superpositions and controlled gates create entanglement.

An important application area of QML is VQCs, which combine quantum computing principles with traditional classifiers to improve the learning process. VQCs enable model optimization by encoding [62]. This study successfully demonstrates data into quantum circuits through feature maps and using parametric quantum circuits. This structure provides a significant advantage in improving classification accuracy by better modeling complex relationships in datasets.

In this study, a simulated VQC architecture on classical hardware is used to evaluate and tune the model performance. This simulation-based approach allows the model to be precisely optimized before running on quantum devices and demonstrates the potential of VQCs to achieve high classification accuracy with fewer features. In conclusion, the transformative properties of QML demonstrate its advanced data processing capabilities, especially in areas that require high-dimensional and complex data analysis.

2.6.1 Variational quantum classifier

VQC is a hybrid model that combines quantum computing with classical ML algorithms. This model is specifically designed to work efficiently with limited resources on small-scale quantum devices and aims to offer high performance in classification problems [63]. The core components of VQC are encoding input data into quantum states, processing through parameterized quantum gates, and updating the parameters using a classical optimization algorithm. The classification steps of the VQC are discussed below.

State preparation refers to encoding classical input data into the quantum system, which forms the foundation for subsequent quantum operations. Specifically, when the input vector $x \in \mathbb{R}^N$, this vector is transferred into an n -qubit quantum system. In this process, amplitude encoding is used. This encoding method represents classical input data as the amplitudes of quantum states in a 2^n -dimensional Hilbert space.

The model circuit is the part that processes the data transferred to the quantum system and performs classification. The input vector $\varphi(x)$ is transformed by the model circuit using a unitary operator U_θ :

$$\varphi' = U_\theta \varphi(x) \quad (6)$$

Each quantum gate is represented by four parameters in the form $G(\alpha, \beta, \gamma, \phi)$:

$$G(\alpha, \beta, \gamma, \phi) = e^{i\phi} \begin{pmatrix} e^{i\beta} \cos \alpha & e^{i\gamma} \sin \alpha \\ -e^{-i\gamma} \sin \alpha & e^{-i\beta} \cos \alpha \end{pmatrix} \quad (7)$$

These parameters are learned and affect the performance of the model.

Read Out and Postprocessing explains the process of measurement and postprocessing after the quantum circuit is executed. Once the quantum circuit $\varphi' = U_\theta \varphi(x)$ is run, the probability of measuring the first qubit is determined as follows:

$$p(q_0 = 1, x; \theta) = \sum_{k=2^{n-1}+1}^{2^n} |(U_\theta \varphi(x))_k|^2 \quad (8)$$

The circuit is executed S times to estimate this probability accurately, and measurements are taken for each execution. Using the samples s_1, \dots, s_S , the probability is estimated. This process is known as Bernoulli parameter estimation.

After the measurements, classical postprocessing begins. In this step, a learnable bias term b is added to obtain a continuous output:

$$\pi(x; \theta, b) = p(q_0 = 1, x, \theta) + b \quad (9)$$

Finally, this continuous value is applied to a threshold function, and the binary classification output is obtained as follows:

$$f(x; \theta, b) = \begin{cases} 1 & \text{if } \pi(x; \theta, b) > 0.5 \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

Table 3 Hardware specifications of the computer

Specifications	
CPU	AMD Ryzen 7 5800H
GPU	NVIDIA GeForce RTX 3050
RAM	16GB
Operating system	Windows 10 Pro

In this study, we used a dataset of 50,000 European credit card transactions to estimate fraud. The dataset is imbalanced, making it difficult to detect fraudulent transactions. To address this issue, we applied advanced data augmentation and balancing methods. PSO, ASO, ACO, and traditional K-Best multiple feature selection algorithms are used to improve the classification process and enhance the fraud detection performance.

Data processing: Given the unbalanced nature of the dataset, the scarcity of fraudulent transactions representing the minority class was a factor that could degrade the performance of the model. To address this imbalance, the minority data was augmented using the SMOTE-ENN method. Then the RUS method was applied to balance the data set. As a result, the dataset was balanced with 25,000 regular transactions and 25,000 fraudulent transactions.

The dataset was carefully separated for the training and testing phases. 37,500 data were used for training, while the remaining 12,500 data were used in the testing phase to evaluate the accuracy and generalization capabilities of the classification algorithms. This test data is critical to measure the real-world performance of the model and determine its success in fraud detection.

Feature selection: The feature selection algorithms used in the study used different data points to optimize the performance of the model. The features selected by each algorithm are listed as follows:

The ACO algorithm selected 29 features: “Time, V1, V2, V3, V4, V5, V6, V7, V8, V9, V10, V11, V12, V13, V14, V15, V16, V17, V18, V19, V20, V21, V22, V23, V24, V25, V26, V28, Amount”. The ASO algorithm has selected 20 features: “V1, V2, V4, V5, V8, V9, V10, V11, V13, V14, V15, V16, V18, V20, V21, V23, V25, V27, V28, Amount”. With the K-Best method, 23 features were identified: “Time, V1, V2, V3, V4, V5, V6, V7, V8, V9, V10, V11, V12, V14, V16, V17, V18, V19, V20, V21, V24, V27, V28”. The PSO algorithm selected the following 23 features: “Time, V1, V4, V5, V6, V7, V8, V9, V10, V11, V12, V13, V14, V15, V17, V18, V20, V22, V23, V24, V25, V26, V28”. The features chosen by these algorithms directly affected the classification performance. Feature selection allowed the model to work more efficiently and faster by filtering unnecessary or irrelevant information during model training.

Classification: The VQC was used in the classification phase. Quantum Machine Learning techniques have the potential to produce more effective results in high-dimensional data spaces than classical ML methods. VQC analyzed complex patterns using quantum computing power to achieve high accuracy in fraud detection. By enabling fast and accurate data processing, quantum computing technology has significantly improved performance, especially in critical areas such as fraud detection.

3 Experimental results

3.1 Experimental setup

In this study, we actively utilized the PennyLane framework [64]. PennyLane is a cross-platform Python library for quantum computing, QML, and quantum chemistry. We employed VQC during the model training, and all computations were performed on a classical computer. The specifications of the computer used are presented in Table 3.

3.2 Performance metrics

The study employs accuracy as the primary performance metric, calculated by determining the ratio of correctly classified instances to the total number of instances. This approach allows for a comprehensive evaluation of the model’s effectiveness. Utilizing the European Cardholder Dataset, the model is designed to distinguish precisely between legitimate and fraudulent activities.

- True positives (TP): The number of samples that the model correctly predicts as positive
- True negatives (TN): The number of samples that the model correctly predicts as negative
- False positives (FP): The number of samples for which the model incorrectly predicts as positive
- False negatives (FN): The number of samples that the model incorrectly predicts as negative

- Total number of samples: Total number of samples (sum of TP, TN, FP and FN)

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$

$$\text{Precision} = \frac{TP}{TP + FP}$$

$$\text{Recall} = \frac{TP}{TP + FN}$$

$$\text{F1 Score} = \frac{2 * \text{Precision} * \text{Recall}}{\text{Precision} + \text{Recall}}$$

3.3 Evaluation of results

This section presents the performance outcomes from the different feature selection methods applied to the European credit card dataset using the VQC. Each scenario highlights the classifier’s ability to distinguish between fraudulent and non-fraudulent transactions, with attention given to the precision, recall, and overall accuracy metrics.

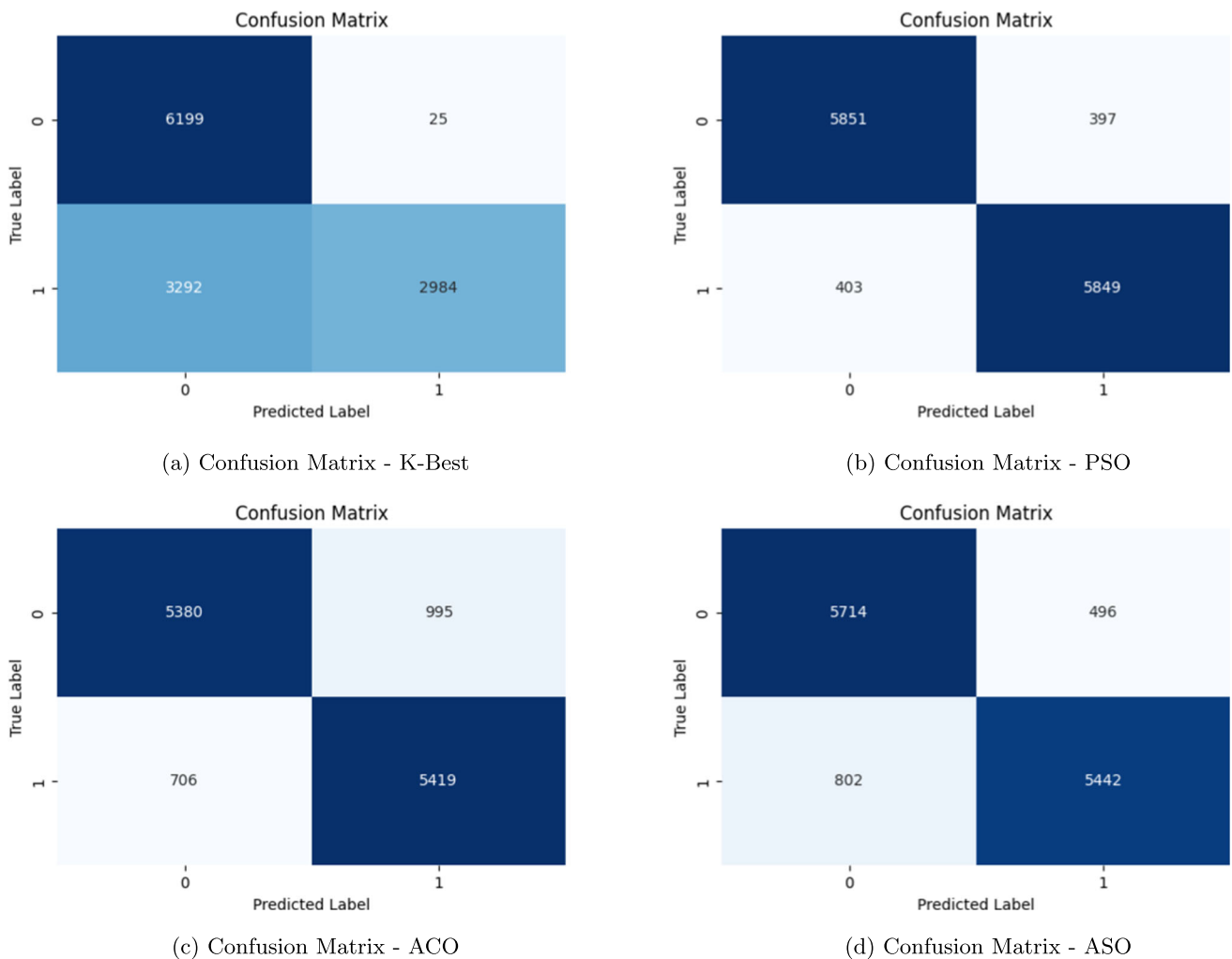


Fig. 2 Confusion matrices for different algorithms

Table 4 The VQC classification performance metrics obtained with each feature selection method

Method	Accuracy	Precision	Recall	F1 score
K-Best	0.8448	0.9917	0.4757	0.6441
PSO	0.9454	0.9364	0.9355	0.9359
ACO	0.8970	0.8449	0.8847	0.8644
ASO	0.8912	0.9164	0.8715	0.8934

3.3.1 Scenario 1: traditional feature selection (K-Best) with VQC

In this scenario, K-Best, a traditional feature selection method, was used. The confusion matrix indicates that out of 50,000 total entries, the model correctly identified 6199 non-fraudulent transactions and 2984 fraudulent transactions. However, there were 25 false positives (non-fraudulent classified as fraudulent) and 3292 false negatives (fraudulent classified as non-fraudulent).

The accuracy achieved in this scenario is 84.48%. While K-Best shows high precision (99.17%) in identifying fraudulent transactions, its recall rate is limited at 47.57% (Fig. 2a). This suggests that further improvements in feature selection are required to enhance the recall performance.

3.3.2 Scenario 2: meta-heuristic feature selection with VQC

The meta-heuristic algorithms PSO, ASO, and ACO were applied for feature selection, and their performances were compared against the traditional K-Best approach.

PSO-based feature selection:

With PSO, the model correctly identified 5851 non-fraudulent transactions and 5849 fraudulent transactions, with 397 false positives and 403 false negatives. The overall accuracy in this scenario is 94.54%, with precision at 93.64% and recall at 93.55%, indicating a balanced performance between precision and recall (Fig. 2b).

ACO-based feature selection:

For ACO, the confusion matrix shows 5380 true negatives and 5419 true positives, achieving an accuracy of 89.70%. However, the false positive rate is higher (16.12%), and the recall is 88.47% (Fig. 2c). Although ACO shows competitive results in identifying fraudulent transactions, its performance is lower than that of PSO.

ASO-based feature selection:

In the case of ASO, the model correctly classified 5714 non-fraudulent transactions and 5442 fraudulent transactions, resulting in an accuracy of 89.12%. Precision stands at 91.64%, while recall is 87.15% (Fig. 2d). ASO delivers a balanced performance, though slightly lower than PSO in terms of accuracy and F1 score.

The VQC classification performance metrics obtained with each feature selection method are shown in detail in Table 4.

4 Conclusion and discussion

This study successfully demonstrates an innovative hybrid approach that combines QML with meta-heuristic feature selection algorithms to address the problem of credit card fraud detection. Advanced data balancing techniques such as SMOTE-ENN and Random Under Sampler are used to improve model performance and accuracy, especially for datasets with severe class imbalance such as the European Cardholder Dataset. Meta-heuristic algorithms such as PSO, ACO, and ASO were also evaluated along with the traditional K-Best method. The results show that the integration of PSO with VQC provides the highest performance with an accuracy of 94.54%. This high accuracy rate emphasizes the effectiveness of the integration of meta-heuristic algorithms with quantum classifiers on complex and high-dimensional data sets. The fact that PSO outperforms the other methods, especially in both precision and recall rates, demonstrates the potential of QML for applications in critical areas such as financial fraud. The results of the study demonstrate the speed and accuracy advantages of QML compared to traditional ML methods, proving that it offers more advanced data processing and classification capabilities in applications to enhance financial security. However, one important limitation of the current work is the reliance on quantum simulators, which do not fully capture the noise, decoherence, and execution constraints present in real quantum hardware. Therefore, the observed results, while promising, may not entirely reflect the performance of the system in deployment scenarios. In addition, the simulators used in this study were constrained by limited qubit availability and gate depth, which restrict the scalability of the model. These constraints imply that larger scale fraud detection datasets or more complex circuits may require future validation using real quantum processors.

Tests with IBM's real quantum computers are expected to provide higher accuracy with more qubits utilization and computational performance. However, access to IBM quantum computers is limited and users can perform a limited number of operations in a certain period of time, which is another factor affecting the application processes. In future studies, it is recommended to test quantum models on real hardware with larger data sets. Moreover, future investigations should incorporate aspects such as model interpretability, fairness, and adversarial robustness, particularly important in financial fraud contexts where decisions must be transparent, ethical, and secure. The integration of Explainable AI (XAI) approaches with QML could offer avenue for enhancing interpretability and regulatory compliance, especially for deployment in high-stakes domains such as finance. Furthermore, with the development of fault-tolerant quantum devices, the performance of QML algorithms should be optimized. Such improvements will further increase the application potential of QML in areas that require high accuracy and performance, such as financial fraud detection. In conclusion, this study makes an important contribution to the literature on how the hybrid use of QML and meta-heuristics can provide higher accuracy and efficiency in data-intensive and critical areas such as credit card fraud detection.

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Author contribution statement

M.Y.K. developed the software, performed the general analyses, and contributed to writing the manuscript. F.A. developed the software, conducted analyses, and contributed to manuscript writing. C.B. also carried out analyses and contributed to manuscript writing. All authors reviewed and approved the final version of the manuscript.

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Data Availability This study utilizes the 'Credit Card Fraud Detection' dataset, made available by Worldline and the Machine Learning Group of Université Libre de Bruxelles, hosted on Kaggle. The dataset includes anonymized transaction data, with fraud instances labeled, and is accessible for non-commercial research purposes under Kaggle's terms.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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