The large numbers hypothesis in cosmology

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In 1937, P.A.M. Dirac suggested the idea that the dimensionless constants of physics must be in relation with the epoch (age of the universe expressed in atomic units). From this hypothesis, known as Large Numbers Hypothesis or Dirac's Principle, he built a cosmological model in 1938 and abandoned it.

Following this principle, P. Jordan developed a series of articles, translated by us, based on the conservation of the dimensionless numbers coincidence. He suggested a model of matter creation to counterbalance the expansion of the universe.

Surprisingly, in the seventies, Dirac came back to his Large Numbers Hypothesis and published a new cosmological model, based on a description of the universe using two metrics.

We intend to review and present the historical development of the Large Numbers Hypothesis and its consequences in cosmology through the works of these two famous authors.

Keywords: Cosmology - Large number hypothesis - History of science.

1. Dirac's Principle

In a short letter to the editor in *Nature*[1], P.A.M. Dirac, following Eddington's work on dimensionless numbers[2], noticed a coincidence between the constants used in cosmology and enunciated his Large Number Hypothesis. Indeed, it could be observed that the ratio between the Coulombian and the Newtonian gravitational forces between an electron and a proton is about 10^{39}; the ratio between the masses of the universe and of a proton is about 10^{78}. These two large numbers need different types of explanations because they are not physically linked. But, if you add the coincidence that the age of the universe, according to the contemporary cosmological models, expressed in atomic units, so-called the epoch, is 10^{39}; it seems logical to put the two previous large numbers in relation with the epoch. It is what Dirac did: "*This suggests that the above-mentioned large numbers are to be regarded not as constants, but as simple functions of our present epoch, expressed in atomic units.*"[1]

This principle has two direct consequences. First, the number of protons and electrons has to increase like the square of the epoch, conserving the null electrical charge of the universe. Secondly, the gravitational constant can not be constant anymore and must decrease with time.

Dirac concluded his letter by a brief paragraph about cosmological applications of his principle, which he studied in a later article, as it will be shown in the next section.

2. Dirac's Cosmology of 1938

In 1938, Dirac published a paper in which he suggested a cosmological model based on the Large Numbers Hypothesis[3]. He rewrote his principle as "*Any two of the very large dimensionless numbers occurring in Nature are connected by a simple mathematical relation, in which the coefficients are of the order of magnitude unity.*"[3]

With this hypothesis, he tackled one of the main problems of cosmology, the determination of the form of $f(t)$, similar to the current scale factor, giving the recession law of galaxies, since any cosmological model must explain Hubble's observations. Doing so, he arrived at the possibility of creation or annihilation of protons and neutrons assuming that the effect will be so faint that it could not be detected in laboratory. However, Dirac noted that "*However, such a spontaneous creation or annihilation of matter is so difficult to fit in with our present theoretical ideas in physics as not to be worth considering, unless a definite need for it should appear, which has not happened so far, since we can build up a quite consistent theory of cosmology without it.*"[3]

Dirac also studied the curvature of the slice of three-dimensional surfaces given for each value of the epoch, or t-space. The curvature cannot be positive, because, in this case, the mass of the universe is a very large number and will be constant, thanks to the assumption of mass conservation. This is in contradiction with his fundamental principle so it should be ruled out. The case of a negative curvature can also be excluded: working in a sphere of radius equal to the radius of curvature of the t-space, the mass contained in this sphere will not evolve with time which contradicts Dirac's principle. Dirac concluded that "*We are thus left with the case of zero-curvature, or flat t-space, as the only one consistent with our fundamental principle and with conservation of mass.*"[3]

The article finished with this summary: "*It is proposed that all the very large dimensionless numbers which can be constructed from the important natural constants of cosmology and atomic theory are connected by simple mathematical relations involving coefficients of the order of magnitude unity. The main consequences of this assumption are investigated and it is found that a satisfactory theory of cosmology can be built up from it.*"[3]

3. Jordan's work

From 1937, Pascual Jordan developed a parallel work based on Eddington's study of dimensionless numbers[2] and Dirac's idea that very large numbers could be expressed in relation with the epoch. Jordan's work has been published in a series of articles[4, 5, 6]. We worked on our own translation of them.

Like Eddington, Jordan hoped to find the way to unify quantum mechanics and genral relativity by finding the relation between their two characteristic constants $\hbar$ and c. Following Dirac's reasoning, Jordan reached the conclusion that the gravitational constant cannot be constant with respect to the time and that matter must be created.

To have a continuous and spontaneous matter creation process, Jordan considered the possible creation of stars. These stars must have the good radius and mass ratio to counterbalance their mass energy with their own gravitational energy. So that, according to him, the energy cost of this creation is null. Jordan found an argument in favour of his theory of star creation in the observation of younger and older stars.

4. Jordan and Hoyle

The history has very often ignored Jordan's German pre-World War II model. In 1948, two articles, due to Hermann Bondi and Thomas Gold for the first[7] and to Fred Hoyle for the second[8], founded the Steady State Theory. Max Born seemed to see some similarities between Jordan's work and Hoyle's model[8]. Therefore, he invited Pascual Jordan to publish in English in the prestigious review *Nature*[9].

However, Hoyle's and Jordan's models are really different. If both of them referred to Dirac's work, they did not develop it in the same way. Jordan worked with the dimensionless constants and their variations when Hoyle modified Einstein's equations to describe a universe with a constant density of matter. And, to create matter, the former considered spontaneous appearance of stars while the latter suggested creation of hydrogen atoms.

That is why Jordan finished his comparison between their models with: "*Several decisive ideas of Hoyle's are in full harmony with my own theory [...] But there are also considerable differences between Hoyle's theory and my own.*"[9]

5. Dirac's Cosmology of 1973

Surprisingly, Dirac used a communication at the Pontifical Academy of Science on evolutionary cosmology[10] to come back to his cosmological model with a matter creation process. He published two other articles[11,12] on this subject.

In this series of papers, Dirac studied two ways to create matter: "*A: Matter is created uniformly throughout space, and hence mainly in intergalactic space. B: Matter is created where it already exists, in proportion to the amount existing.*"[10] Thereafter, he called them *additive and multiplicative creation*[12].

According to his Large Numbers Hypothesis, the gravitational constant must vary. To reconcile this idea with the successful Einstein's theory of gravitation, Dirac suggested the use of two metrics: Einstein's one ds_E and ds_A, measured by atomic apparatus. From that, he built two cosmological models waiting observations to come to make the distinction between the two, as Shapiro's time delay experiment.

In the conclusion, Dirac wrote: "*The foregoing work is all founded on the Large Numbers Hypothesis, in which I have great confidence.*"[12]

Conclusion

The present paper described the historical development of cosmological models based on Large Numbers Hypothesis and reviewed the work of two renowned physicists who built cosmological models on this hypothesis. If this hypothesis is now considered as mere numerology and close to pseudo-science, it is interesting to study its past applications in physics. For a review on controversies about the Big Bang theory and the Steady State theory, we refer, among others, to Helge Kragh's work[13]. Our work gives us the opportunity to illustrate the fact that the Steady State theory was not the only one in competition with the Big Bang Theory and, moreover, not the only one to suggest a process of continuous creation of matter.

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