SLAC–PUB–6527
June 1994
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Depolarization in the SLC Collider Arcs*

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Abstract

In the 1993 running cycle of the Stanford Linear Collider, electron spin polarization measurements with a Møller polarimeter at the end of the linac and a Compton polarimeter near the interaction point (IP) indicated a relative polarization loss of up to 20% across the arc. We present calculations of the depolarizing effects where variations in energy, energy spread and transverse emittance as well as changes in orbit and initial spin orientation are taken into account. We compare our results with measurements and conclude that, in standard operating conditions, the relative polarization loss is only 3±2%.

1. DEPOLARIZATION DUE TO INITIAL LINAC ENERGY SPREAD

The motion of the spin expectation value of the beam, is given by the BMT equation [1],

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S} . \tag{1}$$

The spin $\vec{S}$ rotates around the magnetic field $\vec{\Omega}$ in the rest system of the electron. If an electron is deflected in a transverse magnetic field by an angle θ, the spin is rotated around the field axis by

$$\phi = a\gamma \cdot \theta \ , \tag{2}$$

where a is the anomalous momentum of the electron and γ the Lorentz factor.

The spin of an off-energy particle ($\delta \equiv \Delta E/E_0$) is rotated by an angle of $\Delta\phi$ with respect to the longitudinal spin of the on-energy particle, so its contribution to the longitudinal component of beam polarization is

$$P_z(\delta) = P_0 \cos(\Delta\phi) . \tag{3}$$

The SLC runs at an energy chosen for peak rate of Z^0 production — 45.64 GeV at the IP. At this energy the spin tunes and betatron tunes are equal and therefore in resonance in the collider arcs. This resonance has been exploited to control the spin orientation at the IP by means of vertical orbit bumps in the arc [2].

Figure 1 shows the projection of the spin vector of an off-energy electron (0.3%) onto the spin vector of an on-energy electron (*i.e.* the cosine of the angle between spin vectors). For horizontal initial spin orientation and a perfect arc orbit, the relative angle between the two particles continually increases along the arc until the arc bend field reverses sign (s = 400 m). After this reverse bend section, the relative angle decreases again as the two spin vectors realign at a point ~2/3 through the arc (s = 850 m). The last 1/3 of the arc spreads the spin vectors again. However, if the same two electrons are transported through the arc with an initial vertical betatron oscillation of ~0.3 mm, the spin vectors never realign again and eventually *depolarize* even more than before. This simple model demonstrates the depolarization dependence on arc orbit.

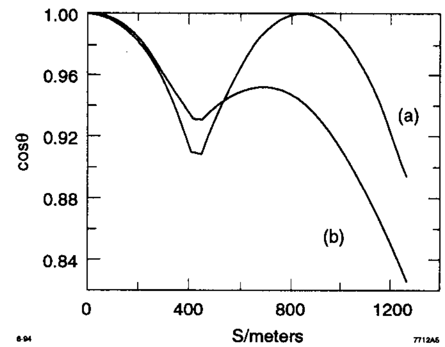

Figure 1. Projection of an off-energy (0.3%) electron spin vector onto that of an on-energy electron along the arc for *a*) a perfect orbit, and *b*) a vertical betatron oscillation. Initial spin orientations are horizontal.

The strong spin sensitivity to arc orbit errors produces a fairly complex, non-planar electron spin rotation through the arc. Since the absolute orbit is not precisely known, the actual evolution of these rotations along the arc are not known. Therefore, although the net horizontal bend angle of the arc is $\pi/2$, the effective bend angle, θ, is written as

$$\theta = f\pi/2 \ , \tag{4}$$

with f used to express the effective net bend deviation from the nominal $\pi/2$. We approximate the polarization contribution of an off energy particle with

$$P_z(\delta) \approx P_0 \cos(a\gamma_0 \delta\theta) \ , \tag{5}$$

or, using (4)

$$P_z(\delta) \approx P_0 \cos(a\gamma_0 \delta f\pi/2) . \tag{6}$$

This approximation is possible because, although the many rotations incurred along the full arc are non-planar, the small additional rotations induced by energy deviations at the scale of the beam energy spread (~0.2% rms) are performed approximately in a plane. This has been verified with measurements and also through detailed spin transport simulations which include varying arc orbits and other errors.

* Work supported by Department of Energy contracts DE-AC03-76SF00515 and DE-AC05-84ER40150

Presented at the Fourth European Particle Accelerator Conference (EPAC'94), London, England, June 27-July 1, 1994

The effective net bend deviation, f, is measured by varying the electron beam energy and recording the longitudinal IP polarization, $P_z(\delta)$, at each energy point. This measurement is performed at low electron beam current ($\sim 1\times10^{10}$) in order to minimize energy spread (<0.1% rms). Figure 2 below shows these measurements for two different initial linac spin orientations [3].

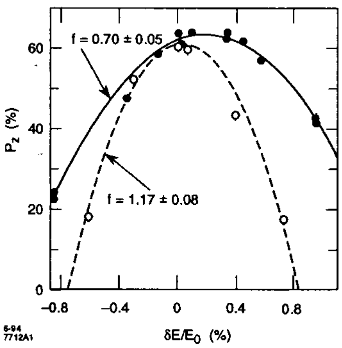

Figure 2. Longitudinal polarization vs. energy deviation in two conditions: *a*) arc orbit and initial vertical spin orientation of 1993 colliding conditions, *b*) longitudinal initial spin orientation with arc orbit to rotate IP spin orientation back into longitudinal plane. The data is used to fit for f. Solid curves are best fits.

Using (6) and the measurement of f, the mean IP polarization can be calculated by integrating over the particle energy distribution, $N(\delta)$, which is assumed Gaussian here.

$$<P_z> = P_0\int N(\delta)\cdot\cos(a\gamma_0\delta\theta)\, d\delta$$

$$= P_0 e^{-(a\gamma_0\sigma_\delta f\pi/2)^2/2} \qquad (7)$$

Using (7) and the following SLC parameters which reflect typical 1993 colliding conditions,

$$\begin{aligned} a\gamma_0 &= 105\ (\overline{E}_{arc} = 46.2\text{ GeV}) \\ \sigma_\delta &= 0.20 \pm 0.05\%\text{ rms} \\ f &= 0.70 \pm 0.05 \end{aligned} \qquad (8)$$

the relative depolarization due to a Gaussian linac energy distribution is

$$1 - \frac{\langle P_z \rangle}{P_0} = 0.03 \pm 0.02\ . \qquad (9)$$

If a uniform linac energy distribution is used in (7) the resulting depolarization is numerically very similar. However, the presence of large low energy beam tails have been observed to impact the net depolarization by up to a few percent. Detailed measurements of these effects, including variations in energy tail collimation over long term are discussed elsewhere [5],[6].

Measurements are made to confirm this result by varying the linac rms energy spread and recording longitudinal IP polarization. Figure 3 shows these measurements overlaid with the expected analytical curves from (7) done for both vertical and longitudinal initial spin orientation at the end of the linac.

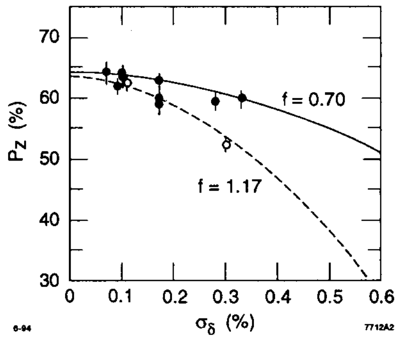

Fig 3. Measured longitudinal IP beam polarization vs. energy spread for two conditions: *a*) vertical initial spin orientation at end-of-linac, *b*) longitudinal initial spin. Solid curves are the analytical model using (7) and measurements of f in figure 2.

Figure 3 and (7) show the importance of the initial spin orientation. An initial vertical spin orientation produces less depolarization since off energy particles in the arc horizontal bending fields simply rotate around the vertical axis. Therefore, it is desirable to maintain a vertical spin for as much of the arc transport as possible. The result is a smaller value of f and a higher IP polarization degree. In this respect, the spin-betatron tune resonance of the arc is fortuitous. By switching off the two upstream spin rotator solenoids and using the arc orbit to orient the spin, we have chosen the optimal end-of-linac spin orientation (vertical) to minimize depolarization due to energy spread in the arcs. With no resonance, the necessary end-of-linac spin orientation would be oriented in the X/Z plane producing f=1.0 and a depolarization, from (7), of 5%.

Simulations were used to study the effect of arc orbit errors on the value of f. A computer code was generated which tracks single electrons through the SLC arc, including magnet misalignments and betatron oscillations, where the spin dynamics are calculated correctly without the approximations of (5) and (6).

Twenty different arc misalignment seeds were run which produce 1.1 mm rms arc orbits in both planes and simulated measurements of f were made just as in figure 2. Figure 4 below shows a histogram of these f values over the twenty misalignment seeds. The results show that a value for f of 0.7 is highly likely given realistic arc orbit errors.

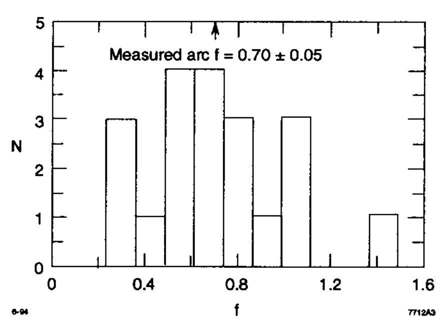

Fig 4. Histogram of simulated f measurements over twenty seeds of random arc orbits with 1.1 mm rms. The measured value of f=0.7 appears to be highly likely.

Figure 4 also indicates that it may be possible to find an arc orbit which simultaneously produces a smaller value of f (less depolarization) while still maintaining a longitudinal IP polarization. This may be done by choosing the front arc orbit for minimum f (maintaining vertical polarization through the front arc) and then varying the back arc orbit to rotate the IP spin into the longitudinal direction. This has been confirmed in simulations where a minimum value of f = 0.237 was achieved while still maintaining longitudinal IP polarization using the back arc orbit. Since the depolarization given by (7) has an approximate quadratic dependence on f, the net depolarization in this case is reduced from 3.0% to 0.3%. However, since the details of the actual spin rotations through the arc are unknown, this procedure applied to the real machine would be reduced to blind trials.

2. DEPOLARIZATION DUE TO SYNCHROTRON RADIATION

A second type of energy spread related depolarization is due to synchrotron radiation within the arc. This additional energy spread is generated along each of the 460 bend/quadrupole magnets within the arc and therefore cannot be analytically treated in the same way as incoming linac energy spread. At an IP energy of 45.64 GeV (46.67 GeV at end-of-linac) the additional rms energy spread accumulated through the arc due to synchrotron radiation is expected to be 0.073%. Monte Carlo estimations of this effect have been made by tracking 100 radiating electrons through the arc and calculating the resulting average polarization magnitude. Five different seeds of random arc orbit errors, all of which have f≈0.7, produce a mean IP depolarization over the five seeds of 0.32% with a 0.15% rms variation. The actual depolarization due to synchrotron radiation depends on the details of the spin transport through the arc which are not known. However, these Monte Carlo results suggest that the effect is small compared to the effect of incoming energy spread.

3. DEPOLARIZATION DUE TO FINITE EMITTANCE

The final IP spin orientation of a single electron is sensitive to its actual trajectory through the arc, especially in the vertical plane. Therefore, it is expected that a finite emittance beam should be somwhat depolarized through the arc. Monte Carlo studies are used to evaluate this depolarization. Figure 5 shows tracking results using 100 electrons where the initial emittance is varied. Some actual measurements are included for comparison, however the emittance increase necessary for a measurable change is large enough to make these measurements difficult — vertical beam size in the arcs is ~15 μm rms, while a ~500 μm oscillation is required to rotate the spin vector by $\pi/2$.

For typical 1993 SLC flat beam emittances of $\gamma\varepsilon_x$ = 45 mm-mrad, $\gamma\varepsilon_y$ = 8 mm-mrad, the net relative depolarization is expected to be ~0.4%.

4. SOKOLOV AND TERNOV EFFECT

The Sokolov and Ternov depolarization [7] should be extremely small (~1×10^{-7}). A storage ring with energy and bend radius of the arc would have a polarization rise time of ~20 seconds. Beam traverses the arc in a few micro-seconds.

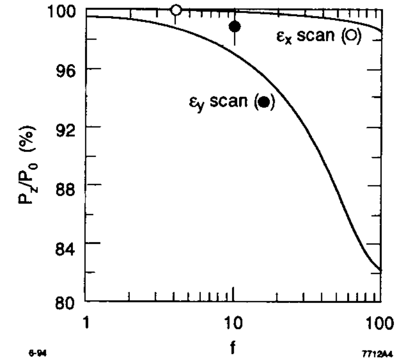

Fig 5. Relative IP polarization versus initial electron beam emittances (solid curve represents vertical emittance scan, dashed curve is horizontal emittance scan). Measurements are included.

5. CONCLUSIONS

All of the depolarizing mechanisms discussed here are small compared to depolarization due to initial linac energy spread. Since these different effects are also not correlated, we neglect all but this dominant depolarization and conclude that for the nominal colliding conditions of 1993 the net SLC arc depolarization is 3±2%. Furthermore, the existence of a spin-betatron resonance has given us the ability to use the arc orbit as a spin rotator and has therefore allowed the injection of a vertical spin orientation which is the most resilient to these depolarizing effects. The possibility exists that arc depolarization may be further minimized by judicious choice of arc orbit, however this is realistically a time consuming, if not difficult task to accomplish.

6. ACKNOWLEDGMENTS

We thank Franz-Josef Decker for many helpful suggestions concerning arc depolarization, and Ross King for initially pointing out that depolarization can be orbit dependent. We also thank Mike Fero, Peter Rowson, Bruce Schumm, Morris Swartz, and Mike Woods for many helpful discussions as well as many fast, accurate Compton polarimeter measurements.

7. REFERENCES

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