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Nuclear structure studies in the 0s1d shell applied to astrophysical rp-process calculations

W. A. Richter[1,2], B. Alex Brown[3]

[1] *iThemba LABS, P.O.Box 722, Somerset West 7129, South Africa*

[2] *Department of Physics, University of the Western Cape, Private Bag X17, Bellville 7535, South Africa and*

[3] *Department of Physics and Astronomy, and National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824-1321, USA*

We present results for levels in ^{30}S (the mirror of nucleus ^{30}S) that are used in rp reaction rate calculations. As the properties of only a few levels in ^{30}S are known, most are determined from the Isobaric Mass Multiplet Equation and the binding energies of the T=1 analog states. Where the analog states are not known the levels are calculated with the sd-shell interactions USDA and USDB. The gamma-decay lifetimes and ^{29}P to ^{30}S spectroscopic factors are also calculated from USDA and USDB, and together with experimental information on the levels of excited states are used to determine the ^{29}P(p,γ)^{30}S reaction rates. Some new results on the ^{35}Ar(p,γ)^{36}K reaction are also presented.

PACS numbers: 26.30.-k, 21.60.Cs, 21.10.Sf, 21.10.Tg

I. INTRODUCTION

In many cases levels of nuclei participating in rp processes have not been measured and one has to rely on theory to estimate the reaction rates. The Isobaric Mass Multiplet Equation (IMME) affords a reliable method of obtaining levels in the final T=1 nucleus of a (p,γ) reaction in terms of the isobaric analog partners and a coefficient c that can be calculated [1]. The lifetimes and the spectroscopic factors entering into the gamma widths and proton widths of the final nucleus are calculated from the sd-shell interactions USDA and USDB [2].

II. PROCEDURE FOR DETERMINING ^{30}S ENERGY LEVELS.

There are three different sources for the energies of ^{30}S that are input into the reaction rate calculations: 1) well-established experimental energies 2) predicted levels based on the IMME to calculate the expected energy of levels in ^{30}S by using the measured binding energies of the T=1 partners and a theoretical value of the c-coefficient of the IMME [3] 3) level energies calculated with the sd-shell interactions USDA and USDB.

The method used for 2) is explained in Ref. [1]. According to the IMME

$$B = a + bT_z + cT_z^2, \quad (1)$$

where B is the binding energy of a state.

Inserting the values of $T_z = 0$, 1 and -1, one obtains three equations, and rearranging one obtains for ^{30}S

$$B_{th}(^{30}\mathrm{S}) = 2B_{exp}(^{30}\mathrm{P}) - B_{exp}(^{30}\mathrm{Si}) + 2c_{th}. \quad (2)$$

c_{th} can be calculated from

$$c_{th} = [B_{th}(^{30}\mathrm{S}) + B_{th}(^{30}\mathrm{Si})]/2 - B_{th}(^{30}\mathrm{P}). \quad (3)$$

For the calculation of the b- and c-coefficients of the IMME we use the USDB Hamiltonian [2] for the charge-independent part and add the Coulomb, charge-dependent and charge-asymmetric nuclear Hamiltonian obtained by Ormand and Brown for the *sd* shell [3]. For the nuclei considered in [3], A=18-22 and A=34-39, the 42 b-coefficients were reproduced with an rms deviation of 27 keV and the 26 c-coefficients were reproduced with an rms deviation of 9 keV. There is considerable state-dependence in the c-coefficients (ranging in values from 130 keV to 350 keV) that is nicely reproduced by the calculations (see Fig. 9 in [3]).

In Fig. (1) values of c from experiment and theory are compared for the lowest few states in ^{30}S ordered according to increasing experimental energy. The experimental values are obtained for states where all three members of the multiplet are known. A very good correspondence can be seen, the largest deviations being less than 10 keV. As in Ref. [3] there is significant state dependence with c values from experiment ranging from about 180 keV to 275 keV. This IMME method was used in

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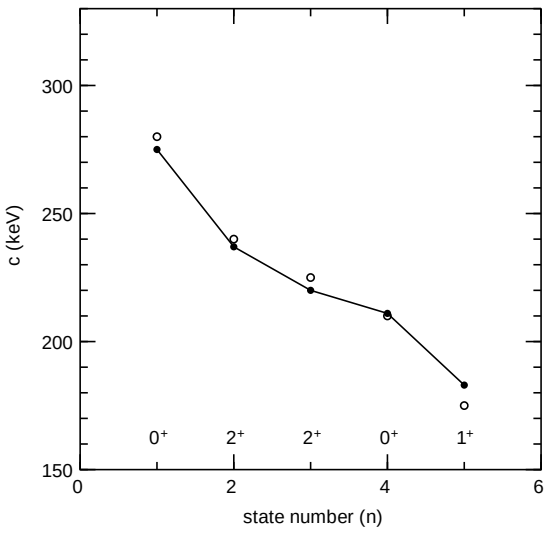

FIG. 1: c-coefficients from the isobaric mass multiplet equation (IMME: $E = a + bT_z + cT_z^2$) versus state number (in order of increasing energy) in ^{30}S based on experimental energies (closed circles) and energies calculated from USDB (open circles).

[4] for the T=1 states of the odd-odd nuclei with mass 28, 32 and 36 and in Ref. [1] for ^{26}Si.

Where data is not available in ^{30}S to determine the c-coefficient from experiment, a fairly reliable value can be obtained from a theoretical calculation using Eq. (3). The binding energies for states in ^{30}S can be then be obtained from Eq. (2), with experimental values of binding energy for corresponding states in ^{30}Si and ^{30}P (when they are known in both).

In Fig. (2) predicted energies in ^{30}S based on the IMME are compared with experimental excitation energies in the mirror nucleus ^{30}Si. The predicted energies are used in the reaction rate calculations but are supplemented with energies calculated with USDB and USDA where there is insufficient information on the T=1 analog states.

In a recent paper [5] some new levels in ^{30}S have been reported. The level observed at 4.693 MeV and given an assignment of 3^+ agrees very well with our predicted 3^+ level at 4.713 MeV. Also the level observed at 4.814 MeV and given an assignment of 2^+ agrees with our predicted 2^+ level at 4.798 MeV.

III. RESULTS FOR THE REACTION RATE

The resonant reaction rate for capture on a nucleus in an initial state i, $N_A < \sigma v >_{\mathrm{res}\,i}$ for isolated narrow resonances is calculated as a sum over

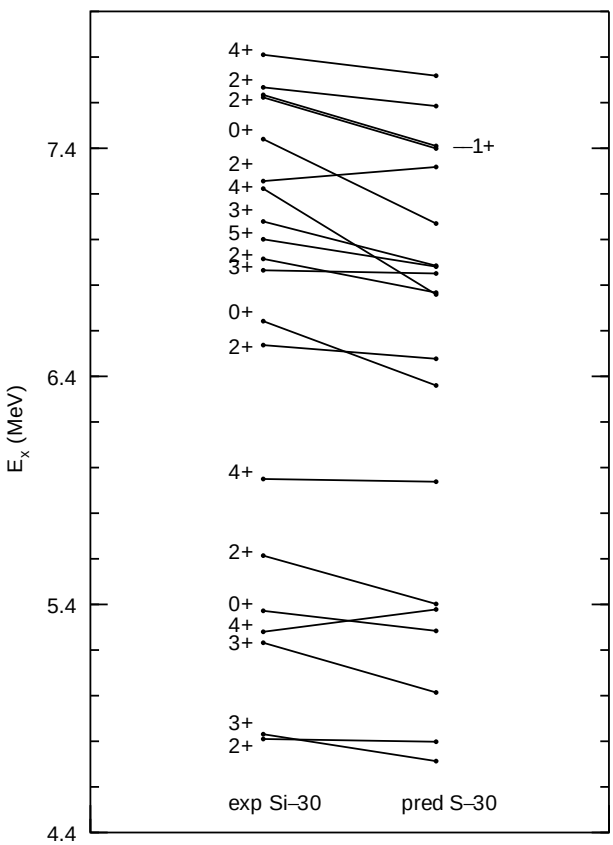

FIG. 2: Predicted excitation energies in ^{30}S above the proton emission threshold and experimental excitation energies in the mirror nucleus ^{30}Si.

all relevant compound nucleus states f above the proton threshold [6]

$$N_A < \sigma v >_{\mathrm{res}\,i} = 1.540 \times 10^{11} (\mu T_9)^{-3/2}$$

$$\times \sum_f \omega\gamma_{if} \; \mathrm{e}^{-\mathrm{E_{res}}/(kT)} \; \mathrm{cm}^3\,\mathrm{s}^{-1}\mathrm{mole}^{-1}. \qquad (4)$$

Here T_9 is the temperature in GigaK, $\mathrm{E}_{res} = \mathrm{E}_f - \mathrm{E}_i$ is the resonance energy in the center of mass system, the resonance strengths in MeV for proton capture are

$$\omega\gamma_{if} = \frac{(2J_f+1)}{(2J_p+1)(2J_i+1)} \frac{\Gamma_{\mathrm{p}\,if}\Gamma_{\gamma f}}{\Gamma_{\mathrm{total}\,f}}. \qquad (5)$$

$\Gamma_{\mathrm{total}\,f} = \Gamma_{\mathrm{p}\,if} + \Gamma_{\gamma f}$ is a total width of the resonance level and J_i, J_p and J_f refer to the target, the proton projectile ($J_p = 1/2$), and states in the final nucleus, respectively. The proton decay width depends exponentially on the resonance energy via the single-particle proton width and can be calculated from the proton spectroscopic factor

C^2S_{if} and the single-particle proton width $\Gamma_{\mathrm{sp}\,if}$ as $\Gamma_{\mathrm{p}\,if} = C^2S_{if}\Gamma_{\mathrm{sp}\,if}$. The single-particle proton widths were calculated from [7]

$$\Gamma_{\mathrm{sp}} = 2\gamma^2 P(\ell, R_c), \tag{6}$$

with $\gamma^2 = \frac{\hbar^2 c^2}{2\mu R_c^2}$ and where the ℓ-dependent channel radius R_c was chosen to match the widths obtained from an exact evaluation of the proton scattering cross section from a Woods-Saxon potential well for $Q = 0.1 - 0.4$ MeV. The simpler model of Eq. (6) matches the results obtained from the scattering cross sections to within about 10%. We use a Coulomb penetration code from Barker [8].

The total rp reaction rates have been calculated for the interactions USDA and USDB. The Q values required were based on measured energies in ^{30}S, and where they were not known values calculated from Eq. (2) were used. Fig. (3) shows the results for the resonance-capture rate obtained using the properties of ^{30}S. The Γ_p and Γ_γ in this case are all based on the USDB Hamiltonian. It is evident that there are several resonances contributing substantially to the rate as the temperature changes.

A similar result for ^{35}Ar(p,γ)^{36}K is shown in Fig. (4). When measurements for negative parity states are not available, one could in principle estimate their effect from a theoretical calculation. However, this is often not practical because of the increase in size of the model space required. An alternative would be to use experimental values of the mirror nucleus. When properties of levels in the final nucleus are uncertain, the crucial parameters of the reaction rate calculations, viz. single-nucleon spectroscopic factors connecting the target and final states, and the lifetimes of the states in the final nucleus are frequently used and can be justified on the basis of isospin symmetry. In view of the correspondence between mirror states for A = 36 it would be reasonable to substitute an experimental value from the mirror nucleus in a case where a calculation is not feasible, as for the 3^- state at 2.468 MeV. In this way the contribution from this level, which lies close to some of the most important resonances, can be taken into account approximately.

IV. UNCERTAINTIES IN THE RESONANT CAPTURE REACTION RATES

A detailed analysis of error sources in the rate calculations has been given in Ref. [1]. A general indication of the variation caused by the use of different interactions can be obtained by comparing the corresponding reaction rates. As an example this is shown in Fig. (5) for the reaction ^{35}Ar(p,γ)^{36}K.

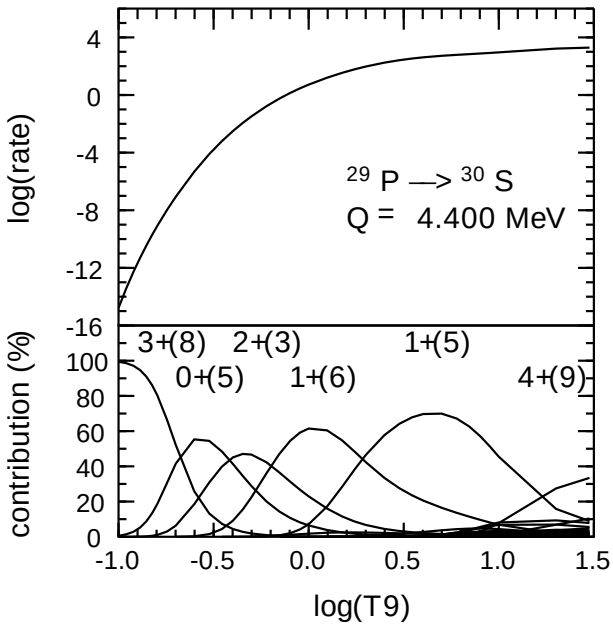

FIG. 3: The total rp reaction rate versus temperature T9 (GigaK) (top panel) and the contribution of each of the final states (lower panel) with USDB. Γ_γ was calculated for ^{30}S levels.

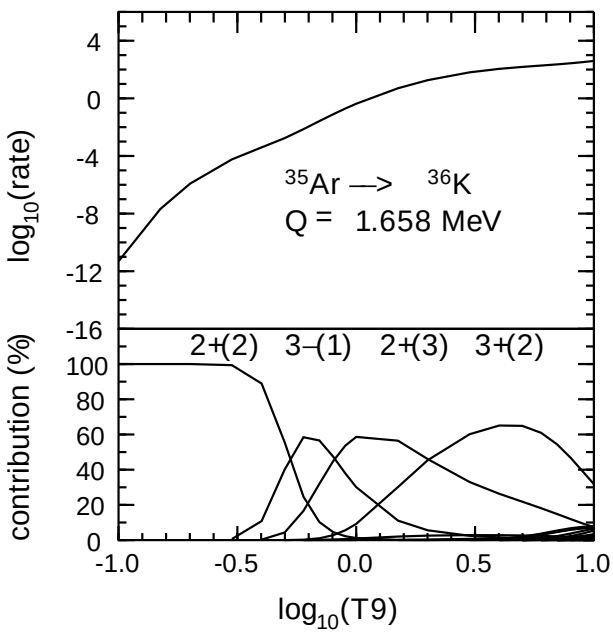

FIG. 4: The total rp reaction rate versus temperature T9 (GigaK) (top panel) and the contribution of each of the final states (lower panel) with USDB. Γ_γ was calculated for ^{36}K levels.

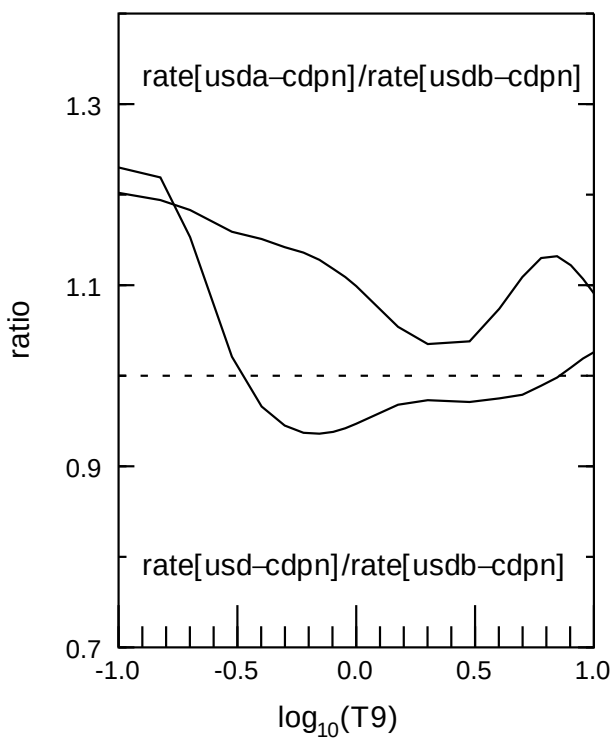

FIG. 5: The total rp reaction rates of USDA versus USDB compared for ^{36}K

V. CONCLUSIONS

Because the calculation of the rp reaction rate for the ^{29}P(p,γ)^{30}S requires a knowledge of the energy levels in ^{30}S, and some levels are uncertain, we have adopted the method of [4] for determining levels which is partly based on experiment and partly on theory. For the experimental part we used well-known binding energies of the T=1 analogue states of ^{30}S. For the theoretical part we used calculated c-coefficients of the isobaric mass multiplet equation. We have demonstrated that a good correspondence between theoretical and experimental values of the c-coefficient for sd-shell nuclei exists. The method leads to a reliable prediction of energy levels in ^{30}S. Using energy values in ^{30}S constrained by our method for the Q values of the proton capture process on ^{29}P, we obtained the required spectroscopic factors and gamma decay lifetimes for rate calculations from shell-model calculations using the new sd-shell interactions USDA and USDB. Our predicted levels for the 3^+ and 2^+ states just above the proton emission threshold agree very well with measurements reported in Ref. [5] and substantiates the assignments made there.

A similar calculation for ^{35}Ar(p,γ)^{36}K was also carried out, where the effect of the low-lying 3^- negative parity state was taken into account approximately by using the measured spectroscopic factor and gamma decay lifetime of the mirror nucleus.

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