Progress Towards Parametric-Resonance Ionization Cooling in the Twin-Helix Channel*

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Abstract. Parametric-resonance Ionization Cooling (PIC) is proposed as the final 6D cooling stage of a high-luminosity muon collider. Combining muon ionization cooling with parametric resonant dynamics could allow an order of magnitude smaller final equilibrium transverse emittance than conventional ionization cooling alone. The same type of cooling channel can be used for Reverse EMittance EXchange (REMEX) to reduce the transverse emittance by another factor of ten. Together, PIC and REMEX can provide two orders of magnitude luminosity increase for a muon collider.

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OVERVIEW OF PARAMETRIC-RESONANCE IONIZATION COOLING

Cooling presents a key challenge for the successful development of both a Higgs factory and muon collider. Because muons are produced as tertiary particles, a beam of a statistically acceptable number of muons will be produced with a large phase space volume. Cooling is required for acceptance into acceleration and storage structures in the machine, and reducing emittance improves luminosity from collisions. Longitudinal cooling is particular important for a Higgs factory. The need for muon cooling is complicated by the short lifetime of the muons, and non-linear effects from the complex magnetic fields of the cooling and transport channels. Ionization cooling techniques offer the best way to accomplish the substantial cooling needed within the limited muon lifetime. Parametric-resonance Ionization Cooling (PIC) is a proposed method for final stage 6D muon cooling that leverages resonance-driven strong focusing with ionization cooling to minimize angular divergence of muons in the beam.

Ionization cooling [1] is achieved by passing a particle through an energy-absorbing material, reducing the particle's momentum in all dimensions while RF fields restore longitudinal momentum. The angular divergence and energy spread are reduced until they reach equilibrium with the stochastic effects of multiple Coulomb scattering and energy straggling. In PIC, a resonance is introduced in a period magnetic channel based on multiples of the betatron oscillation frequency. This allows the channel to reach a new equilibrium [2, 3]. The resonance perturbs the phase-space trajectories of particles at periodic locations along the channel changing their normal elliptical shapes to hyperbolic. At certain periodic focal points, muons in the beam become progressively narrower in position while diverging in angle as they pass down the channel. Without damping, the beam dynamics are not stable and the angular divergence of particles in the beam grows with every period. Placing energy absorbers followed by RF cavities at these focal points allows ionization cooling to limit the growth in angular divergence while maintaining total particle momentum and this stabilizes the beam motion. This resonance also causes a strong reduction of the beam spot size at the absorber locations leading to transverse beam emittance that is about an order of magnitude smaller than without the resonance. The longitudinal emittance is maintained through emittance exchange and shaped wedge absorbers. The absorber locations must be at points of small, but non-zero dispersion. A magnetic channel meeting the requirements for PIC could also be used for Reverse EMittance EXchange (REMEX) [4] by reversing the orientation of the wedge absorbers. This offers the potential of an additional reduction in transverse emittance by about another factor of 10 at the expense of longitudinal emittance.

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One of the key principles for PIC is the correlated optics condition. Under this condition, a stable orbit for particles is maintained with betatron tunes in both the horizontal (λ_x) and vertical (λ_y) planes being low-integer multiples of the period of the dispersion function (λ_D) for the system. The PIC also requires dispersion D such that is small, but non-zero, at the absorber to minimize energy straggling while allowing for emittance exchange to maintain constant longitudinal emittance. Thus, the optics must have correlated values such that $a\lambda_x = b\lambda_y = c\lambda_D$ where *a, b* and *c* are integers.

Because the PIC dynamics are very sensitive to non-linear aberrations from magnetic fringe fields, a solution using helical harmonics [5, 6, 7] has been proposed. To create dispersion in the channel two helical dipole harmonics having equal field strength and equal periods but opposite helicities are superimposed onto each other. Under this configuration, a reference muon maintains a stable orbit within the x-z midplane. A continuous straight quadrupole is superimposed to establish the correlated optics condition. It was demonstrated [6, 7] that a twin-helix channel could meet the correlated optics requirements while offering large dynamic aperture and momentum acceptance.

Beryllium wedge absorbers are added at every other periodic focal point in the channel for ionization cooling, followed shortly by RF cavities to restore and maintain the reference particle momentum. To induce resonances, two uncoupled pairs of helical quadrupole harmonics with very low field strength perturb the orbit at the focal points. For each pair of harmonics, like the primary helical dipole pair, the field strengths and periods are the same, but the harmonics in each pair have opposite helicities. One such pair is used to induce the resonance in the horizontal plane, while a second pair induces the resonance in the vertical plane.

LINEAR MODELING IN COSY

Simulations for PIC have been performed with two different simulation codes [6, 7]. One set of simulations was performed using G4Beamline [8] (G4BL), a Geant-4 toolkit. The other used COSY Infinity [9] (COSY), a differential algebra based code that allow calculation of transfer and aberration maps for particles in the channel to arbitrary order. Because of its ability to turn on non-linear effects of various orders one at a time, COSY offers particular advantages for optimization and aberration correction. Additions to the basic beam physics package used with COSY had to be made to facilitate these simulations. These modifications included:

- Implementation of the magnetic field element for a helical harmonic pair of arbitrary harmonic order with potential for superposition of continuous straight magnetic multipoles of arbitrary order,
- Implementation of a fitting routine to determine the stable reference orbit for muon of a particular energy within the channel
- Implementation of the stochastic processes of multiple scattering and energy straggling in material
- Implementation of a particle tracking method for single particles and basic particle distributions.

Benchmarking was performed with G4BL to verify consistency of results between the two codes despite differences in the simulation methods [10]. With these modifications, simulation of the linear (first order) model of the channel was performed to verify that the simulation results were consistent with theory. The preliminary simulations were done without stochastic effects. A μ^- with momentum of 250 MeV/c was chosen as the reference particle. The period of the helical dipole harmonic was arbitrarily set at 1 meter, and the field strengths for the harmonics and the straight quadrupole were scaled to achieve the correlated optics condition. Resonances were induced with a helical quadrupole harmonic pair (parametric lenses) for each plane that satisfied the correlated optics condition: $\lambda_x = 2\lambda_y = 4\lambda_D$. Simulation, shown in Fig. 2, verified that a test particle offset from the reference orbit in initial position and angle followed a hyperbolic trajectory as it travels down the channel.

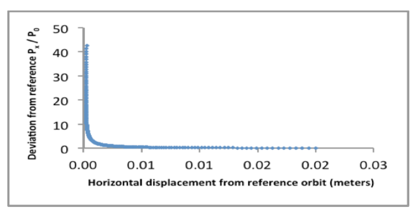

FIGURE 1. The basic twin-helix channel simulated in COSY with parametric lenses. Trajectory of a 250 MeV/c μ^- launched offset in both planes from the reference orbit by 2 cm and 130 mrad is tracked every at every other focal point in the horizontal plane.

Next, beryllium wedge absorbers with a central thickness of 2 cm and a 30% thickness gradient were added at every other focal point. An idealized RF cavity was place 3 cm after each absorber and tuned to restore momentum for the reference particle to maintain its stable orbit. Simulations for the same test particle, Fig. 3, show the effects of ionization cooling with and without using harmonics to induce the PIC resonance condition. With PIC resonances induced, strong focusing causes more reduction in the position offset, and a much greater reduction in total offset after the same number of wedge absorbers.

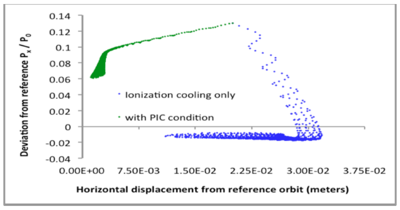

FIGURE 3. The full twin-helix channel simulated in COSY with and without PIC resonance. Trajectory of a 250 MeV/c μ- launched offset in both planes from the reference orbit by 2 cm and 130 mrad is tracked at the center of each wedge absorber in the horizontal plane.

The stochastic effects of multiple scattering and energy straggling were added to the simulations to verify the equilibrium emittance and improved reduction in spot size predicted by theory. To implement multiple scattering, COSY calculates the path length each individual particle takes through the wedge absorber. This parameter, z, as well as the other parameters for that same particle and the absorber, are used to determine standard deviation for the "kick" to particle angle using the PDG formula RMS98 [11] modeling method (1). A random number generated from Gaussian distribution is used to determine the exact kick for the particle, and the result is split via polar angle between the horizontal and vertical plane. The calculated result is then applied to modify particle's coordinates in the tracking subroutine. For energy straggling, a similar approach is used, with the Bohr approximation (2), determining standard deviation for the change in energy [12].

$$\theta_{scatter} = \frac{13.6MeV}{\beta c \rho}\sqrt{\frac{z}{\chi_0}\left(1+.038\ln\left(\frac{z}{\chi_0}\right)\right)}, \qquad (1)$$

$$\Omega^2_{straggling}\left[KeV^2\right] = .26 Z_{absorber} z N_t \left[10^{18} atoms/cm^2\right]. \qquad (2)$$

Where χ_0 represents the radiation length of the absorber material, $Z_{absorber}$ is the absorber material's atomic number, N_t is the atomic density of the absorber. Both processes are repeated for each individual particle every time the particle encounters an absorber in the channel. The results of these simulations for the horizontal plane are shown in Fig. 4 with and without the stochastic effects being included in the simulation. Even with the inclusion of these two important stochastic effects, cooling as predicted by the theory is observed [3, 7].

Simulations were also performed using an uncorrelated distribution of 1000 muons with the following standard deviations from the reference orbit: (1) offset in each plane: 2 cm.; (2) offset in angle in each plane: 130 mrad; (3) energy spread: 1%; and (4) bunch length: 3 cm. The 6D emittance for total distribution was tracked as it travelled through the channel with and without inducing the PIC resonance condition. Fig. 5 shows the cooling factor, a figure of merit determined by dividing the final 6D emittance of the surviving particles in the beam by their initial emittance.

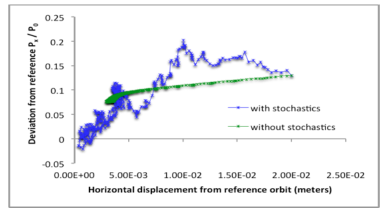

FIGURE 4. The full twin-helix channel simulated in COSY with and without the stochastic effects of multiple scattering and energy straggling. Trajectory of a 250 MeV/c μ- launched offset in both planes from the reference orbit by 2 cm and 130 mrad is tracked at the center of each wedge in the horizontal plane.

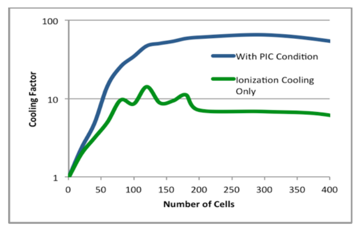

FIGURE 5. Comparison of cooling factor (ratio of initial to final 6D emittance) with and without PIC resonance.

As predicted by theory, 6D cooling with the PIC resonance condition reaches an equilibrium state that beyond that of ionization cooling alone by about a factor of 10.

PROGRESS TOWARDS ABERRATION CORRECTION AND OPTIMIZATION

The baseline simulations described above provide an important tool for optimizing the PIC cooling channel. This linear model simulates the efficiency of the cooling channel where all aberrations have been perfectly corrected. Since muon beams can have a very large initial angular and energy spread, aberrations in the system dependent on these parameters can dramatically impact the final spot size of the beam.

To illustrate this, consider the progress towards aberration correction and optimization of the twin-helix channel. Through separate simulations, a preferred helical dipole harmonic period (λ_D) of 20 cm was chosen for a reference momentum of 250 MeV/c, and the magnetic field strengths of the helical dipole and straight quadrupole components scaled accordingly [4]. With COSY, the largest nonlinear aberrations in the system can be identified order by order, as well as their effects given a specified range in initial particle coordinates. This allows determination of a sensitivity of a system's optic to a range of initial beam parameters. For the twin-helix channel, Table 1 lists the largest 2nd and 3rd order aberrations affecting final spot size at the period focal points in the channel. The aberration (x|aa), for example, shows how final horizontal position of the particle changes as a function of the square of its initial angle (p_x/p_0) in the horizontal plane. Similarly b refers to initial angle (p_y/p_0) in the vertical plane.

TABLE 1. Largest 2nd and 3rd order aberrations affecting spot size for the λ_D=20 cm twin-helix.

Aberration	Magnitude [mm]
(x|aa)	1.5
(x|aδ)	2.1
(x|aaa)	-17.8
(x|abb)	-6.1
(y|aab)	6.1
(y|bbb)	1.2

Studies are currently ongoing to correct these and other significant higher order aberrations in the twin-helix channel using continuous magnetic fields, including higher order helical harmonic pairs and straight multipole fields. In all cases, the correlated optics condition must also be maintained, and the reference orbit must be recalculated since these higher order magnetic fields can modify the orbit of the reference particle. Field strength, phase offset, helicity and harmonic number provide a number of variable parameters for the system.

One such correction scheme minimizing all 2nd and 3rd order aberrations that contribute to deviation in the final position of the particle at each wedge absorber uses a straight octopole field, two pairs of helical sextupole harmonics and two pairs of helical octopole harmonics. Two pairs of helical quadrupole harmonics are also are used to maintain the correlated optics in the channel. Fig. 6 shows a 3rd order simulation of this system after 20 cells.

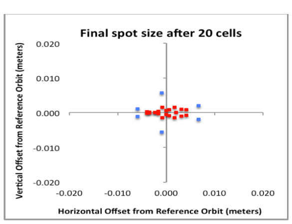

FIGURE 6. Tracking for concentric cones, with angular deviation of up to 120 mrads, of 250 MeV/c muons launched on reference orbit in COSY with non-linear effects through 3rd order and stochastic effects.

Particles shown in blue survive in the channel without corrections, while those shown in red show effects of the correcting magnetic fields. Survivability of muons has dramatically increased as well as focusing of the beam. Aberrations beyond 3rd order still need to be corrected.

CONCLUSIONS AND FUTURE WORK

PIC combines muon ionization cooling with parametric-resonance dynamics to allow final equilibrium transverse beam emittance that is an order of magnitude smaller than those achievable with conventional ionization cooling alone. Linear simulations including stochastic effects have verified the predictions of PIC theory. Using the same magnetic channel, REMEX could allow reduction in transverse emittance by another factor of ten. Thus, PIC and REMEX together provide the potential to increase luminosity by two orders of magnitude.

A twin-helix magnetic channel with correlated optics has been developed for PIC. A basic model of a PIC channel with absorbers and RF cavities has been simulated in G4BL and COSY. Linear simulations in COSY have confirmed the model's validity with stochastic effects included. Compensation of beam aberrations is a challenging aspect of this channel and will be required for complete demonstration of PIC. Progress has been made on this problem and ongoing efforts continue.

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