# Hairy black holes in $N = 2$ gauged supergravity

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We construct black holes with scalar hair in a wide class of four-dimensional $N = 2$ Fayet-Iliopoulos gauged supergravity theories that are characterized by a prepotential containing one free parameter. Considering the truncated model in which only a single real scalar survives, the theory is reduced to the Einstein-scalar system with a potential. Our solution is static, admits maximally symmetric horizons, asymptotically tends to the AdS space corresponding to the extremum of superpotential, but is disconnected with the Schwarzschild-AdS family. Our solution displays fall-off behaviors different from the standard one, due to the fact that the mass parameter at the SUSY vacuum is given by $m^2 = -2\ell^{-2}$. Nevertheless, we identify a well-defined mass in our spacetime, following the prescription of Hertog and Maeda. Our solution shows qualitatively the same thermodynamic behavior as the Schwarzschild-AdS black hole but the entropy is always smaller for a given mass and the AdS radius. We find that our spherical black hole is unstable against radial perturbations.

*Keywords*: Black holes; classical theories of gravity.

## 1. Introduction

A novel 'no hair conjecture' proposed by Ruffini and Wheeler cast a lot of insights into physics of black holes in our world. Unlike the Einstein-Maxwell system without a cosmological constant in which uniqueness thermos of stationary black holes have been established, we do not know much about the spectrum of black holes that are asymptotic to the AdS space. AdS black holes with scalar hair are now of primary importance in the context of gauge/gravity correspondence and applications to condensed matter physics. This is a principal motivation of our present attempt to constructing a black hole solution in the context of supergravity.

This paper presents a short summary of the Ref. 3, in which we constructed an interesting class of exact hairy black hole solution in $N = 2$ gauged supergravity. We explore interesting physical aspects of black-hole geometry. In particular, horizon conditions, thermodynamics and dynamical (in)stability are discussed in detail.

## 2. Hairy black holes

### 2.1. *Model*

We consider the $N = 2$, $D = 4$ gauged supergravity coupled to abelian vector multiplets. We focus on a model with a single vector multiplet and with a prepotential given by

$$F(X) = -\frac{i}{4}(X^0)^n(X^1)^{2-n}\,, \tag{1}$$

where $n$ is a real parameter. This model is obtained by a truncation of the STU theory. Assuming that the scalar field parametrizing the special Kähler manifold is real ($z = \bar{z}$) and truncating further to the model where the electromagnetic fields are absent, the action now reduces to the Einstein-scalar system

$$S = \int \left[ \frac{R}{2} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right] \sqrt{-g} \, \mathrm{d}^4 x \,, \tag{2}$$

where the potential $V$ can be expressed in terms of the real superpotential $W$ as

$$V = 4 \left[ 2(\partial_\phi W)^2 - 3W^2 \right] \,, \qquad W(\phi) = g_1 e^{n\lambda_n \phi/2} + g_0 e^{(n-2)\lambda_n \phi/2} \,. \tag{3}$$

Here $\lambda_n = \sqrt{2/(n(2-n))}$ and the range of $n$ ($0 < n < 2$) comes from the restriction $\mathrm{Im}\mathcal{N} < 0$. $g_0, g_1$ are coupling constants which we shall assume to be positive in what follows.

The potential (3) allows at most two extrema,

$$e^{\lambda_n \phi_1} = \frac{g_0(2-n)}{g_1 n} \,, \qquad e^{\lambda_n \phi_2} = \frac{g_0(2-n)(1-2n)}{g_1(3-2n)n} \,. \tag{4}$$

The critical point $\phi = \phi_1$ is a supersymmetric AdS vacuum, since it also extremizes the superpotential. It is notable that the mass parameter at $\phi = \phi_1$ is given by

$$m^2 = -2\ell^{-2} \,, \tag{5}$$

where $\ell = \rho_0/(2\sqrt{2} g_0)$ is the AdS curvature and we have introduced $\rho_0 \equiv \frac{n}{\sqrt{2}} (\frac{(2-n)g_0}{n g_1})^{1-n/2}$. This is exactly the value for the conformally coupled scalar field in AdS. It is important to remark that the mass (5) at the vacuum $\phi_1$ lies in the Breitenlohner-Freedman (BF) range 1

$$m^2_{\mathrm{BF}} \le m^2 \le m^2_{\mathrm{BF}} + \frac{1}{\ell^2} \,. \tag{6}$$

By requiring the conservation and the positivity of suitable energy functional, Breitenlohner and Freedman 1 found that the slowly decaying mode of the scalar field is also normalizable when the mass parameter lies in this range. It was later shown by Ishibashi and Wald 2 that this is also a necessary condition for stability of the scalar field. They also demonstrated that the self-adjoint extension of Hamiltonian operator is specified by a single parameter $\alpha$ corresponding to possible boundary conditions. Let $r$ denote the standard radial coordinate of AdS. Then the free scalar field $\phi$ propagating in AdS behaves at infinity as

$$\phi \sim \frac{\phi_-(x^i)}{r^{\lambda_-}} + \frac{\phi_+(x^i)}{r^{\lambda_+}} \,, \qquad \phi_+ = \alpha \phi_-^{\lambda_+/\lambda_-} \,, \tag{7}$$

where $\lambda_\pm = \frac{1}{2}(3 \pm \sqrt{9 + 4m^2\ell^2})$. In our model, $\lambda_+ = 2$ and $\lambda_- = 1$. The parameter $\alpha$ plays a crucial role when evaluating the mass of the spacetime.

### 2.2. *Black hole solution*

The exact hairy black hole that we found is static and admitting a maximally symmetric horizon. The metric is given by

$$\mathrm{d}s^2 = -\frac{e^{2\psi}}{f_1^n f_2^{2-n}}\mathrm{d}t^2 + \frac{f_1^n f_2^{2-n}}{e^{2\psi}}\mathrm{d}r^2 + f_1^n f_2^{2-n}\mathrm{d}\Sigma_k^2\,, \tag{8}$$

where $\mathrm{d}\Sigma_k^2$ is the metric of maximally symmetric space with a sectional curvature $k = 0, \pm 1$, and

$$\begin{aligned} f_1 &= \frac{n}{\sqrt{2}}\left(r + \frac{2\beta}{n}\right), \qquad f_2 = \frac{2-n}{\sqrt{2}}\frac{g_0}{g_1}\left(r - \frac{2\beta}{2-n}\right), \\ e^{2\psi} &= 8g_0^2\left(r + \frac{2\beta}{n}\right)\left(r - \frac{2\beta}{2-n}\right)\left(r^2 - \frac{4(1-n)}{n(2-n)}\beta r + 4\frac{5n^2-10n+4}{n^2(2-n)^2}\beta^2 + \frac{k}{8g_0^2}\right), \\ e^{\lambda_n \phi} &= \frac{g_0(2-n)}{g_1 n}\,\frac{r - 2\beta/(2-n)}{r + 2\beta/n}, \end{aligned} \tag{9}$$

As described in detail in Ref. 3, the metric (8) admits a parameter range under which the event horizon exist for each horizon topology $k = 0, \pm 1$. Hence, (8) indeed corresponds to the hairy black hole.

### 2.3. *Mass*

Defining a areal radius by $\rho \equiv \sqrt{f_1^n f_2^{2-n}}$, the metric (8) asymptotically behaves as

$$\mathrm{d}s^2 \simeq -\left(k + \frac{\rho^2}{\ell^2} - \frac{2\mu_1}{\rho}\right)\mathrm{d}\tau^2 + \left(k + \gamma + \frac{\rho^2}{\ell^2} - \frac{2\mu_2}{\rho}\right)^{-1}\mathrm{d}\rho^2 + \rho^2\mathrm{d}\Sigma_k^2\,, \tag{10}$$

where $\tau = \rho_0^{-1}t$, $\gamma \equiv 32g_0^2\beta^2/(n(2-n))$ and

$$\mu_1 = \frac{1}{12}\rho_0\lambda_n^6(n-1)\beta[3kn^2(n-2)^2 + 128g_0^2\beta^2(3-2n)(1-2n)]\,, \tag{11a}$$

$$\mu_2 = \frac{1}{12}\rho_0\lambda_n^6(n-1)\beta[3kn^2(n-2)^2 + 128g_0^2\beta^2(5n^2-10n+3)]\,. \tag{11b}$$

The scalar field behaves as (7) with $\alpha = \frac{1}{2}(1-n)\lambda_n$. As one sees from (10), the metric displays the fall-off behavior which is different from the standard one with $\gamma = 0$. This is precisely because the mass of the scalar field lies in the BF range. Hence, one cannot use the standard mass definition valid only in the Dirichlet boundary condition ($\phi_- = 0$) for computing conserved quantities associated with the asymptotic AdS symmetries.

According to the prescription proposed by Hertog and Maeda 4, the standard definition of conserved quantities in AdS space should be modified due to the contribution of slowly decaying scalar field. Amazingly, the divergent contributions coming from the metric and the scalar field are precisely cancelled out, giving rise to the finite result. Generalizing their argument to the topological cases, we find that the mass is given by

$$M = \frac{\Sigma_k \mu_1}{4\pi}\,. \tag{12}$$

This value picks out the Coulomb part of the spacetime $\Psi_2 = -\mu_1/\rho^3 + \mathcal{O}(1/\rho^4)$, corresponding to the electric part of the Weyl tensor.

### 2.4. *Thermodynamics*

Let us finally discuss the thermodynamics of our hairy black hole. Letting $r_+$ denote the locus of event horizon, then the area and the surface gravity are given by

$$A = \Sigma_k f_1^n(r_+) f_2^{2-n}(r_+)\,, \qquad \kappa = \sqrt{-\frac{1}{2}\nabla_\mu \xi_\nu \nabla^\mu \xi^\nu}\bigg|_{r=r_+}\,, \tag{13}$$

where $\xi = \rho_0 \partial_t$ is the generator of the Killing horizon. It is easy to show that these quantities fulfill the 1st law of black hole thermodynamics

$$\delta M = \frac{\kappa}{8\pi}\delta A\,. \tag{14}$$

For the fixed mass, the area of our hairy black hole is always smaller than the Schwarzschild-AdS black hole (see figure 2 of Ref. 3). This implies that our black hole is unstable. Indeed, our numerical computation shows that the hairy black hole is unstable (at least in certain range of parameter) under the spherically symmetric gravitational perturbations.

It is interesting to explore the implication of this instability in the context of holography. Constructing an exact rotating black hole admitting only the helical symmetry is also an intriguing future work.

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