universe

IMPACT FACTOR 2.9

CITESCORE 3.6

Article

On the Apparent Discretization of Spacetime and Its Connection with the Cosmological Constant

Jaume Giné and Giuseppe Gaetano Luciano

MDPI

https://doi.org/10.3390/universe10030142 *universe*

MDPI

Article

On theApparent Discretization of Spacetime and Its Connection with the Cosmological Constant

Jaume Giné [1] and Giuseppe Gaetano Luciano [2,*]

[1] Departament de Matemàtica, Universitat de Lleida, Av. Jaume II, 69, 25001 Lleida, Spain; jaume.gine@udl.cat
[2] Department of Chemistry, Physics and Environmental and Soil Sciences, Escola Politecninca Superior, Universitat de Lleida, Av. Jaume II, 69, 25001 Lleida, Spain
* Correspondence: giuseppegaetano.luciano@udl.cat

Abstract: The emergence of a minimal observable length of order of the Planck scale is a prediction of many quantum theories of gravity. However, the question arises as to whether this is a real fundamental length affecting nature in all of its facets, including spacetime. In this work, we show that the quantum measurement process implies the existence of a minimal measurable length and consequently the apparent discretization of spacetime. The obtained result is used to infer the value of zero-point energy in the universe, which is found to be in good agreement with the observed cosmological constant. This potentially offers some hints towards the resolution of the cosmological constant problem.

Keywords: uncertainty principle; discrete space-time; quantum fluctuations; cosmological constant

1. Introduction

Gravity is a universal force in nature, although it is the least known at the quantum level. As a matter of fact, a complete theory of quantum gravity is still far from being developed, notwithstanding several decades of research and numerous tentative models, such as string theory [1,2], M-theory [3], brane-world scenario [4], loop quantum gravity [5], and asymptotic safety [6] (see also [7] for a recent review of quantum gravity phenomenology). Within the framework of quantum gravity, the quantization of spacetime is expected to emerge naturally. In fact, continuum space and time as described by Einstein's General Relativity are thought to break down at very small scales, generating a fluctuating and non-smooth structure where spacetime is poorly defined locally. The ensuing background is then characterized by a sea of quantum fluctuations that define and perturb the structure of spacetime itself.

The relation between quantum fluctuations and the texture of spacetime has been largely investigated from different perspectives in recent years. For instance, in [8,9] decoherence mechanisms from a fluctuating minimal length have been discussed along with their potential implications on cavity optomechanics experiments. On the other hand, in [10–13] it has been proposed that at the Planck scale, quantum gravity fluctuations and the structure of spacetime could be related to Einstein–Podolski–Rosen (EPR) [14,15] entangled states through the equivalence with Einstein–Rosen (ER) [16] wormholes (ER = EPR conjecture). However, no clear effect has been revealed so far, indicating that fluctuations might occur at even smaller scales or that we still do not understand the interactions between photons and gravitons.

Starting from the above premises, in Section 2 of the present work we argue that the quantum measurement process implies the existence of a fundamental measurable length at the Planck scale and consequently the apparent discretization of spacetime. It is necessary to note from the outset that we do not intend to provide any rigorous proof of this, since it would require knowledge from a quantum theory of gravity that is not yet available; rather, we aim at outlining some plausibility arguments, which could hopefully set the

check for updates

Citation: Giné, J.; Luciano, G.G. On the Apparent Discretization of Spacetime and Its Connection with the Cosmological Constant. *Universe* **2024**, *10*, 142. https://doi.org/10.3390/universe10030142

Academic Editors: Janusz Garecki and Lorenzo Iorio

Received: 18 December 2023
Revised: 30 January 2024
Accepted: 12 March 2024
Published: 14 March 2024

stage toward developing such a framework. As an application of our study, in Section 3 we infer the value of zero-point energy in the universe, which is found to be very close to the observed cosmological constant. Conclusions and outlook are finally summarized in Section 4.

2. The Minimal Measurable Length

In this section, we argue why spacetime should be considered as apparently discrete. We show that there exists a minimal measurable length which is essentially the Planck length and study the consequences of this result. Also, we anticipate that, although many quantum models of gravity and discussions related to the Planck scale suggest the possibility of discrete time structures, the problem of "quantum time" is far from being understood, as existing paradigms and ideas are still speculative. In our model, we shall essentially provide a semiclassical treatment of time, wherein the only quantum-like ingredient is the time–energy uncertainty relation (see Equation (14) below). Clearly, more research which goes beyond our heuristic arguments is needed to understand the nature of time at the quantum level.

In the same way as the Heisenberg Uncertainty Principle (HUP), the discreteness of spacetime is nontrivially intertwined with the measurement problem in quantum mechanics. Rigorously, the study of quantum measurement processes should involve the usage of projection-valued measures of positive-operator-valued measures derived from the spectral theorem for quantum observables. However, it is known that the spectral theorem may face challenges when applied in the context of quantum gravity. For instance, one such issue concerns the nature of spacetime: while the conventional spectral theorem deals with operator on Hilbert spaces, in quantum gravity questions arise about the fundamental nature of the spacetime manifold, which may not be accurately represented as a Hilbert space. Furthermore, a modification of the standard spectral theorem may be required to accommodate non-commutative geometries in some non-commutative models of quantum gravity. Addressing these issues involves exploring more sophisticated frameworks. This goes far beyond our preliminary analysis, which is essentially based upon semiclassical arguments and order of magnitude estimates.

To understand the problem of quantum measurement in a naive way, we can follow a heuristic approach and think of how we carry out measurements in classical mechanics. Suppose we want to measure the length of a certain object by using a common ruler. If the object is larger than the ruler, we can measure its length by moving the ruler in consecutive positions along the object and keep track of the number of times we iterate this process (or, alternatively, we might use a longer ruler). But if the object is very tiny, we must take a ruler whose unit of measure is smaller than (or at most equal to) the length we want to measure, since otherwise this task cannot be achieved.

On the other hand, it is well-known that in the quantum framework the interactions between bodies occur through the action of other particles called "mediator" bosons. For the electromagnetic interaction this particle is the photon, while for the gravitational field it is expected to be the graviton. Let us then consider a central mass M that interacts with a test body m in a classical view of the interaction. We assume that the two objects are separated by a distance R (which we take to be of the same order as the radius of the central body). In order for the mass m to escape the gravitational field of M, its kinetic energy must be equal to or greater than its gravitational potential energy i.e.,

$$\frac{1}{2}mv^2 \geq \frac{GMm}{R}. \tag{1}$$

From this equation we can derive the escape velocity $v = \sqrt{\frac{2GM}{R}}$ as the minimum speed at which the test mass m has to travel to move away indefinitely from the gravity source M.

By extending the above paradigm to a relativistic picture, we know that there is an upper limit for the speed at which signal carrying information can travel through the space, which is the speed of light c. Therefore, the smallest distance from which the test body can escape is

$$R \geq \frac{2GM}{v^2} \overset{v \to c}{\to} r_s = \frac{2GM}{c^2}. \tag{2}$$

This corresponds to the radius defining the event horizon of a Schwarzschild black hole, which is a coordinate singularity that appears in General Relativity. Nothing can escape the gravity of a black hole, except for the Hawking radiation arising from quantum fluctuations near the event horizon.

Now, the measure theory in quantum mechanics is well understood and based on shooting particles (essentially photons) toward a target to determine its position and size. But, as it happens classically, the size of the probe must be at most equal to that of the target, for otherwise it is impossible to localize the latter.

For massive particles, the characteristic length scale is roughly given by their de Broglie wavelength

$$\lambda = \frac{h}{p}, \tag{3}$$

where h is the Planck constant and $p = m\gamma v = \frac{mv}{\sqrt{1-v^2/c^2}}$ denotes the momentum of the particle. It is clear that for velocities v much less than the speed of light c, we can approximate $p = mv$, so that $\lambda = h/mv$. Notice that Equation (3) resembles the corresponding formula for massless photons

$$\lambda = \frac{hc}{E}, \tag{4}$$

where E is the energy of the photon and we have used the relation $p = E/c$. From this equation, we can see that smaller wavelengths correspond to higher energies and vice versa. Consequently, the smaller the size we aim to probe, the higher the energy we need.

Bearing the above reasoning in mind, let us consider a particle of size r and suppose we want to localize it in the space. Toward this end, we need photons of wavelengths λ_b satisfying

$$\lambda_b \lesssim r. \tag{5}$$

In order for these photons to be detected by the observer after hitting the target, their energy E_b must be less than or equal to the energy of the particle we want to localize, i.e.,[1]

$$E_b \equiv m_b c^2 \lesssim E_p = \sqrt{m^2c^4 + p^2c^2}, \tag{6}$$

where we have defined $m_b \equiv E_b/c^2 = p_b/c$ as the equivalent mass of the photon and E_p is the energy of the particle. Therefore, in the limit case where $p \to 0$, we obtain $m_b \lesssim m$.

Now we try to answer the following question: what is the smallest distance we can measure practically? To address this issue, let us consider the smallest mass that an object compressed inside the smallest radius can have. We denote this mass by m_s and the smallest radius that can have this object is of course the Schwarzschild radius of a black hole with equivalent mass m_s, i.e.,

$$r_s = \frac{2Gm_s}{c^2}. \tag{7}$$

Based on Equation (5), it is clear that to measure this distance r_s we need photons with wavelength λ_b obeying

$$\lambda_b = \frac{h}{m_b c} \lesssim r_s = \frac{2Gm_s}{c^2}. \tag{8}$$

Moreover, from Equation (6) we have $m_b \lesssim m_s$ and consequently using Equation (8) we obtain

$$\frac{h}{m_s c} \lesssim \frac{h}{m_b c} \lesssim r_s = \frac{2Gm_s}{c^2}. \tag{9}$$

Therefore, we have

$$m_s \gtrsim \sqrt{\frac{hc}{2G}} \gtrsim \sqrt{\frac{\hbar c}{G}} = m_P \approx 2.18 \times 10^{-8}\,\mathrm{Kg}, \tag{10}$$

where m_P is the Planck mass. The obtained equation implies that the Planck mass is the mass of the smallest measurable black hole. Notice that this does not mean that the smallest measurable mass is m_P. In fact, all elementary particles have masses much smaller than m_P.

We can now derive the smallest measurable distance. From Equations (7) and (8), we figure out that it is given by the radius of the event horizon of a black hole with equivalent mass m_P. More specifically, for the case when $m_b \simeq m_P$ in Equation (8), we obtain

$$r_s \gtrsim \frac{h}{m_P c} = \sqrt{\frac{hG}{c^3}} = \ell_P \approx 1.616 \times 10^{-33}\,\mathrm{cm}, \tag{11}$$

where ℓ_P is the Planck length. To summarize, our reasoning was as follows: we started by considering the Schwarzschild radius r_s associated to the mass m_s of a given object (See Equation (7)). As is well known, r_s represents the size to which the object needs to be compressed in order to become a black hole. Clearly, this radius defines the event horizon of the black hole. Theoretically, once an object crosses the event horizon, it cannot send information back out, making the inner region inaccessible to external observation. In this sense, r_s is the minimum accessible distance that can be probed by measurements. Next, from Equation (10), we figured out that the minimum mass that such a black hole-like object can have is just the Planck mass m_p. Accordingly, from Equation (7), the minimum accessible distance is (up to a numerical factor, which is irrelevant for our purpose of estimating the orders of magnitude) the Schwarzschild radius of a black hole having the minimum mass m_p. Although this a well-known result, we would like to point out that here it has been derived in a way alternative to other approaches presented in the literature.

Therefore, we do not know if spacetime is discrete and perhaps we will never know that, but for practical purposes and due to the peculiarities of the measurement process in quantum mechanics, it behaves as if it were discrete, with the minimal measurable distance being given by the Planck length. The fact that there exists such an elementary length implies that at scales smaller than ℓ_P, we cannot know what structure the spacetime has, since we cannot access it. From this perspective, our claim differs conceptually from other results in prior literature, where the Planck scale is typically assumed (rather than inferred) as a characteristic cutoff scale. A detailed discussion of whether a minimal length is physical appears in the recent analysis of [17]. In that context, it is claimed that the emergence of a minimal observable length in the framework of generalized uncertainty relations is physical, in the sense that it is not altered by different representations of the physical operators obeying the deformed algebra.

Another important implication of the above achievement is that quantum fluctuations, which in principle could originate at any point in a space, actually appear as if they were created at discrete points on the Planck scale. Therefore, the points where quantum fluctuations can be generated are (at least) at a distance ℓ_P apart from each other. We shall see below that this feature has non-trivial implications on the cosmological constant problem. Moreover, we have to take into account only those fluctuations with wavelength $\lambda \gtrsim \ell_P$. Indeed, assuming that the uncertainty in the localization of any quantum fluctuation is $\Delta x \simeq \ell_P$, for those fluctuations that saturate the Heisenberg Uncertainty Principle, we have

$$\Delta x \Delta p \simeq \frac{\hbar}{2}, \tag{12}$$

which in turn implies

$$\Delta p \simeq \frac{\hbar}{2\Delta x} \simeq \frac{\hbar}{2\ell_P} = \frac{m_p c}{2}. \tag{13}$$

At this stage, it is worth noting that, for our analysis, we are essentially taking inspiration from some semiclassical computations, like those in [18–20], where Δx is interpreted as the width of a given space region, ΔE as the energy fluctuations of the quantum vacuum in this region, and so on. Strictly speaking, a thorough analysis should involve defining such quantities in terms of variances of quantum observables relative to a specific quantum state. This task, however, can only be accomplished upon proper reformulation of our study in terms of quantum operators (see the discussion at the beginning of Section 2).

In the usual (Casimir-like) picture of a quantum vacuum, the existence of quantum fluctuations means that the vacuum is not really empty. In fact, it is filled with virtual particles–antiparticles pairs that continuously originate from fluctuations and annihilate shortly afterwards. The lifetime Δt of such virtual pairs can be roughly estimated from the time–energy uncertainty relation

$$\Delta E \Delta t \simeq \frac{\hbar}{2}. \tag{14}$$

Of course, from this equation we observe that the higher the energy ΔE of the pair, the shorter its lifetime, and vice versa. In particular, in the case of photons (which are expected to provide the dominant contribution to the vacuum energy), Equation (13) implies

$$\Delta E = c\Delta p \simeq \frac{m_p c^2}{2}\,. \tag{15}$$

By plugging into Equation (14), we obtain

$$\Delta t \simeq \frac{\hbar}{m_p c^2} = \frac{\ell_P}{c} = t_P, \tag{16}$$

where t_P denotes the Planck time. We recall that this is the time it takes light to traverse a distance of one Planck length in a vacuum, and is approximately

$$t_P \approx 5.39 \times 10^{-44} s\,. \tag{17}$$

Therefore, the Planck time can be naturally derived from the time–energy uncertainty relation Equation (14) as corresponding to the lifetime of vacuum fluctuations with localization Δx of order of the Planck scale. It is clear that, if the wavelength of the virtual particle is less than ℓ_P, then the scale over which it can be localized is within the Planck length. Hence, the unique quantum fluctuations which spread beyond the cube of side ℓ_P centered on the point where the fluctuation appears are those satisfying $\lambda \gtrsim \ell_P$.

Before proceeding further, we would like to remark that the above considerations on vacuum polarization are the insight and result of perturbative quantum field theory. The extension to gravity and spacetime could be in principle challenging, due to known issues with renormalizability. In the absence of a fundamental description that comprises all four interactions, such arguments can be extrapolated to gravity as a guideline in the quest for a quantum formulation of Einstein's theory.

Before moving on, we would like to draw attention to the fact that, although a completely quantum description of gravity is still missing, it is commonly believed that gravitational effects should somehow affect the basic principles of quantum theory. One of the most eloquent examples is the alleged modification of the HUP approaching the Planck scale. In fact, the standard Heisenberg relation does not include corrections arising from the gravitational interaction.

In most quantum gravity proposals to address the Planck scale, natural cutoffs appear as intrinsic characteristics of quantum spacetime. Mathematically, these are a result of the compactness of the corresponding symplectic manifold, as can be seen in [21]. These cutoffs are encoded as fundamental length scales [22–29], which can be a new source of uncertainty. In particular, the generalized uncertainty principle (GUP) takes into account gravitational uncertainty of position and momentum related to the existence of a minimal length of the order of the Planck length ℓ_P [30–35] and/or a maximal momentum [36,37]. Discreteness of space from GUP has been largely explored in [38].

On the other hand, the dual of the GUP is the extended uncertainty principle (EUP), which considers the geometrical aspects of the spacetime curvature on large scales. Specifically, it embeds the uncertainty related to the background spacetime manifested by a maximal horizon scale (typically the cosmological event horizon r_h) and/or by a minimal momentum scale (see for instance [39–41]).

Both the GUP and EUP can be combined to give rise to the so-called Generalized Extended Uncertainty Principle (GEUP)

$$\Delta p \Delta x \geq \frac{\hbar}{2}\left[1 + \alpha\left(\frac{\ell_P \Delta p}{\hbar}\right)^2 + \beta\left(\frac{\Delta x}{r_h}\right)^2\right], \tag{18}$$

where α, β are (dimensionless) deformation parameters [26]. When $\alpha = 0$ or $\beta = 0$, we recover the EUP and GUP extensions, respectively.

3. Discreteness of Spacetime and Cosmological Constant

In this section, we exploit the above result on the discretization of spacetime to estimate the cosmological constant. To this aim, we first make a digression to review the derivation of the Unruh effect [42] proposed in [43]. The relation of the quantum fluctuations with the cosmological constant problem has been historically studied [44]. Also, in [45] it has been argued that GUP models in their current form are unable to adequately address the cosmological constant problem. In this regard, we emphasize that the solution here proposed is not directly connected with any modification of the Heisenberg relation, but it is rather based on the conjecture that spacetime is apparently granulated at Planck scale and that only points at Planck distance resonate, contributing to the total zero-point energy of the universe.

3.1. The Unruh Effect

The Unruh effect is a characteristic prediction of quantum theory, which states that from the point of view of a uniformly accelerating (Rindler) observer, empty space appears as a thermal bath of particles at a temperature proportional to the proper acceleration, i.e.,

$$T = \frac{\hbar a}{2\pi c k_B}, \tag{19}$$

where k_B is the Boltzmann constant. This effect is due to the particular geometry of Rindler space induced by the uniformly accelerated motion in Special Relativity, which leads to the appearance of an event horizon for the accelerated observer.

Although the original derivation of the Unruh effect was carried out within the framework of quantum field theory [42], the Unruh temperature (19) can be heuristically deduced from the HUP in a simple way [18,43]. Indeed, from Equation (12), we have

$$\Delta x \Delta E \simeq \frac{\hbar c}{2} \tag{20}$$

where we have considered that the major contribution to Unruh radiation comes from photons, for which $\Delta E = c\Delta p$. In passing, we would like to mention that an intuitive derivation of the Unruh effect has been proposed in [46] by using the Doppler shift method for both a scalar field in one spatial dimension and a spin-1/2 Dirac field. For an alternative approach based on the coupling between the Unruh–de Witt detector and an external field, one can resort to the analysis contained in [47].

Now, by taking into account that $\Delta x \simeq \pi d$, where $d = c^2/a$ is the distance of the observer from the Rindler horizon [43], we have

$$\Delta E \simeq \frac{\hbar c}{2\pi d} = \frac{\hbar a}{2\pi c}. \quad (21)$$

Finally, considering that the Unruh radiation is thermalized, i.e., $\Delta E = k_B T$, we obtain $T \simeq \hbar a/(2\pi c k_B)$, which coincides with the definition (19). Notice that the above reasoning can be readily generalized to three spatial dimensions, following the same reasoning as in Section 3.4 below. Furthermore, the same expressions for the vacuum energy and Unruh temperature are obtained in the case of massive fields, as argued in [48] and the subsequent extension to mixed fields [49].

In [28], the modified Unruh effect in the presence of a minimal length was addressed in both quantum mechanics and field theory. In spite of the different approaches, it was shown that corrections to the Unruh temperature are consistent in the two frameworks (up to some numerical factor). It is then expected that our considerations in quantum mechanics might continue to hold valid in quantum field theory, although a universally accepted implementation of a minimal length has not yet been formulated in the latter theory.

3.2. Cosmological Constant

The cosmological constant problem (known also as vacuum catastrophe) is the disagreement between the observed value of the vacuum energy density (i.e., the tiny value of the cosmological constant) and the theoretical large value of the zero-point energy predicted by quantum field theory. The challenge is to determine the value of Λ in Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \quad (22)$$

in such a way that the ensuing dynamic model is consistent with the observed accelerated expansion of the present universe, considering that Λ has the same effect as an intrinsic vacuum energy density with negative pressure.

3.3. Analysis in the Cosmological Horizon

As is well-known, accelerating spatial expansion creates cosmological horizons, i.e., distances beyond which we cannot exchange information. In particular, in [50–52] it has been argued the Hubble horizon is responsible for a damping of Unruh radiation, giving rise to a Hubble-scale Casimir-like effect for any observer at any spacetime point. The energy of fluctuations coming from this horizon at distance $r_u = c/H_0$, where H_0 is the present value of the Hubble parameter, can be estimated by applying the same reasoning as for the Unruh radiation in Equation (21), yielding

$$\Delta E \simeq \frac{\hbar c}{2\Delta x}. \quad (23)$$

Now, we take into account that the spacetime is apparently granulated at Planck scale and each point of Planck size is a resonant point of the spacetime. Hence, in a linear distance r_u, the number of resonant points is $N_0 \simeq r_u/\ell_P$. Next, we use the relation $\Delta x \simeq 2\pi r_u = 2\pi N_0 \ell_P$ [50–52], where the factor 2 is due to the fact that the cosmic horizon appears isotropically [53], obtaining

$$\Delta E \simeq \frac{\hbar c}{4\pi N_0 \ell_P}. \quad (24)$$

Then, the contribution of any resonant point of the cosmological horizon to the vacuum energy is

$$\Delta E_i = \frac{\Delta E}{N_0} \simeq \frac{\hbar c}{4\pi N_0^2 \ell_P} \simeq \frac{\hbar c \ell_P}{4\pi r_u^2}. \quad (25)$$

3.4. Analysis in the Bulk of the Universe

We can alternatively estimate the energy per resonant point from the cosmological constant. The cosmological constant is equivalent to the intrinsic density energy of a vacuum through the equation

$$\rho_{vac} = \frac{\Lambda c^4}{8\pi G}. \tag{26}$$

Therefore, the total vacuum energy is

$$E_T = \rho_{vac}\, V = \rho_{vac}\, \frac{4}{3}\pi r_u^3, \tag{27}$$

where V is the volume enclosed by the cosmological horizon. Since the number of resonant points in the entire universe is $N = \frac{4}{3}\pi r_u^3/\ell_P^3$ (not to be confused with N_0 introduced above), the energy of each resonant point is found to be

$$\frac{E_T}{N} = \rho_{vac}\, \ell_P^3. \tag{28}$$

Finally, we observe that any resonant point of the universe must resonate with the same energy, because whether the point is on the cosmological horizon or inside it only depends on the location of the observer. Note that a fixed point in the universe defines the cosmological horizon associated to this point. We then equate Equations (25) and (28) to obtain

$$\Lambda = \frac{2G\hbar}{c^3 \ell_P^2 r_u^2} = \frac{2}{r_u^2} = \frac{2H_0^2}{c^2}. \tag{29}$$

The obtained result is to be compared with the well-known expression

$$\Lambda = 3\Omega_\Lambda \frac{H_0^2}{c^2}, \tag{30}$$

where Ω_Λ is the ratio between the energy density due to the cosmological constant and the critical density of the universe. Using the value known for $\Omega_\Lambda \simeq 0.69$ [54], this leads to $\Lambda \simeq 2.07 H_0^2/c^2$, which is in good agreement with Equation (29).

4. Discussion and Conclusions

An important feature of most of the existing models of quantum gravity is the emergence of a minimal length at the Planck scale and, consequently, the quantization of space and time. Indeed, according to these models, continuum space and time as described by Einstein's General Relativity break down at very small scales, generating a fluctuating and non-smooth structure where spacetime is poorly defined locally. Although a minimum length seems inconsistent with the Special Theory of Relativity, the problem of a Lorentz invariant scale has been preliminarily addressed in [55–57].

Most of the recent works on the discreteness of spacetime assume the existence of a minimal length and a modified uncertainty principle [58–60], while exploring the related implications. On the other hand, much effort has been devoted to construct a discrete curved spacetime using the Causal Set approach to Quantum Gravity (See [61,62]). However, measurements of such a prediction have been mostly unsuccessful up to now [63,64].

In this work, it has been argued that quantum measurement implies the existence of a minimal length at the Planck scale and, accordingly, the apparent discretization of spacetime. This is in line with the recent result of [17]. Therefore, regardless of whether spacetime is genuinely discrete, it appears as such due to the peculiarities of the measurement process in quantum mechanics. As a consequence, quantum fluctuations that pop out of a vacuum behave as if they were created at discrete points on the Planck scale. This result has been exploited to infer the zero-point energy in the universe, which turns out to be very close to the observed value of the cosmological constant. We once more emphasize that our considerations and results are essentially based on heuristic reasoning. Hence, although the present analysis does not pretend to address the question of minimal length from a very fundamental perspective, it still provides hints towards understanding the intrinsic resolution of spacetime that should emerge in the fundamental theory of quantum gravity.

Further consequences of the existence of a minimal measurable length deserve to be investigated. For instance, in [65,66] it has been shown that minimal length systems can be described in terms of non-extensive statistics. Specifically, the emergence of an elementary length scale would enter into the phase space structure by modifying the elementary cell volume and giving rise to a non-Gaussian (Tsallis-like) statistics. Thus, it is interesting to analyze deeper this connection in order to see whether minimal length-like effects can be measured in non-extensive systems, such as self-gravitating stellar systems, black holes, dissipative systems, or polymer chains. Work along this and other directions is presently under consideration and will be presented elsewhere.

Author Contributions: Conceptualization, J.G. and G.G.L.; formal analysis, J.G. and G.G.L.; investigation, J.G. and G.G.L.; writing—original draft preparation, J.G. and G.G.L.; writing—review and editing, G.G.L. All authors have read and agreed to the published version of the manuscript.

Funding: J.G. is partially supported by the Agencia Estatal de Investigación grant PID2020-113758GB-I00 and an AGAUR (Generalitat de Catalunya) grant number 2021SGR 01618. G.G.L. acknowledges the Spanish "Ministerio de Universidades" for the awarded Maria Zambrano fellowship and funding received from the European Union—NextGenerationEU.

Data Availability Statement: No new data were created or analyzed in this study. Data sharing is not applicable to this article.

Acknowledgments: G.G.L. acknowledges the participation in the COST Association Action CA18108 "Quantum Gravity Phenomenology in the Multimessenger Approach".

Conflicts of Interest: The authors declare no conflicts of interest.

Note

[1] For our purposes of estimating the order of magnitude of the minimal length, it is enough to resort to semiclassical arguments. Clearly, a more rigorous derivation would require considerations from relativistic scattering theory, in which collisions between particles are examined using the principles of special relativity, considering that the particles involved might be moving at velocity close to the speed of light. Since this framework often employs techniques from quantum field theory to describe the scattering amplitudes and cross-sections of particles, we reserve for future investigation exploring whether our result is affected in this context.

References

1. Green, M.B.; Schwarz, J.H.; Witten, E. *Superstring Theory. Volume 1: Introduction*; Cambridge Monographs on Mathematical Physics; Cambridge Univeristy Press: Cambridge, UK; New York, NY, USA, 1988.
2. Mukhi, S. String theory: A perspective over the last 25 years. *Class. Quant. Grav.* **2011**, *28*, 153001. [CrossRef]
3. Duff, M.J. M theory (The Theory formerly known as strings). *Int. J. Mod. Phys. A* **1996**, *11*, 5623–5642. [CrossRef]
4. Kakushadze, Z.; Tye, S.H.H. Brane world. *Nucl. Phys. B* **1999**, *548*, 180–204. [CrossRef]
5. Rovelli, C. Loop quantum gravity. *Living Rev. Rel.* **1998**, *1*, 1. [CrossRef] [PubMed]
6. Weinberg, S. Ultraviolet Divergences in Quantum Theories of Gravitation. In *General Relativity: An Einstein Centenary Survey*; Cambridge Univeristy Press: Cambridge, UK, 1980; pp. 790–831.
7. Addazi, A.; Alvarez-Muniz, J.; Batista, R.A.; Amelino-Camelia, G.; Antonelli, V.; Arzano, M.; Asorey, M.; Atteia, J.-L.; Bahamonde, S.; Bajardi, F.; et al. Quantum gravity phenomenology at the dawn of the multi-messenger era—A review. *Prog. Part. Nucl. Phys.* **2022**, *125*, 103948. [CrossRef]
8. Khodadi, M.; Nozari, K.; Dey, S.; Bhat, A.; Faizal, M. A new bound on polymer quantization via an opto-mechanical setup. *Sci. Rep.* **2018**, *8*, 1659. [CrossRef]
9. Petruzziello, L.; Illuminati, F. Quantum gravitational decoherence from fluctuating minimal length and deformation parameter at the Planck scale. *Nat. Commun.* **2021**, *12*, 4449. [CrossRef]
10. Maldacena, J.M. The Large N limit of superconformal field theories and supergravity. *Adv. Theor. Math. Phys.* **1998**, *2*, 231–252. [CrossRef]
11. Susskind, L. Copenhagen vs Everett, Teleportation, and ER = EPR. *Fortsch. Phys.* **2016**, *64*, 551–564. [CrossRef]

12. Van Raamsdonk, M. Building up spacetime with quantum entanglement. *Gen. Rel. Grav.* **2010**, *42*, 2323–2329. [CrossRef]
13. Tamburini, F.; Licata, I. General Relativistic Wormhole Connections from Planck-Scales and the ER = EPR Conjecture. *Entropy* **2019**, *22*, 3–18. [CrossRef] [PubMed]
14. Einstein, A.; Podolsky, B.; Rosen, N. Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? *Phys. Rev.* **1935**, *47*, 777–780. [CrossRef]
15. Horodecki, R.; Horodecki, P.; Horodecki, M.; Horodecki, K. Quantum entanglement. *Rev. Mod. Phys.* **2009**, *81*, 865–942. [CrossRef]
16. Einstein, A.; Rosen, N. The Particle Problem in the General Theory of Relativity. *Phys. Rev.* **1935**, *48*, 73–77. [CrossRef]
17. Bosso, P.; Petruzziello, L.; Wagner, F. The minimal length is physical. *Phys. Lett. B* **2022**, *834*, 137415. [CrossRef]
18. Scardigli, F. Some heuristic semiclassical derivations of the Planck length, the Hawking effect and the Unruh effect . *Nuovo Cim. B* **1995**, *110*, 1029–1034. [CrossRef]
19. Lambiase, G.; Scardigli, F. Lorentz violation and generalized uncertainty principle. *Phys. Rev. D* **2018**, *97*, 075003. [CrossRef]
20. Blasone, M.; Lambiase, G.; Luciano, G.G.; Petruzziello, L.; Scardigli, F. Heuristic derivation of Casimir effect in minimal length theories. *Int. J. Mod. Phys. D* **2020**, *29*, 2050011. [CrossRef]
21. Nozari, K.; Gorji, M.A.; Hosseinzadeh, V.; Vakili, B. Natural Cutoffs via Compact Symplectic Manifolds. *Class. Quant. Grav.* **2016**, *33*, 025009. [CrossRef]
22. Amati, D.; Ciafaloni, M.; Veneziano, G. Superstring Collisions at Planckian Energies. *Phys. Lett. B* **1987**, *197*, 81. [CrossRef]
23. Konishi, K.; Paffuti, G.; Provero, P. Minimum Physical Length and the Generalized Uncertainty Principle in String Theory. *Phys. Lett. B* **1990**, *234*, 276–284. [CrossRef]
24. Maggiore, M. The Algebraic structure of the generalized uncertainty principle. *Phys. Lett. B* **1993**, *319*, 83–86. [CrossRef]
25. Capozziello, S.; Lambiase, G.; Scarpetta, G. Generalized uncertainty principle from quantum geometry. *Int. J. Theor. Phys.* **2000**, *39*, 15–22. [CrossRef]
26. Kempf, A.; Mangano, G.; Mann, R.B. Hilbert space representation of the minimal length uncertainty relation. *Phys. Rev. D* **1995**, *52*, 1108–1118. [CrossRef] [PubMed]
27. Scardigli, F. Generalized uncertainty principle in quantum gravity from micro-black hole Gedanken experiment. *Phys. Lett. B* **1999**, *452*, 39–44. [CrossRef]
28. Scardigli, F.; Blasone, M.; Luciano, G.; Casadio, R. Modified Unruh effect from Generalized Uncertainty Principle. *Eur. Phys. J. C* **2018**, *78*, 728. [CrossRef] [PubMed]
29. Bosso, P.; Luciano, G.G.; Petruzziello, L.; Wagner, F. 30 years in: Quo vadis generalized uncertainty principle? *Class. Quant. Grav.* **2023**, *40*, 195014. [CrossRef]
30. Hossenfelder, S.; Bleicher, M.; Hofmann, S.; Ruppert, J.; Scherer, S.; Stoecker, H. Collider signatures in the Planck regime. *Phys. Lett. B* **2003**, *575*, 85–99. [CrossRef]
31. Buoninfante, L.; Luciano, G.G.; Petruzziello, L. Generalized Uncertainty Principle and Corpuscular Gravity. *Eur. Phys. J. C* **2019**, *79*, 663. [CrossRef]
32. Scardigli, F. The deformation parameter of the generalized uncertainty principle. *J. Phys. Conf. Ser.* **2019**, *1275*, 012004. [CrossRef]
33. Luciano, G.G. Tsallis statistics and generalized uncertainty principle. *Eur. Phys. J. C* **2021**, *81*, 672. [CrossRef]
34. Luciano, G.G.; Petruzziello, L. Generalized uncertainty principle and its implications on geometric phases in quantum mechanics. *Eur. Phys. J. Plus* **2021**, *136*, 179. [CrossRef]
35. Iorio, A.; Ivetić, B.; Mignemi, S.; Pais, P. Three "layers" of graphene monolayer and their analog generalized uncertainty principles. *Phys. Rev. D* **2022**, *106*, 116011. [CrossRef]
36. Nenmeli, V.; Shankaranarayanan, S.; Todorinov, V.; Das, S. Maximal momentum GUP leads to quadratic gravity. *Phys. Lett. B* **2021**, *821*, 136621. [CrossRef]
37. Petruzziello, L. Generalized uncertainty principle with maximal observable momentum and no minimal length indeterminacy. *Class. Quant. Grav.* **2021**, *38*, 135005. [CrossRef]
38. Ali, A.F.; Das, S.; Vagenas, E.C. Discreteness of Space from the Generalized Uncertainty Principle. *Phys. Lett. B* **2009**, *678*, 497–499. [CrossRef]
39. Park, M.I. The Generalized Uncertainty Principle in (A)dS Space and the Modification of Hawking Temperature from the Minimal Length. *Phys. Lett. B* **2008**, *659*, 698–702. [CrossRef]
40. Mignemi, S. Extended uncertainty principle and the geometry of (anti)-de Sitter space. *Mod. Phys. Lett. A* **2010**, *25*, 1697–1703. [CrossRef]
41. Giné, J.; Luciano, G.G. Modified inertia from extended uncertainty principle(s) and its relation to MoND. *Eur. Phys. J. C* **2020**, *80*, 1039. [CrossRef]
42. Unruh, W.G. Notes on black hole evaporation. *Phys. Rev. D* **1976**, *14*, 870. [CrossRef]
43. Giné, J. Hawking effect and Unruh effect from the uncertainty principle. *EPL* **2018**, *121*, 10001. [CrossRef]
44. Giné, J. Quantum fluctuations and the slow accelerating expansion of the Universe. *Europhys. Lett.* **2019**, *125*, 50002. [CrossRef]
45. Bishop, M.; Contreras, J.; Martin, P.; Singleton, D. Comments on the cosmological constant in generalized uncertainty models. *Front. Astron. Space Sci.* **2022**, *9*, 978898. [CrossRef]
46. Alsing, P.M.; Milonni, P.W. Simplified derivation of the Hawking-Unruh temperature for an accelerated observer in vacuum. *Am. J. Phys.* **2004**, *72*, 1524–1529. [CrossRef]

47. Birrell, N.D.; Davies, P.C.W. *Quantum Fields in Curved Space*; Cambridge Monographs on Mathematical Physics; Cambridge University Press: Cambridge, UK, 1984. [CrossRef]
48. Takagi, S. Vacuum Noise and Stress Induced by Uniform Acceleration: Hawking-Unruh Effect in Rindler Manifold of Arbitrary Dimension. *Prog. Theor. Phys. Suppl.* **1986**, *88*, 1–142. [CrossRef]
49. Blasone, M.; Lambiase, G.; Luciano, G.G. Nonthermal signature of the Unruh effect in field mixing. *Phys. Rev. D* **2017**, *96*, 025023. [CrossRef]
50. McCulloch, M.E. Modelling the Pioneer anomaly as modified inertia. *Mon. Not. R. Astron. Soc.* **2007**, *376*, 338–342. [CrossRef]
51. Giné, J.; Mcculloch, M.E. Inertial mass from Unruh temperatures. *Mod. Phys. Lett. A* **2016**, *31*, 1650107. [CrossRef]
52. Giné, J.; Luciano, G.G. Modeling inertia through the interaction with quantum fluctuations. *Results Phys.* **2021**, *28*, 104543. [CrossRef]
53. Giné, J. Modified Hawking effect from generalized uncertainty principle. *Commun. Theor. Phys.* **2021**, *73*, 015201. [CrossRef]
54. Particle Data Group; Zyla, P.A.; Barnett, R.M.; Beringer, J.; Dahl, O.; Dwyer, D.A.; Groom, D.E.; Lin, C.-J.; Lugovsky, K.S.; Pianori, E.; et al. Review of Particle Physics. *PTEP* **2020**, *2020*, 083C01. [CrossRef]
55. Yamamoto, H. Discrete Space-Time and Lorentz Invariance. *Nucl. Phys. B Proc. Suppl.* **1989**, *6*, 154–156. [CrossRef]
56. Dowker, F.; Henson, J.; Sorkin, R.D. Quantum gravity phenomenology, Lorentz invariance and discreteness. *Mod. Phys. Lett. A* **2004**, *19*, 1829–1840. [CrossRef]
57. Bosso, P.; Das, S. Lorentz invariant mass and length scales. *Int. J. Mod. Phys. D* **2019**, *28*, 1950068. [CrossRef]
58. Johnston, S. Particle propagators on discrete spacetime. *Class. Quant. Grav.* **2008**, *25*, 202001. [CrossRef]
59. Das, A.; Das, S.; Vagenas, E.C. Discreteness of Space from GUP in Strong Gravitational Fields. *Phys. Lett. B* **2020**, *809*, 135772. [CrossRef]
60. Das, S.; Vagenas, E.C.; Ali, A.F. Discreteness of Space from GUP II: Relativistic Wave Equations. *Phys. Lett. B* **2010**, *690*, 407–412. Erratum in *Phys. Lett. B* **2010**, *692*, 342. [CrossRef]
61. Surya, S. The causal set approach to quantum gravity. *Living Rev. Rel.* **2019**, *22*, 5. [CrossRef]
62. Dowker, F. Causal sets as discrete spacetime. *Contemp. Phys.* **2006**, *47*, 1–9. [CrossRef]
63. Brun, T.A.; Mlodinow, L. Detecting discrete spacetime via matter interferometry. *Phys. Rev. D* **2019**, *99*, 015012. [CrossRef]
64. Richardson, J.W.; Kwon, O.; Gustafson, H.R.; Hogan, C.; Kamai, B.L.; McCuller, L.P.; Meyer, S.S.; Stoughton, C.; Tomlin, R.E.; Weiss, R. Interferometric Constraints on Spacelike Coherent Rotational Fluctuations. *Phys. Rev. Lett.* **2021**, *126*, 241301. [CrossRef] [PubMed]
65. Shababi, H.; Ourabah, K. Non-Gaussian statistics from the generalized uncertainty principle. *Eur. Phys. J. Plus* **2020**, *135*, 697. [CrossRef]
66. Jizba, P.; Lambiase, G.; Luciano, G.G.; Petruzziello, L. Decoherence limit of quantum systems obeying generalized uncertainty principle: New paradigm for Tsallis thermostatistics. *Phys. Rev. D* **2022**, *105*, L121501. [CrossRef]