

ASPECTS OF THE THEORY AND PHENOMENOLOGY OF  
TWIN HIGGS MODELS

by

Christopher Allen Krenke

---

A Dissertation Submitted to the Faculty of the

DEPARTMENT OF PHYSICS

In Partial Fulfillment of the Requirements  
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

2009

THE UNIVERSITY OF ARIZONA  
GRADUATE COLLEGE

As members of the Dissertation Committee, we certify that we have read the dissertation prepared by Christopher Allen Krenke entitled Aspects of the Theory and Phenomenology of Twin Higgs Models and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

\_\_\_\_\_  
Zackaria Chacko

Date: 28 April 2009

\_\_\_\_\_  
Keith R. Dienes

Date: 28 April 2009

\_\_\_\_\_  
Shufang Su

Date: 28 April 2009

\_\_\_\_\_  
Mike A. Shupe

Date: 28 April 2009

\_\_\_\_\_  
Bira van Kolck

Date: 28 April 2009

Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copies of the dissertation to the Graduate College.

I hereby certify that I have read this dissertation prepared under my direction and recommend that it be accepted as fulfilling the dissertation requirement.

\_\_\_\_\_  
Dissertation Director: Zackaria Chacko

Date: 28 April 2009

\_\_\_\_\_  
Dissertation Director: Keith R. Dienes

Date: 28 April 2009

## STATEMENT BY AUTHOR

This dissertation has been submitted in partial fulfillment of requirements for an advanced degree at the University of Arizona and is deposited in the University Library to be made available to borrowers under rules of the Library.

Brief quotations from this dissertation are allowable without special permission, provided that accurate acknowledgment of source is made. Requests for permission for extended quotation from or reproduction of this manuscript in whole or in part may be granted by the head of the major department or the Dean of the Graduate College when in his or her judgment the proposed use of the material is in the interests of scholarship. In all other instances, however, permission must be obtained from the author.

SIGNED: Christopher Allen Krenke

## ACKNOWLEDGEMENTS

It is a great pleasure to thank my advisor Professor Zackaria Chacko for mentoring me through my graduate studies. I am truly grateful for everything he has taught me. I also owe a great deal of thanks to Dr. Hock-Seng Goh and Dr. Takemichi Okui for their patient instruction.

To Professor Keith Dienes, thank you for your guidance and support. My gratitude also goes to the staff who have all helped me along the way. In particular, a very special thanks goes to Lisa Shapouri, whom I endlessly inconvenienced.

Finally, I thank my family and friends for their love and support.

## DEDICATION

*To Mom, Dad, Michael & Steven*

## TABLE OF CONTENTS

LIST OF FIGURES . . . . .	8
LIST OF TABLES . . . . .	9
ABSTRACT . . . . .	10
CHAPTER 1 INTRODUCTION . . . . .	11
1.1 The Planck-Weak Hierarchy . . . . .	11
1.2 The Little Hierarchy Problem . . . . .	11
1.3 Twin Higgs Theories . . . . .	13
1.3.1 The Twin Mechanism . . . . .	13
1.3.2 The Mirror Twin Higgs . . . . .	17
1.3.3 The Left-Right Twin Higgs . . . . .	24
1.4 Dissertation Format . . . . .	43
CHAPTER 2 THE PRESENT STUDY . . . . .	45
2.1 The Little Twin Higgs . . . . .	45
2.2 Lepton Number Violating Signals of the Top Partners in the LRTH . . . . .	50
REFERENCES . . . . .	53
APPENDIX A A LITTLE TWIN HIGGS MODEL . . . . .	56
A.1 Introduction . . . . .	57
A.2 Construction of the Model . . . . .	60
A.2.1 Quartic for the Left-Right Model . . . . .	63
A.2.2 SU(4) Invariant Top Yukawa Interaction . . . . .	65
A.3 Radiative Corrections and EW Symmetry Breaking . . . . .	66
A.4 Mirror Model . . . . .	71
A.5 Conclusion . . . . .	73
APPENDIX B LEPTON NUMBER VIOLATING SIGNALS IN THE LRTH . . . . .	78
B.1 Introduction . . . . .	78
B.2 Left-right Twin Higgs model . . . . .	81
B.2.1 Matter Content . . . . .	81
B.2.2 Phenomenology . . . . .	83
B.3 Neutrino Mass Seesaw at the TeV Scale . . . . .	85
B.3.1 Constraints on Majorana Right-handed Neutrinos . . . . .	87

TABLE OF CONTENTS – *Continued*

B.4	$M = 0$ Phenomenology . . . . .	88
B.4.1	$W_R$ Search . . . . .	88
B.4.2	$T_H$ Search: $m_{\nu_R} > m_{T_H}$ . . . . .	90
B.4.3	$T_H$ Search: $m_{\nu_R} < m_{T_H}$ . . . . .	91
B.5	Conclusion . . . . .	91
B.6	Acknowledgments . . . . .	92

## LIST OF FIGURES

1.1	Cancellation of quadratic divergences in the LRTH. . . . .	29
1.2	Mass spectrum of the left-right twin Higgs model. . . . .	35
1.3	Cross section for $W_H$ and $Z_H$ production in the LRTH model. . . . .	36
1.4	Decay branching fractions for $W_H$ and $Z_H$ in the LRTH model. . . . .	37
1.5	$Z_H$ resonance present in the invariant mass distribution of $e^+e^-$ . . . . .	38
1.6	Diagrams contributing to single heavy top production in the LRTH. . . . .	39
1.7	Single heavy top $T_H$ production and decays . . . . .	39
1.8	Leptonic decay channel of the heavy top $T_H$ . . . . .	40
1.9	Decay branching fractions for $\phi^\pm$ as a function of $M$ . . . . .	41
1.10	$\Omega h^2$ vs. $m_{\tilde{S}}$ (Low mass regime). . . . .	42
1.11	$\Omega h^2$ vs. $m_{\tilde{S}}$ (High mass regime). . . . .	42
2.1	Evolution of twin Higgs models . . . . .	52
B.1	Possible $T_H$ decay ( $M=0$ ) . . . . .	84
B.2	$W_R$ production and decay via $\nu_R$ . . . . .	89
B.3	Production of $\nu_R l^\pm$ and $T_H bll$ . . . . .	90



## LIST OF TABLES

1.1	Higgs mass and tuning in mirror twin Higgs models. . . . .	23
1.2	Higgs mass and tuning in the minimal LRTH model. . . . .	33
A.1	Higgs mass and tuning in the little twin Higgs model. . . . .	72
A.2	Higgs mass and tuning in the mirror model with a tree level quartic. .	74

# ABSTRACT

We begin by reviewing the basic theory and phenomenology of twin Higgs models. In these theories, the Higgs arises as a pseudo-Nambu-Goldstone boson of a spontaneously broken global symmetry. A discrete symmetry restricts the form of the radiatively generated Higgs potential such that dimensionful terms respect this global symmetry. The Higgs mass is then protected from receiving quadratically divergent contributions, allowing natural electroweak symmetry breaking up to a cutoff scale of about 10 TeV.

We then show how to incorporate a tree level quartic into the left-right twin Higgs. The addition of such a term results in a substantial reduction in the fine tuning compared to that of the original twin Higgs. We do this by extending the symmetry of the theory to include two  $Z_2$  symmetries, each of which is sufficient to protect Higgs mass from receiving quadratically divergent corrections. Although both parities are broken explicitly, the symmetries that protect Higgs mass from getting a quadratically divergent mass are broken only collectively. Therefore, the Higgs mass parameter is free from quadratic divergences to one loop.

Finally, we consider the collider signatures of the left-right twin Higgs in the limit that the right-handed neutrino mass is less than the right-handed gauge boson mass. In this limit, which has not been considered previously, new leptonic decay channels open up. This allows the discovery of the right-handed gauge boson  $W_R$  and the heavy top partner  $T_H$ , which are responsible for canceling the one-loop quadratic divergences of the Higgs mass. Half of these events contain same-sign leptons without missing energy, which have no SM background. These signals may be used to complement other collider searches, and in certain regions of parameter space, may be the only way to observe the particles responsible for natural electroweak symmetry breaking in the left-right twin Higgs.

## CHAPTER 1

### INTRODUCTION

#### 1.1 The Planck-Weak Hierarchy

The standard model (SM) of particle physics describes most observations in nature to very high precision. However, the model is not without its flaws. The majority of the universe is composed of non-luminous, uncharged “dark matter,” which cannot be accounted for in the SM. The SM also does not explain neutrino masses or generate a sufficient baryon asymmetry. But perhaps most importantly, the SM suffers from the “hierarchy problem.” The SM contains a scalar particle known as the Higgs boson, whose mass depends very sensitively on short distance physics. Without a fine-tuning of parameters the Higgs would naturally be heavy, of order the cutoff of the theory, which for the SM is the Planck scale, the scale where gravity gets strong. However, theoretical consistency of the SM requires a Higgs that is much lighter than this, with mass less than or of order a TeV. Thus, the SM is either fine tuned, or new physics must emerge at the TeV scale to stabilize the Higgs mass. New physics at the TeV scale is an exciting possibility, since the Large Hadron Collider (LHC) will begin to probe this energy regime in the very near future.

#### 1.2 The Little Hierarchy Problem

Even though the Higgs mass is only theoretically constrained to be less than or of order a TeV, precision electroweak tests predict a Higgs mass in the SM that is lighter than 200 GeV. On the other hand, non-renormalizable operators that contribute to precision electroweak observables must be suppressed by a scale that is greater than about 5 TeV. Specifically, the scale  $\Lambda$  that appears in operators such as

$$\frac{\bar{D}^2 H^\dagger D^2 H}{\Lambda^2} - \frac{|H^\dagger D_\mu H|^2}{\Lambda^2}, \quad (1.1)$$

which we expect to arise when we integrate out new physics, is greater than 5 TeV. This observation suggests that new physics will not show itself until scales of at least 5 TeV. However, without significant fine-tuning, quantum corrections from a cutoff scale of order 5 TeV will generate a Higgs mass much greater than 200 GeV. To resolve this “little hierarchy,” we expect new physics with mass less than or of order a TeV, which stabilizes the Higgs mass, but does not contribute significantly to precision electroweak observables. Models that address the little hierarchy should be considered possible low energy effective theories for the ultraviolet physics that cures the Planck-weak “big hierarchy” problem. This is an active area of research and many interesting ideas have been put forth.

Models that address the hierarchy problem typically invoke a new symmetry that eliminates the Higgs mass parameter’s quadratic sensitivity to the cutoff. This is the case, for example, in supersymmetry. The graphs that contribute to the quadratically divergent part of the Higgs mass are cancelled by new contributions from partners of the SM particles, known as “superpartners.” The role of the new symmetry, in this case supersymmetry, is to ensure the necessary relationships between couplings so that the cancellation goes through exactly.

Another possible explanation for the lightness of the Higgs based on a symmetry argument is that it may be the pseudo-Nambu-Goldstone boson of a spontaneously broken global symmetry [1, 2, 3, 4, 5]. Whenever a continuous global symmetry is spontaneously broken, massless particles, known as a Nambu-Goldstone bosons, will always appear in the spectrum of the theory. This fact is known as Goldstone’s Theorem. Moreover, the Nambu-Goldstone bosons possess a shift symmetry that ensures they can only appear derivatively coupled in the Lagrangian.

However, the gauge, Yukawa and self interactions of the SM Higgs explicitly break any shift symmetry. Therefore, the Higgs can only be a “pseudo-Nambu-Goldstone.” Such an explicit breaking of shift symmetry will typically generate a

potential for the Nambu-Goldstone bosons consistent with the remaining symmetries of the theory. In general, this includes quadratically divergent contributions to the Nambu-Goldstone's mass parameter. Therefore, the challenge in describing the Higgs as a Nambu-Goldstone boson is to explain how quadratically divergent terms do not arise in its potential even though it has non-derivative couplings. If this challenge can be met, it is possible for the Nambu-Goldstone bosons, and hence the Higgs, to acquire some mass, while still remaining light compared to the cutoff of the theory. A class of theories that have successfully implemented these ideas are known as little Higgs theories [6, 7, 8, 9, 10, 11].

### 1.3 Twin Higgs Theories

Recently another class of theories, known as twin Higgs theories, have been proposed in which the Higgs is also realized as a pseudo-Nambu-Goldstone boson [12, 13, 14, 15]. Twin Higgs theories are interesting because they provide a novel way to eliminate quadratic divergences of the Higgs mass up to LHC energies, thus, alleviating the little hierarchy problem. In addition, one of these models demonstrates that the new particles responsible for stabilizing the Higgs mass need not be charged under the SM gauge groups. Previously it was assumed that the new states responsible for stabilizing the Higgs mass had to be charged under the familiar SM gauge groups,  $SU(3)_c \times SU(2)_L \times U(1)_Y$ .

#### 1.3.1 The Twin Mechanism

In twin Higgs theories, how is the Higgs protected from receiving quadratically divergent contributions to its mass parameter? For the time being, let us focus only on the gauge interactions. Consider a complex scalar field  $H$ , which transforms as a fundamental under a global  $U(4)$  symmetry

$$H = \begin{pmatrix} H_A \\ H_B \end{pmatrix}. \quad (1.2)$$

$H_A$  and  $H_B$  transform as doublets under the subgroups  $SU(2)_A$  and  $SU(2)_B$ , respectively. A potential for  $H$  which spontaneously breaks the  $U(4)$  global symmetry is given by,

$$V(H) = -m^2 H^\dagger H + \lambda (H^\dagger H)^2. \quad (1.3)$$

This potential gives  $H$  a vacuum expectation value (vev) of  $\langle H \rangle = m/\sqrt{2\lambda}$ , and breaks  $U(4) \rightarrow U(3)$ . This pattern of symmetry breaking yields 7 massless Nambu-Goldstone bosons. A simple way to see why eq. (1.3) does not generate mass for the Nambu-Goldstone bosons is to consider the following parameterization of  $H$ ,

$$H = e^{ih^a t^a/f} \begin{pmatrix} 0 \\ 0 \\ 0 \\ \rho + f \end{pmatrix}, \quad (1.4)$$

where the  $h^a$ 's are the Nambu-Goldstone bosons,  $f$  is the symmetry breaking scale  $m/\sqrt{2\lambda}$ , and the  $t^a$ 's are the broken generators of  $U(4)$ . That is, the broken generators satisfy

$$t^a \begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix} \neq 0. \quad (1.5)$$

It is now clear that in this parameterization only the “radial mode”  $\rho$  has a potential and not the Nambu-Goldstone bosons. Furthermore, any potential that is in  $U(4)$  invariant form will not contribute to the potential of Nambu-Goldstone bosons.

We now gauge an  $SU(2)_A \times SU(2)_B$  subgroup of  $U(4)$ . By gauging these two subgroups we have explicitly broken the global  $U(4)$  symmetry and therefore no

longer expect Goldstone's theorem to apply. However, it is clear that in the limit that the gauge couplings  $g_A$  and  $g_B$  vanish, the  $U(4)$  symmetry of the theory returns. Therefore the contribution of gauge interactions to the potential must be proportional to the  $U(4)$  breaking parameters,  $g_A$  and  $g_B$ . In addition, the theory still possesses an  $SU(2)_A \times SU(2)_B$  symmetry. Therefore gauge loop contributions to the potential above must be of the form,

$$\begin{aligned} \delta V = & \frac{c_1 g_A^2 \Lambda^2}{16\pi^2} H_A^\dagger H_A + \frac{c_1 g_B^2 \Lambda^2}{16\pi^2} H_B^\dagger H_B \\ & + \frac{c_2 g_A^4}{16\pi^2} \log\left(\frac{\Lambda}{g_A f}\right) (H_A^\dagger H_A)^2 + \frac{c_2 g_B^4}{16\pi^2} \log\left(\frac{\Lambda}{g_B f}\right) (H_B^\dagger H_B)^2 + \dots, \end{aligned} \quad (1.6)$$

where  $\Lambda$  is the cutoff of the theory. We see that the explicit breaking of the  $U(4)$  symmetry has generated a contribution to the potential for the Nambu-Goldstone bosons, and hence the SM Higgs, that is quadratically divergent. This is completely expected since Goldstone's theorem only guarantees massless Nambu-Goldstones bosons if the spontaneously broken symmetry is an exact symmetry of the Lagrangian.

Now consider imposing the following discrete  $Z_2$  symmetry that exchanges the A and B-type fields.

$$\begin{aligned} H_A & \leftrightarrow H_B \\ W_A^a & \leftrightarrow W_B^a \end{aligned} \quad (1.7)$$

This “twin symmetry” requires that  $g_A = g_B = g$ . Therefore, the mass terms above eq. (1.6) can be written as

$$\delta V_{mass} = \frac{c_1 g^2 \Lambda^2}{16\pi^2} (H_A^\dagger H_A + H_B^\dagger H_B) = \frac{c_1 g^2 \Lambda^2}{16\pi^2} H^\dagger H. \quad (1.8)$$

Notice that  $\delta V_{mass}$  now has a  $U(4)$  invariant form. Therefore, this quadratically

divergent term does not contribute to the potential for the Nambu-Goldstone bosons, and hence, the SM Higgs. Thus, we have succeeded in removing the quadratically divergent contributions to the Higgs mass and can now hope to achieve natural electroweak symmetry breaking.

The remaining terms in  $\delta V$  give a logarithmically divergent contribution to the Higgs mass,

$$m_h^2 \sim \frac{g^4 f^2}{16\pi^2} \log \left( \frac{\Lambda}{gf} \right). \quad (1.9)$$

In the limit of strong coupling  $\Lambda \sim 4\pi f$ , the Higgs can acquire a weak scale mass for  $f$  of order a TeV.

$$m_h \sim \frac{g^2 f}{4\pi} \quad (1.10)$$

Let us summarize what we have learned. The theory above possesses a discrete symmetry which guarantees that any dimensionful terms in the Higgs potential respect a larger global symmetry. Also, the SM Higgs emerges as a pseudo-Nambu-Goldstone boson associated with the spontaneous breaking of this global symmetry. These two facts are sufficient to ensure that the SM Higgs is protected from receiving quadratically divergent contributions to its mass parameter. We chose to demonstrate these ideas above using a global  $U(4)$  symmetry since it is the simplest group that can contain the  $SU(2) \times U(1)$  of the SM and separately an  $SU(2) \times U(1)$  gauge symmetry for the twin sector.

The challenge now is to create a realistic model utilizing the twin mechanism in which all quadratically divergent contributions to the Higgs mass parameter are absent. To accomplish this task, the twin symmetry must be extended to include all interactions of the SM. Two approaches have been studied in the literature, and are known as the mirror twin Higgs [12] and the left-right twin Higgs [14]. We will review each of these models separately in the following sections.



### 1.3.2 The Mirror Twin Higgs

We are now in a position to review the first realistic model utilizing the twin mechanism, the mirror twin Higgs [12]. The twin symmetry in this model relates the SM fields to those of a “mirror SM,” which has the same field content and interactions as the SM.

Consider two copies of the Standard Model,  $SM_A$  and  $SM_B$ , with a  $Z_2$  symmetry that exchanges all of the fields in  $SM_A$  with those in  $SM_B$ , and vice versa.

$$Z_2 : SM_A \leftrightarrow SM_B \tag{1.11}$$

We identify the fields in  $SM_A$  with those of the SM and the fields of  $SM_B$  with those of a hidden “mirror SM.” If the pattern of symmetry breaking in the Higgs sector is identical to that of the twin mechanism described above,  $U(4) \rightarrow U(3)$ , quadratically divergent contributions to the Higgs potential arising from gauge loops vanish, leaving only logarithmically divergent contributions.

In the SM, there are also quadratically divergent contributions to the Higgs mass arising from fermion loops. However, because the twin symmetry has been extended to include all interactions of the theory, these divergences are also eliminated, leaving only logarithmic divergences. Since the top Yukawa coupling,  $y_t$ , is order one, the largest of the fermionic contributions to the Higgs mass arises from top quark. With a little extra structure, we might be able to improve naturalness further by making the top contribution finite. This can be done by expanding the global symmetry of the top Yukawa coupling to  $SU(6) \times U(4) \times U(1)$ , with the two SM subgroups,  $(SU(3)_c \times SU(2) \times U(1))_{A,B}$  gauged. This is done by introducing the following chiral fermions

$$\begin{aligned}
Q_L &= (\mathbf{6}, \bar{\mathbf{4}}) \\
&= (\mathbf{3}, \mathbf{2}; \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}; \mathbf{3}, \mathbf{2}) + (\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}; \mathbf{3}, \mathbf{1}) \\
&\equiv q_A + q_B + \tilde{q}_A + \tilde{q}_B \\
T_R &= (\bar{\mathbf{6}}, \mathbf{1}) \\
&= (\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}; \bar{\mathbf{3}}, \mathbf{1}) \\
&\equiv t_A + t_B,
\end{aligned} \tag{1.12}$$

or more explicitly

$$Q_L = \begin{pmatrix} q_A & \tilde{q}_B \\ \tilde{q}_A & q_B \end{pmatrix}, \quad T_R = \begin{pmatrix} t_A \\ t_B \end{pmatrix}. \tag{1.13}$$

With this matter content we can write the  $U(4)$  invariant coupling

$$\mathcal{L}_{top} = y_t H Q_L T_R + h.c. \tag{1.14}$$

Notice that in addition to the  $SM_A$  and  $SM_B$  quark doublets,  $q_A$  and  $q_B$ , we also have two exotic quark doublets,  $\tilde{q}_A$  and  $\tilde{q}_B$ , with a mixture of non-trivial A and B quantum numbers. Introduce the two more chiral fermions  $\tilde{q}_{A,B}^c$  with charge assignment opposite to that of the exotic fermions,  $\tilde{q}_{A,B}$ . Then we can write the following  $Z_2$  symmetric mass for the exotic quarks,

$$M(\tilde{q}_A^c \tilde{q}_A + \tilde{q}_B^c \tilde{q}_B). \tag{1.15}$$

This mass term is the only source of  $U(4)$  breaking in the top sector, but it only breaks  $U(4)$  softly. Thus, the top contribution to the Higgs potential will be finite at one loop.

We are now in a position to compute the Higgs potential in more detail. We will do this using the following non-linear parameterization for the seven pseudo-Nambu-Goldstones,

$$H = \exp\left(\frac{i}{f} h^a t^a\right) \begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix} = \exp\left[\frac{i}{f} \begin{pmatrix} 0 & 0 & 0 & h_1 \\ 0 & 0 & 0 & h_2 \\ 0 & 0 & 0 & h_3 \\ h_1^\dagger & h_2^\dagger & h_3^\dagger & h_0 \end{pmatrix}\right] \begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix}, \quad (1.16)$$

where  $h_1, h_2$  and  $h_3$  are complex and  $h_0$  is real. This parameterization of  $H$  describes the dynamics of the Nambu-Goldstone bosons up to the cutoff scale  $\Lambda^1$ . If we assume the theory is strongly coupled at the cutoff scale, we can estimate  $\Lambda \sim 4\pi f$ . In general the effective theory for the Nambu-Goldstones will contain all operators allowed by the  $U(4)$  symmetry suppressed by  $\Lambda$ . However, only  $U(4)$  breaking terms can contribute to the potential for the Nambu-Goldstones, namely the gauge and Yukawa interactions. Of these, the electroweak and top sectors contribute the most and therefore will be studied in detail.

When  $SU(2)_B \times U(1)_B$  is broken to  $U(1)_{Y'}$ , 3 degrees of freedom are eaten by the mirror gauge bosons and correspond to the fields  $h_3$  and  $h_0$ . After removing these fields the parameterization above yields

$$H = f \begin{pmatrix} i \frac{h_1}{|h|} \sin \frac{|h|}{f} \\ i \frac{h_2}{|h|} \sin \frac{|h|}{f} \\ 0 \\ \cos \frac{|h|}{f} \end{pmatrix}, \quad (1.17)$$

where  $h^T = (h_1 \ h_2)^T$  is the uneaten  $SU(2)_A$  doublet, which we identify as the SM

---

<sup>1</sup>One might worry that by using this parameterization of  $H$ , we may not have correctly chosen the vacuum alignment. However, this is not the case. The Nambu-Goldstone bosons  $h$  in eq. (1.16) are free to acquire vevs determined by  $U(4)$  breaking interactions. Therefore, the true vacuum alignment is determined dynamically.

Higgs doublet. Therefore,

$$\begin{aligned} H_A^\dagger H_A &= f^2 \sin^2 \frac{|h|}{f} = h^\dagger h - \frac{(h^\dagger h)^2}{3f^2} + \dots \\ H_B^\dagger H_B &= f^2 \cos^2 \frac{|h|}{f} = f^2 - h^\dagger h + \frac{(h^\dagger h)^2}{3f^2} + \dots \end{aligned} \quad (1.18)$$

We will compute the effective potential for the SM Higgs using the Coleman-Weinberg (CW) effective potential [16]. In the absence of quadratic divergences the effective potential is given by

$$V_{CW} = \pm \frac{1}{64\pi^2} \sum_i M_i^4 \left( \log \frac{\Lambda^2}{M_i^2} + \frac{3}{2} \right), \quad (1.19)$$

where  $M_i$  is the field dependent mass for the  $i$ th field. The sum runs over all degrees of freedom and is positive for fermions and negative for bosons. If the Higgs potential is written as

$$V(h) = m_h^2 h^\dagger h + \lambda_h (h^\dagger h)^2 + \dots, \quad (1.20)$$

it is found that the gauge contribution to the potential is

$$m_h^2|_{gauge} = \frac{6g^2 M_{W_B}^2}{64\pi^2} \left( \log \frac{\Lambda^2}{M_{W_B}^2} + 1 \right) + \frac{3(g^2 + g'^2) M_{Z_B}^2}{64\pi^2} \left( \log \frac{\Lambda^2}{M_{Z_B}^2} + 1 \right), \quad (1.21)$$

where

$$M_{W_B}^2 = \frac{g^2 f^2}{2} M_{Z_B}^2 = \frac{(g^2 + g'^2) f^2}{2}. \quad (1.22)$$

Eq. (1.21) is valid if  $U(1)_{EM}$  in the twin sector is an unbroken symmetry. However,

it is possible that the mirror  $U(1)_{EM}$  is broken giving the mirror photon a mass. This could occur if the mirror hypercharge gauge boson has a mass  $M_B$  which softly breaks the twin symmetry. If  $M_B^2 \gg g'^2 f^2$ , the second term in eq. (1.21) becomes approximately

$$\frac{3g^2 M_{W_B}^2}{64\pi^2} \left( \log \frac{\Lambda^2}{M_{W_B}^2} + 1 \right) + \frac{3g'^2 M_B^2}{64\pi^2} \left( \log \frac{\Lambda^2}{M_B^2} + 1 \right). \quad (1.23)$$

The gauge contributions to the SM Higgs quartic are small and therefore can be neglected. Let us now consider the top sector. To lowest order in  $|h|^2$ , the Higgs dependent masses are

$$\begin{aligned} m_{t_A}^2 &= \frac{y^2 M^2}{M^2 + y^2 f^2} h^\dagger h & m_{T_A}^2 &= M^2 + y^2 f^2 \\ m_{t_B}^2 &= y^2 f^2 & m_{T_B}^2 &= M^2. \end{aligned} \quad (1.24)$$

This leads to a contribution to the Higgs potential given by

$$\begin{aligned} m_h^2|_{top} &= \frac{3}{8\pi^2} \frac{y^2 M^2}{M^2 - y^2 f^2} \left( M^2 \log \frac{m_{T_A}^2}{m_{T_B}^2} - y^2 f^2 \log \frac{m_{T_A}^2}{m_{t_B}^2} \right) \\ \lambda_h|_{top} &= -\frac{m_h^2}{3f^2} + \frac{3}{16\pi^2} \frac{y^4 M^4}{(M^2 + y^2 f^2)^2} \log \frac{m_{T_A}^2}{m_{t_A}^2} \\ &\quad + \frac{3}{16\pi^2} \frac{y^4 M^4 (M^2 + y^2 f^2)}{(M^2 - y^2 f^2)^3} \log \frac{m_{T_B}^2}{m_{t_B}^2} \\ &\quad - \frac{3}{32\pi^2} \left[ \frac{4y^4 M^4}{(M^2 - y^2 f^2)^2} + \frac{y^4 M^4}{(M^2 + y^2 f^2)^2} \right]. \end{aligned} \quad (1.25)$$

In addition, a ‘ $\mu$  term’ which softly breaks the twin symmetry must be added to the theory to achieve the correct pattern of symmetry breaking. This term is given by  $\mu^2 H_A^\dagger H_A$  and therefore according to eq. (1.18) contributes the following to the Higgs potential

$$m_h^2|_\mu = \mu^2 \quad (1.26)$$

$$\lambda_h|_\mu = \frac{\mu^2}{3f^2}. \quad (1.27)$$

Now that the effective potential for the Higgs has been evaluated, we are in a position to compare the fine-tuning in mirror twin Higgs models to that of the SM with a cutoff of  $\Lambda = 5$  TeV. It is convenient to divide these models into two classes, each with distinct phenomenological consequences; those with an extended top sector and those without. For models without an extended top sector, the contribution from the top sector can be found, up to finite terms, by taking the limit  $M \rightarrow \Lambda$  in the formulas above.

To estimate the fine-tuning, suitable values for 4 of the 5 parameters in the theory ( $f, \Lambda \sim 4\pi f, M, M_B, \mu$ ) must be chosen. The resulting potential is then minimized with respect to  $v$ , the SM Higgs vev. The experimental value of  $v$  can then be used to determine the value of the unknown fifth parameter. The fine-tuning is roughly defined as the fractional change in a known observable divided by the fractional change of a parameter in the theory. For example,

$$\text{fine-tuning} \sim \frac{\delta M_Z^2/M_Z^2}{\delta \mu^2/\mu^2} \sim \frac{\partial \log M_Z^2}{\partial \log \mu^2}. \quad (1.28)$$

The tuning at a few points in parameter space for the two minimal mirror twin Higgs models [12] is summarized in table 1.1. For comparison note that the SM with a cutoff of  $\Lambda = 5$  TeV is fine-tuned at the percent level. Due to the absence of quadratic divergences contributing to the Higgs potential, it is clear that the fine-tuning required in twin symmetric models is significantly less than that of the SM, thereby achieving natural electroweak symmetry breaking and stabilizing the Higgs mass up to 5 - 10 TeV.

$\Lambda_{(\text{TeV})}$	$f_{(\text{GeV})}$	$M_{(\text{TeV})}$	$M_{B(\text{TeV})}$	$\mu_{(\text{GeV})}$	$m_h_{(\text{GeV})}$	Tuning
10	800	6	1	239	122	0.134
6	500	5.5	1	145	121	0.378
10	800	-	0	355	166	0.112
6	500	-	0	203	153	0.307

Table 1.1: A summary of the Higgs mass and fine-tuning,  $\partial \log M_Z^2 / \partial \log \mu^2$ , for sample points of parameter space in the two classes of mirror twin Higgs models. The first two lines refer to mirror twin models with an extended top sector. The last two lines refer to the minimal mirror twin models without an extended top sector. In this case,  $M = \Lambda$  and the mirror photon is massless,  $M_B = 0$ .

## Phenomenology

Let us first consider the class of mirror twin Higgs theories with an extended top sector. In this case, there are 4 exotic quarks  $\tilde{q}_{A,B}$  and  $\tilde{q}_{A,B}^c$  with the following quantum numbers under  $[SU(3) \times SU(2) \times U(1)]_{A,B}$

$$\begin{aligned}
\tilde{q}_A &= (\mathbf{3}, \mathbf{1}, 4/3; \mathbf{1}, \mathbf{2}, -1) & \tilde{q}_A^c &= (\bar{\mathbf{3}}, \mathbf{1}, -4/3; \mathbf{1}, \bar{\mathbf{2}}, 1) \\
\tilde{q}_B &= (\mathbf{1}, \mathbf{2}, -1/2; \mathbf{3}, \mathbf{1}, 2/3) & \tilde{q}_B^c &= (\mathbf{1}, \bar{\mathbf{2}}, 1/2; \bar{\mathbf{3}}, \mathbf{1}, -2/3).
\end{aligned} \tag{1.29}$$

These exotic quarks are charged under both  $U(1)_A$  and  $U(1)_B$ , which at one loop leads to kinetic mixing between the photon and its partner, the mirror photon [17]. Since there are severe constraints from big bang nucleosynthesis (BBN) on such mixing, the mirror photon must be heavy, with a mass greater than about 100 GeV.

If the top sector is not extended, there are no longer particles charged under both sets of gauge groups. This fact leads to an absence of kinetic mixing between the photon and its mirror partner up to at least three loop order [12]. Therefore, it is no longer necessary for the mirror photon to be heavy. In fact, it may be possible phenomenologically for the mirror photon to be massless. In this case, if the kinetic mixing term between the photon and mirror photon is small but non-zero, the light exotic fermions may have very small fractional charges.

Other than the possibility of fractionally charged states in the mirror twin Higgs without an extended top sector, the phenomenology of both classes of models are very similar. Each of them predict new light particles that are not charged under the SM gauge groups. Since these particles have no SM charge, they cannot be produced through the SM gauge interactions at colliders. However, these states do couple to the Higgs, and therefore may be produced via Higgs decay. Again, since these particles have no SM charge, it is not possible to detect them directly once they have been produced. On the other hand, it may be possible to detect the invisible decays of the Higgs into the light mirror fermions [18, 19]. Using eq. (1.17) to expand  $H_B$  in the twin symmetric Yukawa interaction  $H_A q_A t_A + H_B q_B t_B$ , gives a term  $\sim f(1 - |h|^2/f^2 + \dots) q_B t_B$ . When the Higgs acquires a vev, there is a coupling of the Higgs to the light quark partners of order  $v/f$ . Therefore the branching fraction for invisible Higgs decays is of order  $v^2/f^2$ .

### 1.3.3 The Left-Right Twin Higgs

We now turn to another possible way to implement the twin mechanism. In the original twin Higgs model, a  $Z_2$  symmetry related each particle in the SM to its corresponding particle in the “mirror” SM. The  $Z_2$  symmetry ensured that dangerous quadratically divergent terms possessed an accidental  $U(4)$  global symmetry and therefore did not contribute to the potential for the SM Higgs. In this model the mirror SM particles are singlets under the SM gauge groups and therefore are only visible as missing energy in collider experiments such as the LHC.

The left-right twin Higgs [14] is a more minimal construction that does not involve approximately doubling the field content of the SM. Instead of using mirror symmetry, the discrete symmetry necessary for the twin mechanism is identified with the parity symmetry associated with left-right symmetric extensions of the SM. In addition to reducing field content, left-right twin Higgs models have far more interesting experimental signatures because the  $Z_2$  exchange symmetry no longer relates SM particles to particles that are singlets under the SM gauge groups. All of the particles in this class of models have SM charge and therefore lead to



interesting collider signals.

The cancellation of quadratic divergences in this class of models is similar to that of the mirror model. Let us again investigate a linear realization of a broken  $U(4)$  symmetry. Consider a complex scalar field  $H$  which transforms as a fundamental under a global  $U(4)$  symmetry. The potential for this field is the same as in eq. (1.3),

$$V(H) = -m^2 H^\dagger H + \lambda (H^\dagger H)^2. \quad (1.30)$$

As before,  $H$  will develop a vev,  $\langle H \rangle = m/\sqrt{2\lambda}$ , that breaks  $U(4) \rightarrow U(3)$  and yields seven massless Nambu-Goldstone bosons.

We now gauge an  $SU(2)_L \times SU(2)_R$  subgroup of  $U(4)$ , which explicitly breaks the global  $U(4)$  symmetry. The  $SU(2)_L$  symmetry is that of the SM while  $SU(2)_R$  corresponds to the right-handed interactions of left-right extensions of the SM.  $H$  now transforms as

$$H = \begin{pmatrix} H_L \\ H_R \end{pmatrix}, \quad (1.31)$$

where  $H_L$  is a doublet of  $SU(2)_L$  and  $H_R$  is a doublet of  $SU(2)_R$ . As before, the Nambu-Goldstones pick up a mass proportional to the explicit breaking,

$$\delta V = \frac{c_1 g_L^2 \Lambda^2}{16\pi^2} H_L^\dagger H_L + \frac{c_1 g_R^2 \Lambda^2}{16\pi^2} H_R^\dagger H_R + \dots \quad (1.32)$$

We now impose a  $Z_2$  parity symmetry that exchanges the left-handed fields with the right-handed fields and vice versa. This symmetry forces the gauge couplings to be equal,  $g_L = g_R$ , and therefore

$$\delta V_{mass} = \frac{c_1 g^2 \Lambda^2}{16\pi^2} (H_L^\dagger H_L + H_R^\dagger H_R) = \frac{c_1 g^2 \Lambda^2}{16\pi^2} H^\dagger H. \quad (1.33)$$

The quadratically divergent terms in  $\delta V$  are now in a  $U(4)$  invariant form and therefore do not contribute to the potential for the Nambu-Goldstone bosons. However, the Nambu-Goldstone bosons do receive logarithmically divergent contributions to the potential that are not  $U(4)$  invariant, which are of the form

$$\delta V \sim \frac{cg^4}{16\pi^2}(|H_L|^4 + |H_R|^4) \log\left(\frac{\Lambda}{gf}\right). \quad (1.34)$$

In the limit of strong coupling,  $\Lambda \sim 4\pi f$ , these terms give a mass to the Nambu-Goldstones of order

$$m_h \sim \frac{g^2 f}{4\pi}, \quad (1.35)$$

which is approximately the weak scale for  $f$  about a TeV.

We now generalize the ideas above to include all interactions of the SM. By making the theory left-right symmetric, the quadratically divergent contributions to the potential for Nambu-Goldstone bosons, and hence the Higgs, have an accidental  $U(4)$  symmetric form. Therefore the Higgs receives at most logarithmically divergent contributions to its potential allowing natural electroweak symmetry breaking. The matter content of the theory is three generations of

$$\begin{aligned} Q_L &= (u_L \ d_L)^T = (\mathbf{2}, \mathbf{1}, \mathbf{1/3}) & L_L &= (\nu_L \ e_L)^T = (\mathbf{2}, \mathbf{1}, -\mathbf{1}) \\ Q_R &= (u_R \ d_R)^T = (\mathbf{1}, \mathbf{2}, \mathbf{1/3}) & L_R &= (\nu_R \ e_R)^T = (\mathbf{1}, \mathbf{2}, -\mathbf{1}) \\ H_L &= (\mathbf{2}, \mathbf{1}, \mathbf{1}) & H_R &= (\mathbf{1}, \mathbf{2}, \mathbf{1}) \end{aligned} \quad (1.36)$$

where the numbers in parentheses indicate the quantum numbers of the fields under  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ . When  $H_R$  acquires a vev,  $SU(2)_R \times U(1)_{B-L}$  is broken down to  $U(1)_Y$  of the SM.

The SM down-type Yukawa couplings arise from non-renormalizable operators of the form

$$\frac{\overline{Q}_R H_R H_L^\dagger Q_L}{\Lambda} + \frac{\overline{L}_R H_R H_L^\dagger L_L}{\Lambda} + h.c., \quad (1.37)$$

while the SM up-type Yukawa couplings arise from non-renormalizable operators of the form

$$\frac{\overline{Q}_R H_R^\dagger H_L Q_L}{\Lambda} + h.c. \quad (1.38)$$

When  $H_R$  acquires a vev, these terms reduce to the well-known Yukawa couplings of the SM. Unfortunately, this method of generating SM Yukawa couplings does not work well in the top sector since the top Yukawa coupling is order one. This problem is remedied by introducing the following vector-like quarks, which transform as

$$T_L = (\mathbf{1}, \mathbf{1}, 4/3) \quad T_R = (\mathbf{1}, \mathbf{1}, 4/3) \quad (1.39)$$

under  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ . We can then write the following left-right symmetric interactions

$$\left( y \overline{Q}_R H_R^\dagger T_L + y \overline{Q}_L H_L^\dagger T_R + M \overline{T}_L T_R \right) + h.c. \quad (1.40)$$

The SM right-handed top quark is then a linear combination of  $T_R$  and the third generation up-type quark in  $Q_R$ , while the left-handed top quark is linear combination of  $T_L$  and the third generation up-type quark in  $Q_L$ . The other two linear combinations are heavy exotic quarks. The parameter  $M$  determines the mixing between the SM left-handed top and  $T_L$  and is constrained by  $Z \rightarrow b\bar{b}$ .

Since the theory is parity symmetric, quadratically divergent contributions to the mass of  $H_L$  and  $H_R$  have the form  $\sim \Lambda^2(|H_L|^2 + |H_R|^2)$ , which has an accidental  $U(4)$  symmetry. Therefore quadratically divergent terms do not contribute to the potential for the Nambu-Goldstones and therefore the Higgs mass. On the other

hand, quantum contributions to the quartic will not be  $U(4)$  invariant and will therefore contribute to the mass of the Higgs. However, these contributions are at most logarithmically divergent and allow for natural electroweak symmetry breaking.

Let us now pause and show diagrammatically how the cancellation of quadratic divergences occurs in this model. In the non-linear parameterization above,  $H_L = h + \dots$  and  $H_R = (0, f - h^\dagger h/2f + \dots)^T$ , where  $h$  is the SM Higgs doublet. The relevant vertices that contribute to gauge loops arise from the following gauge interactions

$$\begin{aligned}
|D_\mu H_L|^2 + |D_\mu H_R|^2 &= g_L^2 H_L^\dagger (W_L^\dagger W_L) H_L + g_R^2 H_R^\dagger (W_R^\dagger W_R) H_R + \dots \\
&= \frac{g_L^2}{4} h^\dagger \sigma^a \sigma^b W_L^a W_L^b h + \frac{g_R^2}{4} (0 \quad f - \frac{h^\dagger h}{2f}) \sigma^a \sigma^b \begin{pmatrix} 0 \\ f - \frac{h^\dagger h}{2f} \end{pmatrix} W_R^a W_R^b + \dots \\
&= \frac{g_L^2}{4} h^\dagger h W_L^a W_L^a + \frac{g_R^2}{4} (f - \frac{h^\dagger h}{2f})^2 W_R^a W_R^a + \dots \\
&= \frac{g_L^2}{4} h^\dagger h W_L^a W_L^a - \frac{g_R^2}{4} h^\dagger h W_R^a W_R^a + \dots
\end{aligned} \tag{1.41}$$

where the  $\sigma^a$ 's are the Pauli matrices and  $W_{L,R} = W_{L,R}^a \frac{\sigma^a}{2}$ . These terms each generate quadratically divergent contributions to the the Higgs mass parameter and are shown in the first two diagrams of Fig. 1.1. However, left-right symmetry guarantees that  $g_L = g_R = g$  and the two diagrams cancel exactly due to the sign difference in eq. (1.41).

How does the cancellation of the top loop go through? The relevant vertices come from the Yukawa interactions

$$\begin{aligned}
&y_L \bar{Q}_L H_L^\dagger T_R + y_R \bar{Q}_R H_R^\dagger T_L + h.c. \\
&\sim y_L \bar{Q}_L h T_R - y_R \bar{Q}_R (f - \frac{h^\dagger h}{2f} + \dots) T_L + h.c. \\
&\sim y_L \bar{Q}_L h T_R - y_R f \bar{Q}_{RU} T_L + y_R \frac{h^\dagger h}{2f} \bar{Q}_{RU} T_L + h.c. + \dots,
\end{aligned} \tag{1.42}$$

where  $Q_{RU}$  is the upper component of  $Q_R$ . The quadratically divergent contributions to the Higgs mass parameter from these terms are shown in the last two diagrams

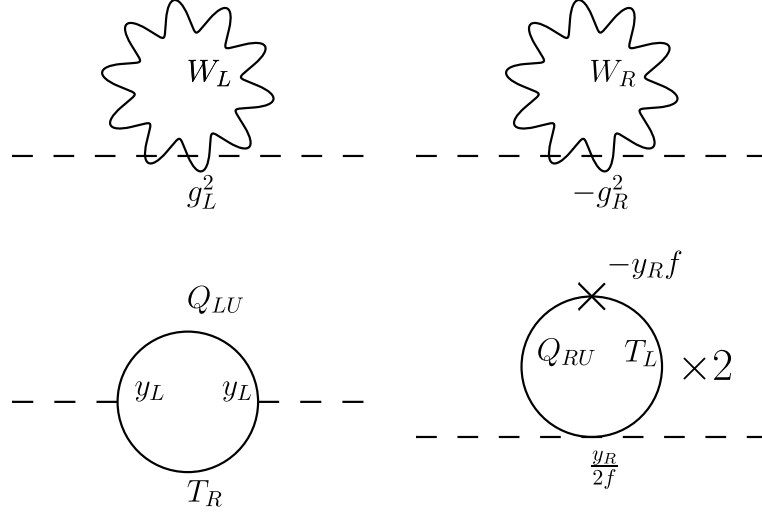


Figure 1.1: Cancellation of quadratic divergences in the left-right twin Higgs model. The top two diagrams are contributions from  $SU(2)_L$  and  $SU(2)_R$  gauge bosons, while the bottom two diagrams are contributions from the top and the heavy top partner.

of Fig. 1.1. The first term gives the usual contribution from the top loop in the SM and is proportional to  $y_L^2$ . The second and third term generate a contribution proportional to  $\frac{y_R}{2f} \times (-y_R f) \times 2 = -y_R^2$ , where the extra factor of two accounts for the fact that there are two such mass insertion diagrams. Imposing left-right symmetry sets  $y_L = y_R$  and these diagrams cancel exactly.

Even though the model described above eliminates the quadratically divergent contributions to Higgs mass, constraints on the parameter  $f$  make the theory somewhat unsatisfactory. Precision electroweak constraints on  $SU(2)_R$  gauge bosons put a lower bound on  $f$  of about 1.6 TeV [20, 21], which begins to reintroduce fine-tuning to the model. Although other solutions to this problem may exist, in the original left-right twin Higgs the problem is remedied in the following way. Note that if  $f$  lies at 2 TeV, the fine-tuning from the gauge sector would still be milder than that of the top sector with  $f \sim 500 - 800$  GeV. Therefore we can solve the problem by raising the effective symmetry breaking scale in the gauge sector

without affecting the top sector. This idea can indeed be accomplished by adding to the theory a new Higgs  $\hat{H}$  that transforms as a fundamental of  $U(4)$  which does not couple to fermions

$$\hat{H} = \begin{pmatrix} \hat{H}_L \\ \hat{H}_R \end{pmatrix}, \quad (1.43)$$

where  $\hat{H}_L$  and  $\hat{H}_R$  transform exactly as  $H_L$  and  $H_R$ . We assume that the potential for  $\hat{H}$  at the scale  $\Lambda$  has the  $U(4)$  invariant form

$$V(\hat{H}) = -\hat{m}^2 \hat{H}^\dagger \hat{H} + \hat{\lambda} (\hat{H}^\dagger \hat{H})^2 \quad (1.44)$$

and that there is no direct coupling between  $H$  and  $\hat{H}$  at this scale. There is now an approximate  $U(4) \times U(4)$  symmetry in the Higgs sector of the theory, with the  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  subgroup gauged. If  $\hat{H}_R$  acquires a vev  $\hat{f} > 2$  TeV breaking  $SU(2)_R \times U(1)_{B-L}$  to  $U(1)_Y$ , then the precision electroweak constraints on the  $SU(2)_R$  gauge bosons of this theory are satisfied. Moreover, the potential for  $H$  is still twin symmetric, allowing for natural electroweak symmetry breaking.

We are now in a position to compute the Higgs potential in more detail. We will do this using the following non-linear parameterizations for the pseudo-Nambu-Goldstones

$$\begin{aligned} H &= \exp\left(\frac{i}{f} h^a t^a\right) \begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix} = \exp\left[\frac{i}{f} \begin{pmatrix} 0 & 0 & 0 & h_1 \\ 0 & 0 & 0 & h_2 \\ 0 & 0 & 0 & h_3 \\ h_1^\dagger & h_2^\dagger & h_3^\dagger & h_0 \end{pmatrix}\right] \begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix} \\ \hat{H} &= \exp\left(\frac{i}{\hat{f}} \hat{h}^a t^a\right) \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hat{f} \end{pmatrix} = \exp\left[\frac{i}{\hat{f}} \begin{pmatrix} 0 & 0 & 0 & \hat{h}_1 \\ 0 & 0 & 0 & \hat{h}_2 \\ 0 & 0 & 0 & \hat{h}_3 \\ \hat{h}_1^\dagger & \hat{h}_2^\dagger & \hat{h}_3^\dagger & \hat{h}_0 \end{pmatrix}\right] \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hat{f} \end{pmatrix}, \end{aligned} \quad (1.45)$$

where  $h_{1\dots 3}$  and  $\hat{h}_{1\dots 3}$  are complex and  $h_0, \hat{h}_0$  are real. This parameterization of  $H$  describes the dynamics of the Nambu-Goldstone bosons up to the cutoff scale  $\Lambda$ . If we assume the theory is strongly coupled at the cutoff scale, we can estimate  $\Lambda \sim 4\pi f$ . However,  $\Lambda$  may be less than this value, for example if the  $U(4) \times U(4)$  symmetry is realized linearly in the UV. In this case,  $\Lambda$  is then the mass of the radial mode. In general the effective theory for the Nambu-Goldstones will contain all operators allowed by the  $U(4)$  symmetry suppressed by  $\Lambda$ . However, only  $U(4)$  breaking terms can contribute to the potential for the Nambu-Goldstones, namely the gauge and Yukawa interactions. Of these, the electroweak and top sectors contribute the most and therefore will be studied in detail.

When  $H_R$  and  $\hat{H}_R$  acquire the vevs  $f$  and  $\hat{f}$ , the gauged  $SU(2)_R \times U(1)_{B-L}$  is broken to  $U(1)_Y$  giving the right-handed gauge bosons masses of order  $gf\hat{f}$ . Of the fourteen Nambu-Goldstone degrees of freedom, three are eaten by the  $W_R$  and  $Z_R$  gauge bosons. As we will see below, the remaining degrees of freedom correspond to the SM Higgs doublet  $h = (h_1^+ \ h_2^0)^T$ , a second  $SU(2)_L$  doublet  $\hat{h} = (\hat{h}_1^+ \ \hat{h}_2^0)^T$ , a neutral Higgs  $\phi^0$  and a pair of charged Higgses  $\phi^\pm$ .

As in the mirror twin Higgs model, the effective potential for the SM Higgs is found by evaluating the CW potential eq. (1.19). It is found that the gauge contributions to the Higgs potential eq. (1.20) are given by

$$\begin{aligned}
m_h^2|_{gauge} &= \frac{3g^2 M_{W_R}^2}{32\pi^2} \left( \log \frac{\Lambda^2}{M_{W_R}^2} + 1 \right) \\
&\quad + \frac{3g^2}{64\pi^2} (2M_{Z_R}^2 - M_{W_R}^2) \left( \log \frac{\Lambda^2}{M_{Z_R}^2} + 1 \right) \\
\lambda_h|_{gauge} &= -\frac{m_h^2|_{gauge}}{3f^2},
\end{aligned} \tag{1.46}$$

where  $M_{W_R}^2 = g^2(f^2 + \hat{f}^2)/2$  and  $M_{Z_R}^2 = (g^2 + g'^2)(f^2 + \hat{f}^2)/2$ . The other gauge contributions to the SM Higgs quartic are small and therefore can be neglected. Let us now consider the top sector. To lowest order in  $|h|^2$ , the Higgs dependent masses

are

$$m_Q^2 = \frac{y^4 f^2}{M^2 + y^2 f^2} h^\dagger h \quad m_T^2 = M^2 + y^2 f^2. \quad (1.47)$$

This leads to a contribution to the Higgs potential from the top sector given by

$$\begin{aligned} m_h^2|_{top} &= -\frac{3}{8\pi^2} y_t^2 m_T^2 \left( \log \frac{\Lambda^2}{m_T^2} + 1 \right) \\ \lambda_h|_{top} &= -\frac{m_h^2|_{top}}{3f^2} + \frac{3}{16\pi^2} \left( y_t^4 \log \frac{m_T^2}{m_Q^2} + 2y^4 \log \frac{\Lambda^2}{m_t^2} \right) \\ &\quad - \frac{3}{32\pi^2} (y_t^4 - 4y^4). \end{aligned} \quad (1.48)$$

In contrast to the mirror twin Higgs, this theory has ‘uneaten’ pseudo-Nambu-Goldstone fields. To ensure that the symmetry breaking pattern above is not spoiled, these fields must all have positive mass squareds. To guarantee that the fields in  $\hat{H}_L$  have a positive mass squared, it is sufficient to add to the potential a term  $\hat{\mu}^2 \hat{H}_L^\dagger \hat{H}_L$ , where  $\mu$  is of order  $f$ . This term breaks left-right symmetry and the approximate  $U(4)$  symmetry for  $\hat{H}$ . Therefore the would-be Nambu-Goldstone bosons in  $\hat{H}_L$  will acquire masses of order  $\hat{\mu}$ . Moreover, since the breaking is soft, it is technically natural for  $\hat{\mu}$  to be smaller than  $\Lambda$ .

Let us now enumerate the Nambu-Goldstone boson degrees of freedom. Eight of the original fourteen degrees of freedom arise from the SM Higgs doublet  $h$  and the second  $SU(2)_L$  doublet  $\hat{h}$  contained in  $H_L$  and  $\hat{H}_L$ , respectively. The six remaining Nambu-Goldstone bosons arise from the fields  $H_R$  and  $\hat{H}_R$ . Of these, three of them are eaten by the right-handed gauge bosons. What about the remaining three? First recall that the six Nambu-Goldstone bosons in  $H_R$  and  $\hat{H}_R$  arise from the spontaneous breaking of the global symmetry  $U(2)_R \times U(2)_{\hat{R}} \rightarrow U(1)_R \times U(1)_{\hat{R}}$ . When  $SU(2)_R \times U(1)_{B-L}$  is gauged, we expect three Nambu-Goldstone bosons associated with the breaking of  $SU(2) \times U(1)_{B-L} \rightarrow U(1)_Y$  to be eaten, while the remaining three acquire mass via their gauge interactions. However, the gauge interactions



preserve a global  $U(1) \times U(1)$  symmetry, with the first  $U(1)$  acting on  $H$ , and the second  $U(1)$  acting on  $\hat{H}$ . Therefore the true symmetry breaking pattern of the theory is  $SU(2) \times U(1) \times U(1) \rightarrow U(1)$ , yielding four exact Nambu-Goldstone bosons. This implies that of the three remaining uneaten Nambu-Goldstone bosons, only two acquire mass via their gauge interactions and one remains massless. Since there is only one massless degree of freedom, this particle must be neutral. We denote this particle by  $\phi^0$ .

Since massless particles with SM strength gauge interactions have not been observed, the neutral Higgs  $\phi^0$  must be given a mass. We can give  $\phi^0$  a mass by adding to the Lagrangian a term  $\mu_R^2 H_R^\dagger \hat{H}_R$ , where  $\mu_R$  is of order 50 - 100 GeV. This term breaks the additional  $U(1)$  symmetry originally protecting  $\phi^0$ , thereby giving it a mass proportional to  $\mu_R$ . Note that this is the only term in the Lagrangian that violates the discrete symmetry  $\hat{H}_R \rightarrow -\hat{H}_R$ . Therefore it is technically natural for  $\mu_R$  to be small.

Now that the effective potential for the Higgs has been evaluated, we are in a position to compare the fine-tuning in the minimal left-right twin Higgs model to that of the SM with a cutoff of  $\Lambda = 5$  TeV. The procedure for estimating the fine-tuning is similar to the mirror twin Higgs case discussed above. The fine-tuning of left-right twin Higgs model for a few points in parameter space is shown in Table 1.2.

$\Lambda_{(\text{TeV})}$	$f_{(\text{GeV})}$	$\hat{f}_{(\text{TeV})}$	$M_{(\text{TeV})}$	$\mu_{R(\text{GeV})}$	$m_{h(\text{GeV})}$	Tuning
10	800	4.29	150	50	174	0.117
6	500	2.27	150	50	172	0.270
5	800	4.68	150	50	155	0.124

Table 1.2: A summary of the Higgs mass and fine-tuning,  $\partial \log M_Z^2 / \partial \log f^2$  at a few sample points in parameter space for the minimal left-right symmetric twin Higgs model. The most significant fine-tuning is occurs when varying the parameter  $f$ .

Recall that the SM with a cutoff of  $\Lambda = 5$  TeV is fine-tuned at the percent level. As in the mirror twin model, the absence of quadratic divergences contributing to the Higgs potential greatly reduces the fine-tuning required in the model, thereby achieving natural electroweak symmetry breaking and stabilizing the Higgs mass up

to 5 - 10 TeV.

## Phenomenology

Unlike the mirror twin Higgs, where the exotic particles only communicate with the SM through the Higgs boson, the left-right twin Higgs contains new particles that are coupled directly to the SM particles. These couplings lead to very rich signals at colliders such as the LHC.

The new particles in the model include the right-handed gauge bosons,  $W_R$  and  $Z_R$ , and a heavy partner of the top quark  $T_H$ , all of which are necessary for canceling the quadratic divergences of the Higgs mass. Three generations of right-handed neutrinos  $\nu_R$  are also present, as required by left-right symmetry. There are also additional Higgs fields arising from the extended Higgs sector.  $\hat{H}_L$  gives rise to a second Higgs doublet,  $\hat{h}^T = (\hat{h}^+, \hat{h}^0)$ , while  $\phi^\pm$  and  $\phi^0$  are the uneaten linear combinations contained in  $H_R$  and  $\hat{H}_R$ . The heavy top mass can be anywhere from 500 GeV to 1.5 TeV depending on the value of  $f$ . The masses of the right-handed gauge bosons depend on the value of  $\hat{f}$ , the larger vev, and are therefore heavier, ranging from about 1 TeV to 5 TeV. The mass of  $\phi^0$  is determined by the  $U(1)$  breaking parameter  $\mu_R$  and is given by  $m_{\phi^0}^2 = \mu_R^2 \hat{f} / f$ . For a reasonable choice of  $\mu_R = 50$  GeV,  $\phi^0$  acquires a mass of about 100 GeV. The mass of the charged Higgs  $\phi^\pm$  depends on  $\mu_R$  in a similar way, but also receives a significant contribution from the CW potential. For  $f = 800$  GeV the mass of the  $\phi^\pm$  is about 200 GeV and increases with  $f$ . The masses of  $\hat{h}_1^\pm$  and  $\hat{h}_2^0$  are determined by  $\mu_R$ ,  $\hat{\mu}$  and contributions from the CW potential. These masses are nearly degenerate and range from about 300 GeV to 1 TeV [22]. The particle spectrum is shown in Fig. 1.2.

Finally, neutrino masses are generated by the following operators: A Dirac mass term arises from an operator of the form

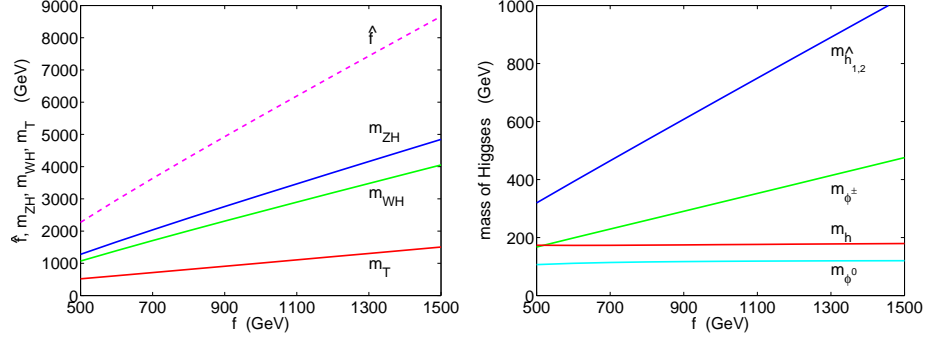


Figure 1.2: Mass spectrum for particles in the left-right twin Higgs model. The plot on the left shows the value of  $\hat{f}$  and the masses of  $Z_H$ ,  $W_H$  and the heavy top,  $T$ . The plot on the right shows the masses of  $\hat{h}_{1,2}$ ,  $\phi^\pm$ ,  $h$ , and  $\phi^0$ . The other parameters are chosen to be  $\Lambda = 4\pi f$ ,  $M = 150$  GeV,  $\mu_R = 50$  GeV and  $\hat{\mu} = f/2$  [22].

$$y_\nu \frac{\bar{L}_R H_R^\dagger H_L L_L}{\Lambda} + h.c. \rightarrow y_\nu \frac{f v}{\Lambda} \nu \nu_R + h.c. = m_D \nu \nu_R, \quad (1.49)$$

while the operator

$$\frac{y_1 (L_R \hat{H}_R \hat{H}_R L_R + L_L \hat{H}_L \hat{H}_L L_L)}{\Lambda} + h.c. \rightarrow y_1 \frac{\hat{f}^2}{\Lambda} \nu_R \nu_R + h.c. \quad (1.50)$$

generates Majorana masses of order  $\hat{f}^2/\Lambda$  and  $v^2/\Lambda$  for  $\nu_R$  and  $\nu_L$ , respectively. For  $y_1 \sim O(1)$ , we get a TeV scale seesaw for  $y_\nu$  of order the electron Yukawa coupling.

What is the discovery potential for the left-right twin Higgs at the LHC? In what follows we assume that  $m_{\nu_R} > m_{W_R}$ . Let us begin with the heavy right-handed gauge bosons,  $Z_R$  and  $W_R$ . The production cross sections for these particles at the LHC were calculated in [22] and are shown in Fig. 1.3. The branching ratios for the decay of right-handed gauge bosons into various final states were also calculated in [22] and are shown in Figure 1.4. The simplest way to discover the  $Z_R$  is through its decay into  $e^+e^-$  or  $\mu^+\mu^-$ . Even though the branching fractions to these leptons

are small, the invariant lepton mass distribution provides a very clean signal that is easily distinguished from the SM background, as shown in Fig. 1.5 [22]. The Drell-Yan production cross section for  $Z_R$  ranges from  $5 \times 10^3$  fb to 2 fb for masses between 1.3 TeV and 5 TeV. The LHC will probably be capable of observing neutral gauge bosons up to a mass of about 5 TeV [23].

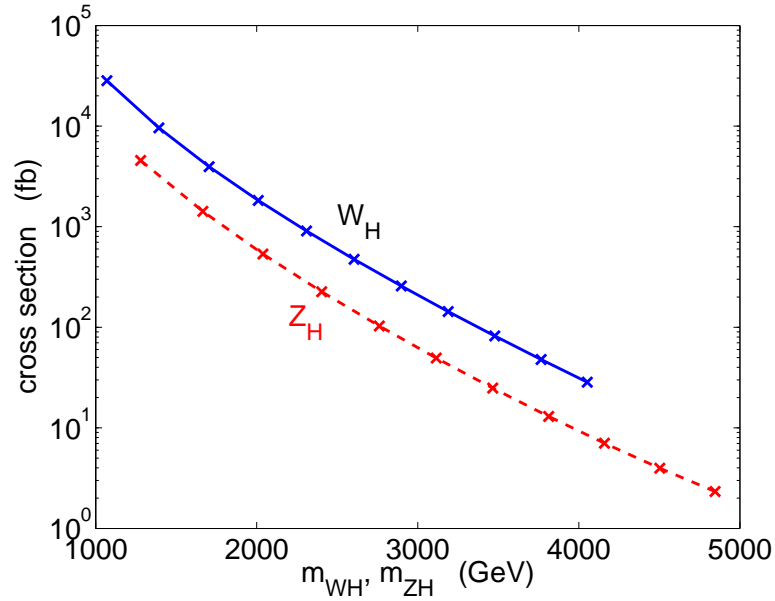


Figure 1.3: Cross section for  $W_H$  and  $Z_H$  Drell-Yan production at the LHC in the LRTH model as a function of gauge boson mass. The crosses correspond to  $f$  values of 500, 600, ..., 1500 GeV [22].

The dominant decay mode for  $W_R$  is into two jets (30 %) [22]. This process has a large QCD background making discovery via this channel very difficult. The  $W_H$  also decays a large fraction of the time (20% - 30%) to a heavy top and a  $b$ -jet [22], as shown in Fig. 1.4. Therefore discovery of the  $W_R$  critically depends on the decays of the heavy top. For a small but reasonable choice of  $M = 150$  GeV, the heavy top decays mostly (more than 70% of the time) [22] to  $\phi^+ b$ . See Fig. 1.7. For this value of  $M$ , the  $\phi^+$  then decays mostly to  $tb$ , as shown in Fig. 1.9. Therefore, the most promising signal is when the top decays leptonically, giving the following decay chain (see Fig. 1.8)

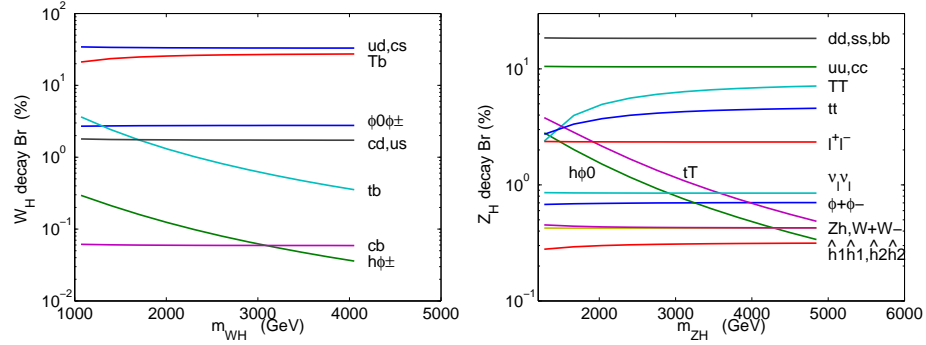


Figure 1.4: Decay branching fractions for the heavy right-handed gauge bosons  $W_H$  and  $Z_H$ . Here we have chosen  $M = 150$  GeV and  $m_{\nu_R} > m_{W_H}$ . Therefore the leptonic decays of  $W_H$  are absent [22].

$$T_H \rightarrow \phi^+ b \rightarrow tbb \rightarrow W^+bbb \rightarrow l^+ \nu bbb. \quad (1.51)$$

The signal is 3  $b$ -jets + charged lepton + missing  $E_T$  with an additional energetic jet, usually a  $b$ -jet. If we assume all of the missing energy came from the neutrino in the decay of a  $W$ , the invariant mass of the charged lepton and neutrino can be used to reconstruct the  $W$ . Once the  $W$  has been reconstructed, we require that the invariant mass of one  $b$ -jet plus the  $W$  gives the top mass. We can then reconstruct the  $\phi^+$  using  $tb$  and the heavy top  $T_H$  using  $\phi^+ b$ . Since the single heavy top production cross section is large, more than 10,000 of these events can be seen with a luminosity of  $10 \text{ fb}^{-1}$  for a heavy top mass around 600 GeV [22]. After reconstructing the heavy top, the invariant mass distribution of the heavy top and energetic  $b$ -jet should provide a signal of  $W_R$ . The SM backgrounds for this process are  $t\bar{t}$ ,  $W + 4$  jets and  $tbj$ , of which  $t\bar{t}$  is dominant [22]. Since one jet in single heavy top production is typically very energetic, a cut on the  $p_T$  of the most energetic jet may be an effective way to reduce the  $t\bar{t}$  background [22].

The dominant production mechanism for  $\phi^+$  is from heavy top decay with a cross section ranging from  $10 \text{ fb} - 6 \times 10^3 \text{ fb}$  [22]. Therefore the procedure described above is also the most effective way to observe the  $\phi^\pm$ . The neutral pseudo-scalar

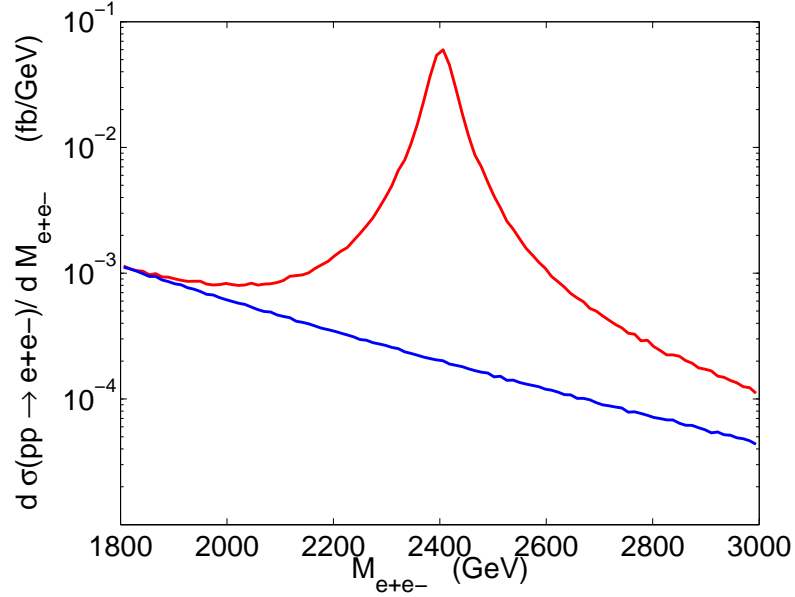


Figure 1.5: Invariant mass distribution of  $e^+e^-$  at the LHC clearly showing a bump at the  $Z_H$  mass. The blue (dark) line is that of the SM background, while the red (light) line is that of the LRTH where  $Z_H$  is produced through a Drell-Yan process. Other model parameters are chosen to be  $f = 800$  GeV,  $M = 150$  GeV, and  $\Lambda = 4\pi f$ . The mass of the  $Z_H$  corresponding to this parameter set is 2403 GeV, with a decay width of  $\Gamma_{Z_H} = 51$  GeV [22].

Higgs  $\phi^0$  is produced mainly through the decay  $W_R \rightarrow \phi^0 \phi^\pm$  with a cross section of 1 fb - 10<sup>3</sup> fb [22].  $\phi^0$  decays dominantly into  $b\bar{b}$ ,  $c\bar{c}$  and  $\tau^+\tau^-$ , which alone is difficult to observe due to the large QCD background. However, the  $\phi^+$  decays most often to  $tb$  and therefore we can trigger on the leptonic decay of the top. The decay chain is then

$$\phi^\pm \phi^0 \rightarrow t b \bar{b} \bar{b} \rightarrow l^\pm \nu b \bar{b} \bar{b} \bar{b}. \quad (1.52)$$

The signal is 4  $b$ -jets + 1 charged lepton + missing  $E_T$ . The  $W$  can be reconstructed if one assumes the charged lepton is the result of a  $W$  decay and that a neutrino is the only source of missing  $E_T$ . Once the  $W$  has been reconstructed, restrict the invariant mass of the  $W$  and two  $b$ -jets to be equal to the mass of the  $\phi^\pm$ ,

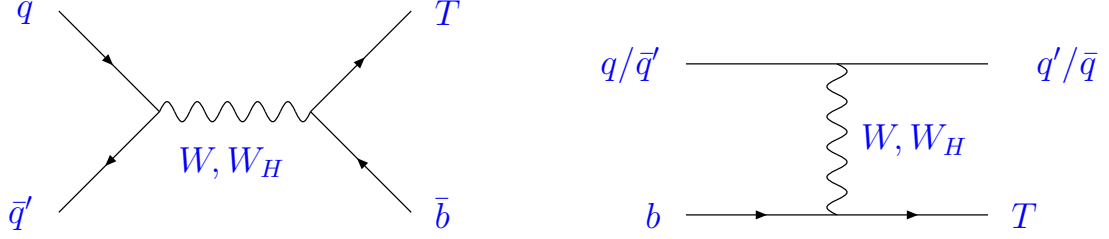


Figure 1.6: Feynman diagrams contributing to single heavy top production [22].

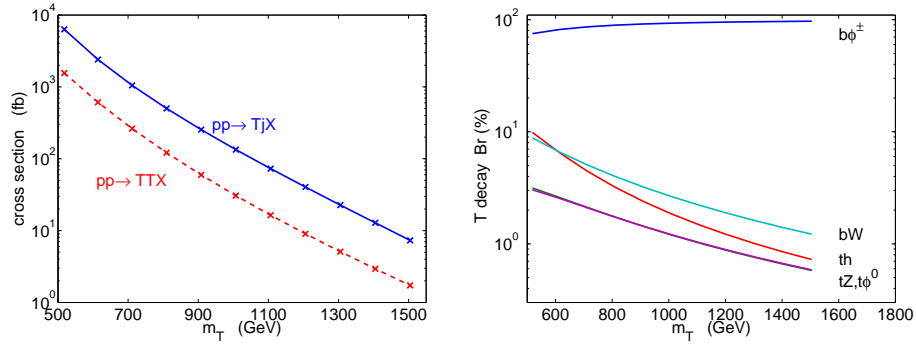


Figure 1.7: The plot on the left shows the single and pair production cross sections for the heavy top  $T_H$  at the LHC. The crosses correspond to  $f$  values of 500, 600, ..., 1500. The plot on the right shows the branching ratios of  $T_H$  decay into various final states [22].

which should be known from heavy top decay. Using the remaining  $b\bar{b}$  pair one can reconstruct the  $\phi^0$ .

What about the additional Higgses  $\hat{h}_1^\pm$  and  $\hat{h}_2^0$ ? These particles interact only with the gauge bosons and have very degenerate masses. A small mass splitting  $\delta m$  of about 100 - 700 MeV is the result of electromagnetic interactions, making the charged  $\hat{h}_1^\pm$  slightly heavier. The  $\hat{h}_2^0$  is stable and a natural dark matter candidate. In colliders the  $\hat{h}_1^\pm$  and  $\hat{h}_2^0$  can only be pair produced through gauge boson exchange. The cross section for this process is small, about 1 fb [22]. Once produced the  $\hat{h}_1^\pm$  can decay into the neutral  $\hat{h}_2^0$  and soft jets or leptons. If  $\delta m > m_\pi$ , the lifetime of  $\hat{h}_1^\pm$  is short and the decay occurs instantly inside the detector. There is no signal in this

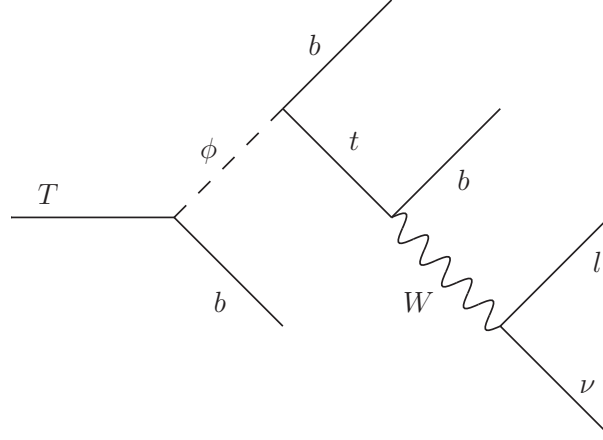


Figure 1.8: Decay chain of the heavy top  $T_H$  resulting in a final state lepton.

case because the soft jets and leptons will be missed and the  $\hat{h}_2^0$  escapes the detector. For  $\delta m \sim m_\pi$  the decay happens more slowly, but inside the detector. In this case, a disappearing track may be observable and provide a signal. For  $\delta m < m_\pi$  the decay occurs outside of the detector. In this case, two charged tracks are present which potentially can be distinguished from muon tracks by ionization rate and time of flight information [22].

The  $\hat{h}_2^0$  is completely stable due to a residual  $\hat{H}_L \rightarrow -\hat{H}_L$  symmetry, has a weak scale mass and only weak interactions. It is therefore a perfect candidate for WIMP dark matter. A study of the relic abundance of dark matter in the left-right twin Higgs model was conducted in [24] and we will now summarize their findings. First decompose the complex scalar  $\hat{h}_2^0$

$$\hat{h}_2^0 = \frac{\hat{S} + i\hat{A}}{\sqrt{2}} \quad (1.53)$$

where  $\hat{S}$  and  $\hat{A}$  are real. To evade direct detection constraints, it is necessary to introduce a small mass splitting between  $\hat{S}$  and  $\hat{A}$  of at least a few hundred MeV. This splitting makes inelastic scattering off of nuclei via a  $Z$  boson kinematically forbidden. Such a splitting can be generated by adding a term  $-\frac{\lambda_5}{2}(H_L^\dagger \hat{H}_L)^2$  to the Higgs potential. When  $\hat{H}_L$  gets a vev of  $\langle \hat{H}_L \rangle = (0 \ v/\sqrt{2})^T$ , this term produces a



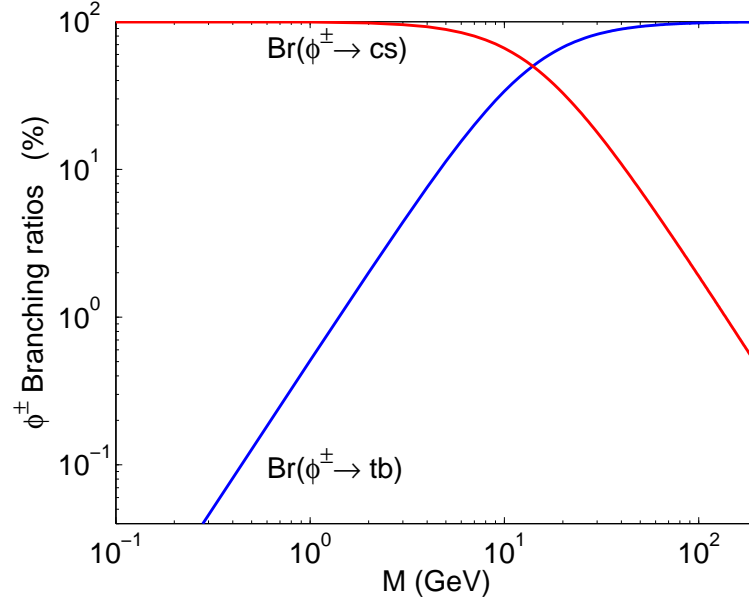


Figure 1.9: Decay branching fraction of  $\phi^\pm$  as a function of  $M$ . Other model parameters are chosen to be  $f = 800$  GeV,  $\mu_R = 50$  GeV and  $\Lambda = 4\pi f$  [22].

mass splitting of

$$m_{\hat{A}}^2 - m_{\hat{S}}^2 = \lambda_5 v^2. \quad (1.54)$$

This term violates left-right symmetry but does not reintroduce quadratic divergences to the Higgs potential at one loop. Note that this term lowers the mass of  $\hat{S}$  below  $\hat{A}$ , making the scalar  $\hat{S}$  the dark matter candidate. This choice is arbitrary and the results below hold for pseudo-scalar dark matter as well. Recall that the mass splitting between  $\hat{h}_1^\pm$  and  $\hat{h}_2^0$  caused by electromagnetic interactions is approximately a few hundred MeV. Defining  $\delta_1 = m_{\hat{h}_1^\pm} - m_{\hat{S}}$  and  $\delta_2 = m_{\hat{A}} - m_{\hat{S}}$ , we have the approximate relation  $\delta_1 \approx 2\delta_2$  for  $\delta_2 \gtrsim 1$  GeV.

Since  $\hat{\mu}$  is a free parameter, it can be used to vary the mass of  $m_{\hat{S}}$  arbitrarily. Therefore the relevant mass parameters in the Higgs sector for a relic abundance study are  $m_{\hat{S}}$ ,  $\delta_1$  and  $\delta_2$ . Although the analysis in [24] considered arbitrary values of  $\delta_1$  and  $\delta_2$ , we will limit our discussion to the case of left-right twin Higgs, where

$\delta_2 \approx 2\delta_1$ . An analysis of the relic density using the program MicrOMEGAs [25] revealed two mass regions of  $m_{\hat{S}}$  that can produce the observed dark matter relic density consistent with WMAP at the  $3\sigma$  level. The first is a low mass region where  $m_{\hat{S}} < 100$  GeV and the second a high mass region where  $400 \text{ GeV} < m_{\hat{S}} < \text{a few TeV}$ .

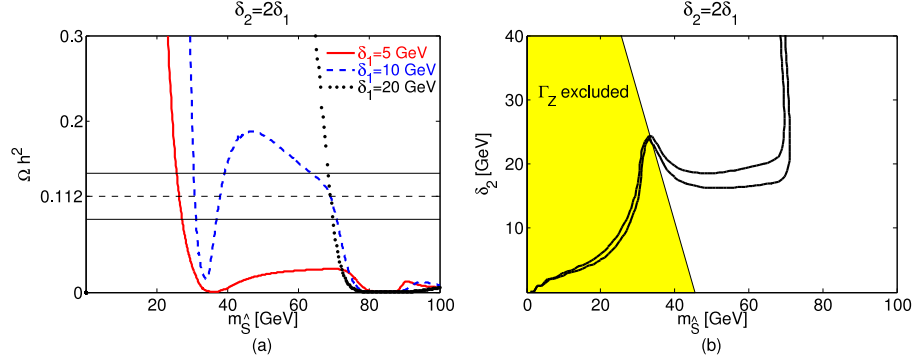


Figure 1.10:  $\Omega h^2$  vs.  $m_{\hat{S}}$  plot (a) and the corresponding relic density contour plot (b) for the LRTH where  $\delta_2 = 2\delta_1$ . The band in plot (a) and the region enclosed by the contours in plot (b) are the WMAP  $3\sigma$  regions [24].

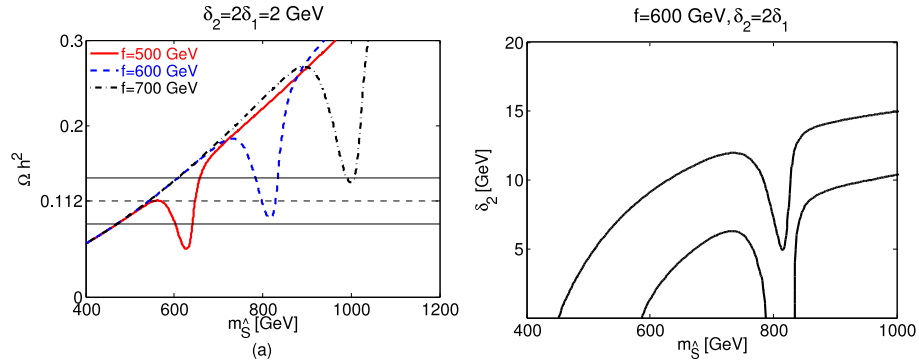


Figure 1.11:  $\Omega h^2$  vs.  $m_{\hat{S}}$  plot (a) for  $f = 500$  GeV (solid curve),  $600$  GeV and  $700$  GeV. The contour plot (b) shows the relic density in the  $m_{\hat{S}}$  vs.  $\delta_2$  plane for  $f = 600$  GeV. The enclosed region corresponds to the WMAP  $3\sigma$  region [24].

Let us first consider the low mass region shown in Fig. 1.10. When  $\delta_2$  is small, coannihilation between  $\hat{S}$  and  $\hat{A}$  needs to be taken into account. When  $m_{\hat{S}} \sim 40$  GeV, the process  $\hat{S}\hat{A} \rightarrow q\bar{q}/l\bar{l}$  via  $Z$  exchange is enhanced due to the  $Z$  pole.

The efficiency of this reaction is the reason for the large dip in the relic density at  $m_{\hat{S}} \sim 40$  GeV. As  $m_{\hat{S}}$  is increased beyond the  $Z$  pole region, the coannihilation cross section decreases, therefore increasing the relic abundance. However, when  $m_{\hat{S}} \sim M_W$  another annihilation channel opens up,  $\hat{S}\hat{S} \rightarrow WW$ , causing efficient annihilation and lowering the relic abundance. When  $\delta_2$  is large, coannihilations are irrelevant and  $Z$  pole region is no longer present. In this case, the only dip in the relic abundance occurs when  $m_{\hat{S}} \sim M_W$ . In the high mass region, shown in Fig. 1.11, the relic abundance begins to increase to an acceptable level allowed by the WMAP data, with the dominant processes still being  $\hat{S}\hat{S} \rightarrow WW/ZZ$ . Later, another dip appears due to the coannihilations between  $\hat{S}$  and  $\hat{A}$  at the  $Z_H$  pole, which occurs when  $m_{\hat{S}} \sim M_{Z_H}/2$ .

#### 1.4 Dissertation Format

The introduction of this work presented the basic theory and phenomenology of two “twin Higgs” models which stabilize the Higgs mass up to a cutoff  $\Lambda = 5 - 10$  TeV. This purpose of this dissertation is two-fold. First we discuss how to implement a tree level quartic without generating a corresponding mass term for the Higgs in the left-right twin Higgs model. As we will see, doing so decreases the fine-tuning of the model significantly. There exists an implementation of this idea in the mirror twin Higgs [13], but this approach does not generalize to the left-right twin Higgs model. Secondly we study the collider signatures of the left-right twin Higgs in the limit that  $m_{\nu_R} < m_{W_R}$ . Previous studies have assumed that  $m_{\nu_R} > m_{W_R}$ , in which case there is a large region of parameter space where it becomes difficult to detect the heavy top quark  $T_H$  and the right-handed gauge boson  $W_R$  due to an all jet final state. These particles are important to observe because they are responsible for the cancellation of quadratic divergences in the left-right twin Higgs. A TeV scale right-handed neutrino will open up exciting lepton number violating decay channels, allowing events containing the heavy top to be triggered on. The body of this thesis contains two appendices consisting of my published work and work submitted for

publication regarding these two topics. The research was conducted by myself and Dr. Hock-Seng Goh, who is a co-author on both works, and under the supervision of Professor Zackaria Chacko. The first appendix is an article demonstrating how to implement the desired type of tree level quartic into the left-right twin Higgs model. The second is a study of the collider signatures of a TeV scale right-handed neutrino in the left-right twin Higgs model. We now briefly summarize these two appendices.

## CHAPTER 2

### THE PRESENT STUDY

#### 2.1 The Little Twin Higgs

Before we begin a discussion of the Little Twin Higgs, let us briefly review how we arrived at this point. The twin mechanism was born with the observation that a discrete twin symmetry in conjunction with a spontaneously broken global symmetry can restrict the form of quadratically divergent terms in such a way that they do not contribute to the Higgs mass. The simplest realistic example of this scenario is the embedding of two  $SU(2) \times U(1)$  gauge symmetries into a global  $U(4)$ , which is spontaneously broken to  $U(3)$ . With this basic setup, there are two directions that have been studied. The twin symmetry can be identified as an exchange symmetry between the SM and a “mirror SM,” or as an exchange symmetry between left-handed SM fields and right-handed SM fields.

In mirror twin Higgs models, an entire copy of the SM is introduced along with a symmetry that exchanges the SM fields with those of the mirror SM. In addition, the top sector may be extended so that its contribution to the Higgs potential is finite at one loop. The fine-tuning in each of these cases is about 10% for a cutoff scale of 10 TeV. Although it was not discussed in the introduction, it is possible to decrease the fine-tuning of these models even further by introducing an additional Higgs field  $\hat{H}$  that acquires a vev which is not aligned with vev of  $H^1$ . In this case, the fine-tuning becomes approximately 30% for a cutoff of 10 TeV.

Left-right twin Higgs models require the introduction of right-handed gauge bosons and right-handed neutrinos. An additional Higgs field must also be introduced to evade constraints on  $SU(2)_R$  gauge bosons. It has been shown that the fine-tuning in the minimal left-right twin Higgs is approximately 10% for a cutoff

---

<sup>1</sup>This possibility will be discussed below.

scale of 10 TeV. We summarize the evolution and fine tuning of twin Higgs models in Fig. 2.1.

Recall that it is possible achieve a fine-tuning in the mirror twin Higgs model of approximately 30% for a 10 TeV cutoff. Is it possible to do as well in the left-right twin Higgs model? To answer this question, we must first understand the origin of the 10% fine-tuning in the left-right twin Higgs.

Consider a simple potential for a complex scalar field  $h$

$$V(h) = m_h^2 h^\dagger h + \lambda_h (h^\dagger h)^2. \quad (2.1)$$

For  $m_h^2 < 0$ ,  $h$  will acquire a vev  $v = \sqrt{|m_h^2|/2\lambda_h}$  and the physical mass  $m_{phys}^2 = 2|m_h^2|$ . Recall that there are many contributions to  $m_h^2$ ,

$$m_h^2 = m^2|_{top} + m^2|_{gauge} + m^2|_{tree} + m^2|_{\lambda}. \quad (2.2)$$

If  $m_h^2$  is negative, the vev squared is then

$$v^2 = \frac{m^2|_{top} + m^2|_{gauge} + m^2|_{tree} + m^2|_{\lambda}}{2\lambda_h}. \quad (2.3)$$

Significant fine tuning results when the left hand side of eq. (2.3) is much smaller than the individual contributions to  $m_h^2$  on the right hand side. However, note that less cancellation is required between the individual contributions of  $m_h^2$  if  $\lambda_h$  can be made large without significantly affecting the  $m_i^2$ 's. In other words, one should try to make the quartic  $\lambda_h$  large without introducing more fine tuning into the numerator of eq. (2.3). In the twin Higgs models described above, both the quartic and mass parameters are determined by the CW potential and are therefore not independent. A simple way to increase naturalness is to introduce a tree level operator that generates a quartic for the Higgs, but does not generate a corresponding mass term.

In this scenario the numerator of eq. (2.3) is not significantly affected when the denominator is made large.

How can one construct such a object? Let us first consider the  $U(4)$  fundamental  $H$  in the mirror twin Higgs.  $|H|^4$  is a  $U(4)$  invariant, so it cannot generate any potential at all for the SM Higgs. What about the mirror symmetric operator  $\lambda(|H_A|^4 + |H_B|^4)$ ? Recall that after removing the eaten fields in  $H$  we have

$$H = f \begin{pmatrix} i \frac{h_1}{|h|} \sin \frac{|h|}{f} \\ i \frac{h_2}{|h|} \sin \frac{|h|}{f} \\ 0 \\ \cos \frac{|h|}{f} \end{pmatrix} \quad (2.4)$$

and

$$\begin{aligned} H_A^\dagger H_A &= f^2 \sin^2 \frac{|h|}{f} = h^\dagger h - \frac{(h^\dagger h)^2}{3f^2} + \dots \\ H_B^\dagger H_B &= f^2 \cos^2 \frac{|h|}{f} = f^2 - h^\dagger h + \frac{(h^\dagger h)^2}{3f^2} + \dots \end{aligned} \quad (2.5)$$

$|H_A|^4$  will not generate a mass for the Higgs, but  $|H_B|^4$  will since it is proportional to  $f^4(1 - h^\dagger h/f^2 + \dots)^2$ , which contains a mass term for the  $h$ . Therefore, to generate the required quartic with a single  $H$  does not seem possible.

One possibility is to misalign the vevs of two different Higgs fields  $H$  and  $\hat{H}$ . Consider a new field  $\hat{H}$  that transforms as a fundamental of  $U(4)$  and acquires the vev  $\langle \hat{H} \rangle = (0 \ 0 \ \hat{f} \ 0)^T$ . Putting the eaten fields back in for completeness we have,

$$H = \begin{pmatrix} ih_1 \\ ih_2 \\ C \\ f + i\phi + \frac{h^\dagger h}{f} \end{pmatrix} + \dots \quad \hat{H} = \begin{pmatrix} i\hat{h}_1 \\ i\hat{h}_2 \\ \hat{f} + i\hat{\phi} + \frac{\hat{h}^\dagger \hat{h}}{\hat{f}} \\ \hat{C} \end{pmatrix} + \dots \quad (2.6)$$

Now consider the mirror symmetric operator  $(|H_A^\dagger \hat{H}_A|^2 + |H_B^\dagger \hat{H}_B|^2)$ . This operator clearly only gives a mass to  $C$  and  $\hat{C}$ , while also generating a quartic term for  $h$  and  $\hat{h}$ . An operator of this form was studied in the case of the mirror twin Higgs [13] and resulted in a significant decrease in the amount of fine tuning required in the model.

Can this technique be incorporated into the left-right twin Higgs? Recall that the symmetry breaking pattern in the left-right twin Higgs is  $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow SU(2)_L \times U(1)_Y$ . With aligned vevs,  $SU(2)_R \times U(1)_{B-L}$  breaks to  $U(1)_Y$ . However, with misaligned vevs, the  $SU(2)_R \times U(1)_Y$  symmetry is broken completely. Without an unbroken  $U(1)$  remaining,  $SU(2)_L$  will also be broken completely, giving the photon a mass. Since a massive photon is unacceptable, this simple operator will not suffice.

What are the properties of the operator we seek? As above, it must generate a quartic for the Higgs but not the corresponding mass term. At the same time, this operator should preserve the same discrete symmetry that protects the Higgs mass from receiving quadratically divergent contributions and does not break  $U(1)_{EM}$ .

We showed in [15] that construction of such an operator is indeed possible. The crucial observation is that there is more than one discrete symmetry which can be identified as twin parity. Consider the operator

$$\lambda \left( |H_R^T \tau_2 \hat{H}_R|^2 + |H_L^\dagger \hat{H}_L|^2 \right). \quad (2.7)$$

This operator is not invariant under the twin parity originally defined in the left-right twin Higgs model. However, it is invariant under an alternative twin parity



$$\begin{aligned}
H_L &\leftrightarrow H_R \\
\hat{H}_L &\leftrightarrow \tau_2 \hat{H}_R^* \\
A_{L\mu}^a T_L^a &\rightarrow A_{R\mu}^a T_R^a \\
Q_L &\rightarrow Q_R^c.
\end{aligned} \tag{2.8}$$

When  $H$  and  $\hat{H}$  acquire the vevs  $\langle H \rangle = (0 \ 0 \ 0 \ f)^T$  and  $\langle \hat{H} \rangle = (0 \ 0 \ 0 \ \hat{f})^T$ , this operator will generate a quartic for the Higgs, but not the corresponding mass term. Roughly speaking, the effect of misaligning the vevs is mimicked by  $\tau_2$ . Moreover, since the vevs are aligned,  $U(1)_{EM}$  is not broken. Most importantly, terms such as  $(|H_L|^2 + |H_R|^2)$  and  $(|\hat{H}_L|^2 + |\hat{H}_R|^2)$  are invariant under the alternative twin parity. Therefore the alternative parity also ensures that quadratically divergent terms have an accidental  $U(4)$  invariant form, thereby protecting the Higgs mass.

The theory now contains two discrete symmetries, with each term in the Lagrangian breaking at most one of them. The quartic above breaks the original twin parity, while the  $U(1)_{B-L}$  gauge interactions break the alternative twin parity. Individually, each of these discrete symmetries is sufficient to ensure that the quadratically divergent contributions to the potential have a  $U(4)$  invariant form, and therefore do not contribute to the potential for the Nambu-Goldstone bosons. However, certain operators not invariant under either discrete symmetry may be generated radiatively. These operators arise from graphs containing at least two vertices, with each vertex breaking one of the discrete symmetries of the theory, so that both symmetries are broken. Any operator that results from such a graph will not be invariant under either discrete symmetry. In this case, quadratic divergences will reappear because there is no longer any symmetry forbidding them. However, this effect is typically postponed to two loops and does not pose a problem if one is only concerned with addressing the little hierarchy problem. This phenomenon is known as collective symmetry breaking and is used extensively in little Higgs theories. Interestingly, we found that this effect is actually postponed to three loops in

the little twin Higgs model.

In the publication that appears in Appendix A, we discuss these and other topics in more detail. We constructed a twin Higgs model based on left-right symmetry with an order one tree level quartic for the SM Higgs. Our analysis showed that electroweak symmetry breaking can happen naturally. For  $\hat{f} = 1.6$  TeV, which is the lower bound from the direct searches on heavy gauge bosons, the fine tuning is found to be about 30% for  $\Lambda = 10$  TeV. We also applied this mechanism to the mirror twin Higgs model and found the fine tuning is about 20% for a 10 TeV cutoff scale. In summary, we showed how to incorporate a tree level quartic into the left-right twin Higgs model, leading to a substantial reduction in the amount of fine-tuning required in this class of theories.

## 2.2 Lepton Number Violating Signals of the Top Partners in the LRTH

To identify the twin mechanism, it is crucial to observe the heavy top partner  $T_H$  and the right-handed gauge boson  $W_R$ . It was shown in the introduction that the most straightforward way to detect both of these particles is through the leptonic decay of the heavy top. However, that discussion was for the case of a small but reasonable value of  $M = 150$  GeV. In the limit that  $M \rightarrow 0$ , this approach fails because the charged Higgses  $\phi^\pm$  decay purely to charm and strange quarks, leading to an all jet final state for heavy top decay. Since there is no lepton in the final state, it is difficult to observe this decay at the LHC. In this scenario, the true mechanism of electroweak symmetry breaking may be beyond the reach of the LHC.

However, in previous studies it was assumed that the right-handed neutrino is heavier than the right-handed gauge boson,  $W_R$ . If the right-handed neutrino is lighter than  $W_R$ , new leptonic decay channels open allowing the discovery of both  $W_R$  and  $T_H$ . In this case, it may be possible to identify the heavy top in a way that is independent of the parameter  $M$ . Moreover, since the right-handed neutrino is Majorana, half of these events are lepton number violating. This fact leads to same-sign dilepton events without missing energy at colliders such as the LHC, which

have no genuine SM background.

In the article submitted for publication in Appendix B, we consider a TeV scale right-handed neutrino in the left-right twin Higgs model such that  $m_{\nu_R} < m_{W_R}$ , and study its collider signatures. We show that this scenario leads to interesting lepton number violating signatures at the LHC. Lepton number violating decays of  $W_R$  should be observable provided that  $W_R$  and  $\nu_R$  are not nearly degenerate. Detection of the heavy top is also possible if  $m_{\nu_R} > m_{T_H}$ . These signals may be used to complement other collider searches, and in the limit that  $M \rightarrow 0$ , may be the only way to observe the particles necessary for the cancellation of quadratic divergences in the left-right twin Higgs.

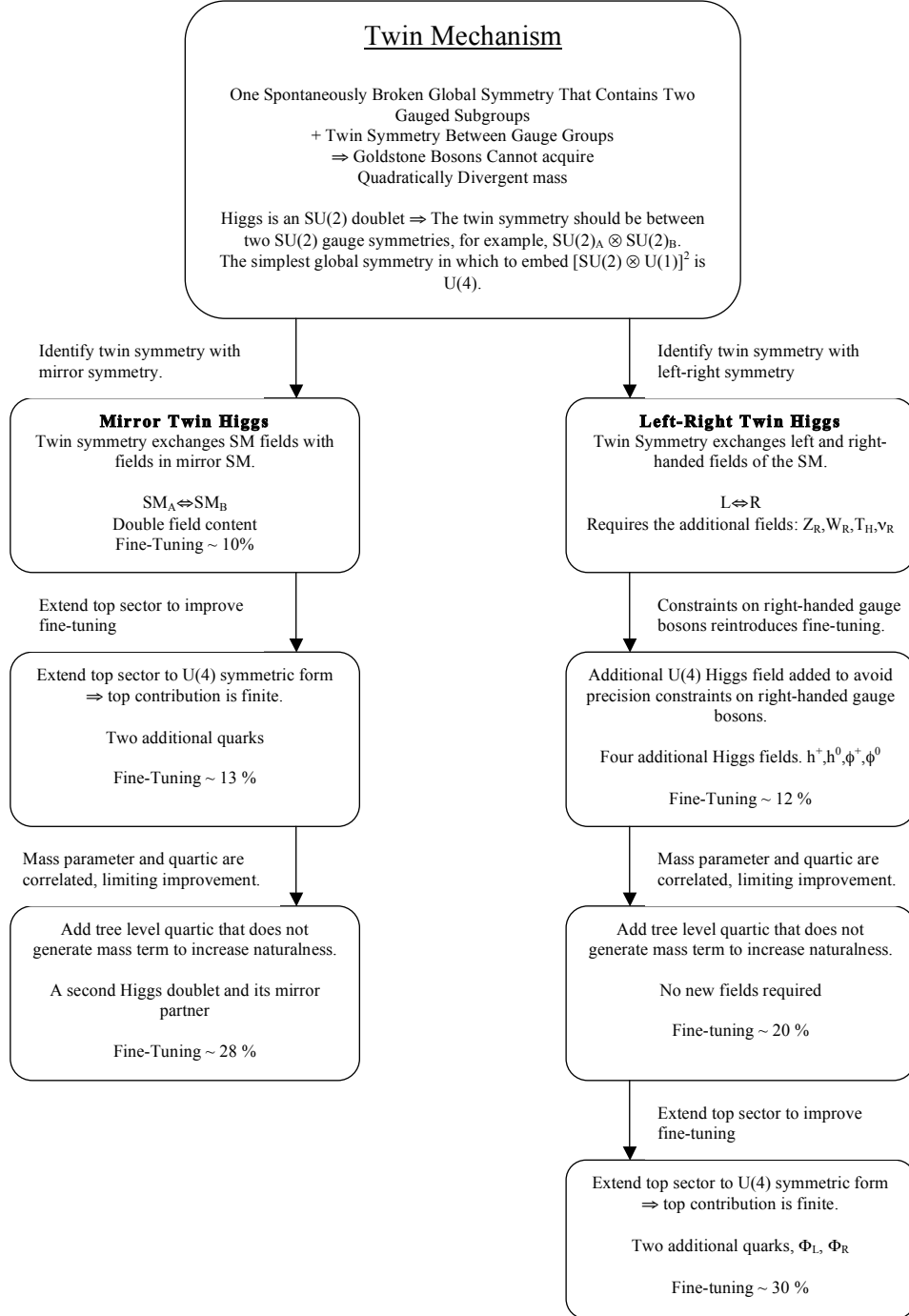


Figure 2.1: Evolution of twin Higgs models. The fine-tuning quoted above is given for a cutoff of 10 TeV.

## REFERENCES

- [1] Howard Georgi and A. Pais. Calculability and Naturalness in Gauge Theories. *Phys. Rev.*, D10:539, 1974.
- [2] Howard Georgi and A. Pais. Vacuum Symmetry and the PseudoGoldstone Phenomenon. *Phys. Rev.*, D12:508, 1975.
- [3] David B. Kaplan and Howard Georgi.  $SU(2) \times U(1)$  Breaking by Vacuum Misalignment. *Phys. Lett.*, B136:183, 1984.
- [4] David B. Kaplan, Howard Georgi, and Savas Dimopoulos. Composite Higgs Scalars. *Phys. Lett.*, B136:187, 1984.
- [5] Howard Georgi and David B. Kaplan. Composite Higgs and Custodial  $SU(2)$ . *Phys. Lett.*, B145:216, 1984.
- [6] Nima Arkani-Hamed, Andrew G. Cohen, and Howard Georgi. Electroweak symmetry breaking from dimensional deconstruction. *Phys. Lett.*, B513:232–240, 2001.
- [7] N. Arkani-Hamed et al. The Minimal Moose for a Little Higgs. *JHEP*, 08:021, 2002.
- [8] N. Arkani-Hamed, A. G. Cohen, E. Katz, and A. E. Nelson. The littlest Higgs. *JHEP*, 07:034, 2002.
- [9] Thomas Gregoire and Jay G. Wacker. Mooses, Topology and Higgs. *JHEP*, 08:019, 2002.
- [10] Ian Low, Witold Skiba, and David Tucker-Smith. Little Higgses from an anti-symmetric condensate. *Phys. Rev.*, D66:072001, 2002.

- [11] David E. Kaplan and Martin Schmaltz. The little Higgs from a simple group. *JHEP*, 10:039, 2003.
- [12] Z. Chacko, Hock-Seng Goh, and Roni Harnik. The twin Higgs: Natural electroweak breaking from mirror symmetry. *Phys. Rev. Lett.*, 96:231802, 2006.
- [13] Z. Chacko, Yasunori Nomura, Michele Papucci, and Gilad Perez. Natural little hierarchy from a partially Goldstone twin Higgs. *JHEP*, 01:126, 2006.
- [14] Z. Chacko, Hock-Seng Goh, and Roni Harnik. A twin Higgs model from left-right symmetry. *JHEP*, 01:108, 2006.
- [15] Hock-Seng Goh and Christopher A. Krenke. A Little Twin Higgs Model. *Phys. Rev.*, D76:115018, 2007.
- [16] Sidney R. Coleman and Erick J. Weinberg. Radiative Corrections as the Origin of Spontaneous Symmetry Breaking. *Phys. Rev.*, D7:1888–1910, 1973.
- [17] Bob Holdom. Searching for epsilon Charges and a New U(1). *Phys. Lett.*, B178:65, 1986.
- [18] Robert Foot, H. Lew, and R. R. Volkas. A Model with fundamental improper space-time symmetries. *Phys. Lett.*, B272:67–70, 1991.
- [19] Robert Foot, H. Lew, and R. R. Volkas. Possible consequences of parity conservation. *Mod. Phys. Lett.*, A7:2567–2574, 1992.
- [20] King-man Cheung. Constraints on electron quark contact interactions and implications to models of leptoquarks and extra  $Z$  bosons. *Phys. Lett.*, B517:167–176, 2001.
- [21] Thomas Appelquist, Bogdan A. Dobrescu, and Adam R. Hopper. Nonexotic neutral gauge bosons. *Phys. Rev.*, D68:035012, 2003.
- [22] Hock-Seng Goh and Shufang Su. Phenomenology of The Left-Right Twin Higgs Model. *Phys. Rev.*, D75:075010, 2007.

- [23] Michael Dittmar, Anne-Sylvie Nicollerat, and Abdelhak Djouadi.  $Z'$  studies at the LHC: An update. *Phys. Lett.*, B583:111–120, 2004.
- [24] Ethan M. Dolle and Shufang Su. Dark Matter in the Left Right Twin Higgs Model. *Phys. Rev.*, D77:075013, 2008.
- [25] G. Belanger, F. Boudjema, A. Pukhov, and A. Semenov. micrOMEGAs2.0: A program to calculate the relic density of dark matter in a generic model. *Comput. Phys. Commun.*, 176:367–382, 2007.

## APPENDIX A

## A LITTLE TWIN HIGGS MODEL

PHYSICAL REVIEW D **76**, 115018 (2007)**Little twin Higgs model**

Hock-Seng Goh and Christopher A. Krenke

*Department of Physics, University of Arizona, Tucson, Arizona 85721, USA*

(Received 6 August 2007; published 27 December 2007)

We present a twin Higgs model based on left-right symmetry with a tree level quartic. This is made possible by extending the symmetry of the model to include two  $Z_2$  parities, each of which is sufficient to protect the Higgs from getting a quadratically divergent mass squared. Although both parities are broken explicitly, the symmetries that protect the Higgs from getting a quadratically divergent mass are broken only collectively. The quadratic divergences of the Higgs mass are thus still protected at one loop. We find that the fine-tuning in this model is reduced substantially compared to the original left-right twin Higgs model. This mechanism can also be applied to the mirror twin Higgs model to get a significant reduction of the fine-tuning, while keeping the mirror photon massless.

DOI: [10.1103/PhysRevD.76.115018](https://doi.org/10.1103/PhysRevD.76.115018)

PACS numbers: 12.60.Cn, 14.80.Cp

**I. INTRODUCTION**

The standard model (SM) is so far the most successful theory that describes physics at energies below the TeV scale. Its predictions are consistent with all precision electroweak (EW) measurements. However, the model is unsatisfactory since the Higgs field, which plays a crucial role in electroweak symmetry breaking, receives quadratically divergent radiative corrections to its mass and thus destabilizes the electroweak scale. Hence, it is unnatural to treat the SM as an effective theory with a cutoff scale much higher than a TeV. On the other hand, the cutoffs of non-renormalizable operators that contribute to precision electroweak measurements are required by experiment to be greater than 5–10 TeV. Such a high cutoff tends to destabilize the electroweak scale and leads to a fine-tuning of a few percent. This problem is known as the little hierarchy problem or the LEP paradox [1].

The idea that the Higgs is a pseudo-Nambu-Goldstone boson (PNGB) corresponding to a spontaneously broken global symmetry was proposed in Refs. [2,3]. Since the mass of a PNGB tends to be lighter than the UV scale, this idea explains why the Higgs is light. However, using this idea to solve the little hierarchy problem is not quite straightforward. A PNGB Higgs by itself is not sufficient since the global symmetry is, by definition, not exact and the couplings that break the global symmetry will still generate a quadratically divergent mass to the Higgs. Thus, the situation is no better than that in the standard model and more structure is needed. The extra structure required to achieve naturalness is the main challenge for model building. One successful mechanism along this line is known as the little Higgs [4,5]. In this class of models, the Higgs mass is protected by two separate global symmetries and every term in the Lagrangian breaks at most one of them. In order to break both global symmetries, radiative corrections to the mass have to involve at least two such terms and thus, quadratic divergences are postponed to two loops. This little Higgs mechanism is also known as collective symmetry breaking. To achieve a

certain level of naturalness, a special operator is also introduced to provide a tree level quartic without generating a tree level mass to the Higgs.

Another mechanism that has been shown to solve the little hierarchy problem is the twin Higgs [6–9] (see also [10–12]). The twin Higgs mechanism is quite different from that of the little Higgs. In twin Higgs models, the Higgs mass is protected by a discrete  $Z_2$ , or twin, symmetry instead of multiple global symmetries. The exact twin symmetry guarantees that all gauge invariant dimensionful terms have, up to all orders in perturbation theory, a form which is invariant under a global  $SU(4)$  symmetry. The mass of the PNGB Higgs is then protected from receiving quadratically divergent contributions. It was shown that this mechanism alleviates the little hierarchy problem to about the 10% level for the cutoff scale  $\Lambda = 10$  TeV without introducing a tree level quartic.

In this class of models where the quadratic divergences are naturally suppressed, one would expect less fine-tuning if the quartic coupling of Higgs  $\lambda$  is large. In the original twin Higgs models, both the squared mass and the quartic for the SM Higgs come from the one-loop Coleman-Weinberg (CW) potential [13]. The quartic coupling is thus not a free parameter and loop suppressed. In order to improve the naturalness, one should try to find a tree level operator that will give the PNGB a quartic coupling without giving it a tree level mass term. In order to not upset the cancellation of radiative corrections, the tree level operator one introduces must preserve the twin parity. To summarize, in order to improve the fine-tuning, the following criteria must be satisfied.

- (i) A tree level operator that generates a quartic for the SM Higgs, but not a mass.
- (ii) This operator must preserve the discrete symmetry that protects the Higgs mass.
- (iii) Reduce as much as possible the mass squared that arises at loop level. For example, reduce top contribution by making the top Yukawa interaction  $SU(4)$  invariant.



# A LITTLE TWIN HIGGS MODEL

Hock-Seng Goh and Christopher A. Krenke

Published in Physical Review D

Copyright 2007 by The American Physical Society

## ABSTRACT

We present a twin Higgs model based on left-right symmetry with a tree level quartic. This is made possible by extending the symmetry of the model to include two  $Z_2$  parities, each of which is sufficient to protect the Higgs from getting a quadratically divergent mass squared. Although both parities are broken explicitly, the symmetries that protect the Higgs from getting a quadratically divergent mass are broken only collectively. The quadratic divergences of the Higgs mass are thus still protected at one loop. We find that the fine tuning in this model is reduced substantially compared to the original left-right twin Higgs model. This mechanism can also be applied to the mirror twin Higgs model to get a significant reduction of the fine tuning, while keeping the mirror photon massless.

## A.1 Introduction

The standard model (SM) is so far the most successful theory that describes physics at energies below the TeV scale. Its predictions are consistent with all precision electroweak measurements. However, the model is unsatisfactory since the Higgs field, which plays a crucial role in electroweak symmetry breaking, receives quadratically divergent radiative corrections to its mass and thus destabilizes the electroweak scale. Hence, it is unnatural to treat the SM as an effective theory with a cutoff scale much higher than a TeV. On the other hand, the cutoffs of nonrenormalizable operators that contribute to precision electroweak measurements are required by experiment to be greater than 5-10 TeV. Such a high cutoff tends to destabilize the

electroweak scale and leads to a fine tuning of a few %. This problem is known as the little hierarchy problem or the LEP paradox [1].

The idea that the Higgs is a pseudo-Nambu-Goldstone boson (PNGB) corresponding to a spontaneously broken global symmetry was proposed in refs. [5, 6]. Since the mass of a PNGB tends to be lighter than the UV scale, this idea explains why the Higgs is light. However, using this idea to solve the little hierarchy problem is not quite straightforward. A PNGB Higgs by itself is not sufficient since the global symmetry is, by definition, not exact and the couplings that break the global symmetry will still generate a quadratically divergent mass to the Higgs. Thus, the situation is no better than that in the standard model and more structure is needed. The extra structure required to achieve naturalness is the main challenge for model building. One successful mechanism along this line is known as the little Higgs [2, 3]. In this class of models, the Higgs mass is protected by two separate global symmetries and every term in the Lagrangian breaks at most one of them. In order to break both global symmetries, radiative corrections to the mass have to involve at least two such terms and thus, quadratic divergences are postponed to two loops. This little Higgs mechanism is also known as collective symmetry breaking. To achieve a certain level of naturalness, a special operator is also introduced to provide a tree level quartic without generating a tree level mass to the Higgs.

Another mechanism that has been shown to solve the little hierarchy problem is the twin Higgs [6, 7, 8, 8] (see also [10, 11, 12]). The twin Higgs mechanism is quite different from that of the little Higgs. In twin Higgs models, the Higgs mass is protected by a discrete  $Z_2$ , or twin, symmetry instead of multiple global symmetries. The exact twin symmetry guarantees that all gauge invariant dimensionful terms have, up to all orders in perturbation theory, a form which is invariant under a global  $SU(4)$  symmetry. The mass of the PNGB Higgs is then protected from receiving quadratically divergent contributions. It was shown that this mechanism alleviates the little hierarchy problem to about the 10% level for the cut off scale  $\Lambda = 10$  TeV without introducing a tree level quartic.

In this class of models where the quadratic divergences are naturally suppressed,

one would expect less fine-tuning if the quartic coupling of Higgs  $\lambda$  is large. In the original twin Higgs models, both the squared mass and the quartic for the SM Higgs come from the one-loop Coleman-Weinberg (CW) potential [13]. The quartic coupling is thus not a free parameter and loop suppressed. In order to improve the naturalness, one should try to find a tree level operator that will give the PNCB a quartic coupling without giving it a tree level mass term. In order to not upset the cancellation of radiative corrections, the tree level operator one introduces must preserve the twin parity. To summarize, in order to improve the fine tuning, the following criteria must be satisfied.

- A tree level operator that generates a quartic for the SM Higgs, but not a mass.
- This operator must preserve the discrete symmetry that protects the Higgs mass.
- Reduce as much as possible the mass squared that arise at loop level. For example, reduce top contribution by making the top Yukawa interaction  $SU(4)$  invariant.

One very simple operator which satisfies the first criterion has been constructed and is used in the twin Higgs model [8]. The basic idea is a mismatched alignment of two vevs. It was shown in ref. [8] that the mirror twin Higgs model [6] improves when this type of tree level quartic is added. The mismatched alignment of the vevs necessarily breaks the mirror  $SU(2) \times U(1)$  gauge symmetry to nothing and so the mirror photon becomes massive. Because of this feature, the mechanism seems difficult to implement in the left-right twin Higgs model [8] since the mismatched vev alignment would break  $U(1)_{EM}$  and the SM photon would become massive.

However, there is actually more than one type of parity which can be identified as a twin parity, i.e. the original twin parity, known also as  $\mathbf{P}$ , and charge conjugation,  $\mathbf{C}$ [14]. Under these parities, scalar and Dirac fermion in the left-right model

transform as

$$\mathbf{P} : \left\{ \begin{array}{l} H_L \rightarrow H_R \\ Q_L \rightarrow Q_R \end{array} \right\} \quad (\text{A.1})$$

$$\mathbf{C} : \left\{ \begin{array}{l} H_L \rightarrow H_R^* \\ Q_L \rightarrow C\bar{Q}_R^T \end{array} \right\} \quad (\text{A.2})$$

In this paper, we show that by using this fact and the idea of collective symmetry breaking, a new type of quartic operator can be constructed. This new quartic has all the properties we mentioned above, but does not break  $U(1)_{EM}$ . Most importantly, it preserves one of the parities that will maintain the cancellation of quadratic divergences to one loop. The quadratic divergences are no longer protected to all orders in perturbation theory as in the original twin Higgs model. However, cancellation to one loop is sufficient to address the little hierarchy problem.

The paper is organized as follows: In section II, we review the twin Higgs mechanism and the left-right twin Higgs model. We then explore the possibilities in introducing a tree level quartic and extend the top sector, making it  $SU(4)$  invariant. In section III, we analyze the radiative corrections and electroweak symmetry breaking. We then apply the same mechanism to the mirror model and reanalyze its naturalness in section IV. In section V, some phenomenology is discussed and our results summarized.

## A.2 Construction of the Model

The scalar field  $H$  in twin Higgs models is in the fundamental representation of a  $U(4)$  global symmetry. After acquiring a vev,  $\langle H \rangle = (0, 0, 0, f)$ ,  $U(4)$  is broken to  $U(3)$ , which yields 7 Goldstone bosons including the standard model (SM) Higgs doublet  $h = (h_1, h_2)$ . The global symmetry is explicitly broken by gauging only a subgroup  $SU(2)_A \times SU(2)_B$  (we ignore  $U(1)$  factors here since they are not relevant to present discussion). Under this gauge symmetry,  $H$  can be represented by  $H = (H_A, H_B)$  where  $H_{A,B}$  transform as doublets of  $SU(2)_{A,B}$ . Since the global symmetry

is broken explicitly by the gauge couplings and the breaking is ‘hard’, masses of the Goldstone bosons will be radiatively generated and be quadratically divergent. However, by imposing the discrete symmetry (twin parity) that interchanges the two gauged  $SU(2)$  symmetries, the quadratic divergences cancel. The simplest way to understand this is the following. First write down the most general gauge invariant mass terms for the linear fields  $H_A$  and  $H_B$

$$\alpha_A H_A^\dagger H_A + \alpha_B H_B^\dagger H_B, \quad (\text{A.3})$$

where  $\alpha_{A,B}$  are not required to be related by the gauge symmetry. After imposing the twin symmetry on all the interactions, however,  $\alpha_A$  is forced to be equal to  $\alpha_B$  and so the form given above is invariant under the global  $U(4)$  transformation. Therefore, this term, which is quadratically divergent, does not contribute to potential of the Goldstone bosons. Higher order terms, the quartic term  $(|H_A|^4 + |H_B|^4)$  for example, can contribute even though they preserve the twin symmetry since twin symmetry does not require these terms to have a  $U(4)$  invariant form. These contributions can have at most logarithmic divergences and so are under theoretical control. Additional interactions such as Yukawa couplings can be added to the theory consistent with the discrete twin symmetry, and the argument above shows that they do not lead to quadratic divergences.

The fine-tuning in twin Higgs theories can be further reduced if there are terms in the Lagrangian which respect the twin symmetry and contribute to the quartic self-coupling of the light pseudo-Goldstone Higgs but not to its mass. In the case of the model discussed above, with a single Higgs field  $H$ , there are no such operators consistent with the symmetries of the theory. However, such terms can be written down in theories with more than one set of Higgs fields. We consider the theory with an extra scalar field  $\hat{H}$ , which has its vev residing in a different direction,  $\langle \hat{H} \rangle = (0, 0, \hat{f}, 0)$  [8]. After the global  $U(4)$  symmetry is spontaneously broken by  $f$  and  $\hat{f}$ , and the massive radial modes are integrated out, we can write down a non-linear sigma model which contains the interactions of the light degrees of freedom. The light fields of the non-linear sigma model can be parametrized as

$$\begin{aligned}
H &= \begin{pmatrix} h_1 \\ h_2 \\ C \\ f + i\phi - \frac{h^\dagger h}{2f} \end{pmatrix} + \dots \\
\hat{H} &= \begin{pmatrix} \hat{h}_1 \\ \hat{h}_2 \\ \hat{f} + i\hat{\phi} - \frac{\hat{h}^\dagger \hat{h}}{2\hat{f}} \\ \hat{C} \end{pmatrix} + \dots
\end{aligned} \tag{A.4}$$

Notice that a quartic term like  $|H|^4$  would give a mass term to the Goldstone boson  $h$  because  $H$  contains a component  $\sim (f - h^2/2f + \dots)$ . The quartic operator  $|H|^4$  therefore contains a term like  $(f - h^2/2f + \dots)^4$ , which gives a mass term for  $h$ . This is why the second Higgs field  $\hat{H}$  is required. With the mismatched alignment of vevs as in eq. (A.4), the operator  $|H^\dagger \hat{H}|^2$  gives mass only to  $C$  and  $\hat{C}$ , and gives rise to a quartic term for  $h$  and  $\hat{h}$  without a corresponding mass term.

The above discussion is general for twin Higgs models. The phenomenological consequences of the additional vev  $\hat{f}$ , however, depend on the model's  $U(1)$  structure. In the mirror twin Higgs model, the gauged subgroup of global  $U(4)$  is  $SU(2)_A \times U(1)_A \times SU(2)_B \times U(1)_B$ . Two identical electroweak gauge symmetries are introduced to two sectors of the model. Sector A is identified with the standard model and sector B is a mirror world of the standard model. An extra scalar multiplet  $\hat{H} = (\hat{H}_A, \hat{H}_B)$  is added to the model in order to implement the above mechanism [8].  $H_B$  and  $\hat{H}_B$  are both singlets under  $SU(2)_A \times U(1)_A$  and have the same nontrivial charge under  $SU(2)_B \times U(1)_B$ . The mismatched vevs thus break the mirror  $SU(2)_B \times U(1)_B$  gauge symmetry to nothing but preserve the entire standard model  $SU(2)_A \times U(1)_A$ . The mirror photon therefore becomes massive, in contrast to the case in the original mirror twin Higgs model where the mirror photon remains massless after  $U(4)$  symmetry is broken.

In the left-right twin Higgs (LRTH) model, the gauged subgroup is that of the left-right model:  $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times \mathbf{P}$ [15]. There are two Higgs fields,

$H = (H_L, H_R)$  and  $\hat{H} = (\hat{H}_L, \hat{H}_R)$ , both of which transform as a fundamental representation under the  $SU(4)$  global symmetry. Under the gauge symmetry, these scalars transform as

$$H_L \text{ and } \hat{H}_L : (\mathbf{2}, \mathbf{1}, \mathbf{1}), \quad H_R \text{ and } \hat{H}_R : (\mathbf{1}, \mathbf{2}, \mathbf{1}) \quad (\text{A.5})$$

In this model, the scalar fields acquire the vevs,  $\langle H_R \rangle = (0, f)$  and  $\langle \hat{H}_R \rangle = (0, \hat{f})$ , which break the  $SU(4)$  global symmetry as well as the gauge symmetry  $SU(2)_R \times U(1)_{B-L}$  down to  $U(1)_Y$  hypercharge. Without introducing any extra scalar fields, can we apply the mismatched mechanism to this model to obtain a tree-level quartic coupling to the pseudo Goldstone Higgs? The previous discussion seems to suggest that we need to change the vev of  $\hat{H}_R$  to  $\langle \hat{H}_R \rangle = (\hat{f}, 0)$ . These new vevs would break  $U(1)_Y$  and hence,  $U(1)_{EM}$ . Therefore, this mechanism can not be applied to the left-right twin Higgs model in its simplest form. The question we would like to answer is whether there exists a different operator or a certain assignment of charges that achieves the same goal, while leaving  $U(1)_{EM}$  unbroken.

#### A.2.1 Quartic for the Left-Right Model

The charge assignment for  $H$  and  $\hat{H}$  given in eq. (A.5) is unique. All other charge assignments which are consistent with the symmetry breaking  $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_{EM}$  and preserve the left-right symmetry are, up to a set of field redefinitions, equivalent to this assignment. With this charge assignment, the vevs that preserve the hypercharge  $U(1)_Y$  is the one given in the original LRTH model:  $\langle H \rangle = (0, 0, 0, f)$  and  $\langle \hat{H} \rangle = (0, 0, 0, \hat{f})$ . In order to have a tree level quartic, we add to the LRTH model the following terms that break the global  $SU(4)$  symmetry

$$\Delta V = \lambda(|(H_R^T \tau_2 \hat{H}_R)|^2 + |(H_L^\dagger \hat{H}_L)|^2). \quad (\text{A.6})$$

These two terms are not symmetric under the twin defined originally in the

LRTH model

$$\begin{aligned} H_L &\leftrightarrow H_R \\ \hat{H}_L &\leftrightarrow \hat{H}_R, \end{aligned} \tag{A.7}$$

where the gauge and matter fields transform as

$$\begin{aligned} A_{L\mu}^a T_L^a &\rightarrow A_{R\mu}^a T_R^a \\ A_{B-L} &\rightarrow A_{B-L} \\ Q_L &\rightarrow Q_R^c, \end{aligned} \tag{A.8}$$

in two-component Weyl notation. However, one can define an alternative twin parity

$$\begin{aligned} H_L &\leftrightarrow H_R \\ \hat{H}_L &\leftrightarrow \tau_2 \hat{H}_R^*, \\ A_{L\mu}^a T_L^a &\rightarrow A_{R\mu}^a T_R^a \\ Q_L &\rightarrow Q_R^c, \end{aligned} \tag{A.9}$$

It can be shown explicitly that the quartic terms given in eq. (A.6) preserve the  $Z_2$  symmetry given in eq. (A.9), which is as powerful as the original twin parity in protecting the Higgs mass from receiving quadratically divergent corrections. All interactions in this model except the  $U(1)_{B-L}$  gauge interaction and the new quartic potential we introduced in eq. (A.6) preserve both of the parities given above. The quartic potential breaks the first parity and the  $U(1)_{B-L}$  breaks the second.

Since every term in this extended LRTH model breaks no more than one parity defined in eqs. (A.7,A.8) and eq. (A.9), quadratically divergent masses of the PNCB can only be generated when both parities are broken collectively. The quadratically divergent contributions to the PNCB masses are generally expected to arise at two loop. However, a more detailed analysis shows that two-loop contributions are also absent, and that contributions begin at three loops. This conclusion can be understood as follows. The leading diagrams that contribute to a quadratically divergent mass have to break both types of parities defined above. Thus, the diagrams must



involve the collection of vertices that break both symmetries. The vertices required are the three point coupling of  $U(1)_{B-L}$  to  $H$ , to  $\hat{H}$  and the quartic coupling proportional to  $\lambda$ . Note that the four point  $U(1)$  coupling to  $H$  actually preserves both parities. With this minimal set of vertices, all diagrams one can construct vanish in the Landau gauge for the same reason that the one loop CW potential vanishes with both gauge and scalar particles running in the loop. Therefore, the leading contribution must involve at least one more vertex, which results in the total number of loops being at least three. Furthermore, the contribution is proportional to  $g_{B-L}^2$  which is parametrically a small number. We have thus succeeded in constructing a tree level quartic without generating a large mass term for the Higgs.

#### A.2.2 SU(4) Invariant Top Yukawa Interaction

Since precision measurements prefer a light Higgs,  $m_h < 200$  GeV [16], a tree level quartic by itself is not as useful as one might hope in addressing the LEP paradox. In order to have a complete solution to the problem, a further suppression of Higgs mass parameter is desirable. An obvious way to achieve this is to extend the top sector to include a  $U(4)$  invariant Yukawa and terms that only break the global symmetry softly. Then, the Higgs potential will receive only a finite contribution from the top sector [6].

The top sector in the original LRTH model contains  $Q_{L,R}$  and  $T_{L,R}$  charged under  $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  as

$$\begin{aligned} Q_L &= (\mathbf{3}, \mathbf{2}, \mathbf{1}, \mathbf{1/3}) & Q_R &= (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{2}}, -\mathbf{1/3}) \\ T_L &= (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{4/3}) & T_R &= (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, -\mathbf{4/3}), \end{aligned} \quad (\text{A.10})$$

where we are using two-component Weyl notation. The gauge invariant top Yukawa terms can then be written down as

$$y(H_R^\dagger \tau_2 Q_R T_L + H_L^T \tau_2 Q_L T_R). \quad (\text{A.11})$$

Without introducing any more extra fields, all other quarks and charged leptons can

get their masses from non-renormalizable operators like

$$\frac{y_u}{\Lambda}(H_R^\dagger \tau_2 Q_R)(H_L^T \tau_2 Q_L) + \frac{y_d}{\Lambda}(H_R^T Q_R)(H_L^\dagger Q_L) \quad (\text{A.12})$$

Due to the smallness of the Yukawa couplings, these non-renormalizable operators will not affect our discussion later of fine tuning. Even after we have modified the Higgs sector by adding a new quartic term eq. (A.6), the charges of the Higgses and their vevs remain the same as that were defined originally and thus these operators remain valid to give masses to light fermions. We will ignore these operators for the rest of this paper.

Notice that neither  $Q_R$  nor  $Q_R^c$ , the complex conjugate of  $Q_R$ , can be combined with  $Q_L$  to form an  $\text{SU}(4)$  multiplet due to the different charges under the gauge or Lorentz groups. To complete the  $\text{SU}(4)$  representation, we need to introduce two extra vector-like quarks

$$\begin{aligned} \Phi_R &= (\mathbf{3}, \mathbf{1}, \mathbf{2}, 1/3) & \Phi_L &= (\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1}, -1/3) \\ \bar{\Phi}_R &= (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{2}, -1/3) & \bar{\Phi}_L &= (\mathbf{3}, \mathbf{2}, \mathbf{1}, 1/3). \end{aligned}$$

$Q_L$  and  $\Phi_R$  form a  $\mathbf{4}$  representation of  $\text{SU}(4)$  and similarly for  $\Phi_L$  and  $Q_R$ . The top Yukawa term then becomes

$$\begin{aligned} \mathcal{L}_{top} &= y(H_L^T \tau_2 \Phi_L + H_R^T \tau_2 Q_R)T_L \\ &+ (H_L^T \tau_2 Q_L + H_R^T \tau_2 \Phi_R)T_R + h.c.. \end{aligned} \quad (\text{A.13})$$

We also add the following soft masses to decouple the extra vector-like quarks,

$$M_R \bar{\Phi}_R \Phi_R + M_L \bar{\Phi}_L \Phi_L + M_0 T_L T_R + h.c.. \quad (\text{A.14})$$

For simplicity, we set  $M_0 = 0$  in the analysis below.

### A.3 Radiative Corrections and EW Symmetry Breaking

In this section we determine the radiative corrections to the pseudo-Goldstone mass and verify that electroweak symmetry is indeed broken by a light Higgs. In partic-

ular, we will compute the CW potential [13] for the light fields, as given by

$$V = \pm \sum_i \frac{1}{64\pi^2} M_i^4 \left( \ln \frac{\Lambda^2}{M_i^2} + \frac{3}{2} \right), \quad (\text{A.15})$$

where the sum is over all degrees of freedom. The sign is positive for fermions and negative for bosons. At one loop the Yukawa couplings, gauge couplings and Higgs self-couplings all contribute separately to the sum, simplifying the calculation. For simplicity, we will work in the context of a model where the symmetry breaking pattern is realized linearly, by the terms

$$\eta(|H|^2 - f^2)^2 + \hat{\eta}(|\hat{H}|^2 - \hat{f}^2)^2. \quad (\text{A.16})$$

We begin by considering the loop contributions from the self-couplings of the scalar fields. Obviously, there can be no  $\eta$  or  $\eta^2$  contribution to the potential of Goldstone bosons since all vertices in the relevant diagrams preserve  $SU(4)$ . Hence, these diagrams will only correct  $\eta$ , a free parameter. Also, to one-loop, the diagrams with one mismatched quartic and one  $SU(4)$  invariant quartic ( $\eta\lambda$  contribution) will only generate corrections to  $\eta$  and  $\lambda$ , both free parameters. This can be understood by the observation that

$$\begin{aligned} & \lambda(|(H_R^T \tau_2 \hat{H}_R)|^2 + |(H_L^\dagger \hat{H}_L)|^2) \\ &= \lambda|H_L^\dagger \hat{H}_L + H_R^\dagger i\tau_2 \hat{H}_R^*|^2 + \lambda|H_L^\dagger \hat{H}_L - H_R^\dagger i\tau_2 \hat{H}_R^*|^2. \end{aligned} \quad (\text{A.17})$$

The first operator is invariant under an  $SU(4)$ , which is also preserved by  $\eta$ , if we arrange  $\hat{H} = (\hat{H}_L, i\tau_2 \hat{H}_R)$ . The same holds for the second if we arrange  $\hat{H} = (\hat{H}_L, -i\tau_2 \hat{H}_R)$ . At one loop, the four-point diagrams that include the  $SU(4)$  invariant quartic can only include one of these operators and thus are invariant under the corresponding  $SU(4)$ . Hence, the combination of the operators above will only correct the tree level parameters  $\eta$  and  $\lambda$ . Therefore, when computing the one loop radiative corrections to quartic terms in the Higgs potential, we can ignore the  $SU(4)$  invariant term given in eq. (A.16).

The effective potential may however contain operators of higher dimensionality involving  $\eta$  arising at one loop, but these operators will make only a finite contribution to the potential of the pseudo-Goldstone bosons. We will therefore neglect this

contribution in our analysis. As mentioned in the previous section, new quadratic contributions could arise from the combination of the quartics above and the  $U(1)$  gauge coupling at the three loop level, which we will also ignore.

The vev that preserves  $U(1)_{EM}$  can be written as

$$\langle H \rangle = f \begin{pmatrix} 0 \\ i \sin x \\ 0 \\ \cos x \end{pmatrix} \quad \langle \hat{H} \rangle = \hat{f} \begin{pmatrix} 0 \\ i \sin \hat{x} \\ 0 \\ \cos \hat{x} \end{pmatrix} \quad (\text{A.18})$$

Expanding the tree level Higgs potential given in eq. (A.6) and keeping only the mass terms we find

$$\begin{aligned} & \lambda \{ |\hat{f} \cos \hat{x} H_{R1} - f \cos x \hat{H}_{R1}|^2 \\ & + |f \sin x \hat{H}_{L2} - \hat{f} \sin \hat{x} H_{L2}^*|^2 \\ & + f \hat{f} \sin x \sin \hat{x} (H_L^\dagger \hat{H}_L + h.c.) \}. \end{aligned} \quad (\text{A.19})$$

For the right-handed fields, obviously three of them are massless and the last one has mass squared  $\lambda(\hat{f}^2 \cos^2 \hat{x} + f^2 \cos^2 x)$ . For the left-handed fields, the eigenvalues are  $\pm \lambda f \hat{f} \sin x \sin \hat{x}$ ,  $\lambda f^2 \sin^2 x$ ,  $\lambda \hat{f}^2 \sin^2 \hat{x}$  and

$$\begin{aligned} & \frac{1}{2} \lambda (\hat{f}^2 \sin^2 \hat{x} + f^2 \sin^2 x) \\ & \pm \sqrt{f^4 \sin^4 x + \hat{f}^4 \sin^4 \hat{x} + 14 \hat{f}^2 f^2 \sin^2 x \sin^2 \hat{x}}. \end{aligned}$$

It is now clear how the quadratically divergent mass terms for the pseudo-Goldstone bosons vanish. The quadratic terms in the one-loop CW potential are proportional to  $\sum_i M_i^2$ . From the masses given above, the trace is not zero but independent of  $x$  and  $\hat{x}$ , which are the two Higgs fields.

Note the presence of a negative mass squared. Once we add soft mass terms (eq. (A.26)), this and all other masses can be made positive.

We now turn our attention to contributions arising from the top Yukawa cou-

pling. The masses of fermions in the top quark sector are given by

$$\begin{aligned} & \frac{1}{2}(f^2 + M^2 \pm \sqrt{(f^2 + M^2)^2 - 4M^2 f^2 \sin^2 x}) \\ & \frac{1}{2}(f^2 + M^2 \pm \sqrt{(f^2 + M^2)^2 - 4M^2 f^2 \cos^2 x}), \end{aligned} \quad (\text{A.20})$$

where we have imposed a left-right symmetry to the soft masses, so  $M_L = M_R = M$ . Again, the sum of  $M_i^2$  is independent of  $x$ .

Finally, we turn our attention to the gauge sector. The masses of the gauge bosons are

$$\begin{aligned} m_{W_H}^2 &= \frac{g_2^2}{2}(f^2 + \hat{f}^2) - m_W^2 \\ m_{Z_H}^2 &\approx \frac{g_1^2 + g_2^2}{2}(f^2 + \hat{f}^2) - \frac{2g_1^2 + g_2^2}{g_1^2 + g_2^2} m_W^2. \end{aligned} \quad (\text{A.21})$$

To quadratic order, the CW potential is

$$V_2^{(1)} = v^2(V_a + V_b \cos^2 \beta), \quad (\text{A.22})$$

where

$$\begin{aligned} V_a &= \frac{1}{32\pi^2} \\ & \left\{ \frac{3}{2} g_2^4 (f^2 + \hat{f}^2) \left( \ln \frac{\Lambda^2}{m_{W_H}^2} + 1 \right) \right. \\ & + 3 \frac{2g_1^2 + g_2^2}{4} g_2^2 (f^2 + \hat{f}^2) \left( \ln \frac{\Lambda^2}{m_{Z_H}^2} + 1 \right) \\ & \left. + 2\lambda^2 (f^2 + \hat{f}^2) \left( \ln \frac{\Lambda^2}{\lambda(f^2 + \hat{f}^2)} + 1 \right) \right\}, \end{aligned} \quad (\text{A.23})$$

$$\begin{aligned} V_b &= \frac{1}{32\pi^2} 12y^2 \frac{M^2}{y^2 f^2 - M^2} \\ & \left( y^2 f^2 \ln \frac{y^2 f^2 + M^2}{y^2 f^2} - M^2 \ln \frac{y^2 f^2 + M^2}{M^2} \right) \end{aligned} \quad (\text{A.24})$$

and

$$v \sin \beta = f \sin x, \quad v \cos \beta = \hat{f} \sin \hat{x}. \quad (\text{A.25})$$

To align the direction of the electro-weak symmetry breaking, we add the following soft mass terms

$$\begin{aligned} V^{(0)} &= m^2 H_L^\dagger H_L + \hat{m}^2 \hat{H}_L^\dagger \hat{H}_L \\ &+ \mu^2 (H^\dagger \hat{H} + h.c.). \end{aligned} \quad (\text{A.26})$$

The first two terms, which are invariant under a global  $SU(2) \times SU(2)$  symmetry, are introduced to tune the scale of electro-weak symmetry breaking. These two terms do not involve the  $H_{RS}$  and are thus insensitive to the alignment of the two vevs. To make sure that the vevs are properly aligned, the last term proportional to  $\mu^2$  is needed. For large and negative  $\mu^2$ , the alignment of the vevs is guaranteed, or equivalently, the squared masses of all the scalar fields in  $H_L$  and  $\hat{H}_L$  are positive. We will take this into account when we analyze the potential numerically later in this section.

Together with the  $SU(4)$  breaking quartic term given in eq. (A.6), the tree level potential is given by

$$\begin{aligned} V^{(0)} &= \lambda v^4 \cos^2 \beta \sin^2 \beta \\ &+ v^2 (m^2 \sin^2 \beta + \hat{m}^2 \cos^2 \beta + 2\mu^2 \sin \beta \cos \beta) \\ &+ 2\mu^2 f \hat{f} \sqrt{A} \end{aligned} \quad (\text{A.27})$$

where  $A = (1 - \frac{v^2}{f^2} \sin^2 \beta)(1 - \frac{v^2}{\hat{f}^2} \cos^2 \beta)$ .

We now minimize the potential  $V = V^{(0)} + V_2^{(1)}$  to find  $v$  and  $\sin \beta$ . The potential  $V$  has the form

$$\begin{aligned} V &= v^2 (a + b \sin^2 \beta + 2\mu^2 \cos \beta \sin \beta) \\ &+ \lambda v^4 \cos^2 \beta \sin^2 \beta, \end{aligned} \quad (\text{A.28})$$

where

$$\begin{aligned} a &= \hat{m}^2 - \mu^2 \frac{f}{\hat{f}} + V_a \\ b &= m^2 - \hat{m}^2 - \frac{\mu^2}{f \hat{f}} (\hat{f}^2 - f^2) + V_b. \end{aligned} \quad (\text{A.29})$$

After minimization, we find

$$\begin{aligned}\sin^2 \beta &= \frac{a}{2a+b} \\ v^2 &= -\frac{2a+b}{\lambda} \left(1 + \frac{\mu^2}{\sqrt{a(a+b)}}\right).\end{aligned}\tag{A.30}$$

The fine tuning is about 13% for  $\hat{f} = 2.0$  TeV and about 18% for  $\hat{f} = 1.6$  TeV, with the feature that  $\lambda$  is much less than 1. Unfortunately, this is not significantly better than the original twin model. Notice that a mass squared is generated at loop level proportional to  $\hat{f}^2 \lambda^2$  (See eq. (A.23)). Since  $\hat{f}$  must be greater than 1.6 TeV to evade the bound from direct  $Z'$  and  $W'$  gauge boson searches [17], the  $\hat{f}^2 \lambda^2$  contribution to the mass squared could be large if we push  $\lambda$  too high, which will tend to increase fine tuning. Thus, a small  $\lambda$  is preferred. However, with a smaller  $\lambda$ , we should account for the one-loop contribution to the quartic, since it may no longer be negligible. The largest loop contribution to the quartic is from the top Yukawa and is given by

$$V_4^{(1)} = \lambda_t v^4 \sin^4 \beta,\tag{A.31}$$

where

$$\begin{aligned}\lambda_t &= \frac{3}{16\pi^2} y^4 \frac{M^4}{m_T^4} \left\{ \ln \frac{m_T^2}{m_t^2} - \frac{1}{2} \right. \\ &\quad \left. + \left( \frac{m_T^2}{2M^2 - m_T^2} \right)^3 \ln \frac{M^2}{m_T^2 - M^2} - 2 \left( \frac{m_T^2}{2M^2 - m_T^2} \right)^2 \right\}\end{aligned}\tag{A.32}$$

and

$$m_T^2 = M^2 + y^2 f^2, \quad m_t^2 = \frac{M^2}{m_T^2} y^2 v^2 \sin^2 \beta.\tag{A.33}$$

After adding eq. (A.31) to eq. (A.28) and repeating the analysis above, we find that a fine tuning of about 30% is easily achieved. Selected points are shown in table (A.1).

#### A.4 Mirror Model

As far as addressing the little hierarchy problem, the mirror twin Higgs model with a tree level quartic [8] provides an improvement over the original mirror model.

$\Lambda_{(\text{TeV})}$	$f_{(\text{GeV})}$	$\hat{f}_{(\text{TeV})}$	$M_{L,R}(\text{TeV})$	$m_h(\text{GeV})$	$\sin^2 \beta$	Tuning
10	800	1.6	4	150/233	0.54	0.30 ( $y$ )
10	800	3.5	4	150/236	0.54	0.10 ( $\hat{f}$ )
20	1600	1.6	4	163/213	0.66	0.11 ( $M$ )
10	800	1.6	10	147/266	0.51	0.19 ( $y$ )
5	800	1.6	4	150/233	0.54	0.30 ( $y$ )
5	800	3.5	4	150/236	0.54	0.16 ( $\hat{f}$ )
10	1600	1.6	4	163/213	0.66	0.11 ( $M$ )

Table A.1: A summary of the Higgs mass and fine tuning,  $\partial \log M_Z^2 / \partial \log f^2$ , for sample points of parameter space. The two values of  $m_h$  correspond to the masses of the two neutral Higgses. The most fine tuned parameter at each point is shown in the parenthesis. At these points, the other parameters are  $\mu^2 = -(150 \text{ GeV})^2$ ,  $\lambda = 0.5$  and  $y = \sqrt{2}$ .

However, as shown in section II, in this theory the mirror photon is necessarily massive. As a consequence, this theory has difficulty in explaining the absence of a mirror electron relic density. In the absence of a massless mirror photon, electrons cannot efficiently annihilate to photons. We now show that using the same mechanism that was discussed in the previous section, this difficulty can be avoided.

The gauge group in the mirror model is  $SU(2)_A \times U(1)_A \times SU(2)_B \times U(1)_B$  which is a subgroup of the global  $U(4)$  symmetry. The scalar fields are  $H$  and  $\hat{H}$  which have the same charge under the gauge group. The top sector is just the SM top Yukawa plus its twin counter part.

$$\mathcal{L}_{top} = y(H_L^T \tau_2 Q_L t_R + H_R^T \tau_2 Q_R t_L) \quad (\text{A.34})$$

We now calculate the CW potential in this model. The masses of heavy gauge bosons are

$$\begin{aligned} m_{W_H}^2 &= \frac{g_2^2}{2}(f^2 + \hat{f}^2) - m_W^2 \\ m_{Z_H}^2 &= \frac{g_1^2 + g_2^2}{2}(f^2 + \hat{f}^2) - \frac{g_1^2 + g_2^2}{g_2^2} m_W^2. \end{aligned} \quad (\text{A.35})$$

For the top sector, up to finite terms which do not significantly alter the fine tuning, we can just take  $M = \Lambda$  to produce the results that correspond to the non- $SU(4)$



invariant top sector. For the Higgs potential, we add the same tree level potential as given in eq. (A.6). The CW potential due to this tree level potential is exactly the same as that obtained in our previous analysis on the left-right model. To quadratic order, the potential is

$$\begin{aligned}
V_2^{(1)} &= \frac{v^2}{32\pi^2} \\
&\{ \frac{3}{2}g_2^4(f^2 + \hat{f}^2)(\ln \frac{\Lambda^2}{m_{WH}^2} + 1) \\
&+ \frac{3}{4}(g_1^2 + g_2^2)^2(f^2 + \hat{f}^2)(\ln \frac{\Lambda^2}{m_{ZH}^2} + 1) \\
&+ 2\lambda^2(f^2 + \hat{f}^2)(\ln \frac{\Lambda^2}{\lambda(f^2 + \hat{f}^2)} + 1) \\
&- 12y^4f^2\sin^2\beta(\ln \frac{\Lambda^2}{y^2f^2} + 1)\}.
\end{aligned} \tag{A.36}$$

The one-loop quartic from the top sector is

$$V_4^{(1)} = \frac{3}{16\pi^2}y^4[\ln \frac{\Lambda^2}{m_t^2} + \ln \frac{\Lambda^2}{m_T^2} + \frac{3}{2}]$$

where

$$m_T^2 = y^2f^2 \quad , \quad m_t^2 = y^2v^2\sin^2\beta. \tag{A.37}$$

We then analyze the effective potential given by  $V = V^{(0)} + V_2^{(1)} + V_4^{(1)}$  as in the previous section. The fine tuning for this model is shown in table (A.2). We see that the results represent an improvement over the mirror model. We expect that further enhancement may be obtained by making the top Yukawa coupling SU(4) invariant as in [6], but we leave this for further work.

## A.5 Conclusion

We have constructed a twin Higgs model based on left-right symmetry with an order one tree level quartic for the light Higgs. The structure of the electroweak symmetry breaking is similar to that of two Higgs doublet model. We analyzed the model and showed electroweak symmetry breaking can happen naturally. For  $\hat{f} = 1.6$  TeV,

$\Lambda_{(\text{TeV})}$	$f = \hat{f}_{(\text{GeV})}$	$\lambda$	$m_h(\text{GeV})$	Tuning
10	800	0.5	178/213	0.16 ( $y$ )
10	800	1	183/213	0.21 ( $y$ )

Table A.2: A summary of the Higgs mass and fine tuning,  $\partial \log v^2 / \partial \log f^2$ , for sample points of parameter space. The two values of  $m_h$  correspond to the masses of the two neutral Higgses. The most fine tuned parameter at each point is shown in the parenthesis. At these points, the other parameters are  $\mu^2 = -(150 \text{ GeV})^2$ ,  $y = 1.2$  and  $\sin^2 \beta = 0.69$ .

which is the lower bound from the direct searches on heavy gauge bosons, the fine tuning is found to be about 30% for  $\Lambda = 10 \text{ TeV}$ . The bound on  $\hat{f}$  gets stronger if we also require the left-right symmetry on the first two generation quarks. The  $K_0\text{-}\bar{K}_0$  mixing puts a very strong constraint on the mass of  $W_H$  which require  $\hat{f} > 3.5 \text{ TeV}$  [18]. In that case, the fine tuning is found to be about 10%.

The phenomenology of the model introduced in section II and III is not significantly different from that of the original left-right twin Higgs model[22, 20, 21]. The extra quarks introduced to complete the  $\text{SU}(4)$  multiplet could have masses of about 4 TeV which is difficult to observe at the LHC. Among these extra quarks there are some with electric charge  $Q = -1/3$ . These new down-type fermions in the model might have sizable contributions to the  $D^0 - \bar{D}^0$  mixing depending on their masses[22]. The current experimental bound can be used to put a bound on the parameter  $M$  in the model. Another difference is that the parity we introduced to make the  $\hat{h}_L$  stable, under which  $\hat{H}_L$  is odd and all other fields are even, is here softly broken by the term  $H_L^\dagger \hat{H}_L$  in eq. (A.26). Hence,  $\hat{H}_L$  is no longer a dark matter candidate and will be produced and decay just like all other scalars in the model. The phenomenology of the scalar sector of the original LRTH model has been studied in ref. [22, 21]. Most of these studies have focused on the scalars in the ‘right-handed’  $H_R$  and  $\hat{H}_R$  since all other scalars in the ‘left handed’ sector other than the SM Higgs do not interact directly with fermions. In both of our new models, for the same reason that  $\hat{H}_L$  is no longer stable, all scalars in the ‘left-handed’ sector can interact with fermions and will behave just like the scalars of two Higgs doublet model. This new phenomenon, probably in combination with some others,

may be used to test the tree level quartic coupling introduced in these twin Higgs models. We leave these studies for future work.

In summary we have shown how to incorporate a tree level quartic into the left-right twin Higgs model, leading to a substantial improvement in fine-tuning. We have further applied this mechanism to the mirror twin Higgs model and established that the fine tuning is about 20% for a 10 TeV cutoff scale.

**Acknowledgments** – We thank Zackaria Chacko for discussions and comments on the draft. C. K and H.S.G are supported by the NSF under grant PHY-0408954.

## REFERENCES

- [1] R. Barbieri and A. Strumia, arXiv:hep-ph/0007265.
- [5] H. Georgi and A. Pais, Phys. Rev. D **10**, 539 (1974); Phys. Rev. D **12**, 508 (1975).
- [6] D. B. Kaplan and H. Georgi, Phys. Lett. B **136**, 183 (1984); D. B. Kaplan, H. Georgi and S. Dimopoulos, Phys. Lett. B **136**, 187 (1984); H. Georgi and D. B. Kaplan, Phys. Lett. B **145**, 216 (1984).
- [2] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B **513**, 232 (2001).
- [3] N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire and J. G. Wacker, JHEP **0208**, 021 (2002); N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP **0207**, 034 (2002); T. Gregoire and J. G. Wacker, JHEP **0208**, 019 (2002); I. Low, W. Skiba and D. Smith, Phys. Rev. D **66**, 072001 (2002); David E. Kaplan and Martin Schmaltz, JHEP **0310**, 039(2003) [arXiv:hep-ph/0302049].
- [6] Z. Chacko, Hock-Seng Goh and Roni Harnik, Phys. Rev. Lett. **96**, 231802 (2006) [arXiv: hep-ph/0506256]
- [7] R. Barbieri, T. Gregoire and L. J. Hall, [arXiv:hep-ph/0509242].
- [8] Z. Chacko, Yasunori Nomura, Michele Papucci and Gilad Perez, JHEP **0601**, 126 (2006) [arXiv:hep-ph/0510273]
- [8] Z. Chacko, Hock-Seng Goh and Roni Harnik, JHEP **0601**, 108 (2006) [arXiv:hep-ph/0512088]
- [10] A. Falkowski, S. Pokorski and M. Schmaltz, Phys. Rev. D **74**, 035003 (2006) [arXiv:hep-ph/0604066]; S. Chang, L. J. Hall and N. Weiner, Phys. Rev. D **75**, 035009 (2007) [arXiv:hep-ph/0604076].

- [11] Gustavo Burdman and Alex Gomes Dias, JHEP **0701**, 041 (2007) [arXiv:hep-ph/0609181].
- [12] R. Foot and R. R. Volkas, arXiv:hep-ph/0610013.
- [13] Sidney R. Coleman and E. Weinberg, Phys. Rev. **D7**, 1888-1910 (1973)
- [14] R. N. Mohapatra, A. Rasin and G. Senjanovic, Phys. Rev. Lett. **79**, 4744 (1997) [arXiv:hep-ph/9707281].
- [15] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 566 (1975); R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 2558 (1975).
- [16] ALEPH Collaboration, arXiv:hep-ex/0511027
- [17] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **69**, 111101 (2004); W. M. Yao *et al.*, J. Physics G **33**, 1 (2006); WWW page URL: <http://pdg.lbl.gov/>
- [18] G. Beall, M. Bander and A. Soni, Phys. Rev. Lett. **48**, 848 (1982).
- [22] H. S. Goh and S. Su, Phys. Rev. D **75**, 075010 (2007) [arXiv:hep-ph/0611015]; AIP Conf. Proc. **903**, 431 (2007) [arXiv:hep-ph/0608330]
- [20] D. W. Jung and J. Y. Lee, arXiv:hep-ph/0701071; Y. B. Liu, H. M. Han and Y. H. Cao, arXiv:hep-ph/0703268.
- [21] Y. B. Liu and J. F. Shen, arXiv:0704.0840[hep-ph]; Y. B. Liu, H. M. Han and Y. H. Cao, arXiv:hep-ph/0701169; D. W. Jung and K. Y. Lee, arXiv:hep-ph/0701087.
- [22] E. Golowich, J. Hewett, S. Pakvasa and A. A. Petov, arXiv:0705.3650[hep-ph].

## APPENDIX B

## LEPTON NUMBER VIOLATING SIGNALS IN THE LRTH

Hock-Seng Goh and Christopher A. Krenke

Submitted to Physical Review D

## ABSTRACT

We study the collider signatures of the left-right twin Higgs in the limit that the right-handed neutrino mass is less than the mass of the right-handed gauge boson. In this limit new leptonic decay chains open up, allowing the particles which cancel the one-loop quadratic divergences of the Higgs, the right-handed gauge bosons and top-partners, to be discovered. Half of these events contain same-sign leptons without missing energy, which have no genuine standard model background and therefore the backgrounds are purely instrumental. These signals may be used to complement other collider searches, and in certain regions of parameter space, may be the only way to observe the particles responsible for natural electroweak symmetry breaking in the left-right twin Higgs.

## B.1 Introduction

The standard model (SM) with a fundamental Higgs field suffers from an extreme sensitivity to short distance physics. If the cutoff of the SM is taken to be the Planck scale, this sensitivity leads to a tremendous fine tuning of the dimensionful parameters in the Higgs potential and a large hierarchy between the weak and Planck scales. If the cutoff of the SM is taken to be about 5 TeV, the minimum allowed by precision electroweak data, an unnatural adjustment of parameters persists and results in a “little hierarchy” [1]. This fact implies that new physics should exist at the TeV scale which is responsible for resolving the little hierarchy problem. This is an interesting observation as the Large Hadron Collider (LHC) at CERN has been

built with the aim of detecting particles with TeV scale masses. Therefore, new physics that is tightly connected to the nature of electroweak symmetry breaking is expected to be within the reach of the LHC. The nature of the new physics that cures the hierarchy problem, or the little hierarchy problem if the ultraviolet cutoff is taken to be less than 5 TeV, is highly constrained. Electroweak precision measurements have imposed very strong bounds on any new physics around a TeV. These constraints pose a great challenge to designing models meant to address the little hierarchy problem.

One class of theories that address the little hierarchy problem is the little Higgs [2, 3, 4]. In these models, the SM Higgs doublet is a pseudo-Nambu-Goldstone boson (NGB) of some spontaneously broken approximate global symmetry in which the SM  $SU(2)$  electroweak symmetry is embedded [5, 6]. The Higgs mass vanishes at tree level due to shift symmetry, but will be generated by radiative corrections when interactions that break the global symmetry, such as gauge and Yukawa interactions, are included. At one-loop, the Higgs mass is protected by multiple approximate global symmetries, which together mimic the full global symmetry. A mass for the Higgs can only be generated at 2-loops than one vertex is involved and the global symmetry is collectively broken. Therefore the Higgs mass is generated at two loops and is only logarithmically sensitive to UV physics. This class of models is able to stabilize the electroweak scale against the UV cutoff up to a scale of about 5 - 10 TeV.

Another class of theories that solves the little hierarchy problem by identifying the Higgs as a pseudo-NGB are twin Higgs models [7, 8, 9]. Instead of protecting the Higgs mass from receiving large radiative corrections by using several approximate global symmetries, twin Higgs theories use a discrete symmetry in combination with an approximate global symmetry to eliminate the quadratic divergences that arise at loop level. Together with the gauge symmetries of the model, the discrete symmetry mimics the effect of a global symmetry, thus stabilizing the Higgs mass.

In the left-right Twin Higgs model [8], the SM gauge symmetry is extended to  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ , which is embedded into a global  $U(4)$  symme-

try. The Higgs arises as a pseudo-Nambu-Goldstone boson (PNGB) when  $U(4)$  is spontaneously broken to  $U(3)$ . An additional  $Z_2$  “twin symmetry” ensures that the quadratic terms in the Higgs potential have an accidental  $U(4)$  symmetry. Since  $U(4)$  invariant terms cannot contribute to the potential for the Goldstones, the Higgs is protected from receiving quadratically divergent contributions to its mass parameter. To evade precision electroweak bounds on  $SU(2)_R$  gauge bosons without significantly affecting naturalness, an additional Higgs field  $\hat{H}$  is introduced that transforms as a fundamental under a new global  $U(4)$ . This addition makes the global symmetry of the theory  $U(4) \times U(4)$ . The new global symmetry does not significantly alter the form of the SM Higgs potential, allowing electroweak symmetry breaking to still happen naturally.

To identify the twin mechanism it is important to observe the heavy top quark partner  $T_H$  and the right-handed gauge boson  $W_R$ . For a reasonable choice of parameters, the most straightforward way to observe both of these particles involves decays of the heavy top quark, which has a channel containing final state leptons that can be used as a trigger. It may be possible to reconstruct these events and observe the heavy top quark at the LHC [10]. However, this decay channel depends on a free parameter  $M$ , which could be very small or zero. In this limit, the heavy top quark can only decay hadronically [10], making these particles very difficult to observe at the LHC due to the large QCD background.

In this paper, we study an alternative way to observe the heavy top quark and the right-handed gauge boson  $W_R$ . If a TeV scale right-handed Majorana neutrino is realized in the left-right twin Higgs such that  $m_{\nu_R} < m_{W_R}$ , new leptonic channels open up that may allow detection of  $W_R$  and  $T_H$  at the LHC. Moreover, because the right-handed neutrino is Majorana, half of these decays are lepton number violating same-sign dilepton events without missing energy, which has no genuine SM background. If  $M$  is small or zero, these lepton number violating signals may be the only way to observe the heavy top quark and  $W_R$  at the LHC.

This paper is organized as follows: In section II, we review the left-right twin Higgs model and discuss its phenomenology in the decoupling case where the pa-



parameter  $M$  is set to zero. In section III, we implement neutrino masses into the model and discuss constraints on a TeV scale right-handed neutrino. We study the collider phenomenology of the model in section IV, focusing on searches for  $W_R$  and the heavy top partner  $T_H$ . We then conclude in section V.

## B.2 Left-right Twin Higgs model

### B.2.1 Matter Content

The fermionic content of left-right twin Higgs model is three generations of

$$\begin{aligned} Q_L = (u, d)_L &= (\mathbf{2}, \mathbf{1}, \mathbf{1/3}) & L_L = (\nu, e)_L &= (\mathbf{2}, \mathbf{1}, -\mathbf{1}) \\ Q_R = (u, d)_R &= (\mathbf{1}, \mathbf{2}, \mathbf{1/3}) & L_R = (\nu, e)_R &= (\mathbf{1}, \mathbf{2}, -\mathbf{1}) \end{aligned} \quad (\text{B.1})$$

where the square brackets indicate the quantum numbers of the corresponding fields under the  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  gauge symmetry of the theory. We see that in addition to the SM fermions the theory includes right-handed neutrinos as required by left-right symmetry. There are two sets of Higgs fields which have quantum numbers [11, 12]

$$\begin{aligned} H_L &= (\mathbf{2}, \mathbf{1}, \mathbf{1}) & H_R &= (\mathbf{1}, \mathbf{2}, \mathbf{1}) \\ \hat{H}_L &= (\mathbf{2}, \mathbf{1}, \mathbf{1}) & \hat{H}_R &= (\mathbf{1}, \mathbf{2}, \mathbf{1}) \end{aligned} \quad (\text{B.2})$$

The reason for introducing the extra set of Higgs fields  $\hat{H}$  is to satisfy precision electroweak constraints on  $SU(2)_R$  gauge bosons. These constraints require the symmetry breaking scale  $f$  of  $SU(2)_R$  to be larger than about 2 TeV [13]. However, for this value of  $f$ , contributions to the Higgs potential from the top sector are very large since the top Yukawa is order one. This effect tends to reintroduce fine tuning to the model, destabilizing the weak scale. By adding an additional Higgs field  $\hat{H}$ , which acquires a vev  $\langle \hat{H} \rangle = \hat{f} \sim 2$  TeV and does not couple to fermions, precision electroweak constraints on  $SU(2)_R$  gauge bosons can be satisfied without affecting the top sector. This arrangement can be justified by imposing a discrete symmetry under which  $\hat{H}$  is odd while all other fields are even. This symmetry allows  $\hat{H}_L$  to

be stable, making it a natural dark matter candidate. It has been shown that this can account for the observed relic abundance of dark matter [14].

The Higgs potential is assumed to have an approximate  $U(4) \times U(4)$  symmetry of which the  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  sub-group is gauged. After breaking the global  $U(4)$  and the gauged  $SU(2)_R$  symmetries, the SM Higgs doublet, which is among the NGBs, has no potential at tree level. However, a potential for the Higgs potential will be radiatively generated at one loop. In this scenario, both the mass and the quartic coupling of the Higgs are loop suppressed. To further reduce fine tuning, a tree level Higgs quartic can be introduced without generating a corresponding tree level mass term for the Higgs, as discussed in [9]. Since the Higgs potential is not relevant to our discussion of neutrino masses or collider signals, we shall not go into a detailed discussion of the Higgs potential.

The down-type Yukawa couplings of the SM emerge from non-renormalizable couplings of the form

$$\left( \frac{\overline{Q}_R H_R H_L^\dagger Q_L + \overline{L}_R H_R H_L^\dagger L_L}{\Lambda} \right) + \text{h.c.} \quad (\text{B.3})$$

The up-type Yukawa couplings of the SM emerge from non-renormalizable couplings of the form

$$\left( \frac{\overline{Q}_R H_R^\dagger H_L Q_L + \text{h.c.}}{\Lambda} \right). \quad (\text{B.4})$$

When the field  $H_R$  acquires a VEV of order  $f$  breaking  $SU(2)_R \times U(1)_{B-L}$  down to  $U(1)_Y$ , these non-renormalizable couplings reduce to the familiar Yukawa couplings of the SM. Unfortunately, this method of generating SM Yukawa couplings does not work well in the top sector since the top Yukawa coupling is order one. This problem is remedied by introducing the following vector like quarks, which transform as

$$T_L = (\mathbf{1}, \mathbf{1}, \mathbf{4/3}) \quad T_R = (\mathbf{1}, \mathbf{1}, \mathbf{4/3}) \quad (\text{B.5})$$

under  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ . We can then write the following left-right symmetric interactions

$$\left( y \overline{Q}_R H_R^\dagger T_L + y \overline{Q}_L H_L^\dagger T_R + M \overline{T}_L T_R \right) + \text{h.c.} \quad (\text{B.6})$$

The right-handed top quark of the SM then emerges as a linear combination of  $T_R$  and the third generation up-type quark in  $Q_R$ , while the orthogonal linear combination is heavy. Provided  $M \lesssim f$  and  $y$  is of order one the physical top Yukawa will then also be of order one.

The parameter  $M$  controls the mixing of the left-handed top with the  $SU(2)_L$  singlet  $T_L$ , and is therefore constrained by  $Z \rightarrow b \bar{b}$ . However, nothing prevents  $M$  from simply being set to zero and therefore this is not a particularly tight constraint. However, the collider phenomenology of this model will depend on the size of this parameter. As we will see below, when  $M$  is small, with  $M = 0$  as a extreme case of this scenario, the heavy top becomes difficult to observe in a hadron collider since it decays dominantly into an all jet final state.

### B.2.2 Phenomenology

The left-right twin Higgs contains many new particles which may be observable at the LHC. The new particles include the right-handed gauge bosons  $W_R$  and  $Z_R$ , a heavy top quark  $T_H$ , a right-handed neutrino  $N$ , and the Higgses  $\hat{h}^T = (\hat{h}^+, \hat{h}^0)$ ,  $\phi^\pm$  and  $\phi^0$ . The gauge boson masses depend on the larger vev  $\hat{f}$  and range from about 1 - 4 TeV, while the heavy top is typically lighter, ranging from 0.5 - 1 TeV. The  $\phi^0$  mass depends on a free parameter in the theory, but is usually taken to be about 100 GeV. The charged Higgs  $\phi^\pm$  mass ranges from about 200 - 400 GeV, while the  $\hat{h}$  mass ranges from about 300 GeV to 1 TeV. The right-handed neutrino mass arises from the operator

$$\frac{(L_R \hat{H}_R \hat{H}_R L_R + L_L \hat{H}_L \hat{H}_L L_L)}{\Lambda} \quad (\text{B.7})$$

and is of order  $\hat{f}^2/\Lambda$ , which is about 1.5 TeV for  $\hat{f} \sim 4$  TeV and  $\Lambda \sim 10$  TeV. We will have more to say about neutrino masses in section B.3.

What are the collider signatures of this model? The  $Z_R$  decays to leptons providing a very clean signal, which may be observable at the LHC [10]. Detection of  $W_R$  however, is more subtle. For now assume that  $m_{\nu_R} > m_{W_R}$  and leptonic decays of the  $W_R$  are kinematically forbidden. This was the scenario studied in [10].

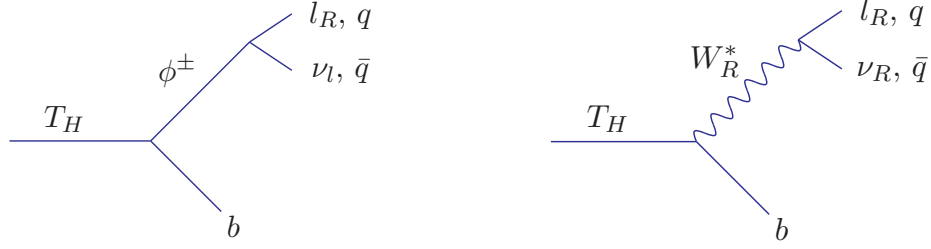


Figure B.1: Possible decays of the heavy top in the limit that  $M = 0$ .

In this case, the  $W_R$  decays a large fraction of the time (20% - 30%) to a heavy top and a  $b$ -jet [10]. Therefore the discovery potential of both the heavy top and  $W_R$  depend critically on how the heavy top decays. The heavy top is produced in association with a  $b$ -quark, with a production cross section of about 500 fb [10]. For a reasonable choice of  $M = 150$  GeV,  $T_H$  decays most often to  $\phi^\pm b$  [10]. For this value of  $M$ , the  $\phi^\pm$  then decays mostly to  $tb$ . It is then possible to trigger on the leptonic decay of the top, giving the following decay chain

$$T_H \rightarrow \phi^\pm b \rightarrow tbb \rightarrow Wbbb \rightarrow l\nu bbb. \quad (\text{B.8})$$

This scenario has been studied and shown to be detectable at the LHC with total luminosity of  $10 \text{ fb}^{-1}$  [10].

### $M = 0$ : The Dark Side of the Model

The discussion above was for  $m_{\nu_R} > m_{W_R}$  and for a small but reasonable value of  $M = 150$  GeV. In this case, the decay of the heavy top had a leptonic final state, which could be used as a trigger. However, the phenomenology changes significantly when  $M$  is very small, less than about 10 GeV. The crucial difference in this case is the decay of  $\phi^\pm$ , which previously decayed to a SM top quark which then decayed leptonically. When  $M = 0$ , the  $\phi^\pm$  decays purely to charm and strange, leading to an all jet final state for heavy top decay.

The reason  $\phi^\pm$  does not decay to the SM top quark can be understood as follows. If  $\phi^\pm$  is thought of as the charged component of  $H_R$ , then  $\phi^\pm$  couples directly only

to  $b_R$  and  $T_L$ , which in the limit that  $M = 0$  are identified as the right-handed SM  $b$  quark and the left-handed heavy top, respectively. When  $M \neq 0$ , mixing between  $T_L$  and  $T_R$  induces a coupling to the SM top quark that is proportional to  $M/f$  for  $M \ll f$ . Therefore, in the limit  $M \rightarrow 0$ ,  $\phi^\pm$  cannot decay to a SM top quark.

What about other decay channels for  $T_H$ ? Two other decay channels are possible and are shown in Fig. B.1. In the first decay channel,  $\phi^\pm$  will dominantly decay to  $q\bar{q}$  because the leptonic decay channel is suppressed by the neutrino Yukawa coupling constant. In the second decay channel,  $\nu_R$  is kinematically unavailable and therefore the leptonic channel is only available through an off shell  $\nu_R$ , which is highly suppressed. Therefore, in the limit that  $M \rightarrow 0$ ,  $T_H$  can only decay hadronically, leading to an all-jet final state for heavy top decay. Detection of the heavy top at the LHC then becomes difficult. In this scenario, the model may become one of those in which the true mechanism of natural electroweak symmetry breaking is beyond the reach of the LHC.

Since the small  $M$  parameter space is large, technically natural, and does not affect the twin mechanism, it is important to examine this possibility more closely. The hope lies in the size of right-handed neutrino mass relative to the mass of the  $W_R$ . If  $m_{\nu_R} < m_{W_R}$ , leptonic decay channels of the heavy top open up and provide a way to observe the heavy top partner that is independent of the parameter  $M$ . As a preliminary, we discuss neutrino mass generation in the left-right twin Higgs model in the next section.

### B.3 Neutrino Mass Seesaw at the TeV Scale

There is more than one way to implement neutrino mass in the left-right twin Higgs model. For a detailed study of neutrino mass generation in this context, see [15]. If lepton number is a good symmetry of the theory, the neutrino masses must be Dirac. In this case, the smallness of the neutrino masses can be understood purely as a result of their small Yukawa couplings. If lepton number is not conserved, the neutrino masses can be Majorana. In this case, the lightness of the SM neutrinos

can be understood as a result of the seesaw mechanism [16]. We will assume that left-right symmetry is exact in the neutrino sector as in all other sectors of the model. The most general collection of operators that generate neutrino masses are the following: Dirac neutrino masses arise from the operators

$$\begin{aligned} y_\nu \bar{L}_R H_R^\dagger H_L L_L / \Lambda + \text{h.c.} &\rightarrow y_\nu \frac{f v}{\Lambda} \nu \nu_R + \text{h.c.} \\ &= m_D \nu \nu_R \end{aligned} \quad (\text{B.9})$$

while the operators

$$y_1 (L_R \hat{H}_R \hat{H}_R L_R + L_L \hat{H}_L \hat{H}_L L_L) / \Lambda + \text{h.c.} \rightarrow y_1 \frac{\hat{f}^2}{\Lambda} \nu_R \nu_R + \text{h.c.} \quad (\text{B.10})$$

$$y_2 (L_R H_R H_R L_R + L_L H_L H_L L_L) / \Lambda + \text{h.c.} \rightarrow y_2 \left( \frac{f^2}{\Lambda} \nu_R \nu_R + \frac{v^2}{\Lambda} \nu_L \nu_L \right) + \text{h.c.} \quad (\text{B.11})$$

generate Majorana masses for the right-handed neutrinos  $\nu_R$  and the left handed neutrinos  $\nu_L$ . One possibility is that we assume lepton number is not violated. In this case, the operators in eq. (B.10) and eq. (B.11) are not present. The light neutrinos  $\nu = (\nu_L, \bar{\nu}_R)$  are Dirac fermions and the small neutrino masses are just the result of small Yukawa couplings, which are around  $10^{-12}$ . The other possibility is that light Majorana neutrinos are generated through a TeV scale seesaw mechanism. If we no longer assume lepton number conservation, all the operators above are allowed. This allows the SM neutrinos to obtain a Majorana mass of the right size if the coupling constant  $y_\nu$  is  $\sim 10^{-5}$ , which is of order the electron Yukawa coupling.

The Dirac neutrino case is straightforward and free of constraints, but the Majorana neutrino case is more subtle. Since the operator in eq. (B.11) gives both  $\nu_R$  and  $\nu_L$  a Majorana mass, this term should be small, i.e.  $y_2 < 10^{-11}$ . To achieve a seesaw with Yukawa couplings of order SM Yukawa couplings, the first term is necessary with  $y_1 \sim O(1)$ . We will follow this possibility from now on, and assume  $y_2 = 0$ . A  $Z_4$  symmetry where  $H$  is neutral may be used to justify this possibility.

### B.3.1 Constraints on Majorana Right-handed Neutrinos

Right-handed neutrinos with masses of order a TeV have recently been studied by several authors [17]. What are the constraints on right-handed neutrinos? There are severe constraints on light degrees of freedom from BBN. However, particles that are heavier than an MeV which do not decay in the era of BBN are completely free of this constraint. Another bound comes from tritium decay, but that also only constrains light particles with masses less than about an MeV. There are stronger restrictions on massive right-handed neutrinos from precision measurements of  $Z$ -decay and single  $\nu_R$  production [18, 19]. However, we will only consider right-handed neutrino masses of order a few hundred GeV, which are free from these constraints.

If the right-handed neutrino is Majorana, the most stringent bound on its mass is from neutrinoless double beta decay [20, 21]. The bound can be approximately expressed as

$$\frac{m_{\nu_R}}{p^2 - m_{\nu_R}^2} \prod_{i=1,2} \frac{V_{i,q} V_{i,l}}{g_2^2} \left( \frac{m_W^2}{m_{x_i}^2} \right) \leq \frac{\text{eV}}{p^2}, \quad (\text{B.12})$$

where  $m_{x_i}$  are masses of the particles that mediate beta decay and  $V_{i,q}$  and  $V_{i,l}$  are the corresponding couplings to quarks and leptons, respectively.  $p$  is the typical energy exchanged in the process, which is of order 100 MeV. For example, in an extension of the SM with only right-handed neutrinos and the seesaw mechanism,  $x_i = W^\pm$  and the couplings are  $V_{i,l} = g_2 \delta_{ss}$  where  $\delta_{ss}$  is the seesaw mixing factor,  $\delta_{ss} \sim \frac{m_D}{m_{\nu_R}} \sim \sqrt{\frac{m_\nu}{m_{\nu_R}}}$ . For a TeV scale right-handed neutrino,  $\delta_{ss} \sim 10^{-7}$ . In general, several diagrams may contribute to neutrinoless double beta decay and various parameters will be constrained by experiment. For the left-right twin Higgs, the diagrams with standard model  $W$  exchange will involve the seesaw mixing factor and therefore are much more suppressed than those with  $W_R$  exchange which, are only suppressed by the mass of the  $W_R$ . Other subdominant diagrams with charged Higgs exchange will also be suppressed. The contribution from  $W_R$  to neutrinoless double beta decay therefore leads to the tightest constraint. Using the experimental bound given in eq. (B.12), we find

$$m_{\nu_R} m_{W_R}^4 \geq 0.4 \text{ TeV}^5. \quad (\text{B.13})$$

The lower bound of the right-handed neutrino mass is then

$$m_{\nu_R} \geq 60 \left( \frac{1.6 \text{ TeV}}{m_{W_R}} \right)^4 \text{ GeV}, \quad (\text{B.14})$$

which is well below the range of mass we will be considering. We will from now on treat the neutrino mass  $m_{\nu_R}$  as a free parameter ranging from 500 - 1500 GeV and study its collider phenomenology.

#### B.4 $M = 0$ Phenomenology

The phenomenology of the left-right twin Higgs has been studied by many authors [10, 22]. We will focus on the limit where the top mixing parameter  $M$  is set to zero. When  $M = 0$  and the right-handed neutrino is heavier than  $W_R$ , some of the new particles including  $W_R$  and heavy top partner  $T_H$  are difficult to detect because their decay channels are dominated by hadronic final states. Here we consider the limit when the right-handed neutrino mass is less than the mass of  $W_R$ . The search for  $W_R$  will then be much more effective due to the opening of leptonic decay channels. Even better, the leptonic decay has a 50% chance of violating lepton number due to the Majorana nature of  $\nu_R$ . The same advantages will also apply to  $T_H$ , but these searches depend on the mass of  $\nu_R$  relative to the mass of  $T_H$ . There are the two possibilities: (i)  $m_{\nu_R} < m_{T_H}$  and (ii)  $m_{\nu_R} > m_{T_H}$ , which we consider separately. In the following analysis we choose the following typical parameter set:  $f = 800 \text{ GeV}$ , which implies  $\hat{f} \approx 4 \text{ TeV}$ ,  $m_{W_R} \approx 1.9 \text{ TeV}$  and  $T_H \approx 780 \text{ GeV}$  for a reasonable choice of soft parameters. A different choice of soft parameters will lead to a different set of masses, so these are not strict mass relations. However, this will not qualitatively affect our conclusions. Let us begin with a discussion of the search for  $W_R$ , which can be done independently from  $T_H$ .

##### B.4.1 $W_R$ Search

The  $W_R$  is dominantly produced via a Drell-Yan process and subsequently decays leptonically to  $\nu_R + l^\pm$  with a branching fraction of about a 10% .  $\nu_R$  then decays to



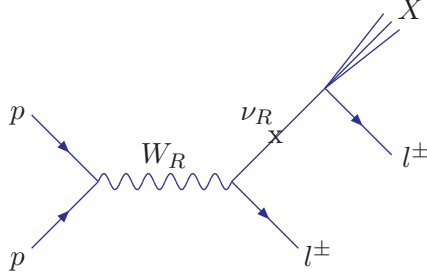


Figure B.2: Diagrammatic view of  $W_R$  production and its lepton number violating decay channel.

$l^\pm + X$  through an off shell  $W_R$  or an on shell charged Higgs  $\phi^\pm$ , as shown in Fig. B.2. Here  $X$  represents any number of final state jets. Due to the fact that  $\phi^\pm$  only decays hadronically and the leptonic decays of the off shell  $W_R$  are kinematically forbidden,  $X$  cannot contain any leptons. As argued in the section B.2.2, this is precisely why the heavy top decays purely hadronically in the decoupling limit when  $\nu_R$  is heavier than  $W_R$ . Since  $\nu_R$  is Majorana, half of these events will contain same sign leptons. The signal is therefore same-sign dilepton  $l^\pm l^\pm X$  events without missing energy, which has no genuine SM background.

The production cross section at the LHC of  $\nu_R + l^\pm$ , where  $l = e, \mu$ , is shown in Fig. B.3 as a function of the right-handed neutrino mass  $m_{\nu_R}$ . Half of these events will be same-sign dilepton lepton events without missing energy. For example, if  $m_{\nu_R} = 1$  TeV, the production cross section is about 300 fb, which leads to approximately 4500 same sign dilepton events with  $30 \text{ fb}^{-1}$  of total luminosity. The invariant mass distribution of all the final state particles should provide a clear signal of  $W_R$ . Furthermore, the invariant mass distribution of the jets plus one lepton should provide a signal of  $\nu_R$ .

As mentioned above, there is no genuine SM background for the same sign lepton signal and so the background at the LHC is purely instrumental. This mostly arises from a mismeasurement of missing energy and/or lepton's charge. The misidentification of the lepton's charge is expected to be around a few percent and the

resolution of measuring missing energy is about a few tens of GeV [23]. The dominant SM background for  $l^\pm l^\pm jj$  is  $pp \rightarrow W^\pm W^\pm W^\pm$ , which is about 0.04 fb after making suitable cuts. [17]. We do not expect these cuts to reduce the  $lljj$  signal and therefore the background should be less than 0.04 fb. Therefore, in this scenario it should be possible to observe the right-handed gauge boson  $W_R$  and the right-handed neutrino  $\nu_R$  at the LHC.

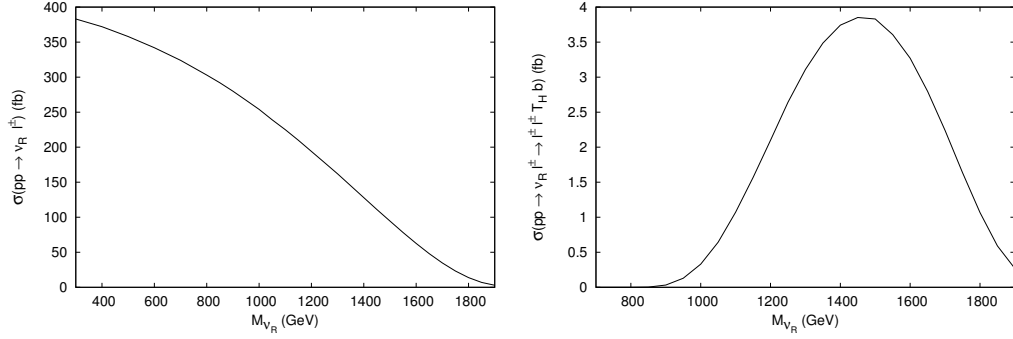


Figure B.3: The left plot shows the production cross section of the right handed neutrino  $\nu_R + l^\pm$  with an associated lepton ( $e^\pm$  or  $\mu^\pm$ ) as a function of  $m_{\nu_R}$ . The right plot shows the production cross section of the heavy top  $T_H$  through the decay of  $\nu_R$  in association with *same-sign* leptons ( $e^\pm$  or  $\mu^\pm$ ) as a function of  $m_{\nu_R}$ .

#### B.4.2 $T_H$ Search: $m_{\nu_R} > m_{T_H}$

In this case,  $T_H$  decays solely to  $b + \phi^\pm$  and results in an all jet final state. However,  $T_H$  can also be produced by the decay of  $\nu_R$  through an off shell  $W_R$ . The process is

$$pp \rightarrow l\nu_R \rightarrow llbT_H \rightarrow llbbjj \quad (\text{B.15})$$

with the cross section

$$\sigma(pp \rightarrow T_H llb) \approx \sigma(pp \rightarrow \nu_R l^\pm) \times Br(\nu_R \rightarrow l^\pm T_H b), \quad (\text{B.16})$$

which is a few fb, as shown in Fig. B.3. By requiring that the leptons be of the same sign and  $b$  tagging, it should be possible to separate these events from background

at the LHC. The invariant mass distribution of the two jets plus one  $b$ -jet should provide a signal of  $T_H$ .

As with the same sign  $lljj$  signal, the background for same sign  $pp \rightarrow llbbjj$  is purely instrumental. Since the background for the  $lljj$  final state can be reduced to less than 0.1 fb, we expect the  $llbbjj$  background to be smaller than 0.1 fb. Therefore, it should be possible to detect the heavy top partner  $T_H$  at the LHC, provided that  $m_{\nu_R} > m_{T_H}$ .

#### B.4.3 $T_H$ Search: $m_{\nu_R} < m_{T_H}$

To observe the leptonic decays of the heavy top in this case, we must look for the decays of  $T_H$  to  $\nu_R$ . Once produced via  $W_R$  decay,  $T_H$  can decay to  $\nu_R + b + l^\pm$  through an off shell  $W_R$ . However, there is another decay channel which does not involve  $\nu_R$ ,  $T_H \rightarrow \phi^\pm + b$ . As discussed above,  $\phi^\pm$  decays to jets, so the signal is either  $pp \rightarrow llbbjj$  or  $pp \rightarrow bbjj$ . The cross sections for these processes are determined by the partial decay width of  $T_H$  to these channels

$$\begin{aligned}\Gamma_{T_H \rightarrow \nu_R l^\pm b} &\sim 10^{-5} \text{ GeV} \\ \Gamma_{T_H \rightarrow b \phi^\pm} &\sim 3 \text{ GeV}.\end{aligned}\tag{B.17}$$

As expected, the two-body decay dominates the decay width, making the branching fraction  $\text{Br}(T_H \rightarrow \nu_R b l^\pm)$  very small. The cross section for  $pp \rightarrow T_H b \rightarrow bbljj$  is then about  $10^{-3}$  fb, which is too small to be observed at the LHC. Therefore, in this case it will not be possible to detect the heavy top partner  $T_H$ .

## B.5 Conclusion

In summary, we have shown that a TeV scale right-handed neutrino in the left-right twin Higgs model leads to interesting lepton number violating signatures for  $W_R$  and  $T_H$  at the LHC, provided that  $m_{\nu_R} < m_{W_R}$ . Lepton number violating decays of right-handed  $W_R$  should be observable provided that  $W_R$  and  $\nu_R$  are not nearly degenerate. Detection of the heavy top is possible if  $m_{\nu_R} > m_{T_H}$ . These signals may

be used to complement other collider searches for  $W_R$  and  $T_H$ . In the limit that  $M \rightarrow 0$ , these signatures may be the only way to observe the particles responsible for natural electroweak symmetry breaking in the left-right twin Higgs.

## B.6 Acknowledgments

We thank Zackaria Chacko for valuable discussions and comments on the draft. C. K is supported by the NSF under grant PHY-0801323. H.S.G is supported by the NSF under grant PHY-04-57315 and by the DOE under grant DE-AC02-05CH11231.

## REFERENCES

- [1] R. Barbieri and A. Strumia, arXiv:hep-ph/0007265.
- [2] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B **513**, 232 (2001)
- [3] N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire and J. G. Wacker, JHEP **0208**, 021 (2002); N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP **0207**, 034 (2002); T. Gregoire and J. G. Wacker, JHEP **0208**, 019 (2002); I. Low, W. Skiba and D. Smith, Phys. Rev. D **66**, 072001 (2002) D. E. Kaplan and M. Schmaltz, JHEP **0310**, 039 (2003).
- [4] H. C. Cheng and I. Low, JHEP **0309**, 051 (2003) [arXiv:hep-ph/0308199]; H. C. Cheng and I. Low, JHEP **0408**, 061 (2004) [arXiv:hep-ph/0405243]; H. C. Cheng, I. Low and L. T. Wang, arXiv:hep-ph/0510225.
- [5] H. Georgi and A. Pais, Phys. Rev. D **10**, 539 (1974); Phys. Rev. D **12**, 508 (1975).
- [6] D. B. Kaplan and H. Georgi, Phys. Lett. B **136**, 183 (1984); D. B. Kaplan, H. Georgi and S. Dimopoulos, Phys. Lett. B **136**, 187 (1984); H. Georgi and D. B. Kaplan, Phys. Lett. B **145**, 216 (1984).
- [7] Z. Chacko, H. S. Goh and R. Harnik, Phys. Rev. Lett. **96**, 231802 (2006) [arXiv:hep-ph/0506256]; R. Barbieri, T. Gregoire and L. J. Hall, arXiv:hep-ph/0509242. Z. Chacko, Y. Nomura, M. Papucci and G. Perez, JHEP **0601**, 126 (2006) [arXiv:hep-ph/0510273].
- [8] Z. Chacko, H. S. Goh and R. Harnik, JHEP **0601**, 108 (2006) [arXiv:hep-ph/0512088];
- [9] H. S. Goh and C. A. Krenke, Phys. Rev. D **76**, 115018 (2007) [arXiv:hep-ph/0707.3650].

- [10] H. S. Goh and S. Su, Phys. Rev. D **75**, 075010 (2007) [arXiv:hep-ph/0611015].
- [11] A. Davidson and K. C. Wali, Phys. Rev. Lett. **59**, 393 (1987); S. Rajpoot, Mod. Phys. Lett. A **2**, 307 (1987) [Erratum-ibid. A **2**, 541 (1987 PHLTA,B191,122.1987)].
- [12] D. Chang and R. N. Mohapatra, Phys. Rev. Lett. **58**, 1600 (1987).
- [13] K. m. Cheung, Phys. Lett. B **517**, 167 (2001) [arXiv:hep-ph/0106251]; T. Appelquist, B. A. Dobrescu and A. R. Hopper, Phys. Rev. D **68**, 035012 (2003) [arXiv:hep-ph/0212073].
- [14] E. M. Dolle and S. Su, Phys. Rev. D **77**, 075013 (2008) [arXiv:hep-ph/0712.1234].
- [15] A. Abada and I. Hidalgo, Phys. Rev. D **77**, 113013 (2008) [arXiv:hep-ph/0711.1238].
- [16] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity* (North-Holland, Amsterdam, 1979);  
T. Yanagida, in *Proc. Workshop on Unified Theories and Baryon Number in the Universe*;  
R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
- [17] T. Han and B. Zhang, Phys. Rev. Lett. **97**, 171804 [arXiv:hep-ph/0604064];  
A. Atre, T. Han, S. Pascoli and B. Zhang, arXiv:0901.3589 [hep-ph].
- [18] O. Adriani *et al.* [L3 Collaboration], Phys. Lett. B **295**, 371 (1992).
- [19] P. Achard *et al.* [L3 Collaboration], Phys. Lett. B **517**, 67 (2001) [arXiv:hep-ex/0107014].
- [20] C. Arnaboldi *et al.*, arXiv:0802.3439 [hep-ex].
- [21] S. Pascoli and S. T. Petcov, arXiv:0711.4993 [hep-ph].

- [22] D. W. Jung and J. Y. Lee, arXiv:hep-ph/0701071; Y. B. Liu, H. M. Han and Y. H. Cao, arXiv:hep-ph/0703268; Y. B. Liu and J. F. Shen, arXiv:hep-ph/0704.0840; Y. B. Liu, H. M. Han and Y. H. Cao, arXiv:hep-ph/0701169; D. W. Jung and K. Y. Lee, arXiv:hep-ph/0701087; W. Ma, C. X. Wei and Y. Z. Wang, arXiv:hep-ph/0905.0597; C. X. Yue, H. D. Yang and W. Ma, arXiv:hep-ph/0903.3720; Y. B. Liu, Y. H. Cao and H. M. Han, Eur. Phys. J. C **53**, 615 (2008) [arXiv:hep-ph/0701169].
- [23] M. D’Alfonso and F. Sarri, “Jet and missing E(T) reconstruction and calibration in ATLAS and CMS,” *Prepared for 2nd Italian Workshop on the Physics of Atlas and CMS, Naples, Italy, 13-15 Oct 2004*; A. Ghezzi and C. Roda, “Jet and missing E(T) reconstruction in ATLAS and CMS,” M. Wielers [ATLAS Collaboration], “Simulation studies of the jet and missing transverse energy performance of the ATLAS calorimeters,” *Prepared for 10th International Conference on Calorimetry in High Energy Physics (CALOR 2002), Pasadena, California, 25-30 Mar 2002*