MODELING HIGHLY COMPRESSIBLE FLOWS IN PIPE NETWORKS USING A GRAPHICAL USER INTERFACE

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ABSTRACT

A highly accurate compressible flow solution engine has been combined with an advanced graphical interface in *AFT Arrow* for Windows. *AFT Arrow* models the flow of real gases, including steam, in complex network piping systems, and also offers heat transfer and system energy balance capabilities. A discussion of the governing equations is given, as well as a summary of the solution method for network pipe systems. An overview of sonic choking in gas piping systems is also given. Several applications are discussed.

NOMENCLATURE

a	=	sonic speed
A	=	cross-sectional flow area of a pipe
D	=	diameter of a pipe
f	=	friction factor
F_{To}	=	Parameter in Equation 9
F_f	=	Parameter in Equation 9
h	=	enthalpy, static
h_o	=	enthalpy, stagnation
L	=	length of a pipe
$\dot{m}$	=	mass flow rate
M	=	Mach Number
P	=	pressure
P_o	=	pressure, stagnation
R	=	gas constant
T	=	temperature, static
T_o	=	temperature, stagnation
V	=	velocity
x	=	length
Z	=	compressibility factor
γ	=	specific heat ratio
ρ	=	density

<u>Subscripts</u>

1	=	Location 1 in pipe
2	=	Location 2 in pipe
i	=	junction at which solution is sought
j	=	junctions with pipes connecting to junction *i*

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INTRODUCTION

Compressible flow modeling in pipe networks is a challenging technical application because of the coupled nature of the governing equations and the large number of parameters involved. For single pipe systems engineers can frequently perform a rough compressible flow calculation for prediction of pressure drop. However, even slight increases in complexity, including branched or looped systems, results in extreme analytical difficulties. If real gas effects are included (which is typical of steam and natural gas systems) the complications compound further.

Because of the analytical difficulties involved, engineers will typically overdesign the piping system using seat-of-the-pants approximations. One serious drawback of overdesigning systems is the increased cost to build and operate the system.

To accurately calculate the pressure drop, flow rates and temperatures in gas systems, a coupled solution method that simultaneously solves all of the relevant equations is required. In addition, a piping network in which flow splits are unknown further requires a matrix method to balance flow and energy in the network.

Still another formidable challenge is that of sonic choking. Sonic choking occurs when the gas cannot flow through a pipe or flow restriction without accelerating past its sonic speed. In such cases the flow "chokes", which means that a shock wave is generated and changes in downstream conditions cannot produce any additional flow through the system.

To assist engineers in designing gas piping systems which encounter the previously mentioned difficulties, *AFT Arrow* for Windows, a compressible flow modeling software product for pipe networks, was developed. *AFT Arrow* addresses all of the relevant phenomena in gas systems including sonic choking.

AFT Arrow incorporates these modeling capabilities within an advanced Windows graphical interface (Figure 1). All modeling is performed with simple drag-and-drop methods, offering a short learning curve and increased ability to identify modeling errors.

FUNDAMENTAL EQUATIONS OF COMPRESSIBLE FLOW

There are five equations that govern compressible flow in piping systems. For each individual pipe these are:

Mass:
$$\frac{d\rho}{\rho} + \frac{dV}{V} = 0 \qquad (1)$$

Momentum:
$$dP + \frac{1}{2}\rho V^2 \frac{f}{D} dx + \rho V dV = 0 \qquad (2)$$

Energy:
$$\dot{m}\; d\left(h + \frac{1}{2}V^2\right) = q \qquad (3)$$

Equation of State:
$$P = Z\rho RT \qquad (4)$$

Mach Number:
$$M = \frac{V}{\sqrt{\gamma ZRT}} \qquad (5)$$

In order to obtain a system mass and energy balance, the following two equations must be satisfied for all branching nodes:

Balance Mass at Branches:
$$\sum_{j=1}^{n} \dot{m}_{ij} = 0 \qquad (6)$$

Balance Energy at Branches:
$$\sum_{j=1}^{n} \dot{m}_{ij}\left(h_{ij} + \frac{1}{2}V_{ij}^{\,2}\right) = 0 \qquad (7)$$

The Length March Method

AFT Arrow implements three independent methods to solve these equations, of which two will be discussed here. For systems that do not approach sonic velocity, the Length March Method (LMM) is optimal. The LMM breaks each pipe into a certain number of sections and solves the governing equations over each section.

Applying calculus and after some algebraic

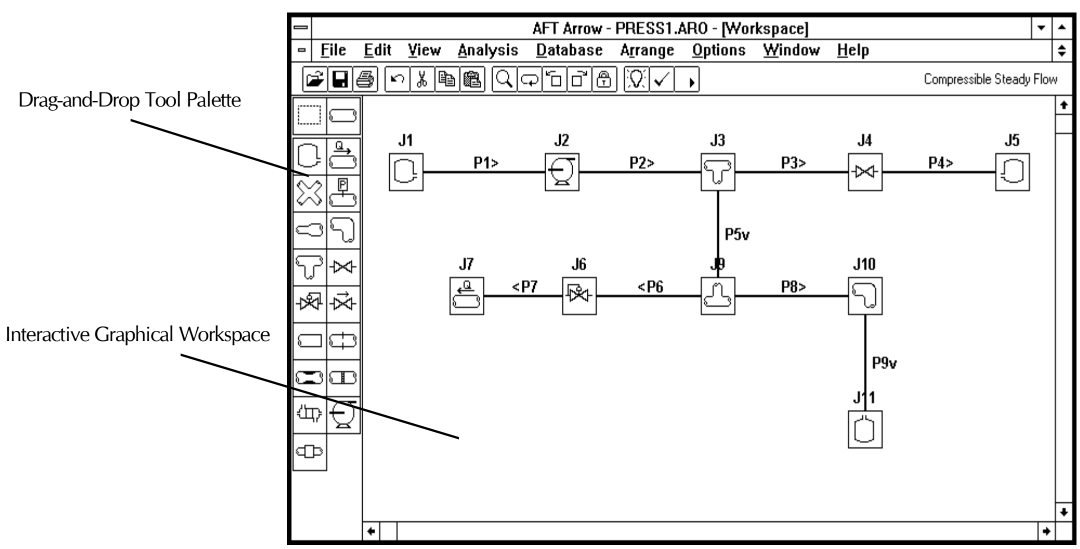

Figure 1. ***AFT Arrow*** **graphical Workspace simplifies model building.**

manipulation, the following equation can obtained which is based on a fixed length step between computing sections.

$$\frac{dP_o}{P_o} = -\frac{\gamma M^2}{2}\left(\frac{fdx}{D} + \frac{dT_o}{T_o} + \frac{dZ}{Z} + \frac{d\gamma}{\gamma}\right)$$

Integration yields:

$$P_{o,2} = P_{o,1} \exp\left[-\frac{\gamma \overline{M}^2}{2}\left(\frac{f}{D}(x_2 - x_1) + \ln\frac{T_{o,2}}{T_{o,1}} + \ln\frac{Z_2}{Z_1} + \ln\frac{\gamma_2}{\gamma_1}\right)\right] \quad (8)$$

Conditions at section 1 are known and the goal is to find conditions at section 2 which satisfies the above equation. Obviously, extensive iteration is required because there are multiple unknowns at section 2 in this equation which are converged upon by applying Equations 1-5 repeatedly.

The logarithmic terms that involve γ and Z account for real gas effects, and have been observed by AFT to be significant for many real gas systems, including steam systems. AFT is not aware of any other published derivations which include these terms and thus account for the real gas effects with complete accuracy.

The Mach March Method

An alternative solution method is optimal for systems that have sonic choking. This method, the Mach March Method (MMM), uses a variable length step based on how fast the Mach Number changes. It takes solution steps over equal Mach Number increments rather than length increments.

Applying calculus and algebra, the following equation can be obtained for the MMM:

$$\frac{dM^2}{M^2} = F_{To}\frac{dT_o}{T_o} + F_f\frac{fdx}{D} + F_{To}\frac{dZ}{Z} + F_{To}\frac{d\gamma}{\gamma}$$

where:

$$F_{To} = \frac{\left(1+\gamma M^2\right)\left(1+\frac{\gamma-1}{2}M^2\right)}{1-M^2}$$

$$F_f = \frac{\gamma M^2\left(1+\frac{\gamma-1}{2}M^2\right)}{1-M^2}$$

Integration yields:

$$x_2 = x_1 + \frac{\ln\frac{M_2{}^2}{M_1{}^2} - \overline{F}_{To}\ln\frac{T_{o,2}}{T_{o,1}} - \overline{F}_{To}\ln\frac{Z_2}{Z_1} - \overline{F}_{To}\ln\frac{\gamma_2}{\gamma_1}}{\overline{F}_f\,\frac{f}{D}} \quad (9)$$

By selecting an increase in Mach Number, say by increments of 0.01 from M_1 to M_2, the distance to x_2 can be computed that is required to obtain this change in Mach Number. Again, heavy iteration is required because there are multiple unknowns at section 2 in this equation which are converged upon by applying equations 1-5 repeatedly.

The MMM is well suited to pipes which have sonic choking, as it is able to accurately follow the rapid acceleration at the end of the pipe and rapidly changing conditions. An example of such an application will be given in later section.

NETWORK SOLUTIONS

To solve Equations 6 and 7 for all junctions in a network, a network solution method is required. *AFT Arrow* uses a modified Netwon-Raphson method to determine the junction pressures and pipe flow rates which satisfy the balance equations.

SONIC CHOKING

Sonic choking is the phenomenon in gas piping systems where the flow conditions reach a Mach Number of 1, and cannot accept any further increase in flow rate. In such cases a lowering of downstream pressure will not produce any additional flow through the system.

A convenient equation can be derived (Saad (1993)) which relates all relevant parameters which affect sonic choking. This equation is based on the continuity equation. The equation shown here is that from Saad (1993), which is for an ideal gas, but corrected for real gas effects.

$$\dot{m} = A\frac{P_o}{\sqrt{T_o}}f(M) \quad (10)$$

$$f(M) = \sqrt{\frac{\gamma}{ZR}}M\left(1+\frac{\gamma-1}{2}M^2\right)^{-(\gamma+1)/[2(\gamma-1)]}$$

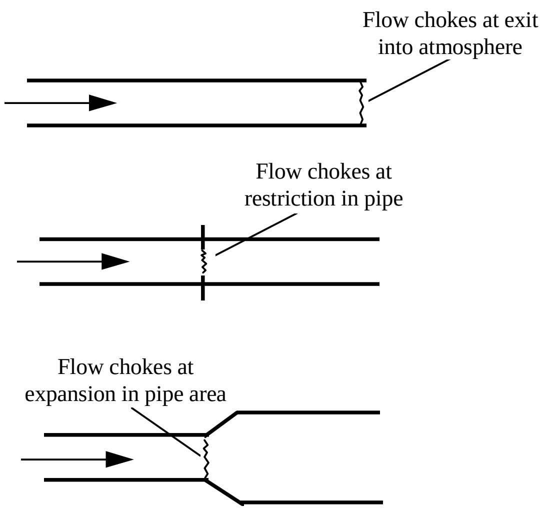

Figure 2. Three types of sonic choking.

Sonic choking can occur at three locations in a piping system (Figure 2):

1. At a pipe exit where it expands into a large tank or the atmosphere.
2. At a restriction in the pipe such as an orifice or valve. In such cases the area *A* becomes the C_dA, which accounts for the vena contracta of the flow which is the true flow area which the fluid must flow through.
3. At an expansion in pipe diameter such as occurs in a true expansion to a larger pipe diameter or a branching section where the total area of the connecting pipes exceeds that of the choked pipe.

In the first case, sonic choking is reached at the end of the pipe because the Mach Number (typically) increases along the pipe length. A shock wave occurs at the pipe exit and a discontinuity in pressure results. Figure 3 depicts the Mach Number variation along a sonically choked 100 foot pipe. The rapid increase in Mach Number near the end is apparent, which is the reason the MMM Method is preferred.

In the second case, the flow chokes because the area for flow through the restriction limits the mass flow rate according to Equation 10. The Mach Number in the adjoining pipes, however, is much less than 1 because of the larger area. A shock wave occurs across the restricted flow area and a discontinuity in pressure results.

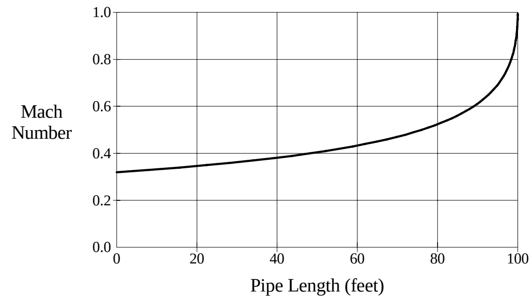

Figure 3. Mach Number variation in a 100 foot sonically choked pipe predicted by *AFT Arrow*.

In the third case, the sonic choking exists at the pipe exit as it enters the expansion. The Mach Number is 1 at this location and a shock wave exists, as well as a discontinuity in pressure. This case is similar in many respects to case 1, but with the difference being that the pressure is not known downstream of the shock wave and must be determined by iteration.

AFT Arrow models sonic choking in piping systems by checking all local solutions against Equation 10. If sonic choking exists, repeated point-and-shoot iteration is required to resolve the flow rate and the pressure drop across the shock waves.

A common but misleading belief in the engineering community is that sonic choking occurs when the pressure drop exceeds some percentage, about 50% for air systems. In reality, sonic choking is affected by the friction losses in the system and is dependent on such losses. Depending on the system, pressure losses much greater than 50% can occur without sonic choking occurring. In addition, real gas and thermal effects will influence the allowable pressure drop before sonic choking occurs.

SONIC CHOKING APPLICATION

Consider the steam system shown in Figure 4a. This system drops in pressure from 500 psia to 100 psia, and sonically chokes at the pipe exit. The *AFT Arrow* predictions are shown in Figures 4b-e for various fluid and thermodynamic parameters. The rapid change of all parameters near the end of the pipe is evident. These results use a real gas model and real enthalpy model for the steam. The predictions indicate that sonic choking will exist for all downstream pressures less than 262.8 psia.

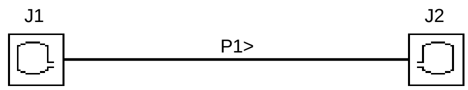

Figure 4a. 500 psi steam system example model.

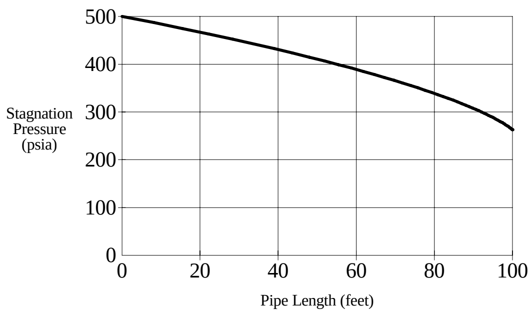

Figure 4b. Stagnation pressure variation in a 100 foot sonically choked pipe predicted by *AFT Arrow*.

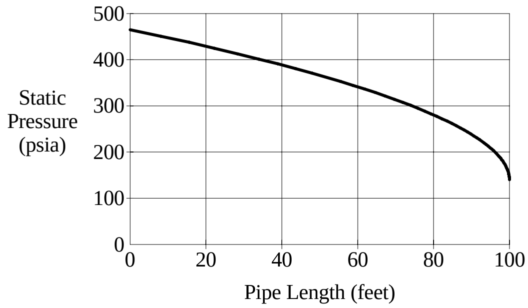

Figure 4c. Static pressure variation in a 100 foot sonically choked pipe predicted by *AFT Arrow*.

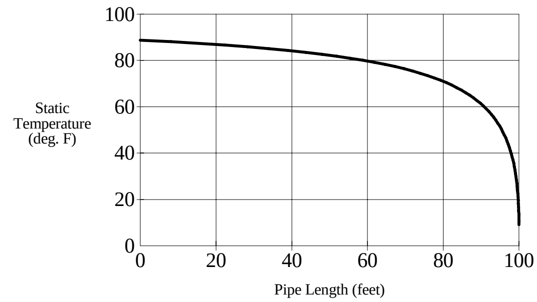

Figure 4d. Static temperature variation in a 100 ft. sonically choked pipe predicted by *AFT Arrow*.

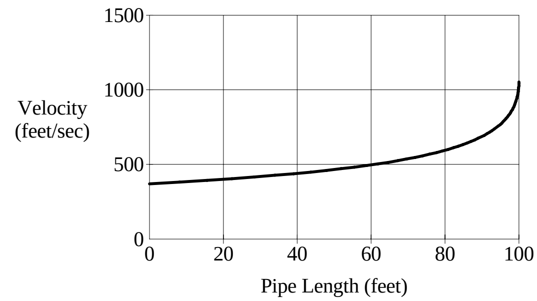

Figure 4e. Velocity variation in a 100 foot sonically choked pipe predicted by *AFT Arrow*.

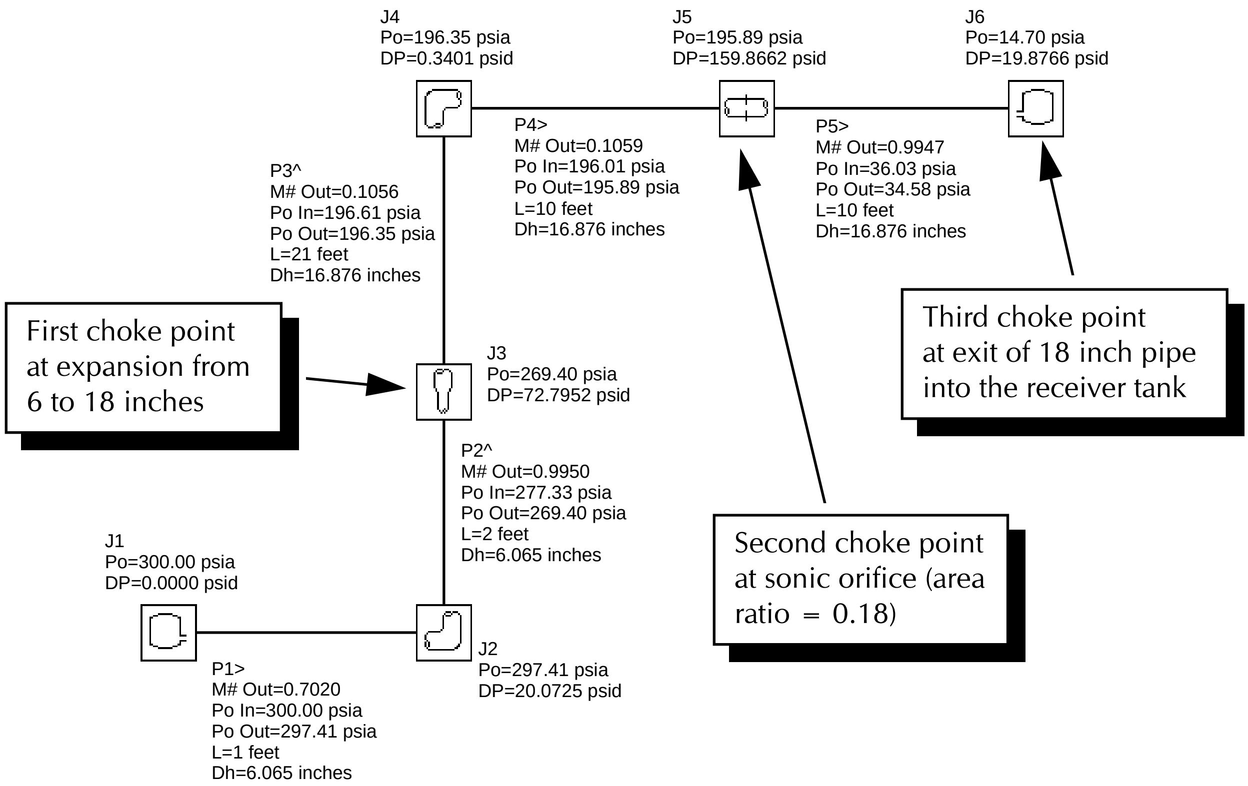

Figure 5. Triple Choke Steam Flow model predictions by *AFT Arrow*.

Agreement with published cases for ideal gases (Saad (1993)) can be shown. Published cases for real gas systems such as steam are difficult to find.

MULTIPLE CHOKING POINTS

It should be noted that sonic choking can exist in any multiple of the above geometries and at multiple locations in the same pipe system. *AFT Arrow's* solution method incorporates this reality. The model shown in Figure 5 is that of a "triple choke steam flow", where sonic choking occurs in three places, as indicated in Figure 5. The pressure drop in the system is as shown in Figure 6.

Sonic choking in multiple locations in a flow path can be understood as follows. Recall that the sonic flow rate will have a unique value given upstream conditions and the piping losses and thermal environment. No matter what happens at the exit boundary condition, the flow rate will not increase or change in any way. However, by changing the *upstream* conditions the sonic flow rate can be changed. This is because the conditions are being changed on the same side as the shock wave and can thus influence the fluid behavior. Figure 7 graphs the supply pressure vs. sonic flow rate for a series of conditions for this system.

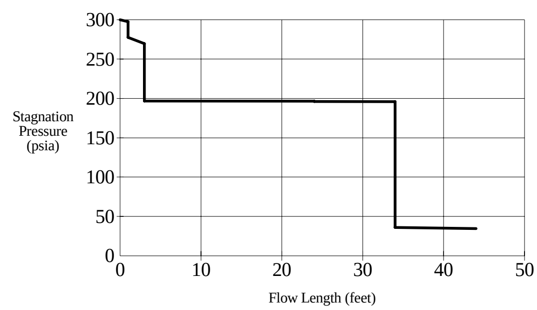

Figure 6. Stagnation pressure drop in Triple Choke Steam Flow model predicted by *AFT Arrow*.

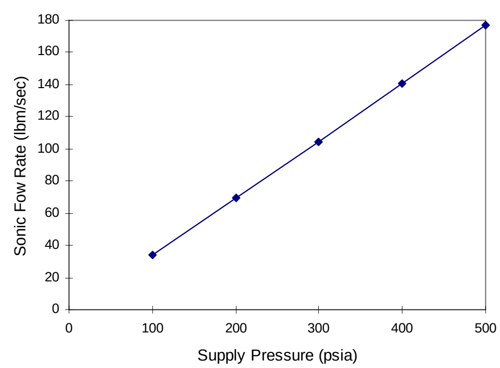

Figure 7. Sonic flow rate vs. supply pressure in Triple Choke Steam Flow model.

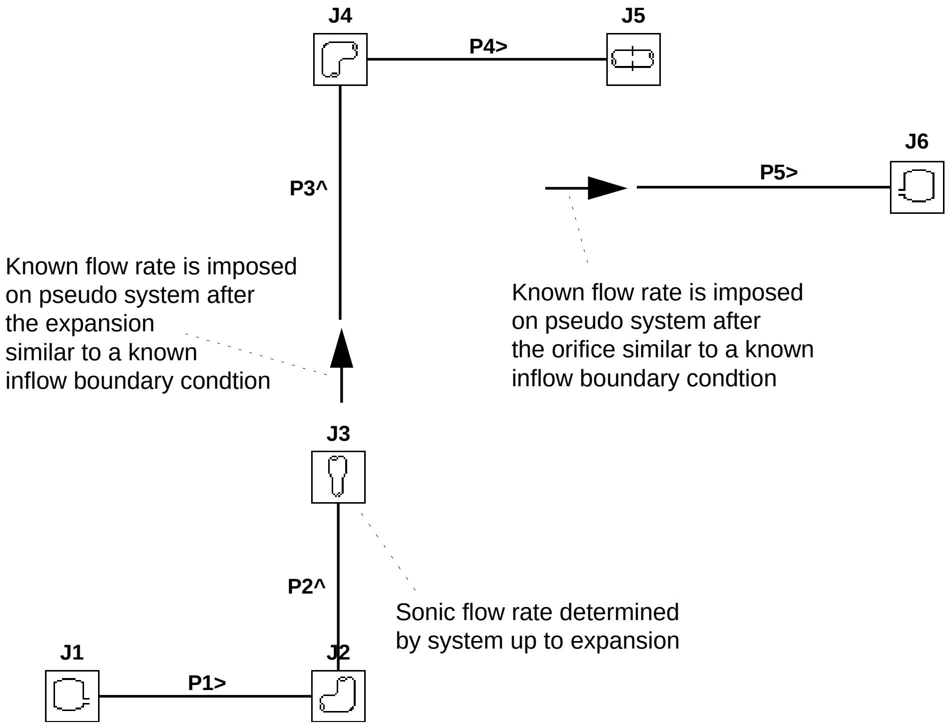

Figure 8. Triple Choke Steam Flow model pseudo system for determining shock wave pressure drops.

To understand multiple sonic choking, consider the system in Figure 5. By iteration on flow rate it is determined that sonic choking will exist at the pipe expansion at junction J3. This fixes the flow rate for the entire system. This flow rate can then be entered as an upstream boundary condition on the remaining pipes. In this way the pressure can be obtained downstream of the shock wave by iteration. After entering the flow rate as an upstream pseudo-boundary condition, it may turn out that sonic choking occurs again, such as it would for any pipe system with an upstream assigned flow rate. In this system the flow again chokes at the restricted area of an orifice at J5. Again the known choked flow rate is used as a pseudo-boundary condition at the entrance to the pipe downstream of the orifice. Solving this pseudo system results in a third choking condition at the exit of pipe P5 as it connects to atmospheric pressure at J6. Again a shock wave occurs as the flow exits the system.

The pseudo system that represents the triple choke system can be depicted as in Figure 8.

GRAPHICAL MODELING

AFT Arrow's compressible flow solver has been incorporated into a highly advanced graphical Windows interface (Figure 1). This makes it very straightforward to learn how to use the software and to verify models. Since all modeling is performed visually, the system layout is readily apparent and it is a simple matter to identify connection errors.

The models are built on the Workspace using drag-and-drop operations from a Palette of tools, which represent various pipe system components. Users can customize each pipe system component, even entering in raw test data and performing a curve fit.

In addition, customizable engineering units and customizable reports can be generated, which further enhances understanding of the pipe system.

APPLICATIONS

AFT Arrow has been successfully applied to numerous gas piping systems, including steam, natural gas, air, carbon monoxide, and high pressure nitrogen, helium, hydrogen and oxygen.

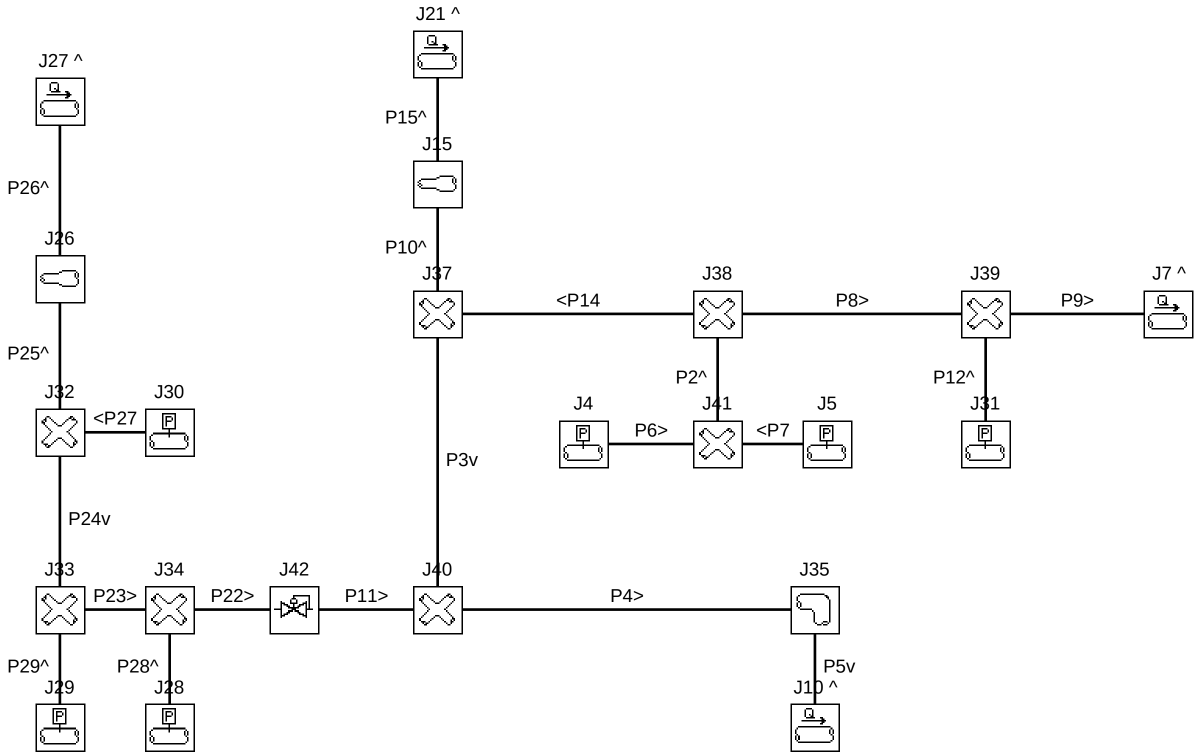

Figure 9. *AFT Arrow* model of 800 psi steam system flowing in a cogeneration piping system.

Some examples of various pipe systems modeled with *AFT Arrow* are shown in Figures 9-11. In Figure 9, a steam distribution system for a cogeneration facility is shown. This system involved determination of delivery pressures at multiple locations around the facility. Operating pressures were 800 psi.

A second application is that shown in Figure 10 of a the fuel supply to a Rolls Royce Avon gas turbine. In this application a gaseous hydrocarbon is supplied around a manifolding system, with the objective being a determination of the delivery temperatures to ascertain the thermal distribution. Unbalanced thermal distribution significantly impacts gas turbine operations and expected lifetime.

A third application is shown in Figure 11. This system is a feedwater heater vent system, and involves 500 psi steam that experiences sonic choking a numerous sonic orifices as it is being delivered to a 100 psi receiver location.

CONCLUSIONS

AFT Arrow offers a highly accurate solution method for real gas compressible flow modeling, including steam and natural gas systems. Previous real world applications have demonstrated its ability to model a wide range of gas piping systems. The graphical interface implementation within Microsoft Windows offers ease-of-use benefits and numerous customizing features making *AFT Arrow* an excellent tool for power system engineers.

REFERENCES

Saad, M.A., *Compressible Fluid Flow*, 2nd Edition, Prentice-Hall, Englewood Cliffs, NJ, 1993.

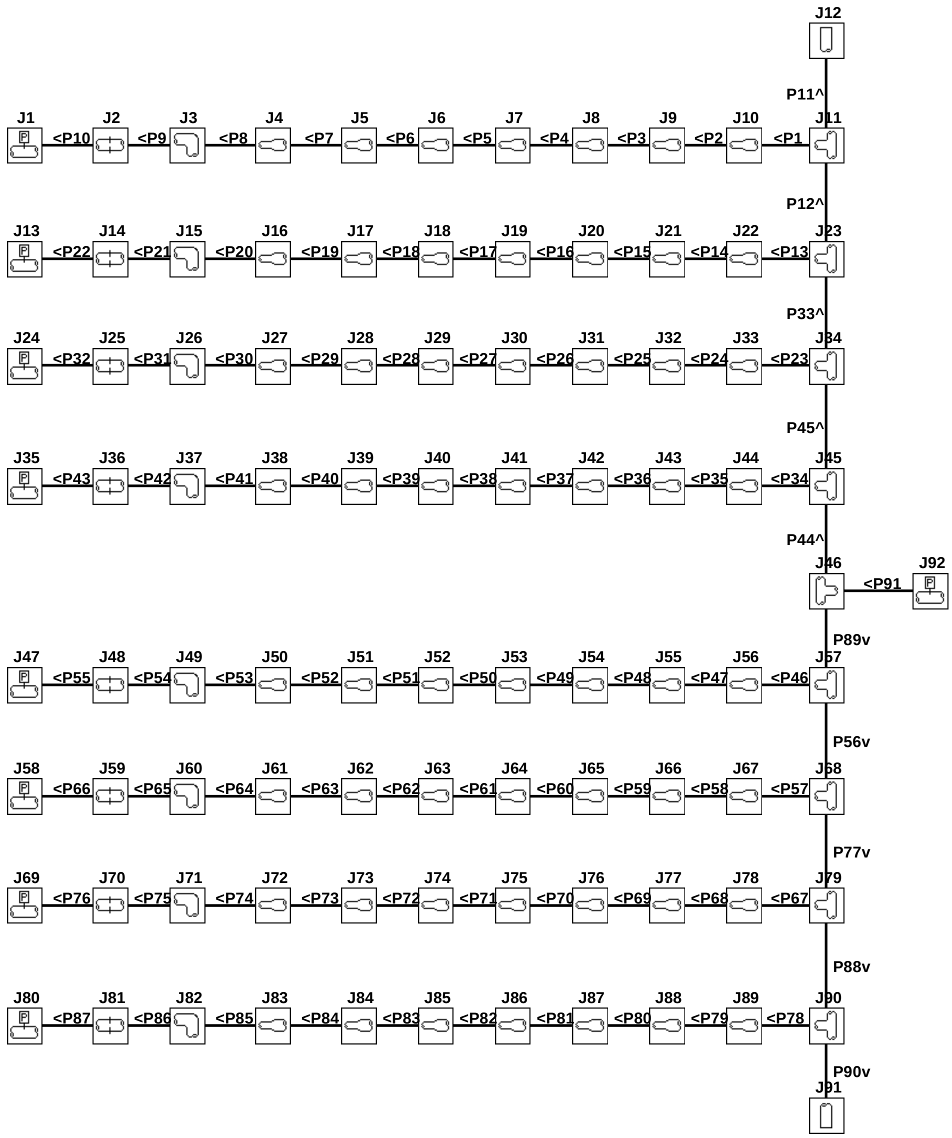

Figure 10. ***AFT Arrow*** **model of fuel manifolding system for a 140 psi Rolls Royce Avon gas turbine.**

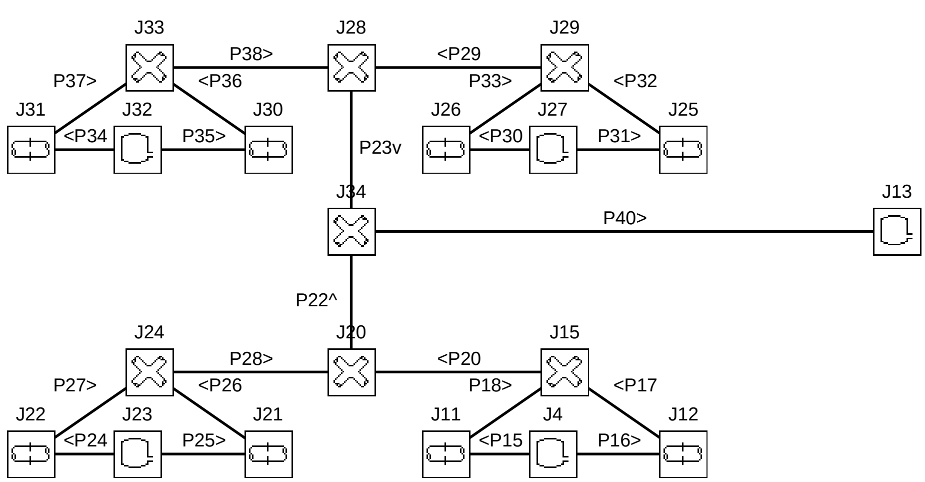

Figure 11. ***AFT Arrow*** **model of 500 psi feedwater heater steam vent system.**