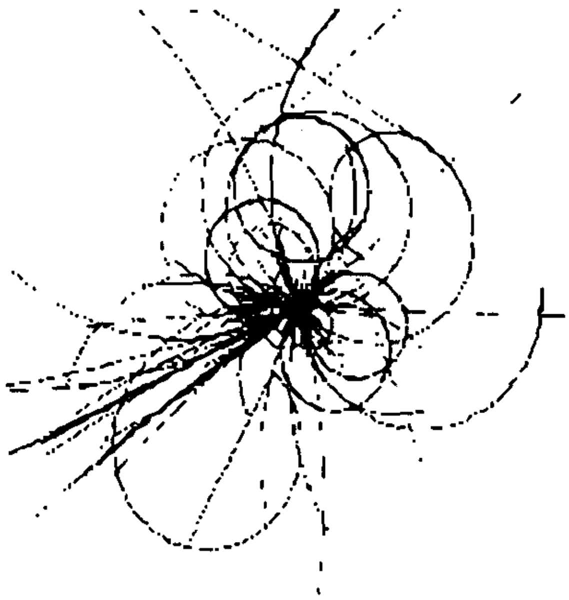
THE SUPERCONDUCTING SUPER COLLIDER LABORATORY



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IMPEDANCE EFFECTS ON DECOHERENCE RATE OF EXPERIMENT 778

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I. THE MODEL

Consider N particles in a coasting beam oscillating with transverse positions $x_i(t,\theta)$ and betatron frequencies $\omega_i/2\pi$. They obey the equation of motion in the lab frame:[1]

$$\left\{\frac{\partial^2}{\partial t^2}+2\omega_0\frac{\partial^2}{\partial t\partial\theta}+\omega_0^2\frac{\partial^2}{\partial\theta^2}+\omega_i^2\right\}x_i(t,\theta)=iC\bar{x}(t,\theta)\ , \tag{1}$$

where $\omega_0/2\pi$ is the revolution frequency, θ is the azimuthal angle around the storage ring, and C represents the driving force:

$$C=\frac{Z_\perp I_0 c^2}{2\pi RE/e}\ , \tag{2}$$

with $Z_\perp$ the transverse impedance, I_0 the beam current, c light velocity, R ring radius, and E particle longitudinal energy. We neglect tune dependence on amplitude here and only consider a tune distribution

$$f(\omega)=\frac{\sigma}{\pi}\frac{1}{(\omega-\omega_\beta)^2+\sigma^2}\ , \tag{3}$$

where $\omega_\beta/2\pi$ is the mean betatron frequency and $\sigma/2\pi$ represents the spread. Solving Eq. (1), the dispersion relation is

$$\Omega'(\Omega'-\omega_\beta+i\sigma)(\Omega'+\omega_\beta+i\sigma)+iC(\Omega'+i\sigma)=0\ , \tag{4}$$

where $\Omega'=\Omega-n\omega_0$ with $\Omega/2\pi$ the coherent betatron frequency and n the azimuthal harmonic corresponding to θ. Equation (1) is solved as an initial value problem with all the particles kicked to an initial displacement of x_n^0. If we further assume that $C/2\omega_\beta$ and σ have the same order of magnitude and they are both much less than ω_β, the solution for the center-of-mass of the particles can be simplified to

$$\begin{aligned}\bar{x}(t,\theta) &= \sum_{n=-\infty}^{\infty} x_n^0 \left\{ \frac{in\omega_0\sigma}{\omega_\beta^2} e^{-in(\omega_0 t-\theta)} \right. \\ &\quad + \frac{n\omega_0+\omega_\beta}{\omega_\beta} e^{-i[(n\omega_0-\omega_\beta)t-n\theta]-(\sigma-C/2\omega_\beta)t} \\ &\quad \left. - \frac{n\omega_0-\omega_\beta}{\omega_\beta} e^{-i[(n\omega_0+\omega_\beta)t-n\theta]-(\sigma+C/2\omega_\beta)t} \right\} . \qquad (5)\end{aligned}$$

It is clear that the effect of the impedance driving force will change the decoherence rate by $C/2\omega_\beta$ only.

The Lorentian distribution (3) has been chosen in favor of the more common Gaussian, because the former leads to a solution with a finite number of terms corresponding to a finite number of zeroes in the dispersion relation (4). On the other hand, a Gaussian distribution will lead to infinite number of terms in the solution (5). Just retaining the term with the highest growth may be meaningless because an infinite sum may lead to anything. However, a Lorentian distribution leads to an exponential decoherence instead of a Gaussian decoherence when a Gaussian distribution is used. A Lorentian distribution is not so well convergent as a Gaussian distribution. This explains why $\bar{x}(t,\theta)$ in Eq. (5) does not vanish when $t \to 0_+$ even if we keep all orders in σ. For a more reasonable description, may be a Lorentian-squared should be used. What we will get are exactly the three exponential terms in Eq. (5), but the coefficients in front of the exponentials may be different. Therefore, for the present investigation, a simple Lorentian is good enough.

II. APPLICATION TO EXPERIMENT 778

We want to know how much the impedance will affect the decoherence rate. In the 1987 run, E778 was performed[2] with bunch intensity $N = 0.5 \times 10^{10}$ per bunch, rms bunch length $\sigma_\ell \sim 15$ cm, rms momentum spread $\sigma_p/p = 1.5 \times 10^{-4}$, energy $E = 150$ GeV, and betatron tune $\nu_\beta \sim 20$. With a chromaticity of 10 units, the betatron-frequency spread is

$$\sigma \sim 450/\text{sec} \qquad \text{or} \qquad \text{decoherence time} \sim 700 \text{ turns} . \qquad (6)$$

The current I_0 in Eq. (2) should be taken as the peak bunch current. Thus, $I_0 = eNc/\sqrt{2\pi}\sigma_\ell = 0.64$ A. For the Tevatron,[3] $Z_\parallel/n \sim 1$ to 5 Ohms from 1 MHz to 0.5

GHz, corresponding to $Z_\perp \sim 2R(Z_\parallel/n)/b^2 \sim 1.6$ to 8.0 MΩ/m, taking the beam pipe radius as $b = 3.5$ cm and Tevatron radius $R = 1$ km. We get

$$\frac{C}{2\omega_\beta} \sim 7.7 \text{ to } 39/\text{sec} \qquad \text{or} \qquad \text{decoherence time } \sim 40000 \text{ to } 8000 \text{ turns}\ , \tag{7}$$

which is at least about an order of magnitude less significant than σ in Eq. (7). Since the natural tune spread σ is small compared with the spread due to anharmonic effects (tune dependence on amplitude),[4] we can therefore safely conclude that the effect of coupling impedance to the observed decoherence rate ($\sim$ 200 turns) is negligible.

III. IMPEDANCE CONTRIBUTION TO OTHER GROWTH RATES

The criterion of transverse microwave stability for a relativistic bunch is[5]

$$|Z_\perp(\omega)| \le \frac{4\sqrt{2\pi}|\eta|}{I_p\bar\beta}\left(\frac{E}{e}\right)\left(\frac{\sigma_p}{p}\right)\left(\frac{\omega}{\omega_0}\right)\ , \tag{8}$$

where η is the frequency dispersion, I_p is the peak current, and $\bar\beta$ is the average beta-function. Using $\eta \sim 0.0028$, $\bar\beta \sim 100$ m, we get at 1 GHz, $|Z_\perp| \le 207$ MΩ/m.

The criterion of transverse mode-crossing stability is[5]

$$|\bar Z_\perp| \le \frac{4\sqrt{\pi}|\eta|}{I_0\bar\beta}\left(\frac{E}{e}\right)\left(\frac{\sigma_p}{p}\right)\ , \tag{9}$$

where I_0 is the *average* current per bunch. We get $|\bar Z_\perp| \le 117$ MΩ/m. It is clear that we are far away from the instability limits of these two types of instabilities.

For the head-tail dipole mode, the growth time in number of turns $\mathcal{N}$ is[6]

$$\mathcal{N} = \frac{2\pi\eta\nu_\beta E/e}{eNc\xi\, \mathcal{R}e\, Z_\perp}\ . \tag{10}$$

The formula is valid if $\mathcal{R}e\, Z_\perp$ is a constant and the betatron chromaticity phase lag $\xi\omega_0\hat\tau/\eta \ll 1$, where $\hat\tau$ is the maximum half bunch length in time. In our case, this phase lag is ~ 0.54 . We get

$$\mathcal{N} = \frac{1.1\times 10^5}{N\xi\, \mathcal{R}e\, Z_\perp}\ , \tag{11}$$

where N is in 10^{10} and $Z_\perp$ in MΩ/m. Putting in $N = 0.5$, $\xi \sim 10$, and $\mathcal{R}e\, Z_\perp \sim 8$, we obtain $\mathcal{N} \sim 2750$ which is an order of magnitude bigger than the observed value. Also in Expt E778, a scanning of ξ from -20 to $+20$ showed no significant change in the decoherence time.[2] We can therefore rule out the influence of any head-tail growth in the experiment.

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